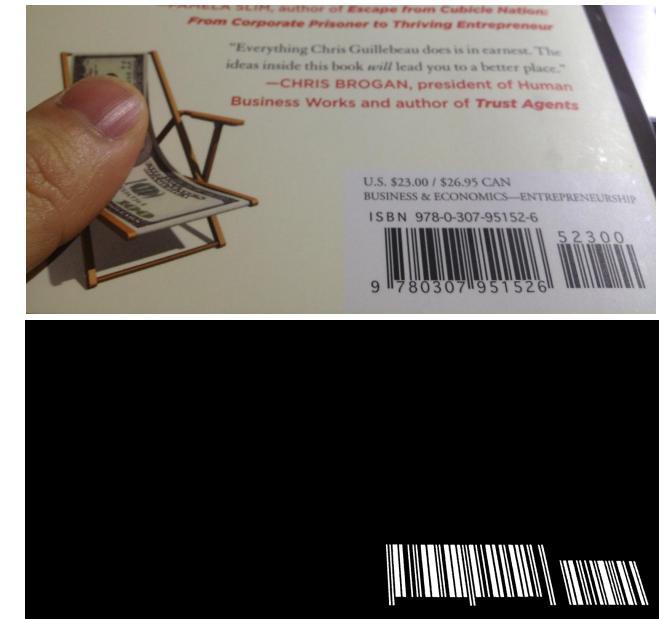
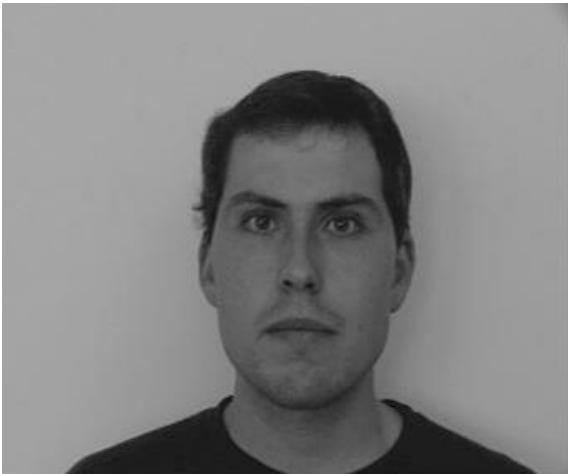


# Image Segmentation

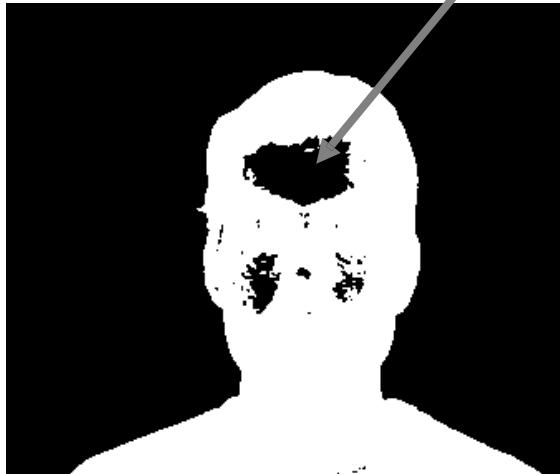
- Gray-level thresholding
- Supervised vs. unsupervised thresholding
- Binarization using Otsu's method
- Locally adaptive thresholding
- Maximally stable extremal regions
- Color-based segmentation
- Region labeling and counting
- Region moments



# Gray-level thresholding

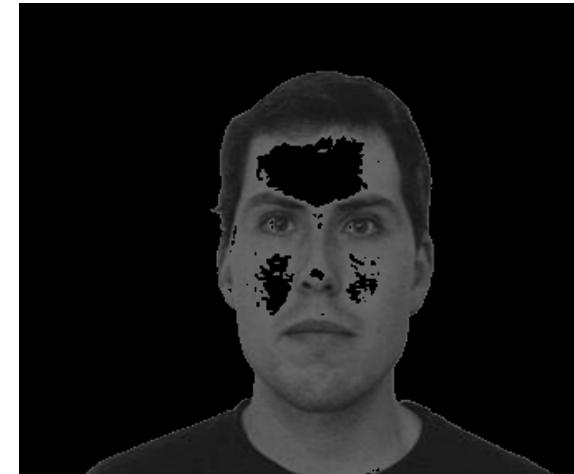


Original image  
*Peter f [x,y]*



Thresholded  
*Peter m [x,y]*

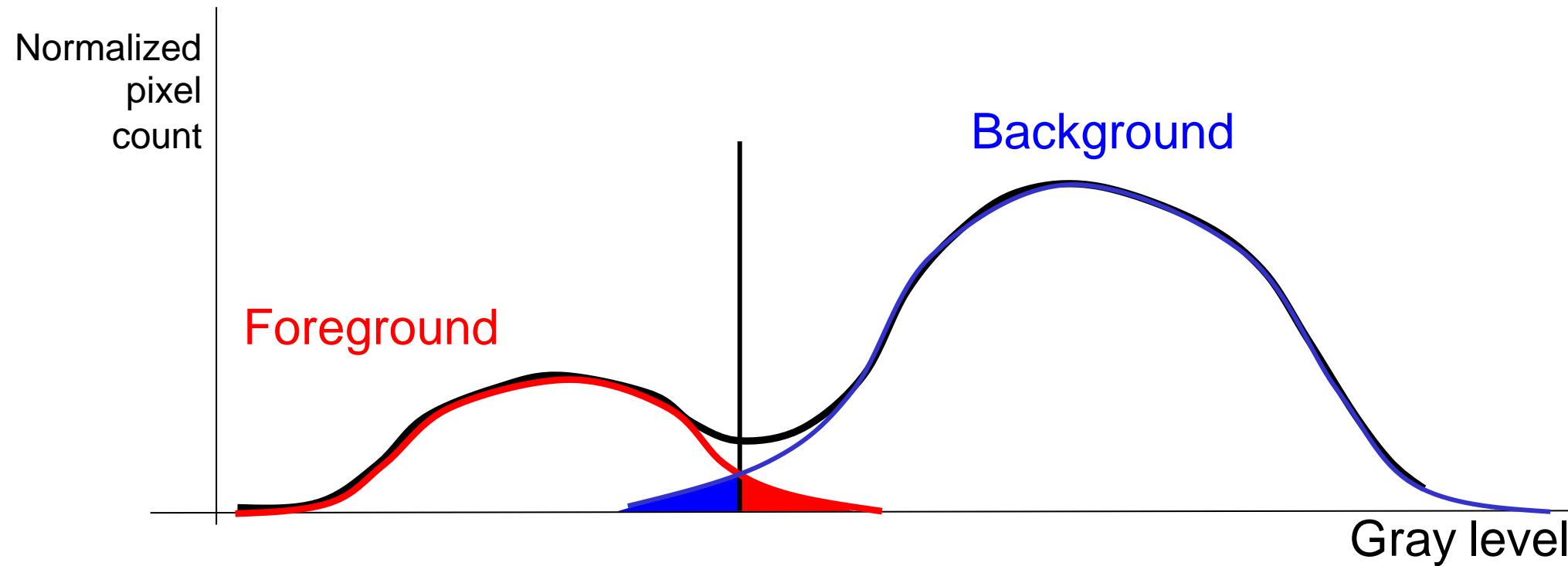
How can holes be filled?



$$f[x,y] \cdot m[x,y]$$



# How to choose the threshold?



# Unsupervised thresholding

- Idea: find threshold  $T$  that minimizes *within-class variance* of both foreground and background (same as k-means)

$$\sigma_{\text{within}}^2(T) = \frac{N_{\text{Fgrnd}}(T)}{N} \sigma_{\text{Fgrnd}}^2(T) + \frac{N_{\text{Bgrnd}}(T)}{N} \sigma_{\text{Bgrnd}}^2(T)$$

- Equivalently, maximize *between-class variance*

$$\begin{aligned}\sigma_{\text{between}}^2(T) &= \sigma^2 - \sigma_{\text{within}}^2(T) \\ &= \left( \frac{1}{N} \sum_{x,y} f^2[x, y] - \mu^2 \right) - \frac{N_{\text{Fgrd}}}{N} \left( \frac{1}{N_{\text{Fgrd}}} \sum_{x,y \in \text{Fgrnd}} f^2[x, y] - \mu_{\text{Fgrnd}}^2 \right) - \frac{N_{\text{Bgrnd}}}{N} \left( \frac{1}{N_{\text{Bgrnd}}} \sum_{x,y \in \text{Bgrnd}} f^2[x, y] - \mu_{\text{Bgrnd}}^2 \right) \\ &= -\mu^2 + \frac{N_{\text{Fgrnd}}}{N} \mu_{\text{Fgrnd}}^2 + \frac{N_{\text{Bgrnd}}}{N} \mu_{\text{Bgrnd}}^2 = \frac{N_{\text{Fgrnd}}}{N} (\mu_{\text{Fgrnd}} - \mu)^2 + \frac{N_{\text{Bgrnd}}}{N} (\mu_{\text{Bgrnd}} - \mu)^2 \\ &= \frac{N_{\text{Fgrnd}}(T) N_{\text{Bgrnd}}(T)}{N^2} (\mu_{\text{Fgrnd}}(T) - \mu_{\text{Bgrnd}}(T))^2\end{aligned}$$

[Otsu, 1979]

# Unsupervised thresholding (cont.)

- Algorithm: Search for threshold  $T$  to maximize

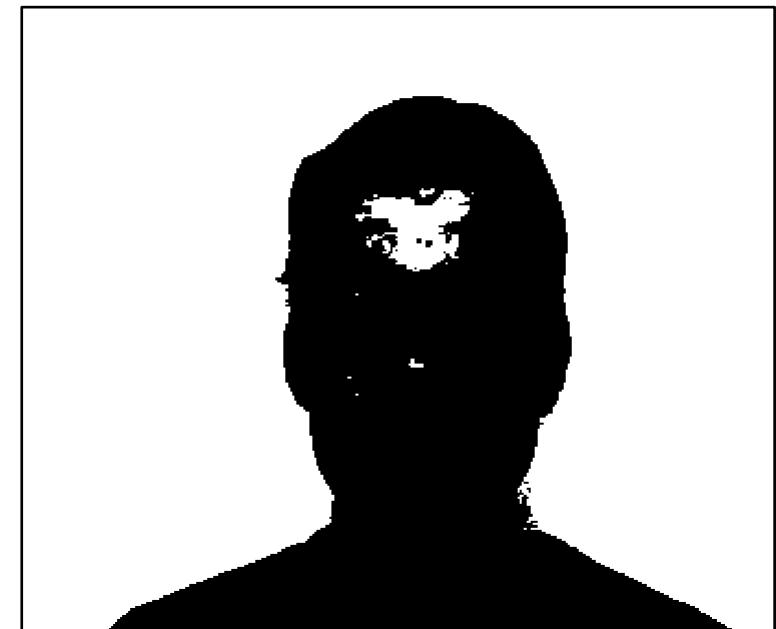
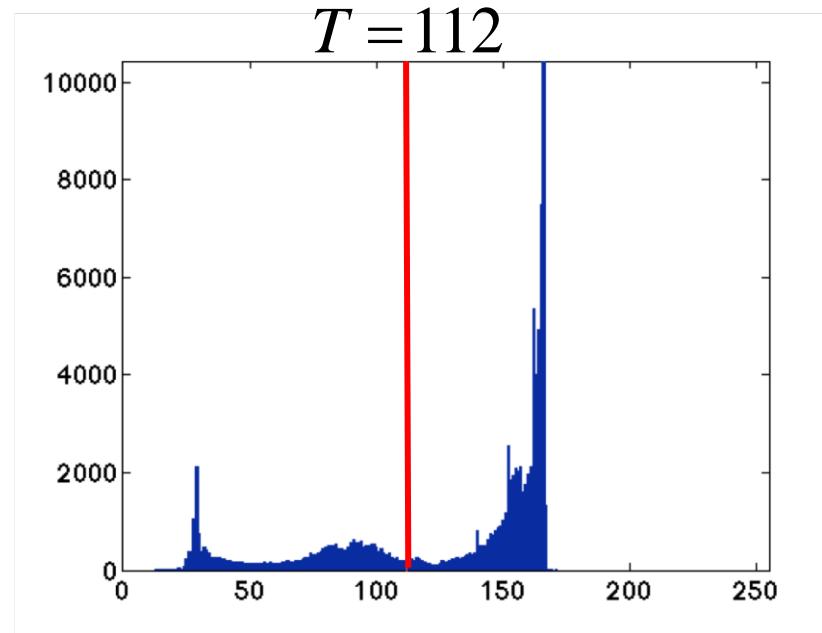
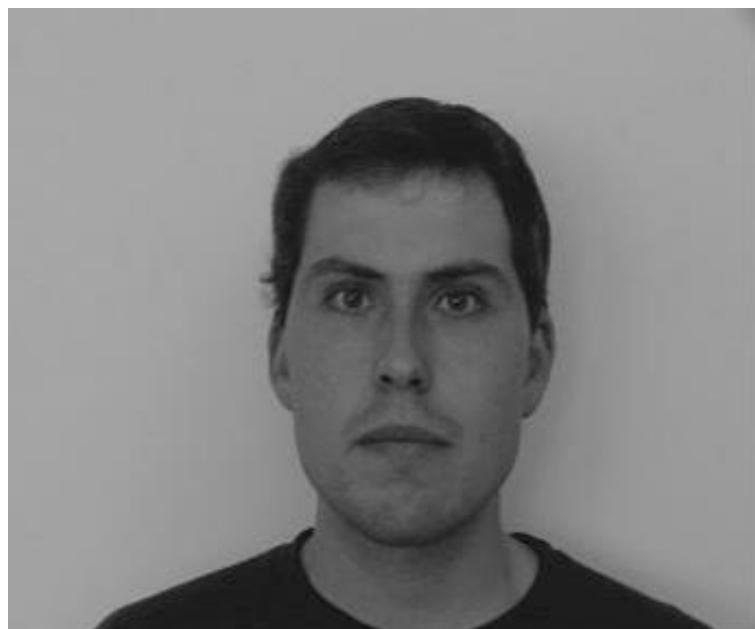
$$\sigma_{\text{between}}^2(T) = \frac{N_{\text{Fgmd}}(T)N_{\text{Bgmd}}(T)}{N^2}(\mu_{\text{Fgmd}}(T) - \mu_{\text{Bgmd}}(T))^2$$

- Useful recursion for sweeping  $T$  across histogram:

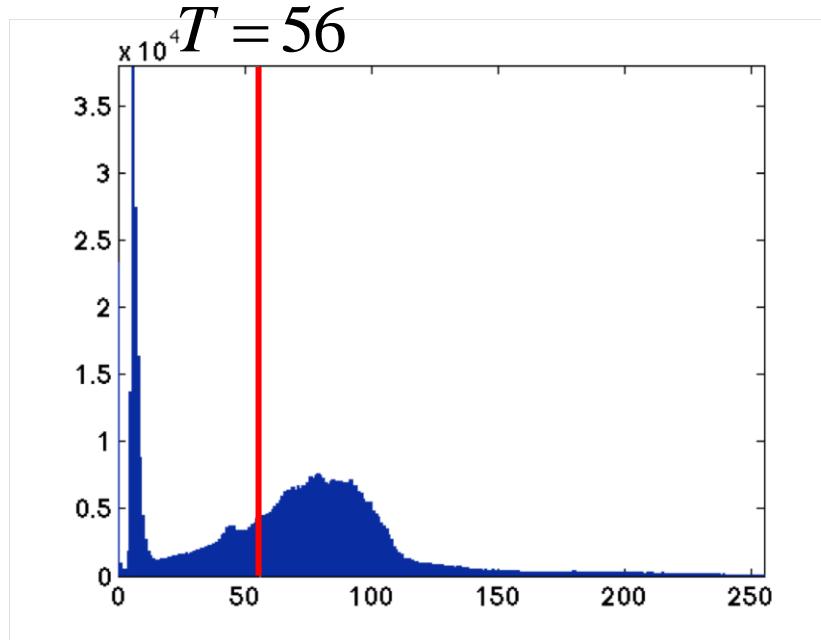
$$N_{\text{Fgmd}}(T+1) = N_{\text{Fgmd}}(T) + n_T$$
$$N_{\text{Bgmd}}(T+1) = N_{\text{Bgmd}}(T) - n_T$$
$$\mu_{\text{Fgmd}}(T+1) = \frac{\mu_{\text{Fgmd}}(T)N_{\text{Fgmd}}(T) + n_T T}{N_{\text{Fgmd}}(T+1)}$$
$$\mu_{\text{Bgmd}}(T+1) = \frac{\mu_{\text{Bgmd}}(T)N_{\text{Bgmd}}(T) - n_T T}{N_{\text{Fgmd}}(T+1)}$$

[Otsu, 1979]

# Unsupervised thresholding (cont.)



# Unsupervised thresholding (cont.)



# Unsupervised thresholding (cont.)

**The Stanford Daily**

Tuesday, September 18, 2012 ♦ 13

**FOOTBALL**

## The winding road ahead

**By SAM FISHER**  
FOOTBALL EDITOR



SIMON WARBY/The Stanford Daily

Andrew Luck may be gone, but with Stanford man's win over USC, the Cardinal put itself in position to achieve beyond the path paved by number 12. You heard right, though it may seem like luck left to do, this 2012 Stanford team showed that it's capable of playing at a national championship level.

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Please see AWARDS, page 15

**Stephen Taylor:** It all starts and ends with Stanford's workhorse. Taylor was everywhere you looked and everywhere against USC. He provided the big plays with a game-tying touchdown on the middle from the defensive line and another score through the air and on the ground. He was pound to pound to wear down the Trojans at the end. He 213 total yards of offense to go with a pair of TDs had fans on both sides forgetting

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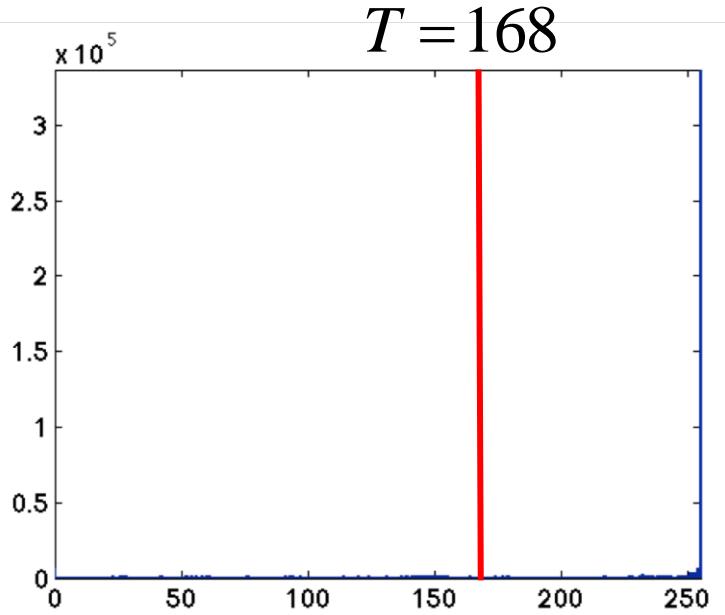
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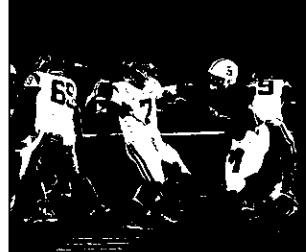
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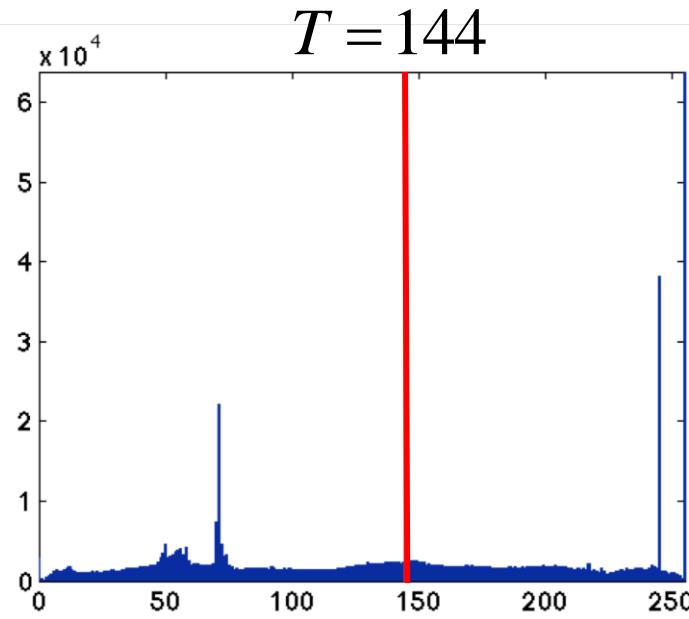
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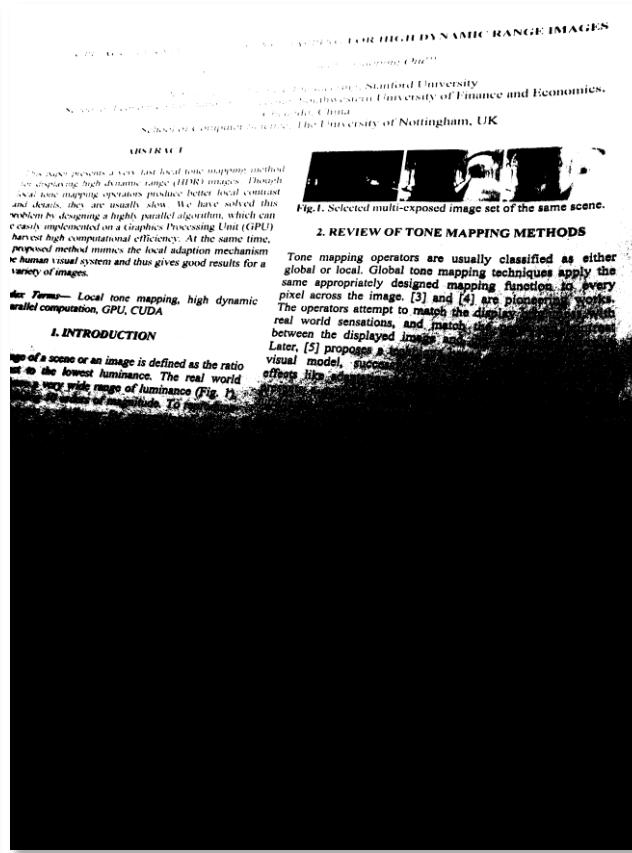
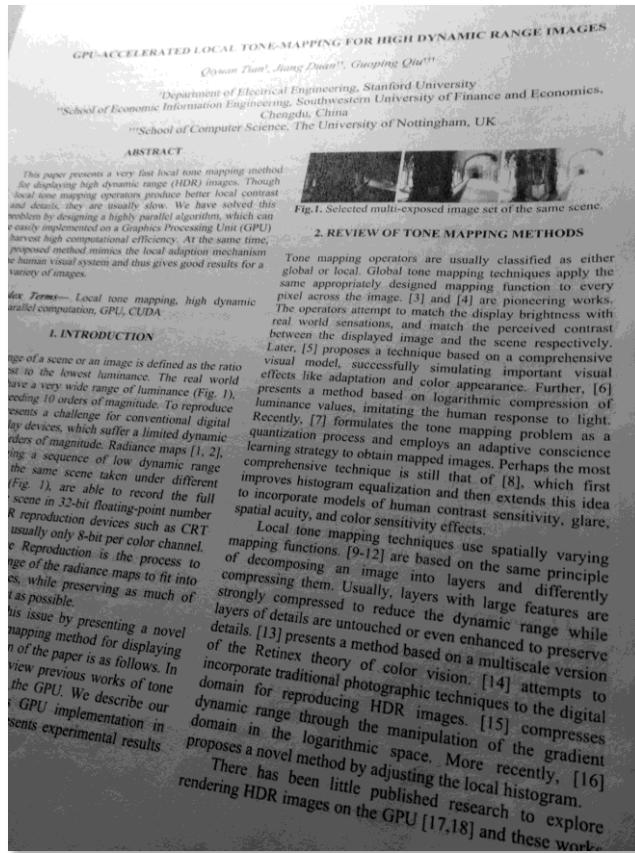


# Unsupervised thresholding (cont.)



# Sometimes, a global threshold does not work

Original image



Thresholded with  
Otsu's Method



# Locally adaptive thresholding

- Slide a window over the image
- For each window position, decide whether to perform thresholding
  - Thresholding should not be performed in uniform areas
  - Use variance or other suitable criterion
- Non-uniform areas: apply Otsu's method (based on local histogram)
- Uniform areas: classify the entire area as foreground or background based on mean value

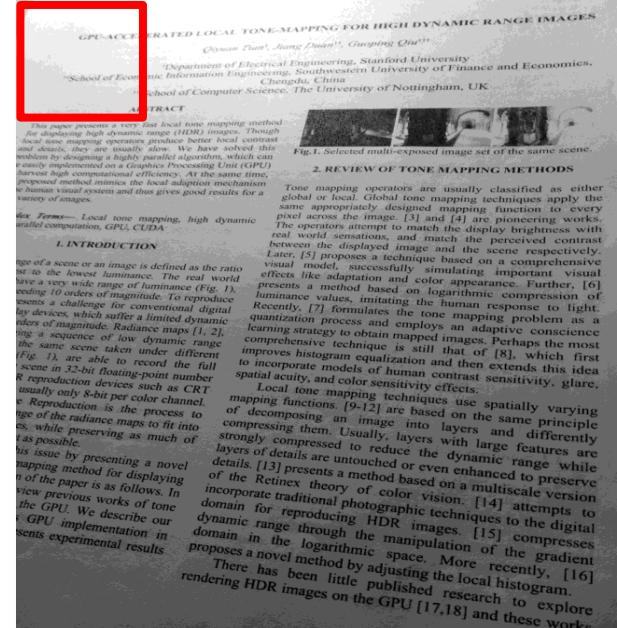


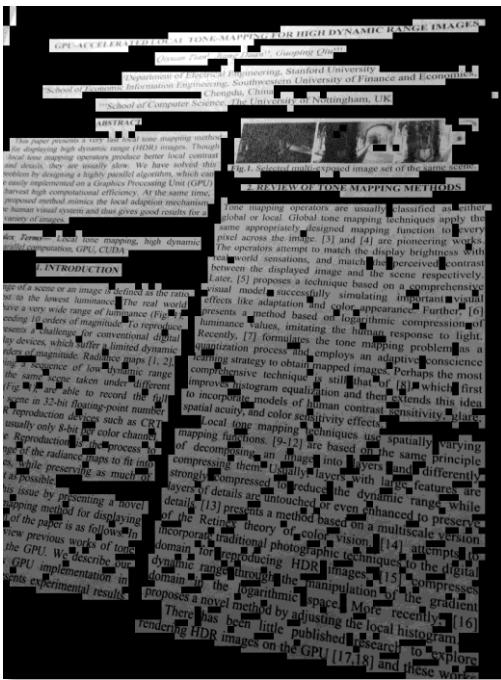
Fig. 1. Selected multi-exposed image set of the same scene.

2. REVIEW OF TONE MAPPING METHODS

Tone mapping operators are usually classified as either global or local. Global tone mapping techniques apply the same appropriately designed mapping function to every pixel across the image. [3] and [4] are pioneering works. The former attempts to match the display brightness with real world sensations, and match the perceived contrast between the displayed image and the scene respectively. Local [5] proposes a technique based on a comprehensive visual model, successfully capturing important visual effects like adaptation and color appearance. Furthermore [6] presents a method based on logarithmic compression of luminance, matching the human response to light. Recently, [7] formulates the tone mapping problem as a quantization process and employs a subjective conscience feedback strategy to obtain mapped images. Perhaps the most comprehensive technique is still that of [8], which first improves histogram equalization and then extends this idea to incorporate models of human contrast sensitivity, glare, spatial acuity, and color sensitivity effects.

Local tone mapping techniques use spatially varying mapping functions. [9-12] are based on the same principle of decomposing an image into layers and differently compressing them. Usually, layers with large features are strongly compressed to reduce the dynamic range while layers of details are untouched or even enhanced to preserve details. [13] presents a method based on a multiscale version of the Retinex theory of color vision. [14] attempts to incorporate traditional photographic techniques to the digital domain for reproducing HDR images. [15] compresses dynamic range through the manipulation of the gradient domain in the logarithmic space. More recently, [16] proposes a novel method by adjusting the local histogram. There has been little published research to explore rendering HDR images on the GPU [17,18] and these works

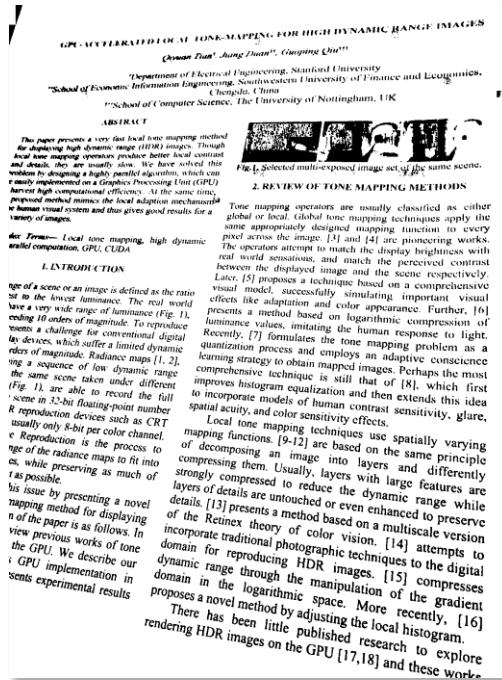
# Locally adaptive thresholding (example)



Non-uniform areas



Local threshold values



Locally thresholded result



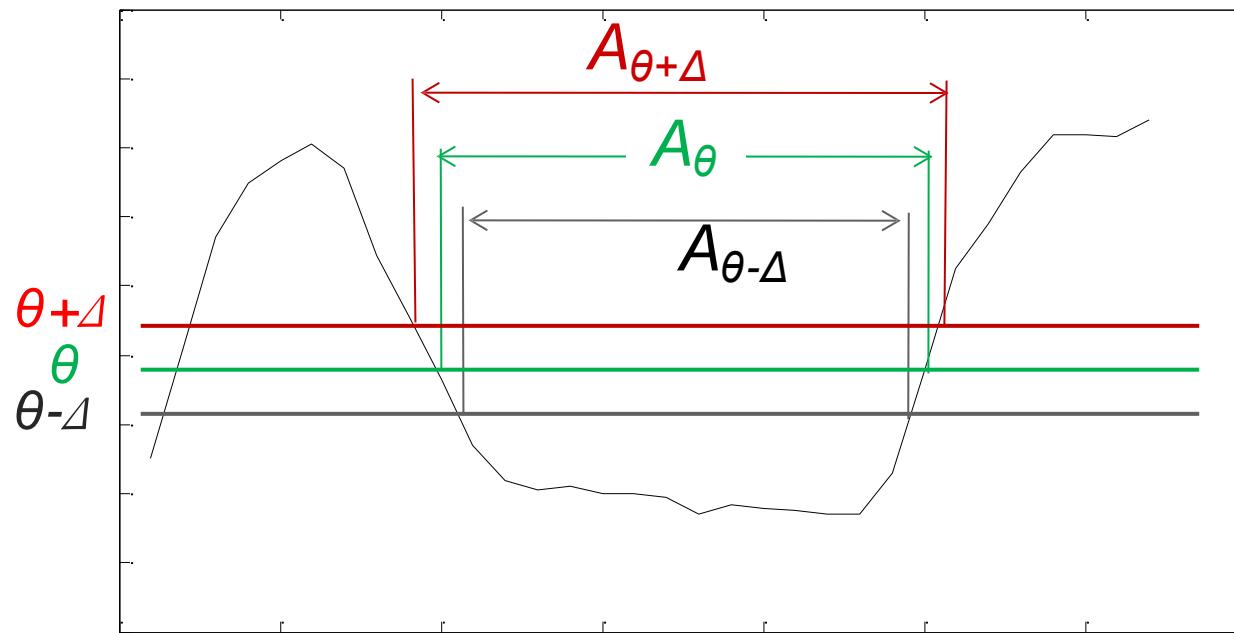
# Maximally stable extremal regions

- Extremal region: any connected region in an image with all pixel values above (or below) a threshold
- Observations:
  - Nested extremal regions result when the threshold is successively raised (or lowered).
  - The nested extremal regions form a “component tree.”
- Key idea: choose thresholds  $\theta$  such that the resulting bright (or dark) extremal regions are nearly constant when these thresholds are perturbed by  $+/-\Delta$

→ “*maximally stable*” *extremal regions (MSER)*

[Matas, Chum, Urba, Pajdla, 2002]

# MSERs: illustration



$$\text{Local minimum of } \left| \frac{A_{\theta-\Delta} - A_{\theta+\Delta}}{A_\theta} \right| \rightarrow \text{MSER}$$

[Matas, Chum, Urba, Pajdla, 2002]

# Level sets of an image

1	1	1	1	1	1	1	1	1	1	1	5	4	4	8
1	7	6	4	2	2	3	3	3	3	1	5	4	4	8
1	7	6	4	2	2	3	3	3	3	1	5	4	4	8
1	7	6	4	2	2	3	3	3	3	1	5	4	4	8
1	7	6	4	2	2	5	5	5	5	1	5	4	4	8
1	6	6	4	2	2	5	5	5	6	1	5	4	4	4
1	6	6	4	2	2	6	6	6	6	1	5	5	5	5
1	4	4	4	2	2	6	6	6	6	1	5	5	5	5
1	1	1	1	1	2	6	1	1	1	1	2	2	2	2
1	8	8	5	1	2	6	1	7	7	1	2	2	2	2
1	8	8	5	1	1	1	1	7	7	1	1	1	1	2
1	8	8	5	5	5	3	3	7	7	1	1	1	1	2
1	8	8	5	5	3	3	3	7	7	7	1	1	1	1
1	8	8	5	5	3	3	3	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

$f[x, y]$

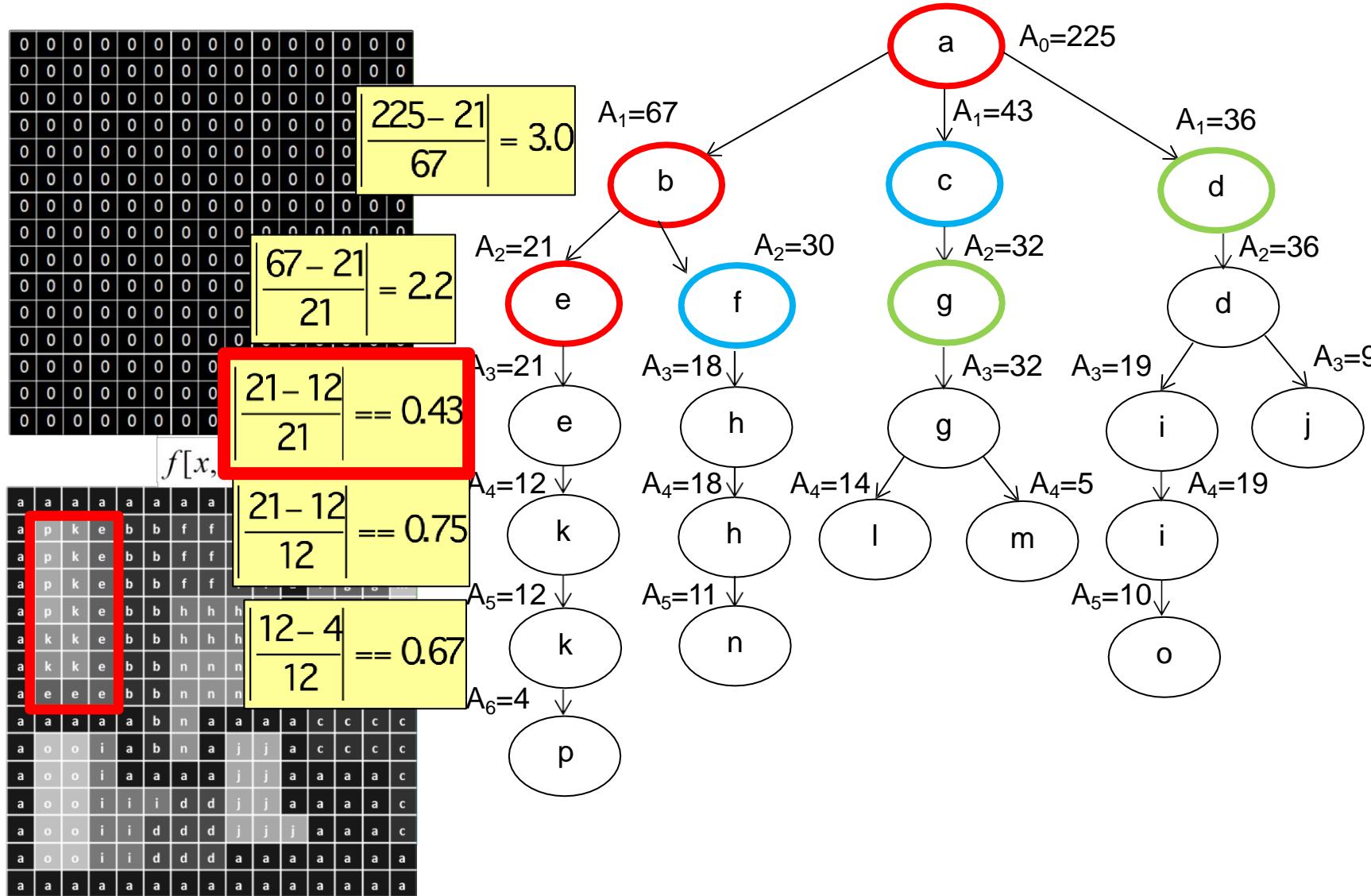
Image

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$f[x, y] > 8$

Level Set

# Component tree of an image



Local minima of sequence

$$\left| \frac{A_{\theta-\Delta} - A_{\theta+\Delta}}{A_\theta} \right|$$

$\theta = \Delta, \Delta + 1, \dots \rightarrow \text{MSERs}$

# MSER: examples



Dark MSERs,  $\Delta=15$



Original image



Bright MSERs,  $\Delta=15$

# MSER: examples



Dark MSERs,  $\Delta=15$

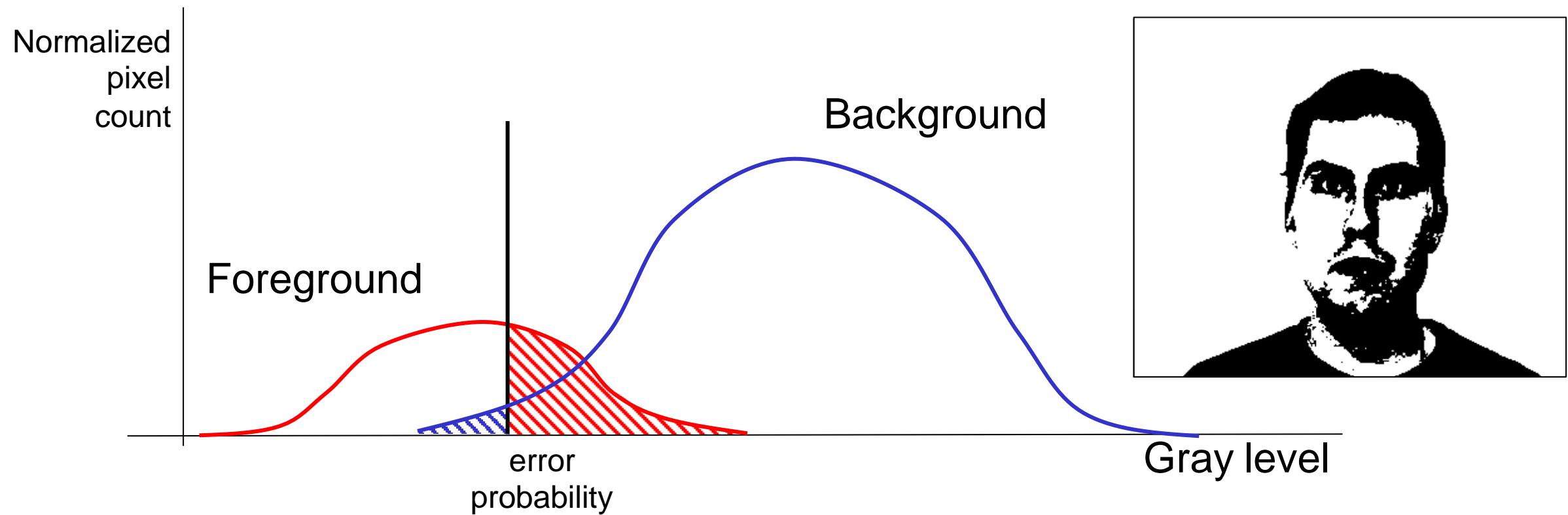


Original image

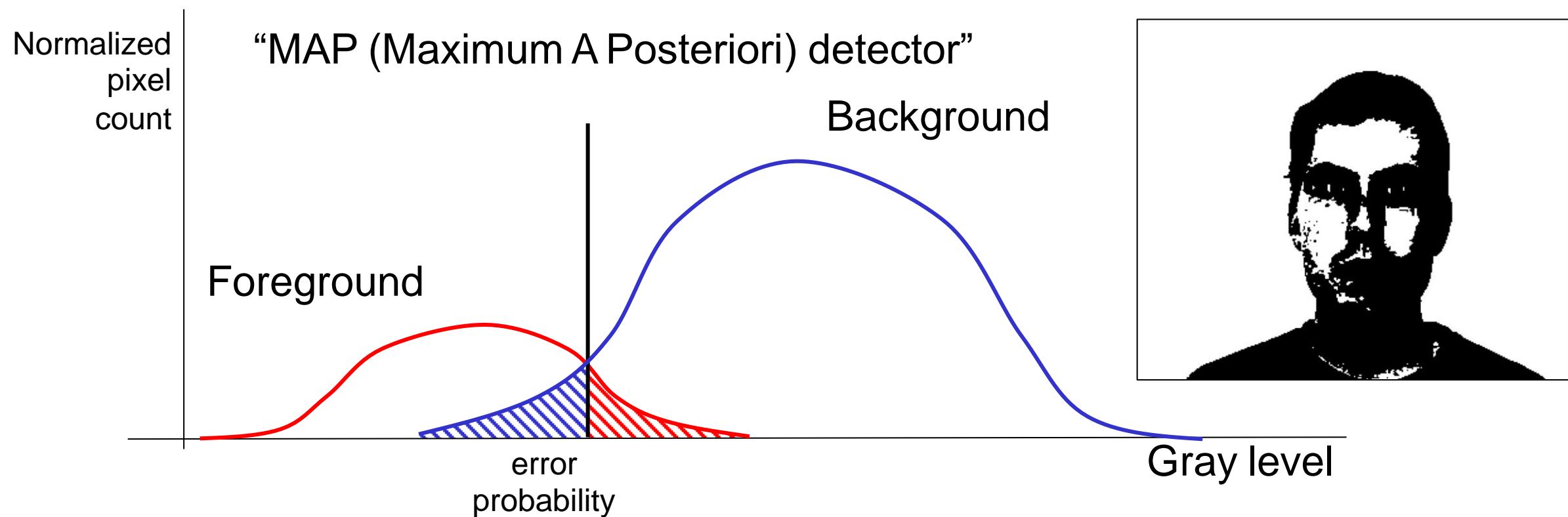


Bright MSERs,  $\Delta=15$

# Supervised thresholding



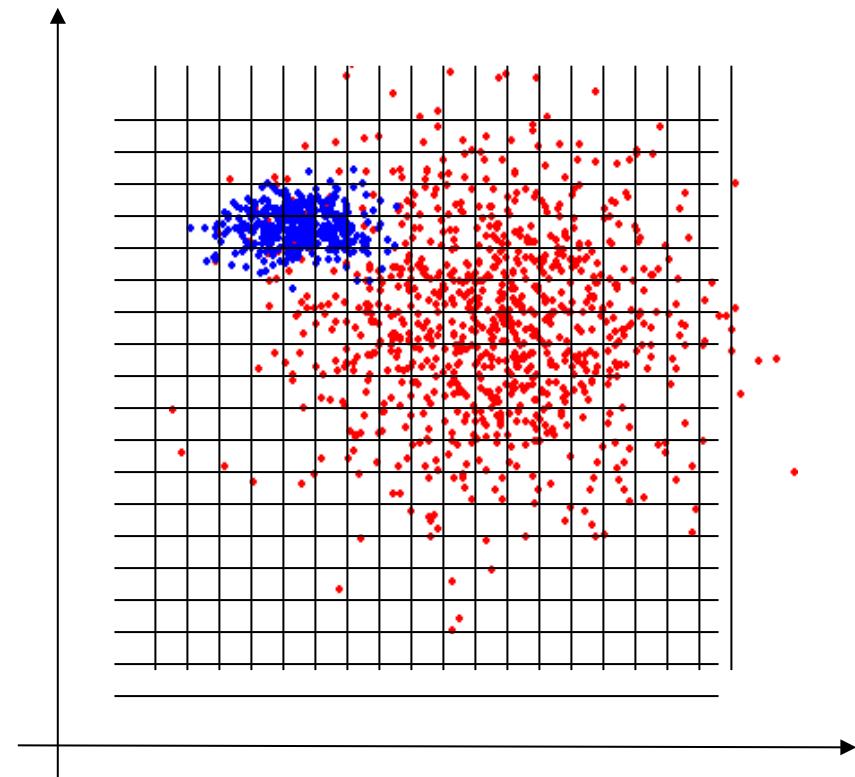
# Supervised thresholding



If errors  $\text{BG} \rightarrow \text{FG}$  and  $\text{FG} \rightarrow \text{BG}$  are associated with different costs:  
“Bayes minimum risk detector” is optimal.

# Multidimensional MAP detector

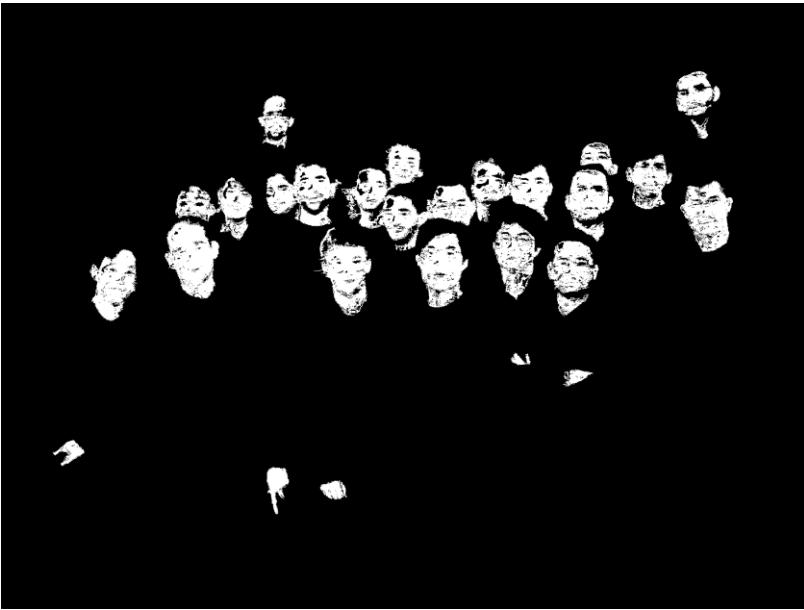
- Training
  - Provide labelled set of training data
  - Subdivide n-dimensional space into small bins
  - Count frequency of occurrence for each bin and class in training set, label bin with most probable class
  - (Propagate class labels to empty bins)
- For test data: identify bin, look up the most probable class



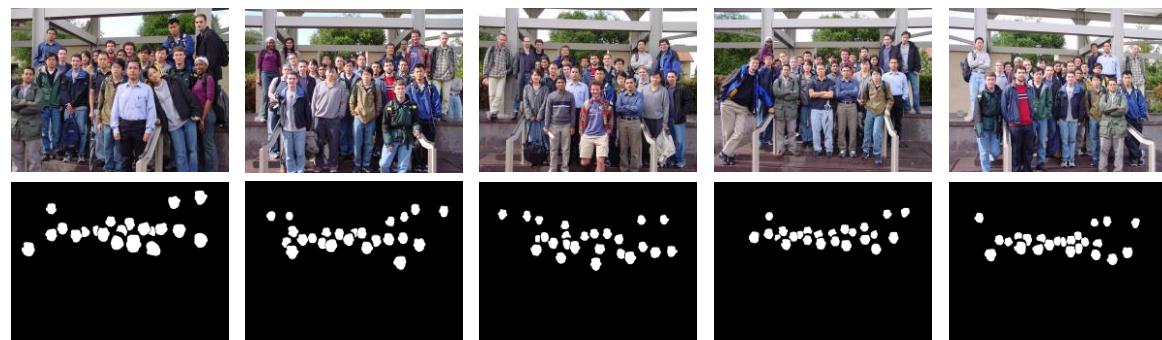
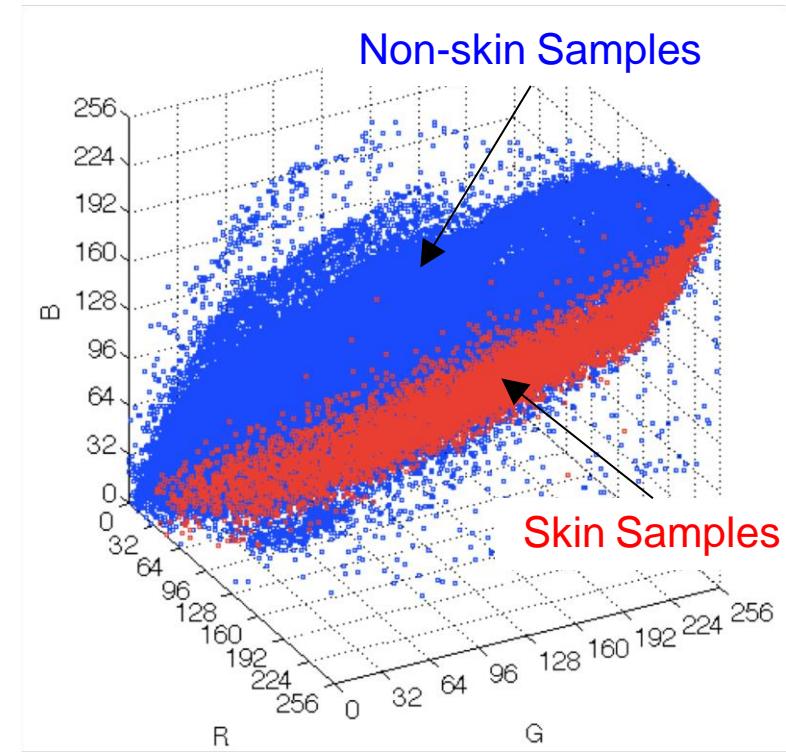
# MAP detector in RGB-space



Original image



Skin color detector



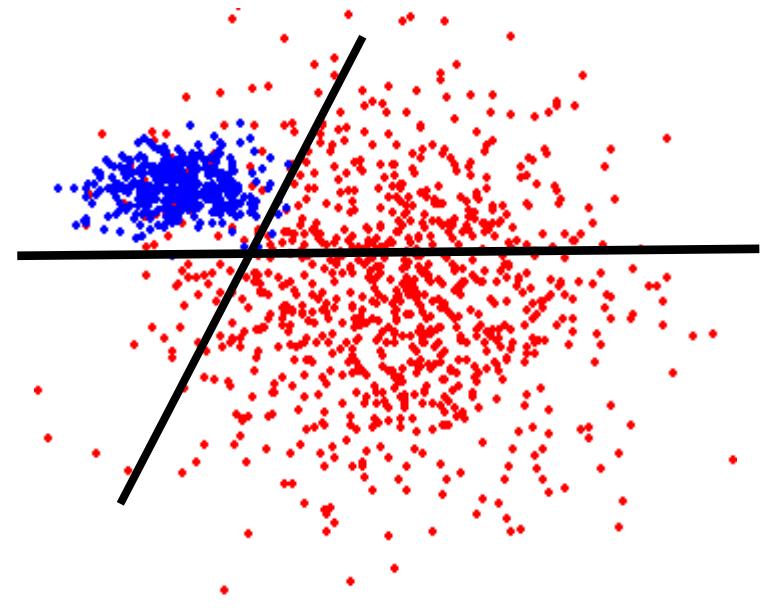
Five training images



# Linear discriminant function

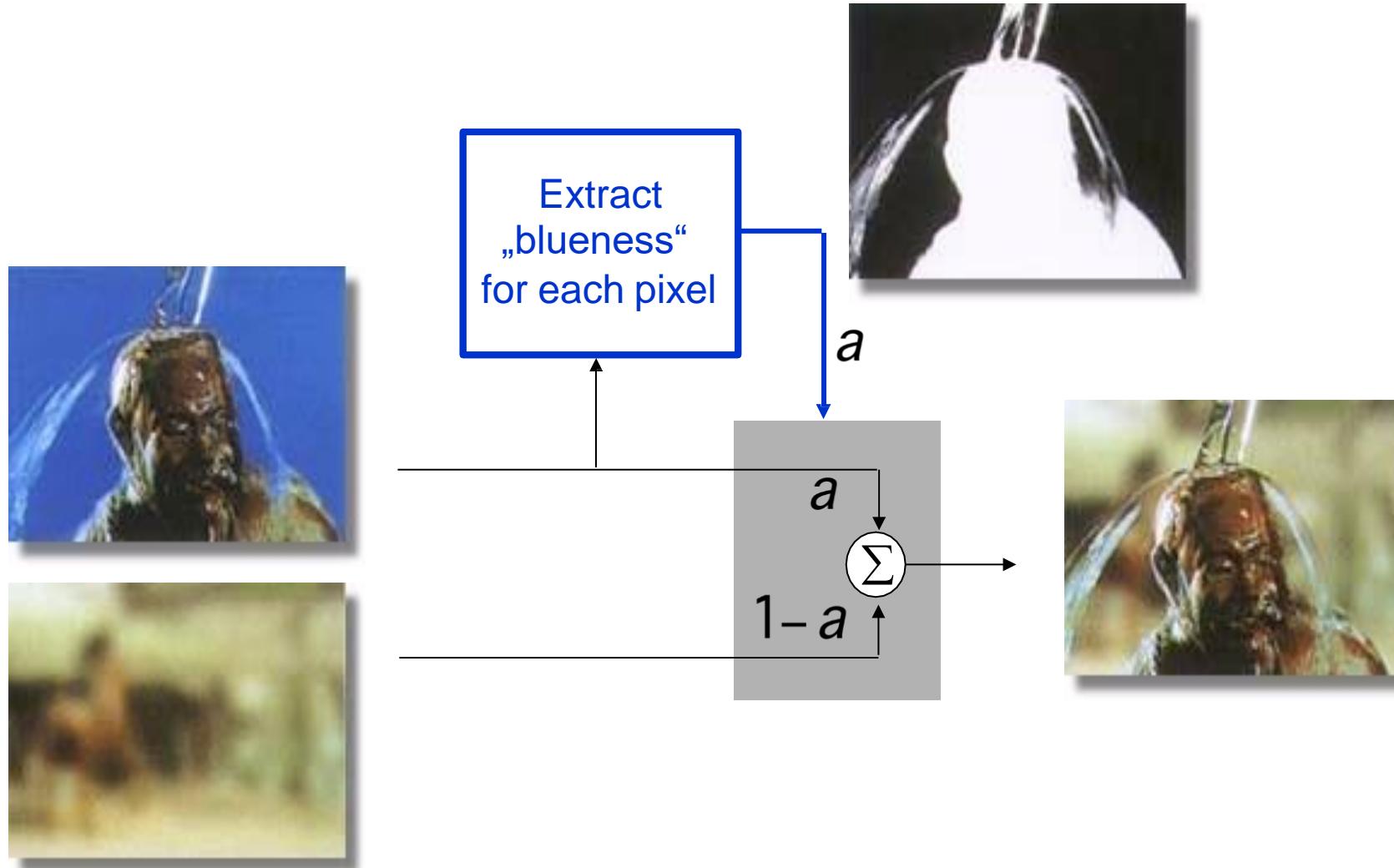
- To segment image with  $n$  components  $f_i, i=1,2,\dots,n$  into two classes, perform test

$$\sum_i w_i f_i + w_0 \geq 0 \ ?$$

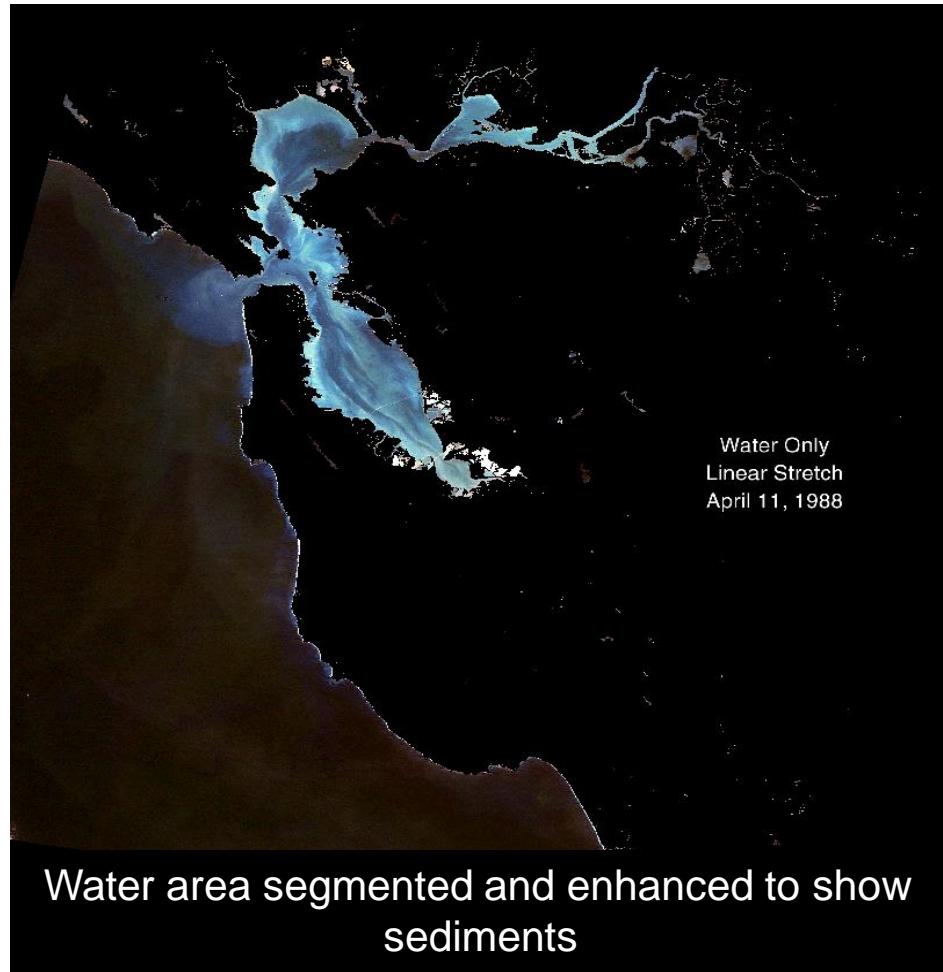
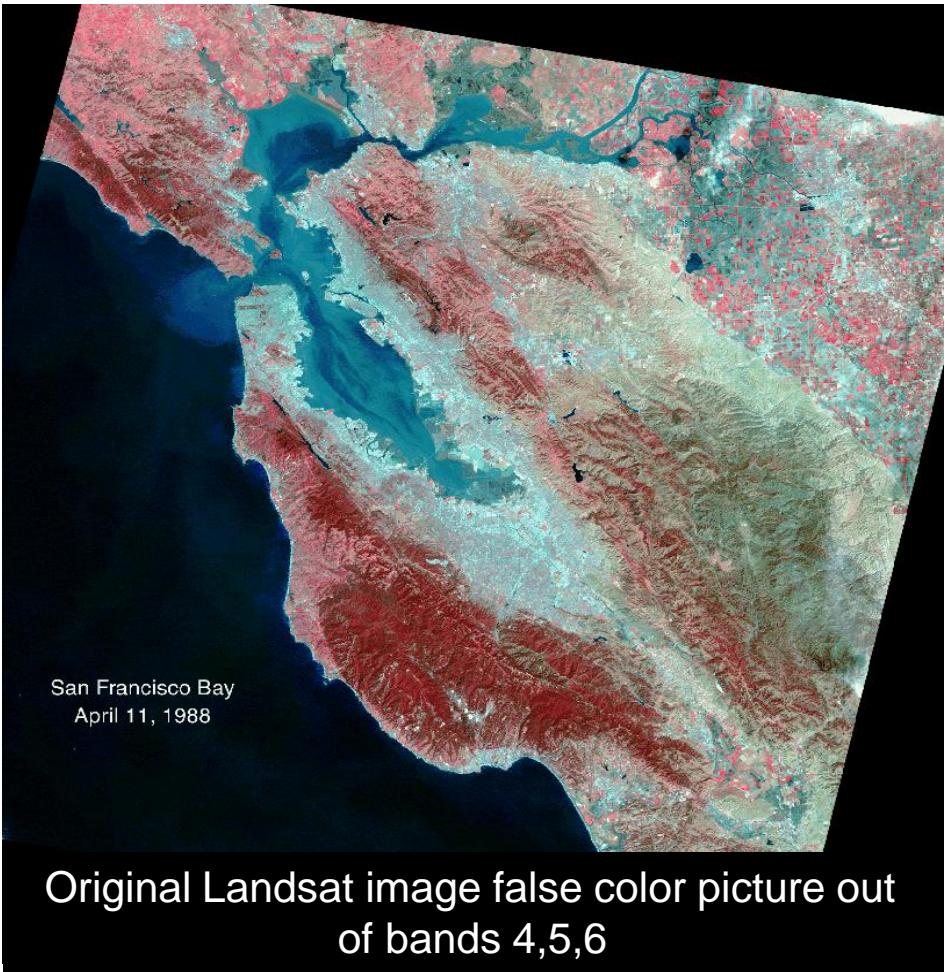


- Categories are separated by hyperplane in  $n$ -space
- Numerous techniques to determine weights  
 $w_i, i=0,1,2,\dots,n$ , see, e.g., [\[Duda, Hart, Stork, 2001\]](#)
- Can be extended to the intersection of several linear discriminant functions
- Can be extended to multiple classes

# Chroma keying



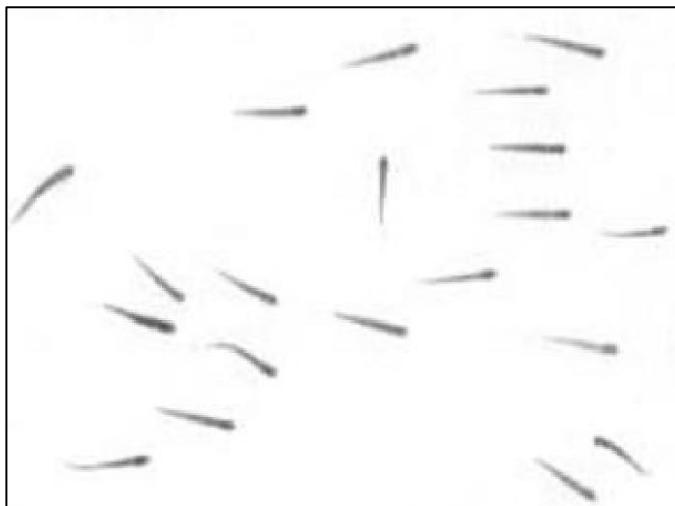
# Landsat image processing



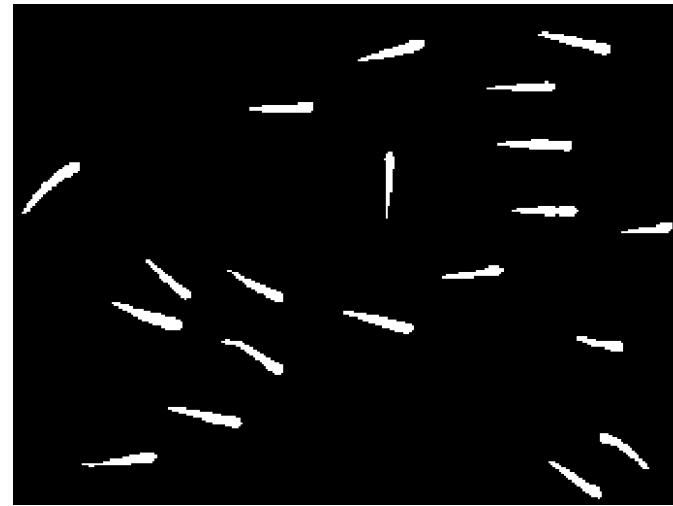
Source: US Geological Survey USGS, <http://sfbay.wr.usgs.gov/>

# Region labeling and counting

- How many fish in this picture?



Original *Fish* image



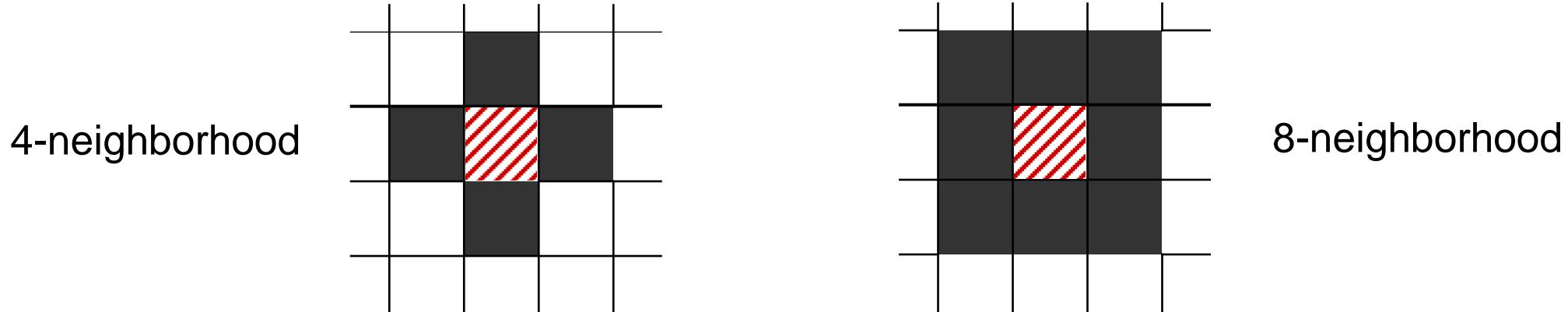
after thresholding

- Which pixels belong to the same object (region labeling)?
- How large is each object (region counting)?



# 4-connected and 8-connected neighborhoods

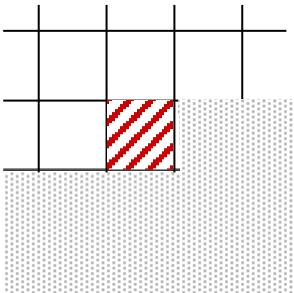
- Definition: a **region** is a set of pixels, where each pixel can be reached from any other pixel in the region by a finite number of steps, with each step starting at a pixel and ending in the neighborhood of the pixel.



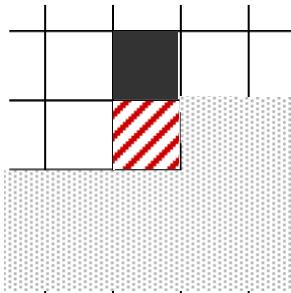
- Typically, either definition leads to the same regions, except when a region is only connected across diagonally adjacent pixels.

# Region labeling algorithm (4-neighborhood)

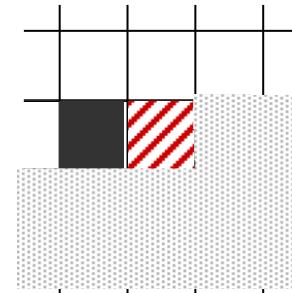
- Loop through all pixels  $f[x,y]$ , left to right, top to bottom
- If  $f[x,y]=0$ , do nothing.
- If  $f[x,y]=1$ , distinguish 4 cases



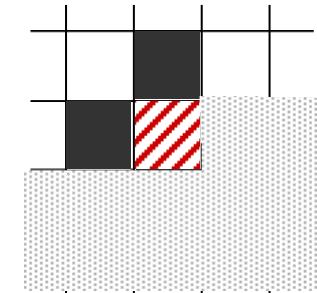
Generate new region label



Copy label from above



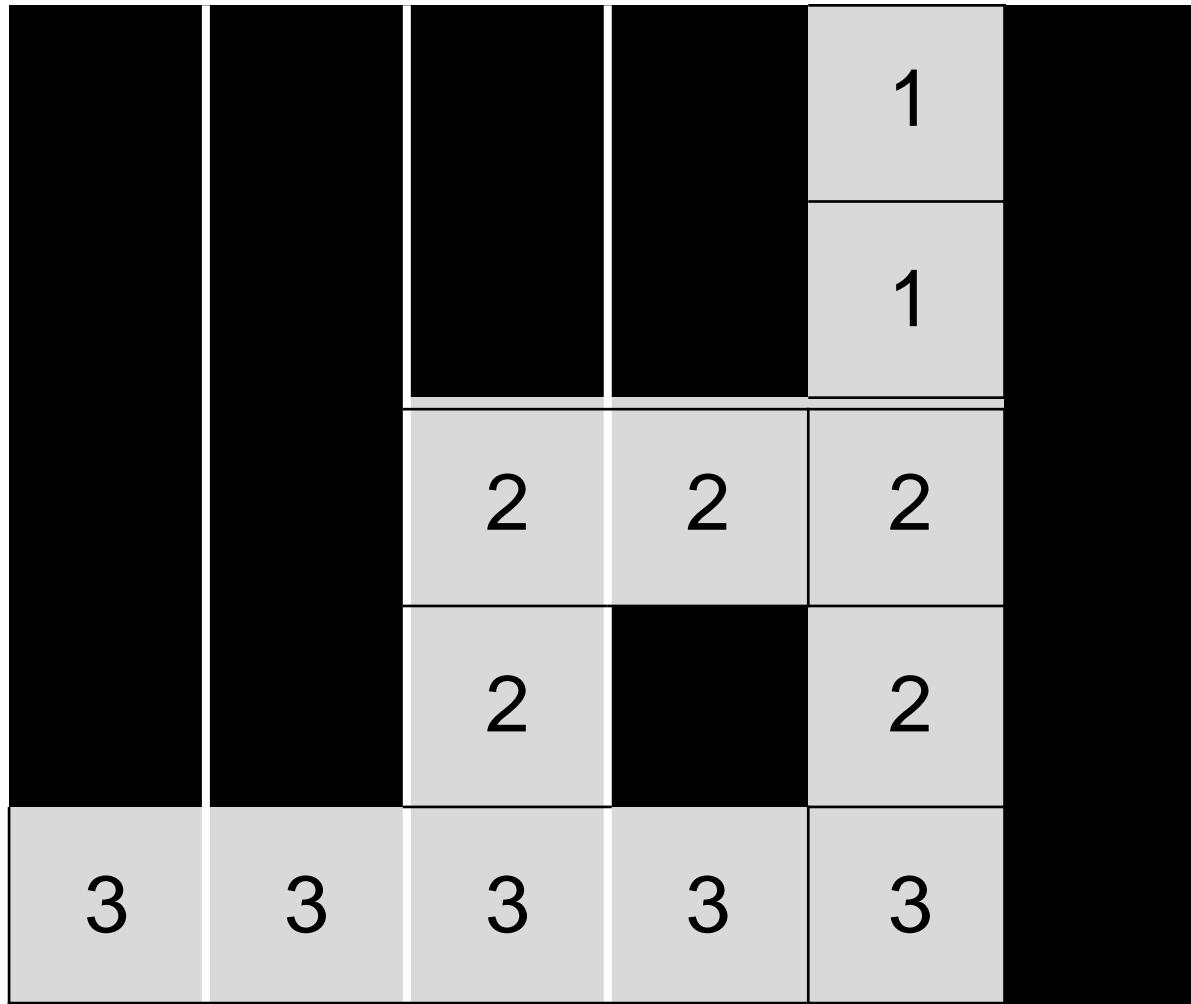
Copy label from the left



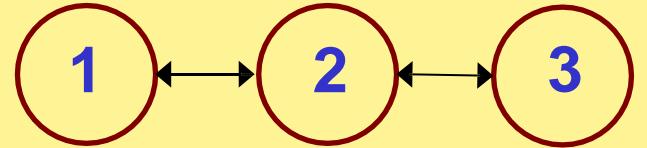
Copy label from the left. If labels above and to the left are different, store equivalence.

- Second pass through image to replace equivalent label by the same label.

# Region labeling example (4-neighborhood)

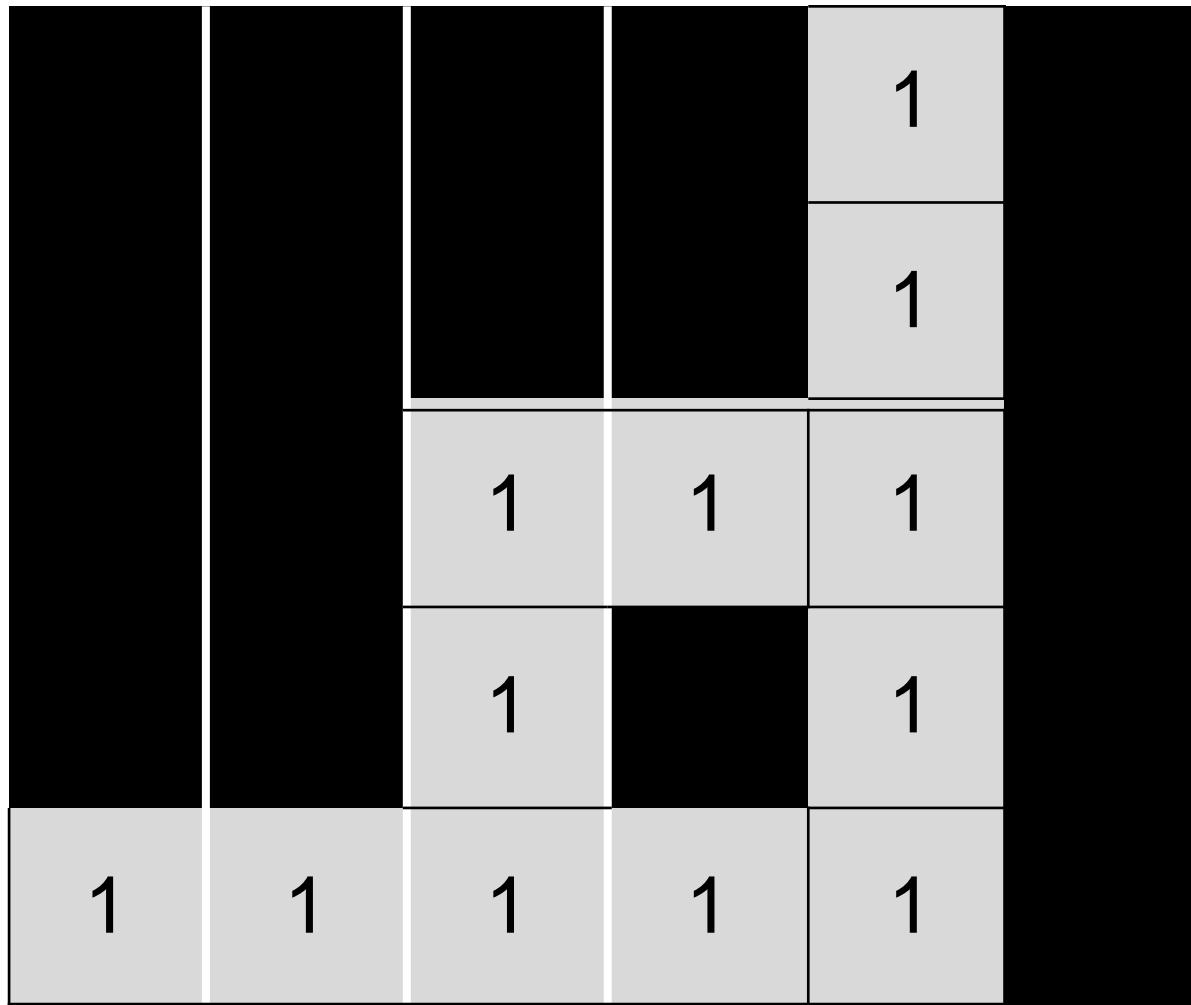


List of Region Labels



*All three labels are equivalent, so merge into single label.*

# Region labeling example (4-neighborhood)



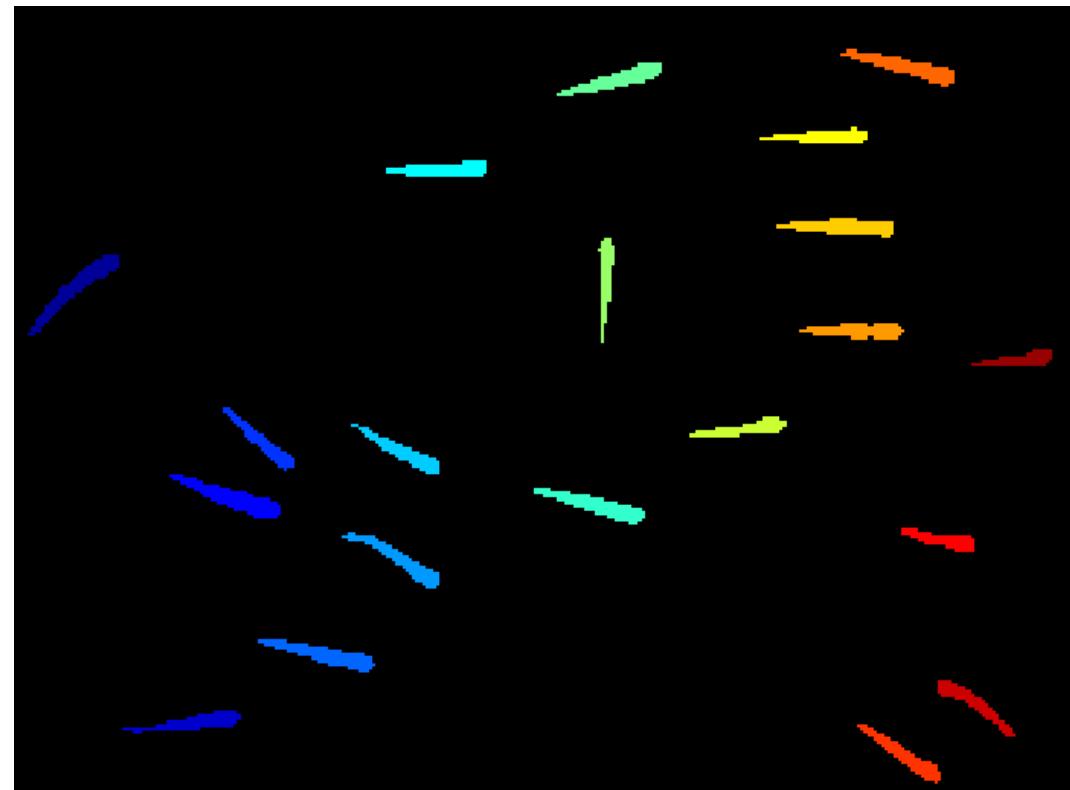
List of Region Labels

1

# Example: region labeling



Thresholded image



20 labeled regions



# Region counting algorithm

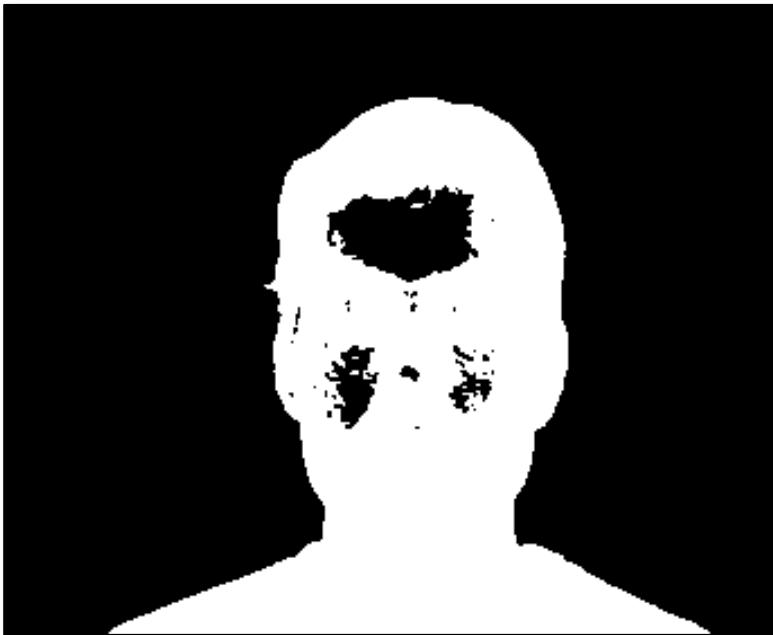
- Measures the size of each region
- Initialize  $counter[label]=0$  for all  $label$
- Loop through all pixels  $f[x,y]$ , left to right, top to bottom
  - If  $f[x,y]=0$ , do nothing.
  - If  $f[x,y]=1$ , increment  $counter[label[x,y]]$

# Small region removal

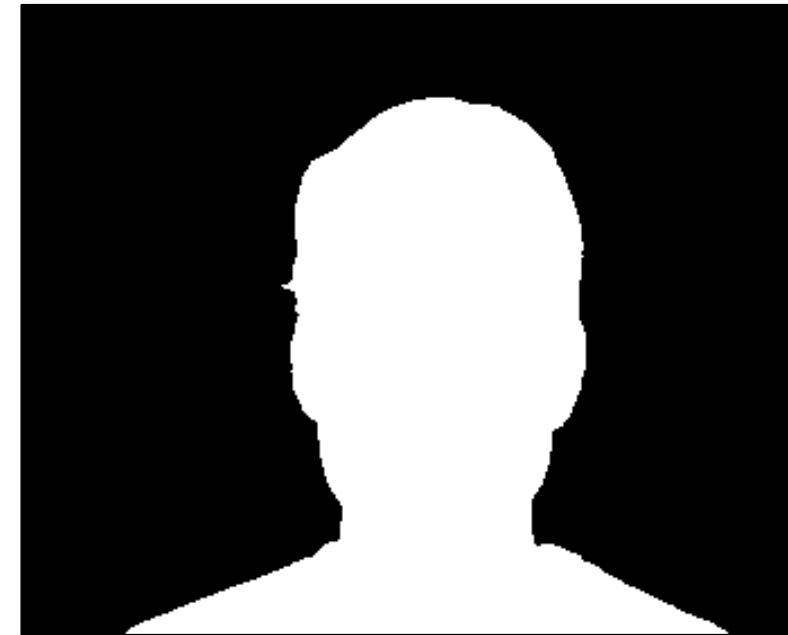
- Loop through all pixels  $f[x,y]$ , left to right, top to bottom
  - If  $f[x,y]=0$ , do nothing.
  - If  $f[x,y]=1$  and  $counter[label[x,y]] < S$ , set  $f[x,y]=0$
- Removes all regions smaller than  $S$  pixels

# Hole filling as dual to small region removal

Mask with holes



After NOT operation, (background)  
region labeling, small region removal,  
and second NOT operation



# Region moments

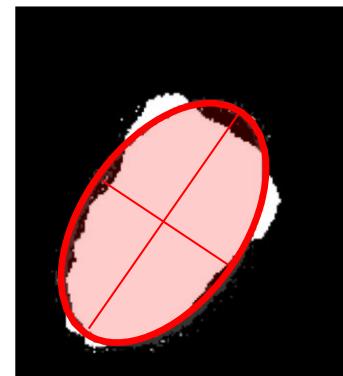
- Raw moments  $M_{pq} = \sum_{x,y \in \text{Region}} x^p y^q$

- Central moments

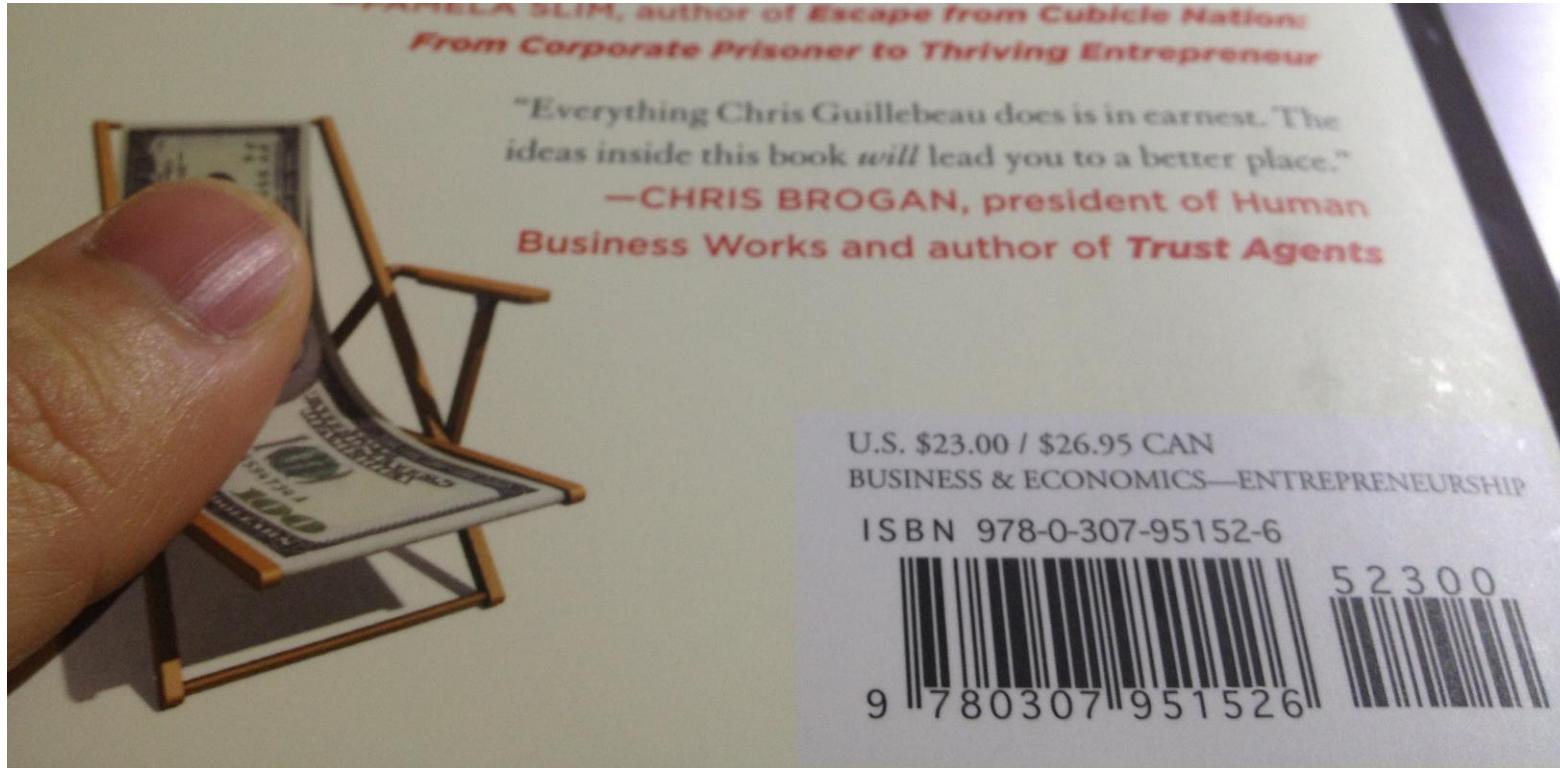
$$\mu_{pq} = \sum_{x,y \in \text{Region}} (x - \bar{x})^p (y - \bar{y})^q \quad \text{with } \bar{x} = \frac{M_{10}}{M_{00}} \text{ and } \bar{y} = \frac{M_{01}}{M_{00}}$$

- Region orientation and eccentricity:  
calculate eigenvectors of covariance  
matrix

$$\begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix}$$



# Example: Detecting bar codes

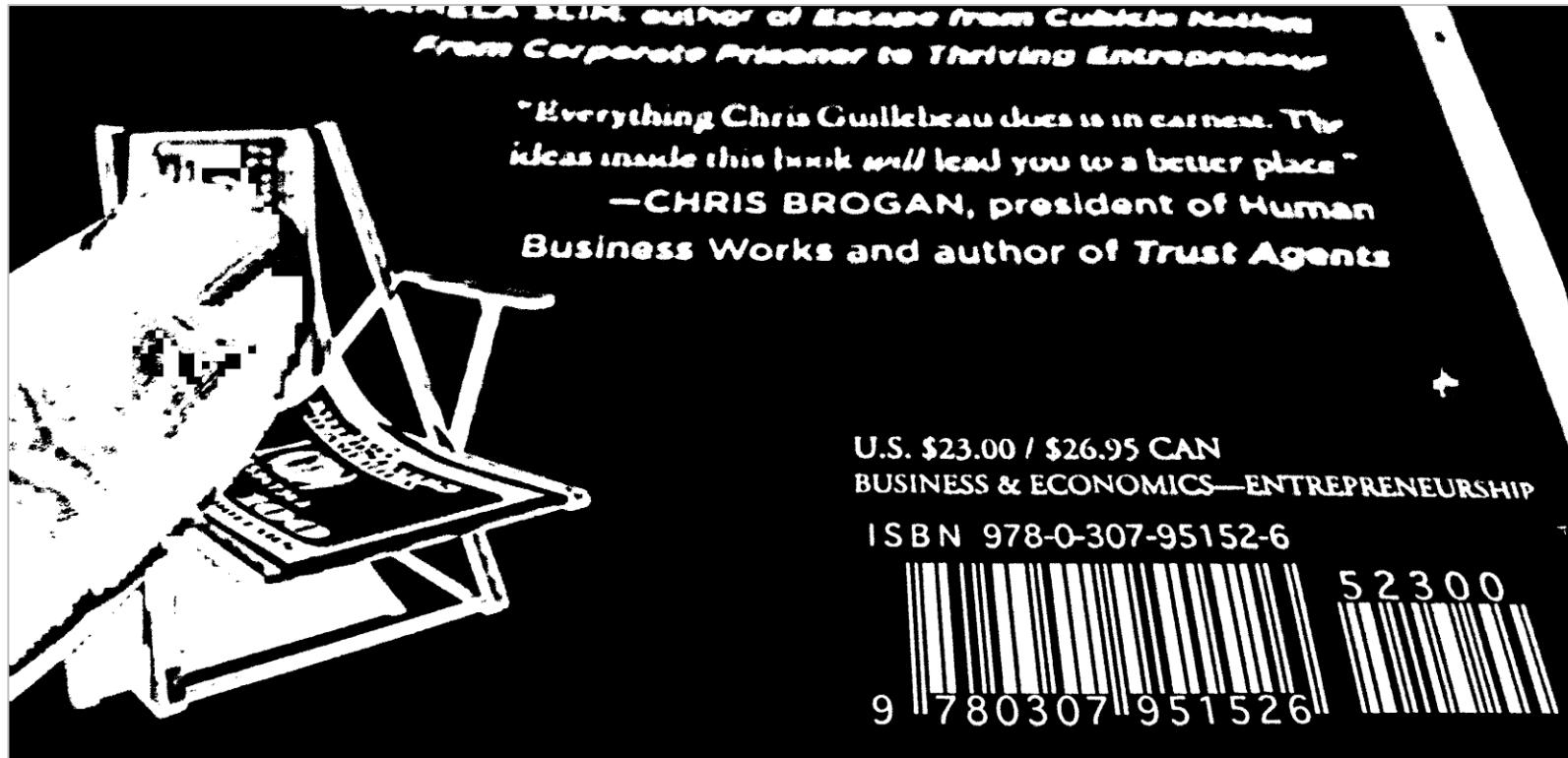


Original Image



# Example: Detecting bar codes

Locally adaptive  
thresholding



# Example: Detecting bar codes

Locally adaptive  
thresholding

Filtering by  
eccentricity

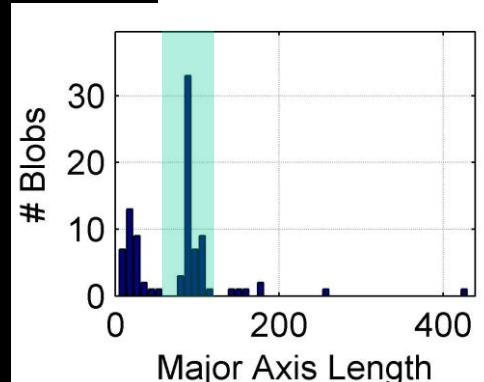


# Example: Detecting bar codes

Locally adaptive  
thresholding

Filtering by  
eccentricity

Filtering by major  
axis length



# Example: Detecting bar codes

Locally adaptive thresholding

Filtering by eccentricity

Filtering by major axis length

Filtering by orientation

