1 Examples of math in Latex

Sets
$F = \{F_1, \dots, F_m\}$
$L = \{\text{edge, non-edge}\}$.

$$f : S \to L$$

$$f = \{f_1, \dots, f_m\}$$

2 A numbered equation on a separate line

Equation 1 is shown below:

$$\mathbf{y^2 = z_i^2} \tag{1}$$

Here's another equation, equation 2:

$$\mathbf{y = z_i + x_i} \tag{2}$$

3 Elements of sets

N sub i: N_i
element of: $i \in S$
arrow $i \in N_j \Leftrightarrow j \in N_i$
$N = \{N_i \quad | \quad \forall i \in S\}$. The pair (S, N)
superscript: x^2

4 Probability

Probability $P(F_i = f_i)$ is abbreviated $P(f_i)$.
The joint event $(F_1 = f_1, \dots, F_m = f_m)$
F is a random Markov field iff:
1) $P(f) > 0, \forall f$
2) $P(f_i | f_{S-\{i\}}) = P(f_i | f_{N_i})$

5 Sum, Prod

$$Z = \sum_{\text{all } f} \exp\left(-\frac{U(f)}{T}\right)$$

$$U(f) = \sum_{c \in C} V_c(f)$$

$$U(f) = \sum_{i \in S} V_1(f_i) + \sum_{i \in S} \sum_{j \in N_i} V_2(f_i, f_j)$$

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$$U(F) = \sum_{i \in S} f_i G_i(f_i) + \sum_{i \in S} \sum_{j \in N_i} \beta_{ij} f_i f_j$$

$$U(f) = \sum_i (f_i - f_{i-1})^2$$

$$U(f) = \sum_{i,j \in N_i} V_2(f_i, f_j)$$

$$U(d|f) = \sum_{i=1}^{m} (f_i - d_i)^2 / 2\sigma^2$$

$$U(f) = \sum_{i \in S, j \in N_i} \lambda(1 - \delta(f_i - f_j))$$

$$p(d|f) = \prod_i p(d_i|f_i)$$

6 Fractions

$$P(f) = \frac{\exp\left(-\frac{U(f)}{T}\right)}{Z}$$

$$P(f_i|f_{Ni}) = \frac{\exp\left[-\left(V_1(f_i) + \sum_{j \in N_i} V_2(f_i, f_j)\right)\right]}{\sum_{f_i \in L} \exp\left[-\left(V_1(f_i) + \sum_{j \in N_i} V_2(f_i, f_j)\right)\right]}$$

Conditional probability for $f_i = 0, 1$

$$P(f_i|f_{Ni}) = \frac{\exp\left(\alpha f_i + \sum_{j \in N_i} \beta f_i f_j\right)}{\sum f_{i \in \{0,1\}} \exp\left(\alpha f_i + \sum_{j \in N_i} \beta f_i f_j\right)}$$

$$p(d|f) = \frac{1}{\prod_{i=1}^{m} \sqrt{2\pi\sigma_i^2}} \exp(-U(d|f))$$

7 Argmin, Argmax

$$f^* = \arg\min_f U(f|d)$$

$$f^* = \arg\max_f \{P(f|d)\} = \arg\max_f \{p(d|f)P(f)\}$$

$$f_i^{(k+1)} = \arg\min_{f_i} V(f_i|d_i, f_{N_i}^{(k)})$$

8 Misc

$d_i = f_i + e_i$, with $e_i \sim N(\mu, \sigma^2)$.
$a \leq x_i \leq b$

$$V_c(f) = \begin{cases} 0 & \text{all sites in } c \text{ have same label} \\ \lambda_c > 0 & \text{otherwise} \end{cases}$$

A special case: Only 2-cliques

$$V_2(f_i, f_j) = \lambda(1 - \delta(f_i - f_j))$$

$g(x) = \min(x^2, \theta)$.
$g(\cdot)$

9 Metropolis sampler (algorithm)

For a given 'temperature' T:

- initialize f
- Repeat
  1. generate $f' \in N(f)$.
     $N(f)$ denotes the vicinity of f: the new configuration f' is obtained by perturbing f. Example: change the label at site i: $f_i \rightarrow f_i'$.
  2. $\Delta U = U(f') - U(f)$
  3. $P = \min\{1, \exp(-\Delta/T)\}$
  4. if random$[0, 1) < P$ then $f := f'$.
     The new configuration f' is always accepted if $\Delta U \leq 0$ and accepted with probability $P = \exp(-\Delta U/T)\}$ if $\Delta U > 0$. This is the occasional 'uphill' move, facilitated by high T.
- until equilibrium
- return f