

Local Search

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Instructor's Information

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Acknowledgment

The slides in this PPT file are composed using the materials supplied by

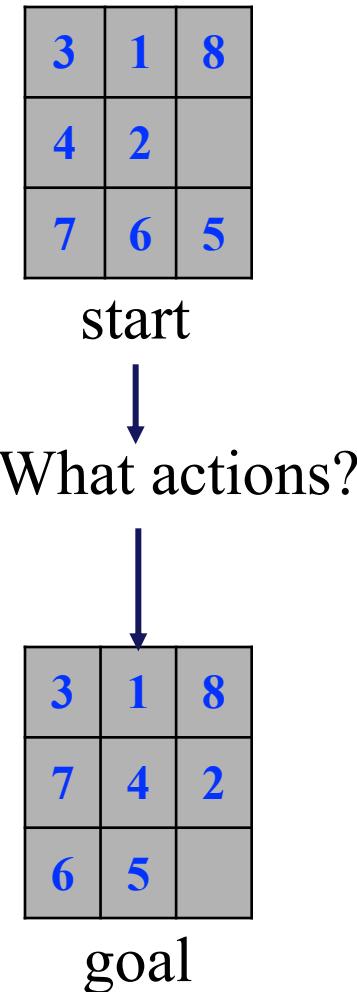
- ☞ **Prof. Stuart Russell and Peter Norvig:** They are currently from University of California, Berkeley. They are also the author of the book “Artificial Intelligence: A Modern Approach”, which is used as the textbook for the course
- ☞ **Prof. Tom Lenaerts,** from Université Libre de Bruxelles

Outline

- ❖ Recap
- ❖ Local Search - Introduction
- ❖ Hill Climbing
- ❖ Simulated Annealing
- ❖ Genetic Algorithms

Recap

- ❖ Previously strategies
 - ☞ Breadth-First Search
 - ☞ Depth-First Search
 - ☞ Uniform-Cost Search
 - ☞ Greedy Best-First Search
 - ☞ A-Star
 - ☞ ...
- ❖ Previously strategies' type of solutions
 - ☞ A typical solution is a **path** from the start to the goal state



Recap

- ❖ Previously strategies
 - ☞ Breadth-First Search
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 - ☞ ...
- ❖ Previously strategies' type of solutions
 - ☞ A typical solution is a **path** from the start to the goal state

start



What actions?



		Q	
Q			
			Q
	Q		

a goal

Local search and optimization

Local Search

- ❖ Local search's type of solutions
 - ☛ NOT a path from the start to the goal state
 - ☛ **Solution: a state, a representation that satisfies users' need**
 - ☛ **The start state???**
 - ✓ Any (a randomly generated state)

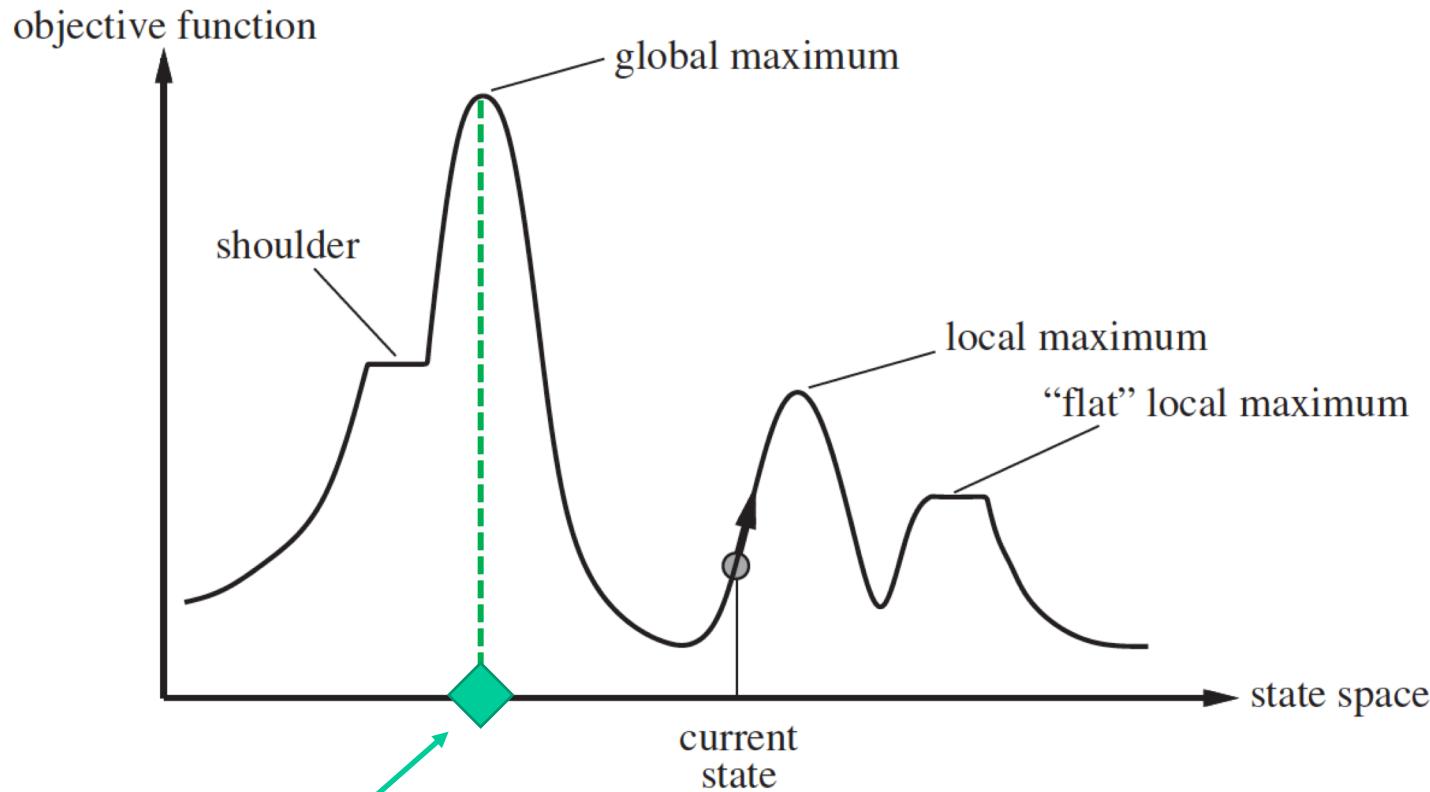
			Q
Q			
			Q
	Q		

a goal

Local search and optimization

- ❖ Local search= use single current state and move to neighboring states.
- ❖ Advantages:
 - ☞ Use very little memory
 - ☞ Find often reasonable solutions in large or infinite state spaces.
- ❖ Are also useful for pure optimization problems.
 - ☞ Find best state according to some *objective function*.
 - ☞ e.g. survival of the fittest as a metaphor for optimization.

Local search and optimization



How can we find this optimal state?

Hill-climbing

Hill-climbing search

- ❖ “is a loop that continuously moves in the direction of increasing value”
 - ☞ It terminates when a peak is reached.
- ❖ Hill climbing does not look ahead of the immediate neighbors of the current state.
- ❖ Hill-climbing chooses randomly among the set of best successors, if there is more than one.
- ❖ Hill-climbing a.k.a. *greedy local search*

Hill-climbing search

function HILL-CLIMBING(*problem*) **return** a state that is a local maximum

input: *problem*, a problem

local variables: *current*, a node.

neighbor, a node.

current \leftarrow MAKE-NODE(INITIAL-STATE[*problem*])

loop do

neighbor \leftarrow a highest valued successor of *current*

if VALUE [*neighbor*] \leq VALUE[*current*] **then return** STATE[*current*]

current \leftarrow *neighbor*

Hill-climbing search

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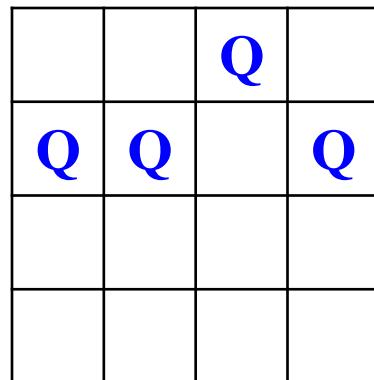
current \leftarrow *neighbor*

We are at a peak already → stop searching

Hill-climbing example

- ❖ 8-queens problem (complete-state formulation).
- ❖ Successor function: move a single queen to another square in the same column.
- ❖ Heuristic function $h(n)$: the number of pairs of queens that are attacking each other (directly or indirectly).

Hill-climbing example



A start state

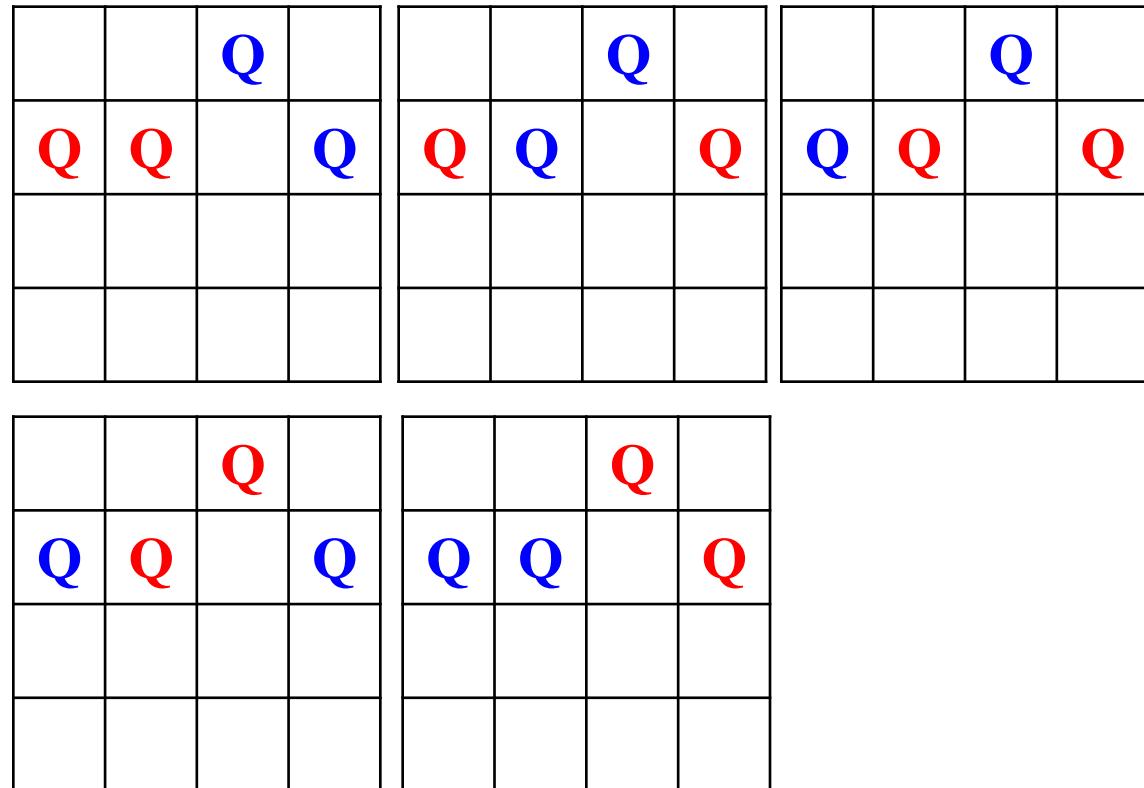
(Assume that we are given this state to start)

Hill-climbing, objective function

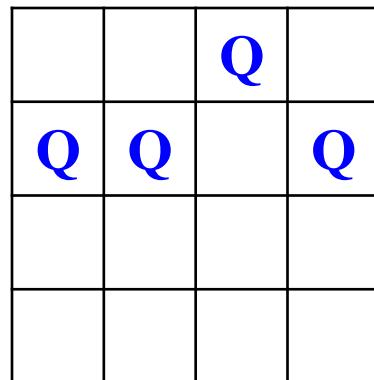
		Q	
Q	Q		Q

A start state

#attackingpairs: 5



Hill-climbing, objective function



A start state

How many neighbours does this have?

Hill-climbing, objective function

	1	2	3	4
4			Q	
3	Q	Q		Q
2				
1				

A start state

How many neighbours does this have?

There are four queens; each moving in a column

Three possible moves for each queens

→ $4 \times 3 = 12$ neighbours (corresponding to 12 blank cells)

Hill-climbing, objective function

		Q	
Q	Q		Q

A start state
#attackingpairs: 5

Move
the first queen to (4,1)

Q		Q	
	Q		Q

A start state
#attackingpairs: 5

Represented

↓

5		Q	
Q	Q		Q

Hill-climbing, step-by-step

1	2	3	4	
4	5	4	Q	3
3	Q	Q	6	Q
2	5	3	5	2
1	3	3	4	3

A start state

#attackingpairs: 5

12 children, the smallest heuristic is 2 (<5)

Best move: move 4th Queen to (2,4)

		Q	
	Q	Q	
			Q

#attackingpairs: 2

Hill-climbing, step-by-step

	1	2	3	4
4	3	3	Q	3
3	Q	Q	4	5
2	4	2	3	Q
1	1	0	3	3

#attackingpairs: 2

12 children, the smallest heuristic is 0
Best move: move 2nd Queen to (1,2)

		Q	
	Q		
			Q

#attackingpairs: 0

Solution is found

The solution can be found after 2 moves

Hill-climbing, step-by-step

3	3	Q	3
Q	Q	4	5
4	2	3	Q
1	0	3	3

#attackingpairs: 2

12 children, the smallest heuristic is 0
→ Make a move to the solution

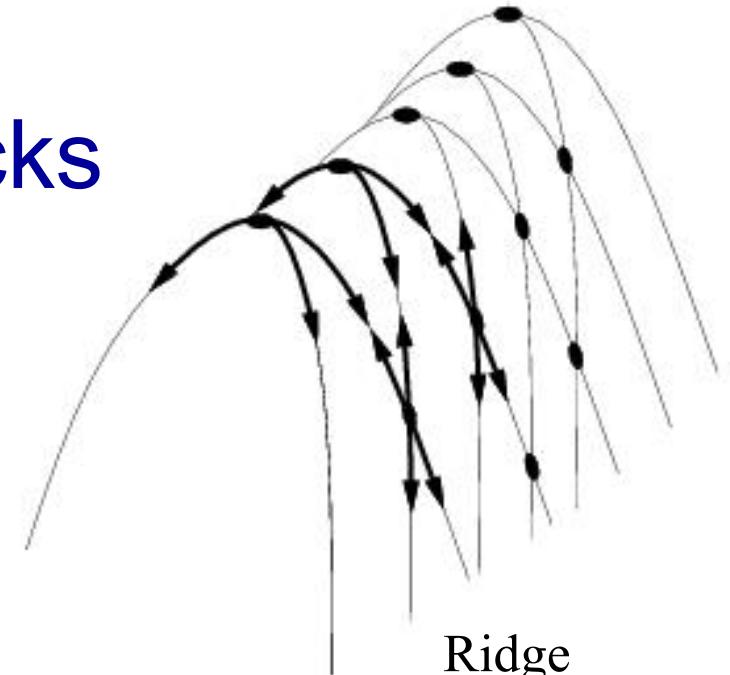
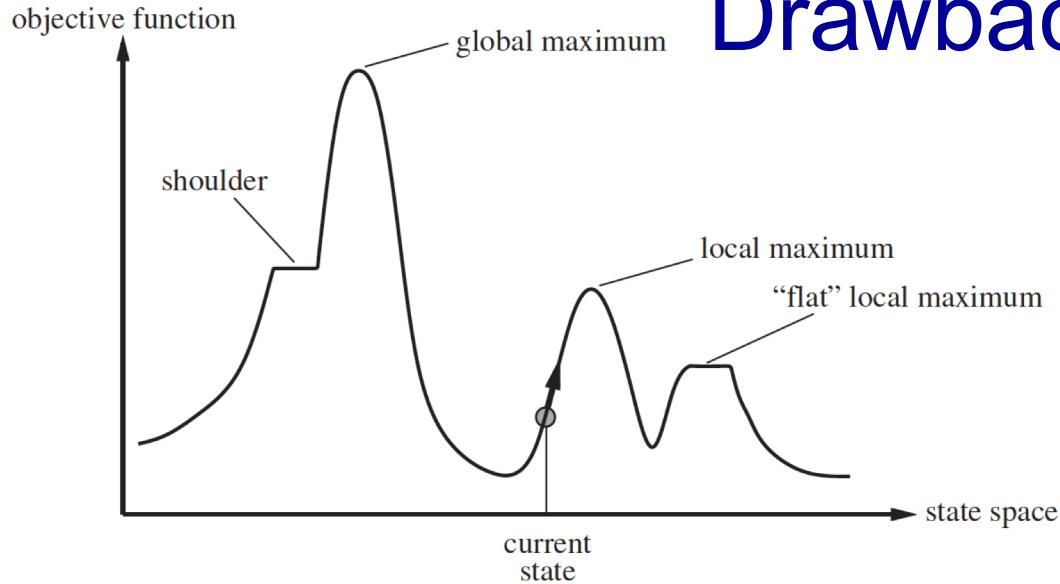
		Q	
Q			
			Q
	Q		

#attackingpairs: 0

Solution is found

The solution can be found after 2 moves
Lucky, most of the cases we can not solve the game
by using simple Hill-Climbing

Drawbacks



- ❖ Ridge = sequence of local maxima difficult for greedy algorithms to navigate
- ❖ Plateaux = an area of the state space where the evaluation function is flat.
- ❖ Gets stuck 86% of the time.

Hill-climbing variations

- ❖ Stochastic hill-climbing
 - ☞ Random selection among the uphill moves.
 - ☞ The selection probability can vary with the steepness of the uphill move.
- ❖ First-choice hill-climbing
 - ☞ cfr. stochastic hill climbing by generating successors randomly until a better one is found.
- ❖ Random-restart hill-climbing
 - ☞ Tries to avoid getting stuck in local maxima.

Simulated annealing

Simulated annealing

- ❖ Escape local maxima by allowing “bad” moves.
 - ☞ Idea: but gradually decrease their size and frequency.
- ❖ Origin; metallurgical annealing
- ❖ Bouncing ball analogy:
 - ☞ Shaking hard (= high temperature).
 - ☞ Shaking less (= lower the temperature).
- ❖ If T decreases slowly enough, best state is reached.
- ❖ Applied for VLSI layout, airline scheduling, etc.

Simulated annealing

```
function SIMULATED-ANNEALING(problem, schedule) return a solution state
  input: problem, a problem
        schedule, a mapping from time to temperature
  local variables: current, a node.
                    next, a node.
                    T, a “temperature” controlling the probability of downward steps

  current  $\leftarrow$  MAKE-NODE(INITIAL-STATE[problem])
  for t  $\leftarrow$  1 to  $\infty$  do
    T  $\leftarrow$  schedule[t]
    if T = 0 then return current
    next  $\leftarrow$  a randomly selected successor of current
     $\Delta E \leftarrow$  VALUE[next] - VALUE[current]
    if  $\Delta E > 0$  then current  $\leftarrow$  next
    else current  $\leftarrow$  next only with probability  $e^{\Delta E/T}$ 
```

Simulated annealing

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Simulated annealing

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```

current \leftarrow MAKE-NODE(INITIAL-STATE[*problem*])

for *t* \leftarrow 1 to ∞ **do**

T \leftarrow *schedule*[*t*]

if *T* = 0 **then return** *current*

next \leftarrow a randomly selected successor of *current*

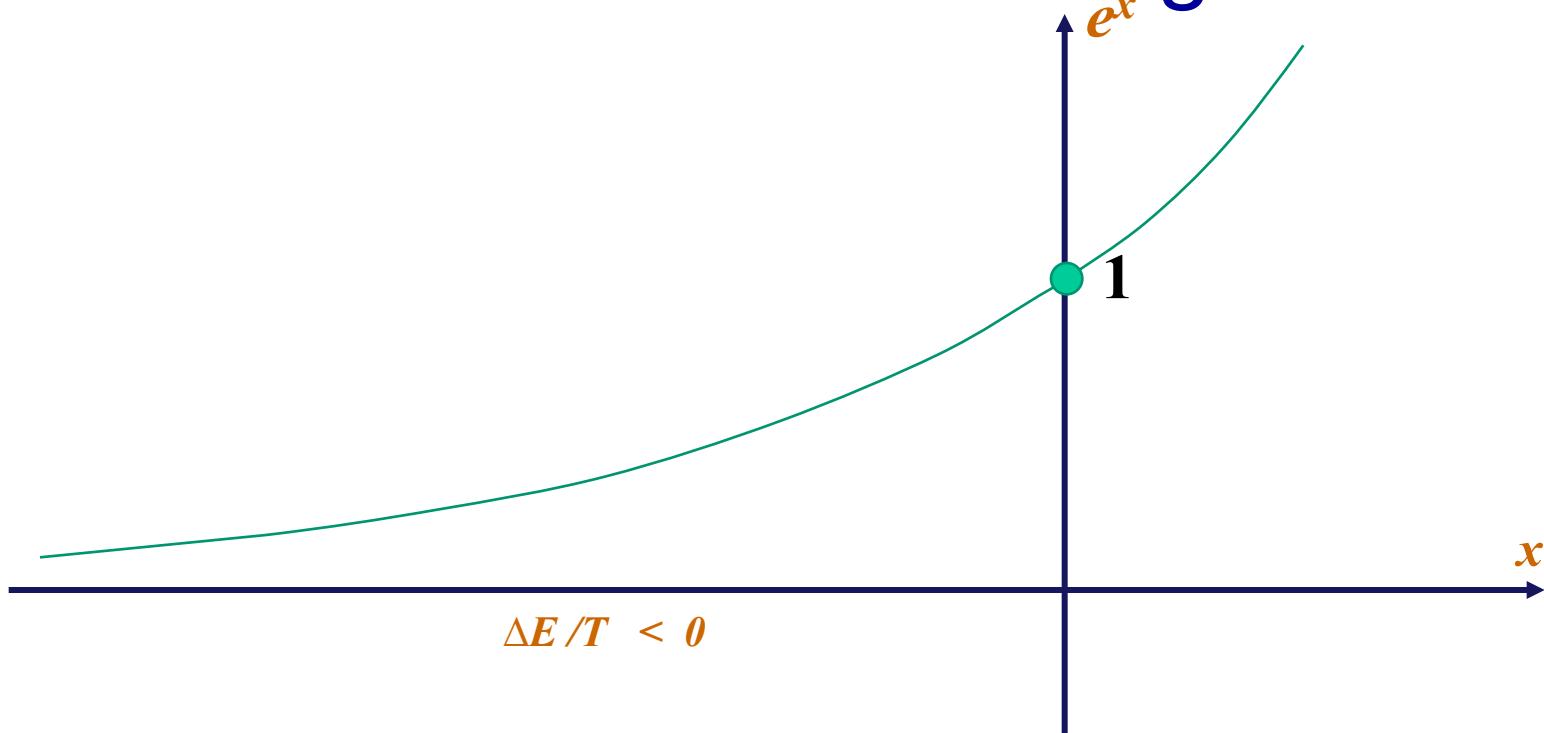
$\Delta E \leftarrow$ VALUE[*next*] - VALUE[*current*]

if $\Delta E > 0$ **then** *current* \leftarrow *next*

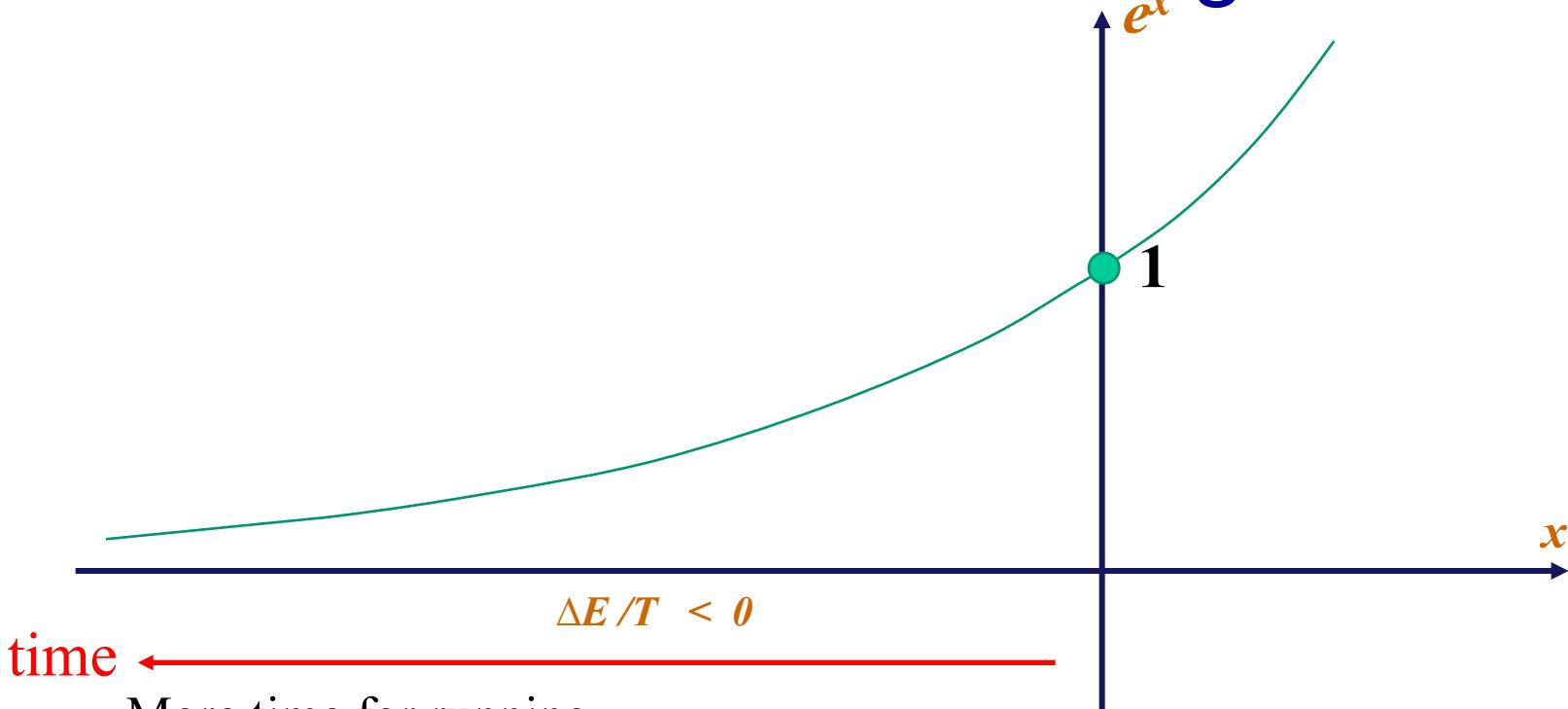
else *current* \leftarrow *next* **only with probability** $e^{\Delta E / T}$

Accept a state worse than the current

Simulated annealing



Simulated annealing



time ←

More time for running

⇒ Temperature (T): smaller and smaller

⇒ $|\Delta E/T|$: larger and larger

⇒ $\Delta E/T$: more negative

⇒ probability for accepting a worse state be smaller and smaller

Local beam search

Local beam search

- ❖ Keep track of k states instead of one
 - ☞ Initially: k random states
 - ☞ Next: determine all successors of k states
 - ☞ If any of successors is goal → finished
 - ☞ Else select k best from successors and repeat.
- ❖ Major difference with random-restart search
 - ☞ Information is shared among k search threads.
- ❖ Can suffer from lack of diversity.
 - ☞ Stochastic variant: choose k successors at proportionally to state success.

Local beam search

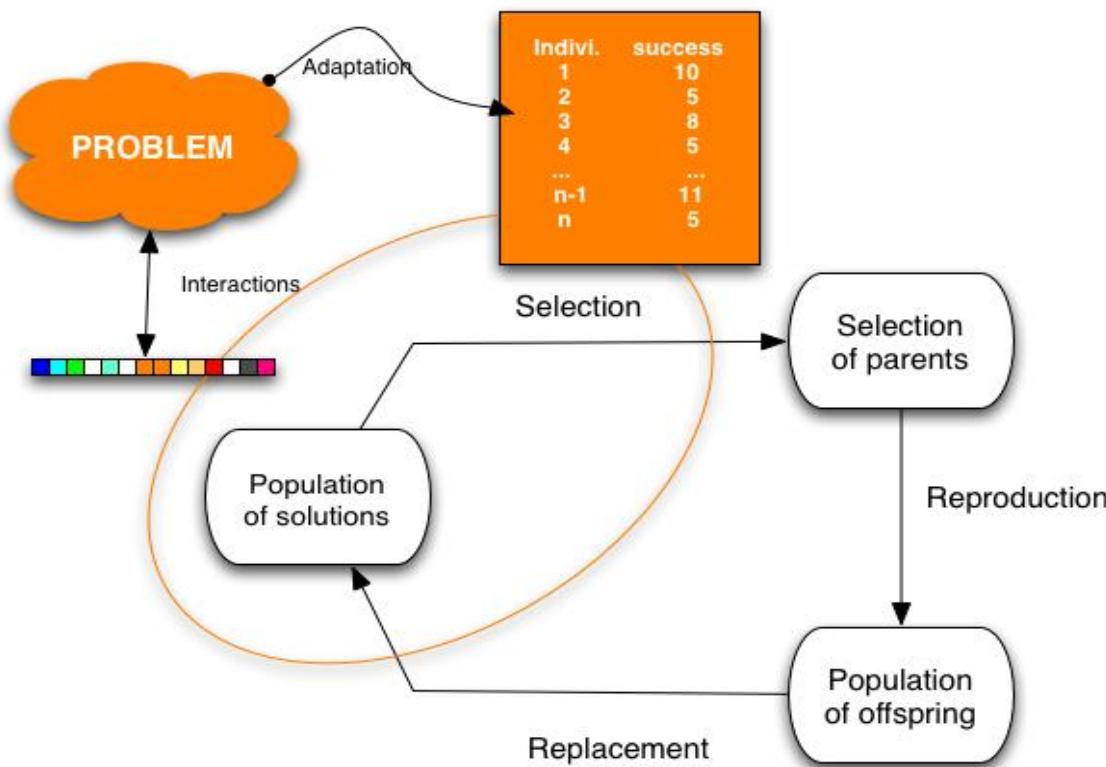
Different to running k instances of Hill-Climbing parallel

- ❖ Keep track of k states instead of one
 - ☞ Initially: k random states
 - ☞ Next: determine all successors of k states
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Genetic algorithms

Genetic algorithms

- ❖ Variant of local beam search with *sexual recombination*.

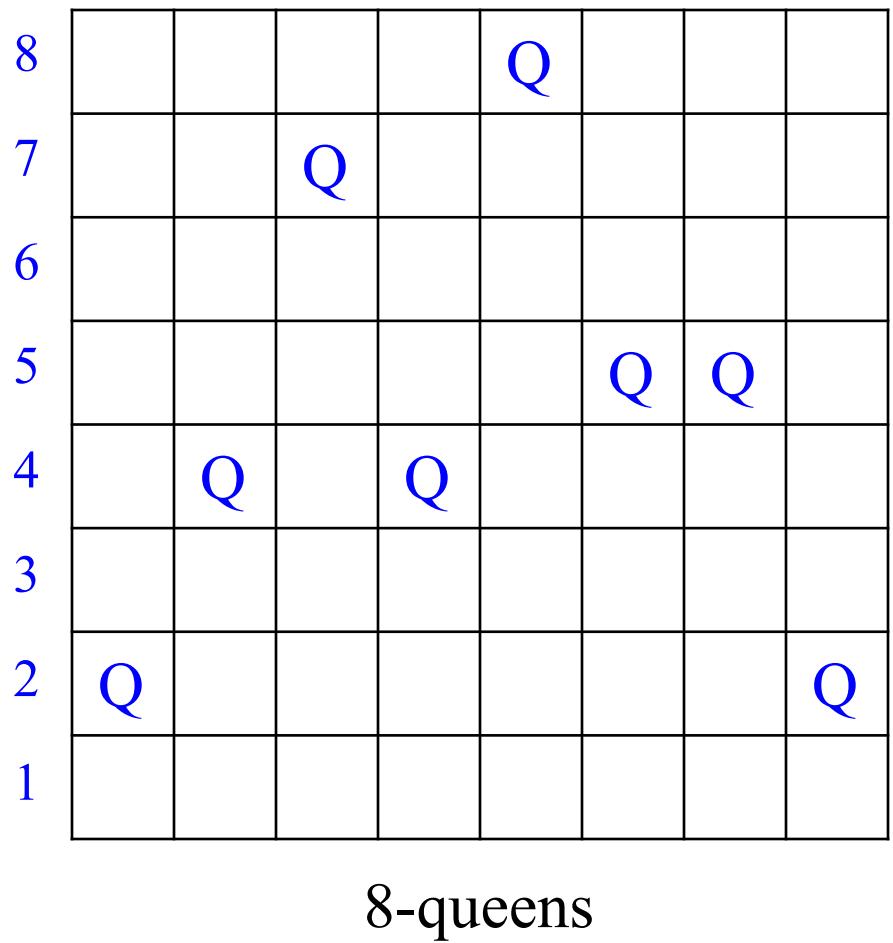


Genetic algorithms

❖ Tasks

- Problem's states => genes: how to?
- Genes: how to measure the goodness/fitness?
- Mating individuals: how to?

Genetic algorithms, example



Percentage having matting:
~ fitness value

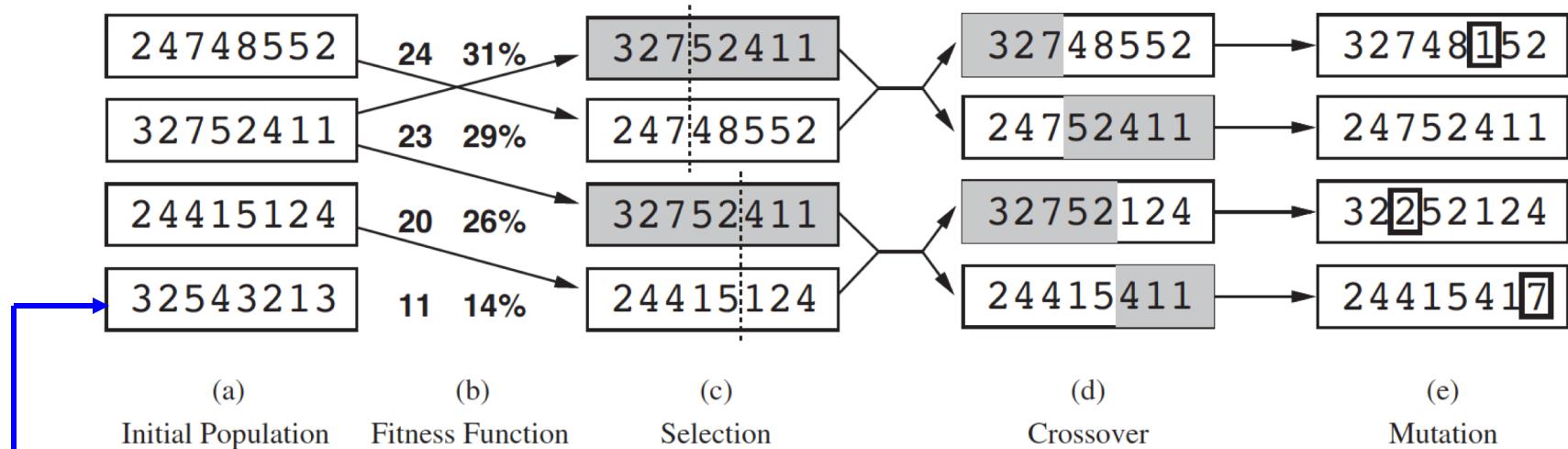
Fitness value:
#nonattacking pairs
 $(n(n-1)/2)$: max

Encoded
as a string

2 4 7 4 8 5 5 2	24	31%
3 2 7 5 2 4 1 1	23	29%
2 4 4 1 5 1 2 4	20	26%
3 2 5 4 3 2 1 3	11	14%

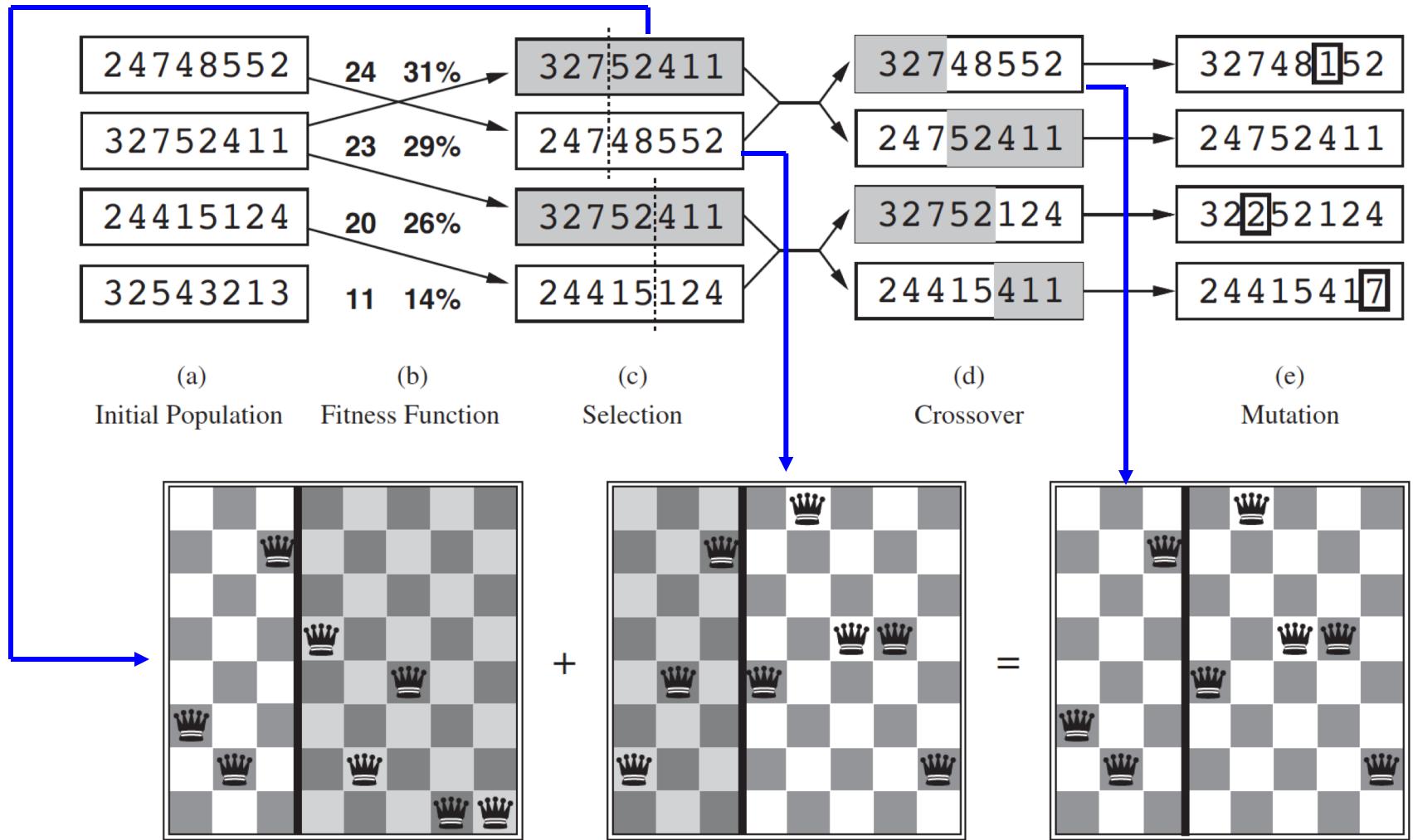
(a) Initial Population (b) Fitness Function

Genetic algorithms, example

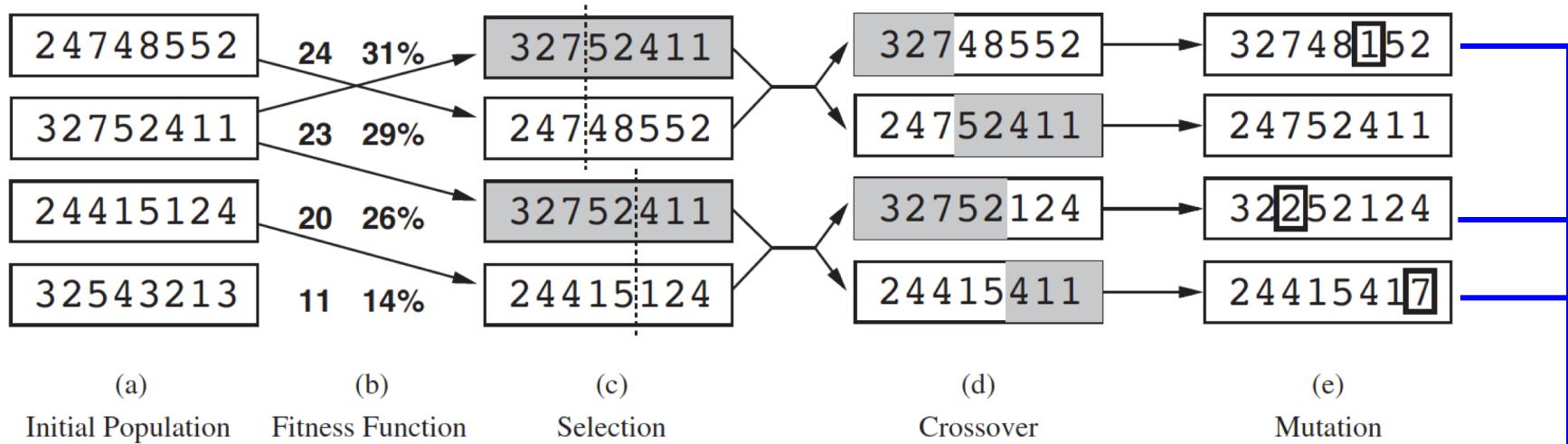


Some individuals may not have any chance for mating!!!

Genetic algorithms, example



Genetic algorithms, example



Have mutations with a small chance (probability)

Genetic algorithm

```
function GENETIC_ALGORITHM(population, FITNESS-FN) return an individual
  input: population, a set of individuals
        FITNESS-FN, a function which determines the quality of the individual
  repeat
    new_population  $\leftarrow$  empty set
    loop for i from 1 to SIZE(population) do
      x  $\leftarrow$  RANDOM_SELECTION(population, FITNESS_FN)
      y  $\leftarrow$  RANDOM_SELECTION(population, FITNESS_FN)
      child  $\leftarrow$  REPRODUCE(x,y)
      if (small random probability) then child  $\leftarrow$  MUTATE(child)
      add child to new_population
    population  $\leftarrow$  new_population
  until some individual is fit enough or enough time has elapsed
  return the best individual
```