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HA: 6
M: 4 (*)
AP: 5

(*) Gleichanteile um Faktor 2 falsch
function, die eine DFT berechnet, fehlt

$$f_T(t) = \pi \cdot \sin\left(\frac{\pi}{4} t\right) - \frac{\pi}{2}$$

1) a) $c_0 = \frac{1}{T} \cdot \int_0^T f(t) \, dt$ (T: Periode)

$$= \frac{1}{2} \cdot \int_0^2 \left(\pi \cdot \sin\left(\frac{\pi}{4} t\right) - \frac{\pi}{2}\right) dt$$

$$= \frac{1}{2} \cdot \left(\int_0^2 \pi \sin\left(\frac{\pi}{4} t\right) dt - \frac{\pi}{2} \int_0^2 dt \right)$$

$$= \frac{1}{2} \cdot \left(\pi \cdot \frac{4}{\pi} \cdot \int_0^2 \sin\left(\frac{\pi}{4} t\right) d\left(\frac{\pi}{4} t\right) - \frac{\pi}{2} \cdot \left[t \right]_0^2 \right)$$

$$= \frac{1}{2} \cdot \left(4 \cdot \left[-\cos\left(\frac{\pi}{4} t\right) \right]_0^2 - \pi \right)$$

$$= \frac{1}{2} \cdot (4 - \pi) = 2 - \frac{\pi}{2}$$

$$\Leftrightarrow c_0 = 2 - \frac{\pi}{2} \approx 0{,}429 \ ✓$$

b) $c_n = \frac{1}{2} \int_0^2 \left(\pi \sin\left(\frac{\pi}{4} t\right) - \frac{\pi}{2}\right) \cdot e^{-in\pi t} \, dt$

$$= \frac{1}{2} \left(\underbrace{\pi \cdot \int_0^2 \sin\left(\frac{\pi}{4} t\right) \cdot e^{-in\pi t} dt}_{I_1} - \underbrace{\frac{\pi}{2} \int_0^2 e^{-in\pi t} dt}_{I_2} \right)$$

$$\cdot\ I_1 = \int_0^2 \sin\left(\frac{\pi}{4}t\right)\cdot e^{-in\pi t}\,dt$$

$*$ Vorgegebene Formel: $\int e^{ax} \sin bx \, dx$

$$= \frac{e^{ax}}{a^2+b^2}\left(a \sin bx - b \cos bx\right)$$

hier anwenden, mit: $x = t$ (variable)

$a = -i.n.\pi$

$b = \frac{\pi}{4}$

$$\Rightarrow I_1 = \left[\frac{e^{-in\pi.t}}{\frac{\pi^2}{16}-(n\pi)^2}\left(-in\pi\cdot\sin\left(\frac{\pi}{4}t\right) - \frac{\pi}{4}\cdot\cos\left(\frac{\pi}{4}t\right)\right)\right]_0^2$$

$$\cancel{= \frac{e^{-2\pi n i}}{\pi^2\left(\frac{1}{16}-n^2\right)}\left(-in\pi \cdot -\frac{\pi}{4}\right) \frac{1}{\pi^2\left(\frac{1}{16}-n^2\right)}} - \frac{1}{\pi^2\left(\frac{1}{16}-n^2\right)}$$

$$= \frac{e^{-2\pi n i}}{\pi^2\left(\frac{1}{16}-n^2\right)}\left(-in\pi\right) - \frac{1}{\pi^2\left(\frac{1}{16}-n^2\right)}\cdot\left(-\frac{\pi}{4}\right)$$

$$= \cancel{e^{-2\pi n i}}\ \frac{-i.n}{\pi\left(\frac{1}{16}-n^2\right)} + \frac{1}{4\pi\left(\frac{1}{16}-n^2\right)}$$

$$= \frac{1-4in}{4\pi\left(\frac{1}{16}-n^2\right)} \quad (2)$$

$$I_2 = \int_0^2 e^{-in\pi t}\, dt$$

$$= \frac{-1}{in\pi} \cdot \int_0^2 e^{-in\pi t} \cdot d(-in\pi t)$$

$$= \frac{-1}{in\pi} \cdot \left[e^{-in\pi t} \right]_0^2$$

$$= \frac{-1}{in\pi} \cdot \left(e^{-2\pi n i} - 1 \right) = 0 \checkmark \quad (3)$$

(2), (3) $\rightarrow$ (1)

$$\Leftrightarrow c_n = \frac{1}{2} \cdot \left(\pi \cdot \frac{1 - 4in}{4\pi\left(\frac{1}{16} - n^2\right)} \right)$$

$$= \frac{1 - 4in}{8\left(\frac{1}{16} - n^2\right)}$$ (✓) Vereinfachen!

$$A_n = 2|c_n| = 2 \cdot \frac{|1 - 4n \cdot i|}{|8| \cdot \left| \frac{1}{16} - n^2 \right|}$$

$$= 2 \cdot \frac{\sqrt{1 + 16n^2}}{8 \cdot \left| \frac{1}{16} - n^2 \right|}$$

$$= \frac{\sqrt{1 + 16n^2}}{4 \cdot \left| \frac{1}{16} - n^2 \right|} = \frac{4\sqrt{16n^2 + 1}}{16n^2 - 1}$$

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c) $A_1 = \frac{\sqrt{1+16}}{4 \cdot \left| \frac{1}{16} - 1 \right|} \approx 1{,}099$

$A_2 \cancel{\approx} = \frac{4\sqrt{65}}{63} \approx 0{,}512$

$A_3 \approx 0{,}337$ $A_4 \approx 0{,}251$ ✓

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