Université Paris-Sud
USTH
Master WEO

practice 1

Markov chains

The goal of this practice is to analyze different Markov processes. Use Matlab to calculate the characteristic matrices and to plot figures. At the end of the practice you must give me a report containing your Matlab code, the answers to the questions, the plots giving the proportion of items in each state as a function of time, the results of your <u>own</u> analyses (using what you have learnt in the lecture) and your conclusions about this Markov process. **The more analyses, comments and conclusions the better**. On your report, write the order you have chosen for the states. If you are finished with one Markov process, you can analyze another one.

Each group chooses a different project:

1. **Lead in tap water**

   We suppose that, in a given city, tap water contains lead (Pb). Lead dissolved in water can come from pipes, but also from weld and lead-rich brass accessories. Lead can damage the brain and the liver. It also fixes to the bones where it replaces calcium atoms. In the brain, the Pb atoms may fixe on NMDA protein. In this case the brain is damaged.

   The Pb atoms enter the body by the digestive system (state D). Then, though the blood veins, the lead atoms are transferred to the right ventricle of the heart (state R) with a time constant $r = 1/2\ \mathrm{s}^{-1}$. From the right ventricle, the atoms go to the lung (state U) with a time constant l. Choose yourself a reasonable value for l. Then Pb goes back to the left ventricle of the heart (state L) with the same time constant l. From the left ventricle, the Pb atoms can go to the brain (state B, time constant $b = 1/3\ \mathrm{s}^{-1}$), or to the bones (state O, time constant $o = 1/4\ \mathrm{s}^{-1}$), or to the liver (state I, time constant o). From brain, bones and liver, the Pb goes back to the right ventricle with time constants $c = 1/4.8\ \mathrm{s}^{-1}$ for the brain, $q = 1/23\ \mathrm{minute}^{-1}$ for the bones and o from the liver. From the liver, the Pb atoms can also be eliminated (state E, time constant $e = 1/45\ \mathrm{minute}^{-1}$) from the body. Finally, the Pb atoms from the brain can get fixed to NMDA molecules (state M, time constant $m = 1/12\ \mathrm{minute}^{-1}$).

   Show that lead atoms accumulate first rapidly in bones then are transfered slowly to elimination and to the NMDA.

2. **Protection of the population from Fukushima fallout**

   After the explosion of the Fukushima nuclear plan, radioactive iodine atoms where released in the air (state A). Some iodine felt on the sea water (state W) with the time constant $w = 1/5\ \mathrm{day}^{-1}$ and possibly eat by fish (state F, time constant $f = 1/3\ \mathrm{year}^{-1}$) which can release back the iodine to the water (time constant $h = 1/10\ \mathrm{day}^{-1}$). Other iodines felt from the air on the ground (state G, with time constant $g = 1/15\ \mathrm{day}^{-1}$) then possibly fixed by plants (state P, time constant $p = 1/7\ \mathrm{day}^{-1}$). The iodines on plants and on the ground can diffuse to the underground (state U) with the same time constant $u = 1\ \mathrm{month}^{-1}$. Human (state H) can absorb iodines by eating fish (time constant $x = 1/3\ \mathrm{month}^{-1}$), by inhaling them from the air (time constant $y = 1/4\ \mathrm{month}^{-1}$) or by eating plants (time constant $z = 1/16\ \mathrm{day}^{-1}$). The iodine inside the human body can be eliminated by urine (state E, time constant $e = 1/2\ \mathrm{day}^{-1}$) or can emit their radioactivity and become stable (state R, time constant $r = 1/15\ \mathrm{day}^{-1}$). Finally, all the iodines in all the other states (A, W, F, G, P, U) can also loose their radioactivity (state S) with the same time constant r.

   The authorities want to know if it is more important to forbid the Japanese to eat plants or to eat fish. To calculate that, you must write the differential equation that gives $\mathrm{dH_P}$ and $\mathrm{dH_F}$ the quantities of radioactivity absorbed by humans between time t and t+dt by eating plants and fish. With Matlab, you can add up all these small quantities to obtain the final values.

3. **bacteria on rau muống plants**

   In the following, all the time constants are given in day^{-1}.

   The water coming from a river is used to irrigate a rau muống fields. Unfortunately, the river contains bacteria (in state R) that can by released to the water on the fields (state W) with time constant $w = 1.1$. The bacteria can flow back to the river with time constant $r = 0.23$ or be absorbed by the rau muống plants (state P) with time constant $y = 0.52$. The bacteria in the rau muống can be eaten by humans (state H, time constant $a = 0.001$) or released back to the field water with time constant $x = 0.01$. The bacteria in the river can flow to the sea (state S) with time constant $s = 2.1$ or be eaten by river fish (state F, time constant $f = 0.21$). Finally, the fish can be eaten by humans with time constant $b = 0.07$.

   We want to know if humans are more contaminated by the rau muống bacteria or by the fish bacteria. Write the differential equation giving $\mathrm{d}H$ as a function of $\mathrm{d}t$. This equation contains two terms: $\mathrm{d}H_\mathrm{P}$ and $\mathrm{d}H_\mathrm{F}$. Write the Matlab code that sums all these terms to calculate the final quantities of bacteria absorbed by humans.

4. **Nitrates**

   Nitrates NO_3^- are used to fertilize fields. Unfortunately, the nitrates tends to flow to rivers where they induce eutrophication, that is an increase the concentration of algae that consume the water oxygen and kill fish. Nitrates are also bad for human health as they prevent blood to carrying efficiently oxygen to the body tissues. In the following, all the time constants are given in day^{-1}.

   At the beginning of March, the nitrates are spread over a field (state F). With time constants $n = 0.02$, $u = 0.015$ and $r = 0.01$, this nitrates migrate towards the neighboring fields (state N), towards underground water source (state U) and towards the river (state R). The nitrates can also come back from the neighboring fields or migrate from the neighboring fields to underground sources and to the river with the same time constants n, u and r. The nitrates from the river and from the underground water may be drunk by humans (state H) with time constants $x = 1.5\ 10^{-6}$ and $y = 0.002$. The nitrates from the river and from the underground water can flow to the sea (state S) with time constants $a = 0.012$ and $b = 0.27$.

   The nitrates are dangerous for the river fish when the proportion is over 0.1. During what approximate period will this happen? What is the proportion of nitrates that remains on the field one year latter?

5. **Hanoi water**

   Here is a simplified description of the rain water-flow in the city of Hanoi. In the following, all the time constants are given in hour^{-1}. The water is brought over Hanoi by the clouds (state C). The rain water falls over impermeable areas (state I) such as roads or buildings with a time constant $i = 10$, over lakes (state L) with a time constant $l = 1$ and over permeable areas (state P) with a time constant $p = 2$. From impermeable areas, the water flows to permeable areas with a time constant $q = 1.5$ or, through the sewers, to the Red river (state R) with a time constant $v = 1.7$. From the lakes, the rain waters flows to the river with time constant $r = 3$. The rain water in the Red river goes to the sea (state S) with a time constant $s = 14$. The rain water felt on the permeable areas flows to the rivers with a time constant $w = 0.1$ or is absorbed underground (state U) with a time constant $u = 18$. Underground water flows to the sea with a time constant $a = 9$ or can be directly used by human (state H) with time constant $y = 0.01$ or can go to the city pumping station (state X, time constant $x = 0.1$) to be used, after treatments, by humans, with a time constant $z = 0.3$.

   For each time step $\mathrm{d}t$, what are the quantities $\mathrm{d}H_\mathrm{U}$ and $\mathrm{d}H_\mathrm{X}$ of underground water and tap water drunk by the inhabitants ? Write the Matlab code to calculate the total quantity of underground and tap water used by the inhabitants. Compare these proportions.

6. **Fragmentation of hydrocarbons**

   Propane C_3H_8 is a pollutant gas that is emitted during combustion. In order to destroy these molecules, one can use a plasma (free gas of ions and electrons). The flow of propane molecules goes through the plasma where their are excited by the electrons. As a consequence, the molecules are sequentially broken into two fragments. Each set of propane fragments is called a *partition*. The goal is to reduce large hydrocarbon molecules into smaller, less polluting ones.

   The fragmentation of propane results into 9 partitions (see figure). Each arrow corresponds to the splitting of one fragment into two smaller ones. The values on the arrows are called "branching ratios" and are the equivalent of time constants.

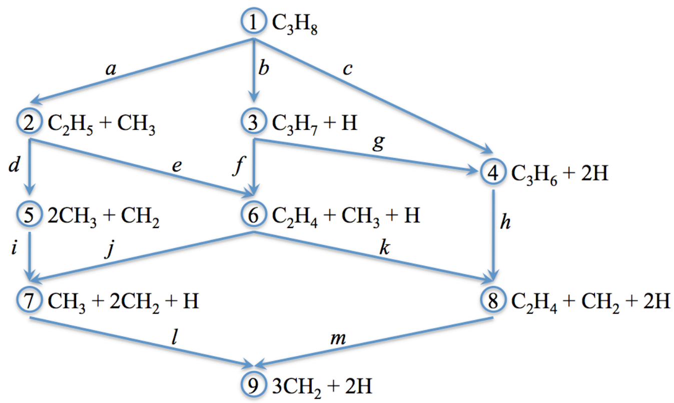

Figure 1: Propane fragmentation

   The values of the time constants are the following: $a = 13,\ b = 23,\ c = 109,\ d = 235,\ e = 456,\ f = 53,\ g = 58,\ h = 1242,\ i = 214,\ j = 733,\ k = 2355,\ l = 1754,\ m = 514.$

   How long the molecules have to spend into the plasma so that one half of the initial propanes are reduced to the 9^{th} partition?

   If we suppose that the fragmentations stop at partitions 7 and 8 (we don't have partition 9), what are the relative proportions of CH_3 and C_2H_4 in the final gas after a very long time in the plasma?

7. **Heavy metals in the sea**

   A steel plant (state P) releases heavy metal atoms in the sea water (state W) with time constant $w = 10$ day^{-1}. These atoms can be absorbed by seaweeds (state S) with time constant $a = 1/1.2$ day^{-1} or by krill (state K, krill are small sea animals like shrimps) with time constant $c = 1/2.3$ day^{-1} or by fish (state F) with time constant $e = 1.2$ day^{-1}. Seaweeds, krill and fish can release the heavy metal atoms back to the water with time constants $b = 1/5$ day^{-1}, $d = 1/1.1$ day^{-1} and $f = 1/0.1$ day^{-1} and can died (state D) with time constants $g = 1$ day^{-1}, $h = 1/10$ day^{-1} and $i = 1/2$ year^{-1}. The krill eat seaweeds (time constant $j = 1/12$ day^{-1}) and fish eat krill (time constants $k = 1/3$ month^{-1}).

   The total mass of seaweeds is 100 times the mass of krill and the mass of krill is 100 times the mass of fish. What can you say about the relative proportions of heavy metals in living seaweeds, krill and fish?

8. **Chernobyl**

   After the explosion of the Chernobyl nuclear plants in 1986, radioactive Cesium (Cs) nuclei where released in the atmosphere (state A). These nuclei could be inhaled by cows (state C) or humans (state H) with the same time constant $x = 1/500$ day^{-1} or fixed on the ground (state G, with time constant $g = 1/5$ day^{-1}) or fixed on vegetables (state V, with time constant $v = 1/20$ day^{-1}). The cesium nuclei that felt on the ground can diffuse to vegetables with time constant $a = 1/30$ day^{-1}. The radioelements in vegetables can be eaten by cows (time constant $c = 1/15$ day^{-1}) or by humans (time constant $z = 1/12$ day^{-1}). Finally, the radioelements in cows can be absorbed by humans with time constant $y = 1/5$ day^{-1} mostly by drinking milk. Finally, all Cs nuclei can loose their radioactivity and become stable (state S) with time constant $s = 1/30$ year^{-1}.

   Calculate dH_A, dH_C, dH_V the quantities of Cesium absorbed by humans between time t and $t+\mathrm{d}t$. Using Matlab, sum these quantities to know what is the main source of contamination for humans: air, milk or vegetable?

9. **Natural radioactivity**

   Radium 226 is the main source of natural radioactivity in the world. It may become dangerous for the humans if the concentration in their house is over 200 mSv, which happens in some granitic areas. The Radium 226 nuclei (state Ra) that is present the ground or from the walls of the houses decays into Radon 222 (state Rn) with time constant $x = 0.1$ minute^{-1}. Radon being a gas, it escapes the ground and vaporizes in the atmosphere of a house. The radon decays into Polonium 218 (state Po8) with time constant $y = 0.26$ day^{-1}. Polonium 218 can decay into lead 214 (state Pb4) with time constant $z = 0.33$ minutes^{-1}. Lead 214 decays into Bismuth 214 (state Bi) with time constant $u = 0.037$ minutes^{-1}. Bismuth 214 decays into Lead 210 (state Pb0) with time constant $v = 0.05$ minutes^{-1}. Lead 210 is a very stable nuclei, that is, it does not decay for a very long time. All radioactive nuclei in the air of the house can be inhaled by the inhabitant (state L) with the same time constant $l = 1$ hour^{-1} or can be reabsorbed by the ground or the wall of the house (state G) with time constant $a = 1$ day^{-1} for Rn and $b = 100$ day^{-1} for Po8, Pb4, Bi and Pb0.

   The nuclei decay by α or β^- radioactivity following:

   $$^{222}\mathrm{Rn} \xrightarrow{\alpha} {}^{218}\mathrm{Po} \xrightarrow{\alpha} {}^{214}\mathrm{Pb} \xrightarrow{\beta^-} {}^{214}\mathrm{Bi} \xrightarrow{\alpha\,+\,\beta^-} {}^{210}\mathrm{Pb}$$

   An α radioactivity, from the nuclei fixed into the lung, induces 5 times more damages to the body than a β radioactivity. A radon nuclei induces three α and two β^- radioactivities, whereas ^{214}Bi induces only one α and one β^- radioactivity.

   For each time step dt, what quantity of Rn nuclei is absorbed by the lung? Write the matlab code to calculate the total quantity of Rn nuclei absorbed by the lung. Same question for Po8, Pb4 and Bi.

   Compare the amount of damages induced by the different types of radioactive nuclei when they are inhaled.