# Nonlinear adaptive filtering algorithms

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**Abstract:**

This paper provides an overview of the Linear Input Filter, Memoryless Polynomial Nonlinearity, and Linear Output Filter (LNL) model, which is a widely used mathematical model to describe various nonlinear systems that exhibit a memoryless, polynomial behavior. The paper focuses on linear-nonlinear-linear (LNL) systems that have finite impulse response (FIR) filters at both the input and output stages and discusses the LNL cascade structure and Cascaded Volterra and FIR filter Structure in detail. Furthermore, the paper presents numerical experiments that demonstrate the effectiveness of these structures in modeling and manipulating nonlinear systems, particularly in echo return loss enhancement (ERLE). The results show that the Cascaded Volterra and FIR filter Structure outperforms the LNL cascade structure in ERLE. Finally, the paper concludes by emphasizing the significance of comprehending the behavior of nonlinear systems and the role of the LNL model in their analysis and control.

## 1. INTRODUCTION

Nonlinear systems are present in various fields of science and engineering, ranging from control systems to signal processing. The mathematical modeling of such systems is a crucial step towards their analysis and control. The Linear Input Filter, Memoryless Polynomial Nonlinearity, and Linear Output Filter (LNL) model is a widely used mathematical model that describes a class of nonlinear systems. The LNL model consists of three components: a linear input filter, a memoryless polynomial nonlinearity, and a linear output filter as shown in Fig.1 [1]. This model is effective in describing a range of nonlinear systems that exhibit a memoryless, polynomial behavior. The LNL model finds applications in various fields such as signal processing, control systems, and other areas where nonlinear systems need to be analyzed and controlled. Additionally, the LNL model serves as a fundamental building block for the design of more complex nonlinear models. In this paper, we present a comprehensive overview of the LNL model.

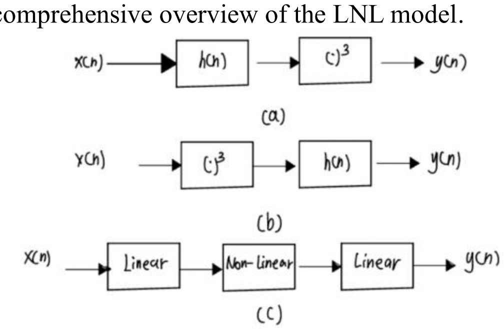


Fig.1 (a)Wiener model, (b)Hammerstein model, and (c) LNL model

## 2. STRUCTURE OF LNL Model

The focus of this work is on linear-nonlinear-linear (LNL) systems that have finite impulse response (FIR) filters at both the input and output stages. To represent these systems, the paper uses two specific models, which are illustrated in Fig.2 and Fig.3. In Fig.2, the nonlinear filter is implemented using a memoryless polynomial filter, while in Fig.3, Filter-1 is a nonlinear filter with memory that can be realized using a Volterra filter. The paper categorizes the configurations of LNL systems and develops one case from each category. This approach enables the paper to investigate the behavior of LNL systems with FIR filters and nonlinear components, which have important applications in signal processing, communication systems, and control engineering.

*2.1 The LNL cascade structure*

This section being discussed presents a joint NLMS (Normalized Least Mean Square) adaptation of the LNL (Linear Nonlinear Linear) structure with a 1IR (one impulse response) output stage, using an output error algorithm [2]. The statement outlines the details of three different filters: FIR filter-I, Polynomial filter, and FIR filter-3. For the FIR filter I, the coefficient vector is denoted as u, and the input vector as x. On the other hand, the Polynomial filter has a coefficient vector v and an input vector y. Lastly, FIR filter-3 has a coefficient vector w and an input vector z. The description of these filters suggests that they are interconnected, possibly through a signal processing system. The reference to Fig.2 , which is likely to depict the three filters and their connections, indicates that this interconnection is a significant aspect of the system. The use of different filter types in combination can offer several advantages, such as improved signal quality and reduced noise, making such systems an essential component of many modern electronic devices.

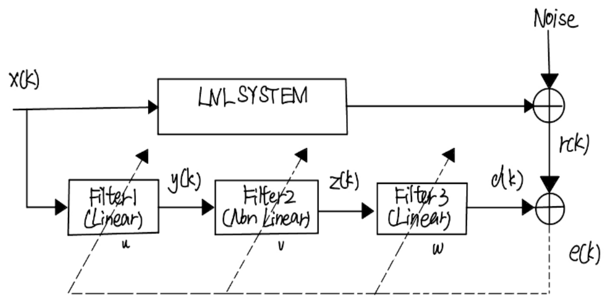


Fig.2 LNL model structures

The output of y[k], z[k], and d[k] are given by:

$$y[k] = u^{T}[k]\, x[k], \tag{1}$$

$$z[k] = v^{T}[k]\, y[k], \tag{2}$$

$$d[k] = w^{T}[k]\, z[k], \tag{3}$$

Where

$$x[k] = [\, x[k], x[k-1], \dots, x[k- M, + l]]^{T}, \tag{4}$$

$$y[k] = [\, y[k], y^{2}[k], \dots, y^{N}[k]]^{T}, \tag{5}$$

$$z[k] = [\, z[k], z[k-1], \dots, z[k- M+1]]^{T} \tag{6}$$

r[k] is the reference signal for the cascade, therefor the error e[k] is given by:

$$e[k] = r[k] - d[k] \tag{7}$$

The gradient $\nabla_u$, $\nabla_v$, $\nabla_w$, of $e^2$, which is the error function, can be computed with respect to the variables u, v and w at instant k as shown in below.

$$\nabla_u e^{2}[k] = -2e[k](\nabla_u\, z[k])w[k], \tag{8}$$

$$\nabla_v e^{2}[k] = -2e[k](\nabla_v\, z[k])w[k], \tag{9}$$

$$\nabla_w e^{2}[k] = -2e[k]\, z[k], \tag{10}$$

Define a $M_1 * M_2$ matrix,

$$X[k] = [b[k]x[k], b[k-1]x[k-1], \dots\dots, b[k-M\_1+l]x[k-M\_2+1]], \tag{11}$$

From equation (2), we have

$$\nabla_u z[k] = X[k] \tag{12}$$

Therefore,

$$\nabla_u e^{2}[k] = -2e[k]\, X[k] \tag{13}$$

The calculation of these gradients is a crucial step in many optimization algorithms. By computing these gradients, the algorithm can determine the direction in which each variable needs to be updated to reduce the error function $e^2$. The specific method for calculating these gradients may vary depending on the optimization algorithm used, but the goal is to find the values of w, v, and u that minimize e.

### *2.2Cascaded Volterra and FIR filter Structure*

Cascaded Volterra and FIR filter structures are popular methods used in signal processing to model and manipulate nonlinear systems. These structures process input signals through multiple stages of filtering and nonlinear processing to produce an output signal as shown in fig.3. This figure discussed system comprises of a Volterra filter and an FIR filter, represented by coefficient vectors v and w, and input vectors y and z respectively. The Volterra filter is nonlinear and uses convolutions to process the input signal, while the FIR filter is linear and uses a finite impulse response. The outputs of these filters can be combined to achieve desired signal processing outcomes, and the design and implementation depend on the specific requirements of the application being modeled. The Cascaded Volterra filter structure can capture the interactions between different frequency components present in the input signal, making them effective in modeling and manipulating nonlinear systems. On the other hand, the FIR filter structure employs a series of linear filters that have a fixed number of taps. While they can only model linear systems, they are still widely used due to their simplicity and efficiency. However, unlike Volterra filters, FIR filters cannot capture interactions between frequency components in the input signal.

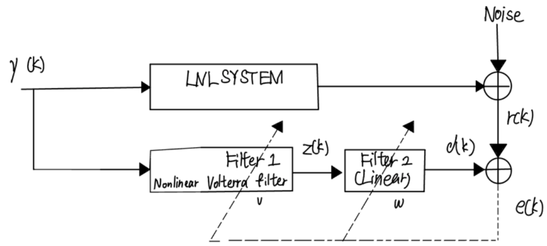


Fig. 3 Cascade of Volterra and FIR filter structures

The output of z[k], and d[k] are given by

$$z[k] = v^T[k]\, y[k], \quad (14)$$

$$d[k] = w^T[k]\, z[k], \quad (15)$$

Where

$$y[k] = [\, y[k], y^2[k], \ldots, y^N[k]]^T, \quad (16)$$

$$z[k] = [\, z[k], z[k-1], \ldots, z[k- M+1]]^T \quad (17)$$

r[k] is the reference signal for the cascade, therefor the error e[k] is given by:

$$e[k] = r[k] - d[k] \quad (18)$$

The gradient $\nabla_v$, $\nabla_w$, of $e^2$ are

$$\nabla_w e^2[k] = -2e[k]\, z[k], \quad (19)$$

$$\nabla_v e^2[k] = -2e[k](\nabla_v\, z[k])w[k], \quad (20)$$

Due to the above relationship,

$$\nabla_u e^2[k] = -2e[k]\, Y[k]w[k] \quad (21)$$

## 3.Echo return loss enhancement (ERLE)

Echo Return Loss Enhancement (ERLE) is a metric that measures the efficacy of an Acoustic Echo Canceller (AEC) system in reducing echo signals during single-talk scenarios. The AEC system's performance can be evaluated by analyzing the magnitude of its error signal (n). In an ideal scenario, the steady-state error signal should have zero magnitude. The ERLE value is defined as the difference between the echo power before and after the AEC processing, normalized by the echo power before processing [4]. The ERLE is expressed in decibels (dB) and is given by the following formula:

$$ERLE = 10log10\, \frac{(E[d^2(n)])}{(E[e^2(n)])} \quad (22)$$

Where $E[d^2(n)]$is the power of the echo signal before the AEC processing and $E[e^2(n)]$ is the power of the echo signal after the AEC processing. A higher ERLE value indicates better performance of the AEC system, as it implies a greater reduction of the echo signal.

## 4.NUMERICAL EXPERIMENTS

This section introduces a numerical experiment that aimed to test cascade structures for the identification of a nonlinear system. The experiment used an echo path model that included a loudspeaker modeled as a Wiener model and an acoustic echo path modeled as a linear filter as shown in Fig.4 . The loudspeaker was modeled with an FIR filter with a memory length of 8 in series cascade with a memoryless nonlinearity of order 5, while the amplifier was modeled by a hyperbolic tangent-like nonlinear function which is f(x) = x - $0.5x^3$ + $0.02x^5$. The linear part of the echo path was modeled using an FIR filter of memory length 64 with a small random number added to each coefficient, and an all-pass filter with two complex conjugate poles and two zeros was used for all simulations. The test signal used was a signal, and a Gaussian noise of variance 0.001 was added to the echo signal.

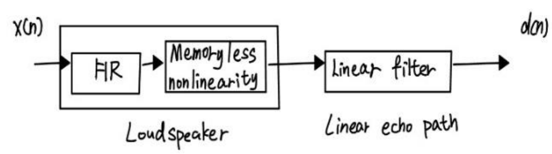


Fig. 4 Model of the echo path

Example 1:

To ensure effective acoustic echo cancellation (AEC), the length of the filter should be at least M = M1 + M2 - 1 = 71, considering the lengths of the linear section in the loudspeaker and the echo path. To experimentation, an FIR filter with a length of 71 was used as a baseline. The experiment was conducted over 40,000 iterations, with the first 20,000 iterations involving de-amplification of the babble input and operation of the loudspeaker in its linear region. In the subsequent 20,000 iterations, the de-amplification was removed, and the loudspeaker was driven into its nonlinear region. The simulation results, as represented in Fig.5, show that the FIR filter was effective in achieving an echo return loss (ERLE) of 3.5dB in the linear region. However, in the nonlinear region, the performance was significantly affected, with the ERLE dropping to only 3.2dB. This is a comparison group in this experiment.

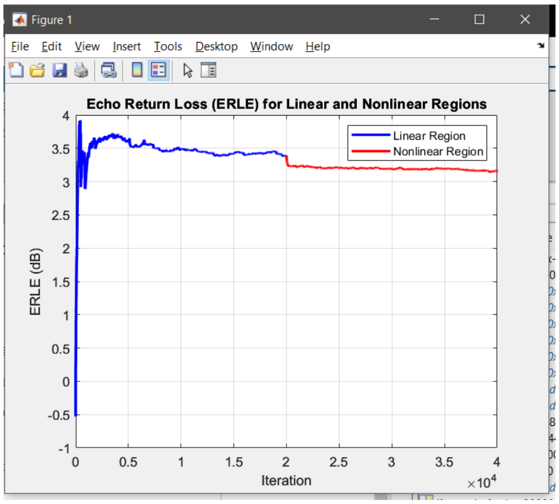


Fig. 5 RELE for linear and nonlinear regions

Example 2

the LNL cascade structure was used for echo cancellation, with an input filter length of M1 = 8 and an output filter length of M2 = 64. The polynomial filter used had 77 taps and a nonlinearity of order N = 5. The experiment converged to approximately 11 dB after 15,000 iterations, as illustrated in Fig. 6's ERLE plot. Also, the filter only reached an ERLE of 12dB in linear region after 1,000 iterations with the output error scheme, indicating that it had converged to a local minimum.

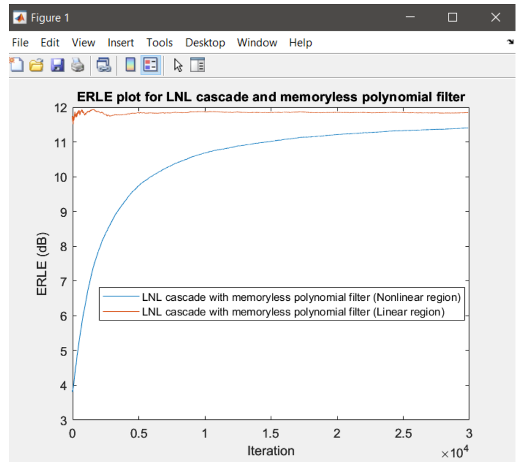


Fig. 6 ERLE plot for LNL cascade and memoryless polynomial filter

Example 3

[1]A series-cascade structure of a Volterra filter and an FIR was used to test the effectiveness of echo cancellation. The Volterra filter had a memory length of M1 = 8 and a nonlinearity order of 3, requiring 164 filter coefficients, while the FIR filter had a length of M2 = 64. The experiment converged to an echo return loss (ERLE) of 23 dB after 1,000 iterations, which was 8 dB higher than the ERLE obtained in the linear region, as demonstrated by **Figure #'s ERLE** plot.

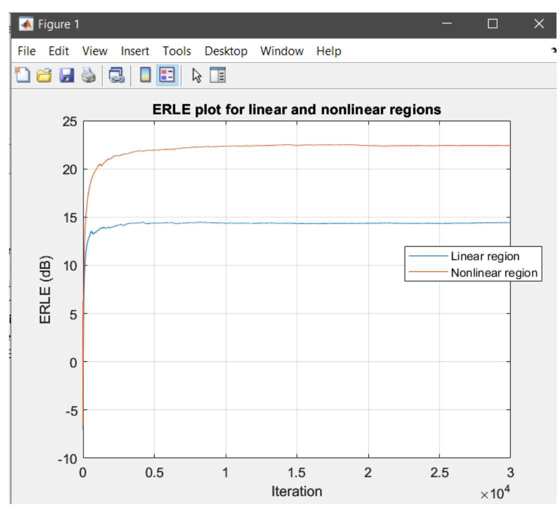


Fig. 7 ERLE plot for linear and nonlinear regions

## 5.CONCLUSIONS

The concepts of The LNL cascade structure and Cascaded Volterra and FIR filter Structure are reviewed and discussed. In the example shown in the simulation, Cascaded Volterra and FIR filter structure has a better performance compared to the Cascaded LNL Structure in echo return loss (ERLE) because of the higher value in both linear region and nonlinear region. On the other hand, the General Cascaded LNL Structure can capture the linear characteristics of the echo path, but it may not efficiently model the nonlinear part of the path.

## 6.REFERENCE


[1] V. Hegde, C. Radhakrishnan, D. J. Krusienski and W. K. Jenkins, "Architectures and algorithms for nonlinear adaptive filters," Conference Record of the Thirty-Sixth Asilomar Conference on Signals, Systems and Computers, 2002., Pacific Grove, CA, USA, 2002, pp. 1015-1018 vol.2, doi: 10.1109/ACSSC.2002.1196937.

[2] C. Radhakrishnan, W. K. Jenkins and A. Garga, "Conjugate gradient algorithm for series cascade nonlinear adaptive filters," The 2004 47th Midwest Symposium on Circuits and Systems, 2004. MWSCAS '04., Hiroshima, Japan, 2004, pp. II-II, doi: 10.1109/MWSCAS.2004.1354084.

[3] X. Y. Gao, W. M. Snelgrove and D. A. Johns, "Nonlinear IIR adaptive filtering using a bilinear structure," IEEE International Symposium on Circuits and Systems,, Portland, OR, USA, 1989, pp. 1740-1743 vol.3, doi: 10.1109/ISCAS.1989.100702.

[4] X. Y. Gao, W. M. Snelgrove and D. A. Johns, "Nonlinear IIR adaptive filtering using a bilinear structure," IEEE International Symposium on Circuits and Systems,, Portland, OR, USA, 1989, pp. 1740-1743 vol.3, doi: 10.1109/ISCAS.1989.100702.


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