5.7 (d) margin of error, $\varepsilon = 0.005$.

$1-\alpha = 0.95$

$\Rightarrow \alpha = 0.05$

$\therefore \alpha/2 = 0.025$

$Z_{\alpha/2} = 1.96$

$p = 0.5 \rightarrow$ worst case scenario

$$n = \left(\frac{Z_{\alpha/2} \cdot \sqrt{p(1-p)}}{\varepsilon} \right)^2$$

$$= 0.25 \times \left(\frac{1.96}{0.005} \right)^2$$

$$= 38416.$$

$\therefore$ To ensure 95% probability of keeping error not exceeding 0.005, number of samples need to be 38416.

(a) - Generate enough samples of poisson distribution with parameters (λ) 3 and 5 respectively. (X, Y)

- To generate poisson samples:
  1) Generate random numbers u_1, u_2 ———
  2) Find largest value K for which $u_1 \times u_2 \times - - - u_K \geq e^{-\lambda}$
  3) Return K.

- check the number of times where $X > Y$.
- Divide it by number of samples which is the desired answer.

(b) solving the probability of royal flush using Monte Carlo simulation:

A royal flush in Poker consists of the Ten, Jack, Queen, King and Ace of the same suit. There are 4 suits, so there are 4 possible royal flushes.

1) For each simulation, draw 5 cards from the standard deck (this follows the uniform distribution)

ii) Run this simulation for enough times.
Refering to (d) → the number of simulations for 0.95 probability with error not exceeding 0.005.

iii) calculate the probability by dividing the number of royal flush occurrence by total simulation count which is the desired result.

by calculation: There are $\binom{52}{5}$ ways to choose 5 cards out of 52.

There are total 4 possible royal flushes.

$\therefore$ probability of royal flush $= \dfrac{4}{\binom{52}{5}}$.

(c) Description of probability calculation using Monte-Carlo Simulation:

Let,
First mechanic, X has service time with exponential distribution $Exp(\lambda_x = 5)$

Second mechanic Y, has service time $Exp(\lambda_y = 20)$.

Probability of being served by first mechanic
$P_x = 1/5 = 0.2$

Probability of being served by second mechanic,
$P_y = 4/5 = 0.8$

By following the steps below, generating samples using Monte Carlo simulation, the probability of service time being more than 35 minutes or $\frac{35}{60} = 0.583$ hours can be calculated:

(1) First determine the number of samples needed to be calculated for maintaining error ϵ under a specific value with probability $(1-\alpha)$

$$\text{number of samples} = 0.25 \times \left(\frac{Z_{\alpha/2}}{\epsilon}\right)^2.$$

(2) Generating samples for the service time in a two step process:

(i) Generate a random number U from std uniform distribution

(ii) if u is less than ~~0.2~~ $P_x = 0.2$ the first mechanic is chosen. Otherwise choose the second mechanic Y having $P_y = 0.8$.

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(3) After the mechanic is chosen calculate samples with $Exp(\lambda x)$ or $Exp(\lambda y)$.

for sample $x_i = -\frac{1}{\lambda x} \ln(1-u)$,

or $x_i = -\frac{1}{\lambda y} \ln(1-u)$

By repeating steps 2 and 3 for n number of times, n samples can be generated.

Finally, in the n samples, count how many times the service time > 0.583 hours, dividing this with n, we will get the desired probability value.

Example sample generation:

(1) From uniform distribution table, $U = 0.6813$. $U > 0.2$ & $U < 0.8$ hence second mechanic with $\lambda y = 20$ is chosen.

(2) $$x = -\frac{1}{\lambda y} \ln(1-u)$$

$$= -\frac{1}{20} \ln(1 - 0.6813)$$

$$= 0.0572$$

a sample would be 0.0572 hours.