A study on the classification of handwritten digits

Laurent Verweijen, Tamis van der Laan

Abstract—**This paper describes our research for using pattern recognition techniques to classify individual digits in bank account numbers and the monetary amount. This paper will describe the steps we have taken to do such a task which consists of preprocessing the data, training the classifier and testing the performance. Matlab was used to perform these steps. Two cases are taken into account. In the first case the pattern recognition system is trained once and then applied. That means that our training set is large. In the second case, the training set is trained for each batch to be processed, which means that our training set is much smaller. We describe how we found the optimal classifiers, how these classifiers work and how we optimized these classifiers to make them perform even better. After doing a lot of tests, we noticed that the best performance for the small dataset was reached by using a nearest mean scaled classifier and for the big dataset the voted perceptron classifier performed best.**

I. Introduction

In our approach Matlab was used to research these problems using the PR toolbox for fast prototyping. This toolbox contains common classifiers used for pattern recognition which include among others the support vector machine and the k-nearest-neighbour classifier. The PR toolbox for Matlab was developed by the Delft Pattern Recognition Group (in 2004 renamed to Quantitative Imaging) at the Faculty of Applied Physics (later Applied Sciences) of Delft University of Technology [4]. We will first discuss how we solved the first case, then the second case and finally we will give some conclusions and recommendations about how this could be applied in a banking system. Prior experience with person recognition [2] led us to the use of Histogram of Oriented Gradients feature extraction in combination with a support vector machine classifier. Eventually the support vector machine would be dropped in favor of a similar less computationally expensive classifier named the voted perceptron classifier. Many other classifiers were also tested. For the second case where the data set was small we found that the nearest nearest mean scaled classifier performed best.

II. Methods

A. Pre-processing

Either before training or classification a preprocessing step is used in order to filter out noise and transform the image in such a way that structure is maintained and the data is prepared for feature selection, extraction and classification. Initially the data that was provided consisted of a set of images from different sizes that each contains a handwritten digit.

Figure 1 shows the pipeline that was used for preprocessing. First noise is reduced by removing very small objects. Then the data is closed in case the image contains gaps. Then the data is skewed so the longest side of the data (side with the most variance) is parallel to the y axis (slant correction). Then the features of our data are extracted and finally principal component analysis is used to reduce the size of the feature set as some features are barely to not used.

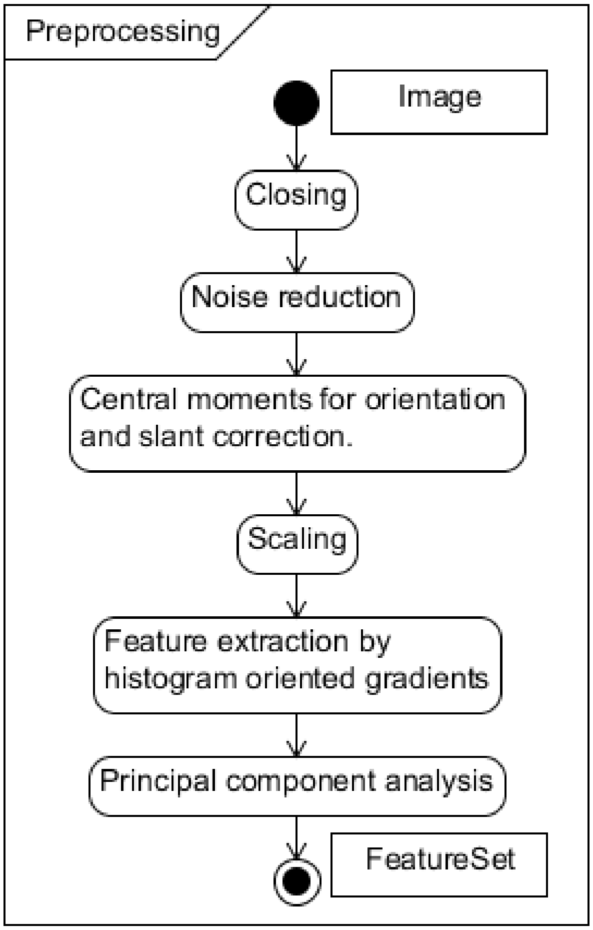

Preprocessing
Image
Closing
Noise reduction
Central moments for orientation and slant correction.
Scaling
Feature extraction by histogram oriented gradients
Principal component analysis
FeatureSet

Fig. 1. Preprocessing pipeline

In table A some examples that show noise reduction and slant correction are shown based on the scheme above.

A.1 Dilation and Erosion

Noise is reduced by first eroding and then dilating the given binary input image using a 2-dimensional diamond shaped kernel. We then wish to close holes or attach objects near each other as parts of numbers may be missing. We do this using a closing operation which consists of a dilation followed by an erosion again using a 2-dimensional diamond shaped kernel.

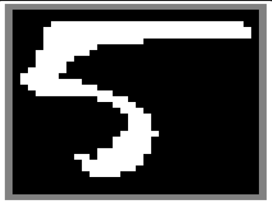
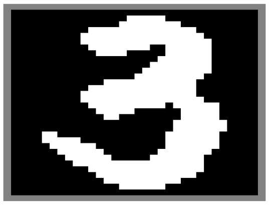
Before	After
	
	

TABLE I

Noise reduction and slant correction applied to our images

A.2 Central Moments for orientation and slant correction

Next we transform the binary input image such that it is centered and oriented in such a way that the longest side of the number is parallel to the y axis. The transformation used is skewing and removes the slanting from the text. This operation will also decrease the variance of the pixels as some pixels such as those in the corners will be activated less often and therefore contribute less to the number recognition system. This decrease in pixel variance propagates through to the Histogram Oriented Features. This in turn allows Principle Component Analysis to decrease the feature set significantly as we threshold out the low variance principal components. This allows for far faster training and classification performance.

We first need to center the image. To do this we find the image centroid, which is simple the mean of the x axis and the y axis:

$$\bar{x} = (x_1 + x_2 + ... + x_n)/n_x \tag{1}$$

$$\bar{y} = (y_1 + y_2 + ... + y_n)/n_y \tag{2}$$

In order to find the orientation of binary numbers we used central moments. Central moments are calculated in the following way and make use of the centroid:

$$\mu_{pq} = \sum_x \sum_y (x - \overline{x})^p (y - \overline{y})^q f(x, y) \tag{3}$$

Next we you use this definition to compute $\mu_{20}, \mu_{21}, \mu_{11}$ which represents the x and y variance and covariance respectively. We then use these central moments to compute the orientation as followed:

$$\Theta = \frac{1}{2} \arctan\left(\frac{2 \cdot \mu_{11}}{\mu_{20} - \mu_{02}}\right) \tag{4}$$

We then skew the image parallel to the y axis and find the bounding box and square it where the sides are equal to the longest previous side such that geometric information is preserved. We leave a one pixel border around the bounding box because the Histogram of Oriented Gradient algorithm (as its name already suggests) makes use of the gradient (first order derivative in both the x and y direction) and thus its three part filter needs the extra one pixel space. The slant correction skew matrix is given by:

$$S = \begin{pmatrix} 1 & 0 \\ \sin(0.5\pi - \theta) & \cos(0.5\pi - \theta) \end{pmatrix} \tag{5}$$

A.3 Scaling

Finally the images are scaled to 32 by 32 pixels, ready for feature selection, extraction and classification.

B. Feature extraction and selection

We used two methods for feature extraction and selection. The first method used is called Histogram Oriented Gradients and is a form of feature extraction and reduction that describes the orientation of the gradient in terms of frequency. The second method used is called principal component analysis and is used to remove those histogram of oriented gradient features with the least variance, and is thus a form of feature selection or dimensionality reduction. But before arriving at these features we first tested and experimented with combinations of other features that are commonly used when dealing with handwriting detection. We list a couple of them below:

1. Using raw pixels as a control
2. Image profiles
3. Image moments such as hu moments
4. Canny edge detector

We however found that none could compete with the histogram of oriented gradients in combination with the classifiers tested.

B.1 Raw pixels

The simplest features that can be taken from an image are the raw pixels. This is also the fastest feature extraction, because the only operation that needs to be performed is the scaling of all images to the same size. We tried this for different image sizes. Most digits got recognized well by using this method. The error rate on a big dataset was about 10%. In particular the digits three and five were often mixed up. We used the raw data as a control to compare features based on classification errors.

B.2 Histogram oriented features (HOG)

Histogram of Oriented Gradients is a feature extraction and selection technique. [3] It makes use of the image gradient, were both the orientation of the gradient and magnitude are used. Due to the binary nature of our input we will not be using the magnitude of the gradient. The first step of the Histogram Oriented Gradients algorithm is to compute the gradient of the image. This is done by taking the first derivative of the image in both the x and y direction. The method used for this is convolution with the following filters:

$$[-1, 0, 1] \text{ and } [-1, 0, 1]^T \tag{6}$$

We now obtain two images containing the x derivative and y derivative respectively. The direction angle of the gradient for each pixel can now be computed using the following formula:

$$\theta = \arctan\left(\frac{\partial f}{\partial y}, \frac{\partial f}{\partial x}\right) \tag{7}$$

The next step is to create cells which will contain histograms. In the original paper the gradient magnitude of each pixel within the range of a given bin is summed to get the final bin value. The original paper is aimed at pedestrian detection however using grey scale images. Because we are using binary images we simply increase the bin width with one if the gradient direction of a pixels lays within the range of the given bin.

This method will result in several bins per cell, the original author used 9 bins per cell and 16 cells. We found through experiment that 9 bins were most effective for our problem as well, but 16 cells were used as opposed to 9 used by the original author. We derived at these constants experimentally by looking at the average and minimum error of all the tested classifiers for each possible combination were 25 cells and 10 bins were chosen as maximums.

Next, the author locally normalizes the cells in order to compensate for illumination and contrast. However we do not implement this step as we are using binary images and thus both the contrast and illumination are already normalized perfectly across our image.

The last step is to put all the bins of each cells into a column vector which has became our feature vector for classification.

To get the histogram of oriented gradients to work in matlab, we downloaded an implementation by O. Ludwig [1]. When we tested his implementation we noticed that we got better results then we got when using raw pixel data.

C. Principal component analysis

The hog descriptor does a good job at both reducing the amount of features needed as well as extracting features that are easier to classify by the bulk of classifiers. However there will still remain features (bins) that are either always empty or are barely used. For this reason we apply Principal Component analysis, which will extract the principal components and allows us to remove those with too little variance (empty to barely filled bins).

Principal component analysis (pca) is a data reduction technique that generates a set of principal components such that as much variance as possible between the features is maintained. In the features which we extracted from the data, some features don't differ as much as other features. This is due to the way the numbers are usually written and the orientation correction we used in a previous step. We therefore use pca to make our feature set smaller, so that our classifier becomes faster.

The mathematical theory behind pca is as follows: Let X be a feature matrix with the rows representing different samples and the columns representing different features. We first subtract the mean of each feature from the columns so that the mean of each column, becomes 0. We call this new matrix $X_{adjusted}$. Then we do an eigenvalue decomposition of the covariance of this matrix. $PDP^{-1} = cov(X_{adjusted})$. By sorting the eigenvectors by corresponding eigenvalues, we can find the L vectors that contribute most to our data. The highest eigenvalues correspond to the eigenvectors that capture most covariance. The multiplication of our data with these eigenvectors $X_{reduced} = P_L X_{adjusted}$ will then become our reduced feature set.[5]

Using principle component analysis we managed to reduce the 144 dimensions obtained from the histogram of oriented gradients feature extraction method to a mere 57 features which translates into a 60% feature reduction! For the large data set we managed to reduce the 144 dimensional feature space to 82 which still signifies a major reduction of 43% of the feature space!

Not only did the principal component analysis method increase performance, it also increased the classification accuracy! This is due to the reduction of useless dimensions and the elimination of problems related to the curse of dimensionality. This was especially so with the nearest scaled mean classifier as it is a density based classifier and thus is especially prone to the curse of dimensionality. On the small data set the classification error of the NSMC decreased with an impressive 80%! The error on the large data set using the voted perceptron classifier also decreased with a percentage of 0.8% which is also a very significant decrease given the large amount of data available.

III. Classification Data Set

Although we initially opted to use the support vector machine, we tested many classifiers in order to find the classifier that would give us the best classification result. It turned out that several classifiers were simply too computationally expensive to test. We used the nist_eval function provided by the client to test all the classifiers. The support vector machine classifier was one of these classifiers together with all neural network based classifiers that were too computationally expensive to subject to rigorous testing and were therefore not considered. We excluded these classifiers as we wish to provide a certain degree of classification certainty to our client. We soon found a similar classifier too the support vector machine called the Voted Perceptron classifier and found this to be the most likely candidate for use in classification. This notion was

Classifier	Error	Execution Time 1.0e-05 *
K-nearest neighbour Classifier	0.1220	0.4634
Linear kernel Classifier	01470	0.4966
Decision tree Classifier	0.8050	0.4635
Fisher's Least Square Linear Classifier	0.1580	0.4635
Linear Bayes Normal Classifier	0.1470	0.4635
Linear classifier built on the KL expansion of the common covariance matrix	0.1470	0.6290
Logistic Linear Classifier	0.1550	0.4966
Naive Bayes classifier	0.5290	0.5959
Nearest Mean Classifier	0.1340	0.4304
Nearest Mean Scaled Classifier	0.1000	0.5959
Linear perceptron classifier	0.3910	0.4966
Polynomial Classification	0.1580	0.4966
Quadratic Bayes Normal Classifier	0.9000	0.4966
Voted perceptron classifier	0.1900	0.4966
Uncorrelated normal based quadratic Bayes classifier	0.2090	0.5297
Random subspace combining classifier	0.9480	0.4966
Linear classifier using PC expansion on the joint data	0.1470	0.5628
Parzen Classifier	0.1130	0.4635

TABLE II

This table shows the error and execution time on the nist_eval function which uses 100 samples per class to test the classifier. The training data consisted of the small data set of 10 samples per class.

confirmed in the testing phase as it yielded the lowest error on the large data set of all classifiers considered. For the small data set however the voted perceptron classifier performed decent but not excellent. We found that the nearest mean scaled classifier performed best on the small data set. For this reason we chose the VPC for case one and the NMSC for case two. The performance of both classifiers were found to be more than fast enough.

A. Classifier Testing Phase

We used the testing function provided by the client to test all the classifiers. We used the default setting of a 100 samples to test on. With exclusion of the support vector machine classifier and any neural network based classifiers. The following two tables show the errors for the small and large data set respectively:

We see that many classifiers meet our criteria. We chose to use the two best classifiers namely the Voted perceptron classifier for the large data set and the Nearest Mean Scaled classifier for the small data set.

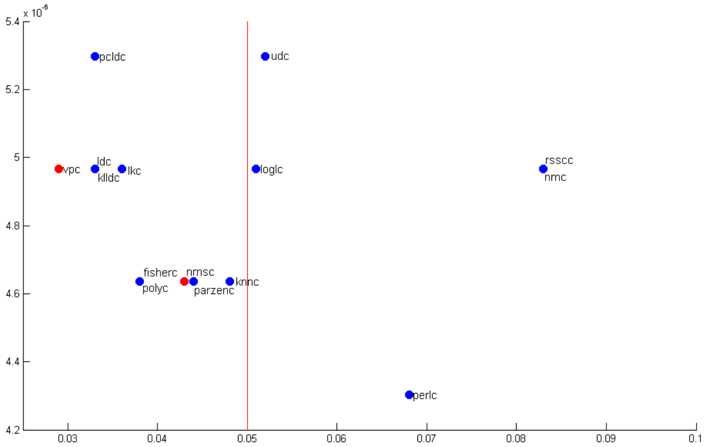


Fig. 2. This table shows the error and execution time on the nist_eval function which uses 100 samples per class to test the classifier. The training data consisted of the small data set of 10 samples per class.The red line shows the 25% maximum error set by the client.

Classifier	Error	Execution Time 1.0e-05 *
K-Nearest Neighbor Classifier	0.0480	0.4635
Linear kernel classifier	0.0360	0.4966
Decision Tree Classifier	0.8030	0.4635
Fisher's Least Square Linear Classifier	0.0380	0.4635
Linear Bayes Normal Classifier	0.0330	0.4966
Linear classifier built on the KL expansion of the common covariance matrix	0.0330	0.4966
Logistic Linear Classifier	0.0510	0.4966
Naive Bayes classifier	0.1130	0.4304
Nearest Mean Classifier	0.0830	0.4966
Nearest Mean Scaled Classifier	0.0430	0.4635
Linear perceptron classifier	0.0680	0.4304
Polynomial Classification	0.0380	0.4635
Quadratic Bayes Normal Classifier	0.1220	0.4966
Voted perceptron classifier	0.0290	0.4966
Uncorrelated normal based quadratic Bayes classifier	0.0520	0.5297
Random subspace combining classifier	0.0830	0.4966
Linear classifier using PC expansion on the joint data.	0.0330	0.5297
Parzen Classifier	0.0440	0.4635

TABLE III

This table shows the error and execution time on the nist_eval function which uses 100 samples per class to test the classifier. The training data consisted of the large data set of 100 samples per class.

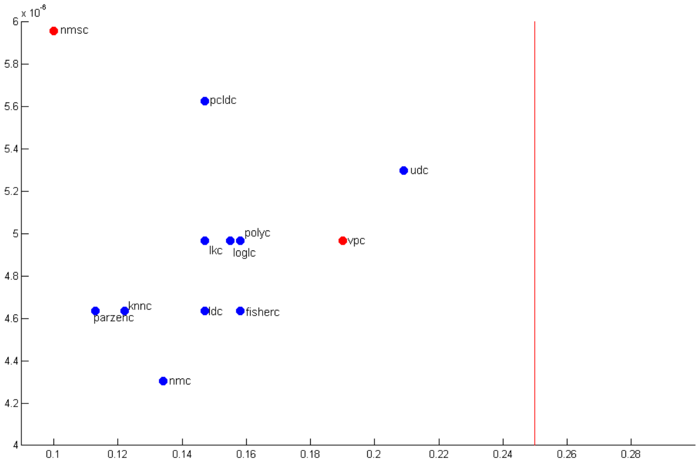


Fig. 3. This table shows the error and execution time on the nist_eval function which uses 100 samples per class to test the classifier. The training data consisted of the large data set of 100 samples per class.The red line shows the 5% maximum error set by the client.

B. *Voted Perceptron Classifier*

As discussed above in the introduction the plan was to treat the number recognition system in the same way as the person recognition system implemented and described by Manabu Sassano [6]. This implementation used a state vector machine for classification. But because this classifier was too heavy for the problem at hand we opted for a simpler but similar classifier, namely the voted perceptron classifier. This study by Manabu Sassano, IJCNLP compared the SVM classifier to the voted perceptron classifier and found them almost equal in terms of accuracy. The advantage however of the Voted Perceptron Classifier is that it is significantly faster in terms of learning and prediction time.

The Voted Perceptron Classifier is very easy to understand, let x denote the feature vector were $||x||$ denotes the amount of features and $y \in \{-1, 1\}$ the corresponding label. The training algorithm simply starts by applying the prediction step which is the simple function $\hat{y} = sign(\overline{v} \cdot \overline{x})$ were $\overline{v}$ is initialized at the beginning as $\overline{v} = \overline{0}$. If the prediction of y is correct i.e. $\hat{y} = y$ then no further steps are taken. If however the prediction is not correct then the training step occurs. This training step takes the form of the following simple equation $\overline{v} = \overline{v} + y\overline{x}$. It has been shown that if the data is linearly separable that the algorithm will make a finite number of mistakes but will converge after a certain amount of cycles to the right answer. If however the data is not linearly separable the algorithm will not converge. Therefore the amount of cycles allowed is bounded. Due to the high performance of the algorithm we choose the maximum amount of cycles to be 1000 which may be far more than needed.

C. *Nearest Mean Scaled Classifier*

The nearest mean scaled classifier is an extension of the nearest mean classifier, so I will describe that one first.

The nearest mean classifier is a classifier that assigns a sample to the class to which its euclidean distance is smallest. So from the training data the mean is taking for each class and assigned to M_C and then it minimizes:

$$(X - M_c)^2 \text{ for } c \in C \tag{8}$$

In which X is the feature vector from the sample that needs to be classified. And C are the classes. $M = M_1, \ldots, M_k$ are the mean vectors of all the classes it could get assigned to. So $M_c = \sum x \in X$ for all $X \in C$

The nearest mean scaling classifier is similar to a nearest mean classifier, except that nmsc makes the assumption that the data is according to a normal distribution, to which the data is scaled first.

D. *Kernel Mapping The data for classification*

The Voted Perceptron Classifier is in essence a linear classifier that can be made non linear by adding extra dimension based on the data itself. The way the data is mapped to extra dimensions is called kernel mapping, where the kernel dictates the structure of the added higher dimension. We experimented with sever proximity based kernels to construct a non linear VPC but they all yielded worse results. This suggests the data to be linear and no kernel mapping is required.

IV. Future work

One thing that could be researched for performance accuracy would be combining classifiers. For example we may be able to increase performance by bootstrapping from the training set and training multiple voted perceptron classifiers. We could then use the majority vote of these classifiers for classification of given unknown samples. In this way accuracy may be increased. We could also construct a general classifier that can classify large and small data sets equally well by combining the VPC and the NMSC. We could again use bootstrapping and a majority vote but this time weigh the vote by the outcome of a function of the sample size. This function could be linear or nonlinear. A sigmoid function might be considered. Another possibility would be to combine the top 5 classifiers of both the small and large datasets respectively.

V. Conclusion and recommendations

We have shown that by using central moments and skewing we can provide decent text slanting correction. We also show that we can filter out noise using erosion and dilation filters. We have also obtained desirable low level error rates within the boundaries set by the client.

From our tests it seems like the more data is used for training the better the results. When 1000 instead of 10 samples per class are used, the error becomes 10 times as small. However this does not mean that we will ever achieve a zero percent error rate. This is because there is a degree of subjectivity in the data, what one person may label a eight may be labeled by others as a six. We thus we see practically identical symbols in more than one class. We can only approximate with a large enough data set what class label the average person would give to a number symbol.

In this sense the system will never reach a zero percent error. Adding extra features also won't solve this problem as ambiguity can still occur. With this insight we recommend using ambiguity rejection were credit cards with ambiguous unclear numbers can be traced back to their owners by employees.

Apart from the problem of ambiguity between classes there is another possible problem which is that symbols could be written down that aren't numbers at all. Symbols that do not belong to any class will show up as outliers, therefore we recommend a form of outlier detection to be applied. This outlier detection mechanism would have to be be applied to the data before principal component analysis as features normally not activated and removed by principal component analysis may now be activated and reveal outliers. After divergent symbols are detected by outlier detection the corresponding credit card can again as above be traced back to their respected owners.

We believe that a single classifier will suffice for classification as a large training set is already available and classification error and time is low. We understand that the choice of classifier is important when dealing with this problem we however belief that the feature extraction method used is more important in order to minimize the classification error. This can be seen in figures two and three which show good performance for many classifiers.

Another thing that might be looked into, are the classifiers based on a neural network. In our experiments, these classifiers performed too slow, but a more efficient implementation and with more time to train them, these classifiers might do pretty well and would automatically return a degree of certainty per number thus having a automatic form of outlier and ambiguity rejection in them.

If no classifiers can be combined, a single method has to be chosen and performance is an issue, then we recommend the Nearest Mean Scaled classifier if the training set is small and Voted Perceptron classifier if the training set is large.

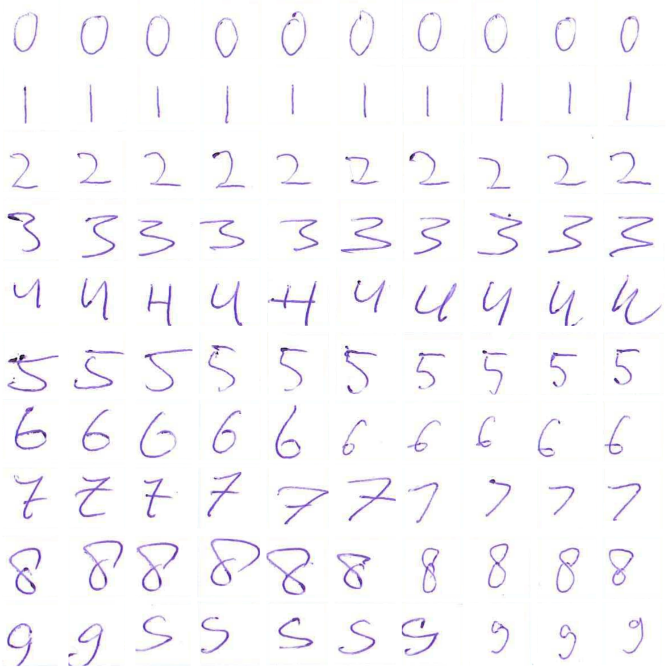


Fig. 4. Our own handwritten digits

Appendix

A. Manual handwritten digits

Eventually, we scanned in our own handwritten digits. Our scan can be seen in figure 4. Because we wanted to test how well it performs on more ambiguous digits as well, we made some of our digits a bit more ugly.

We trained the voted perceptron classifier on 1000 samples for each dataset and then tested it on our own handwriting.

We did a small amount of preprocessing on our scan. First we extracted the individual digits from our handwriting. This was pretty easy because the digits in the figure were well aligned. The image had rgb channels, but we wanted it to be a boolean image, so in order to get a boolean result, we averaged the rgb components, to calculate the strength for each pixel and then we tresholded all the pixels on a value we found by experimentation.

After running the classifier, we found an error rate of 20%. Because not all the digits in our handwriting are as clear as the other ones, we thought it was interesting to make a confusing matrix (in table IV) of our result to detect which digits are the outliers. As a summary, we made a table of error rates (in table V). As can be seen our 7's and 9's are hard for the classifier to recognize, whereas our 1, 2, 5 and 8 are recognized in all cases.

Class	0	1	2	3	4	5	6	7	8	9
Error	1	0	0	3	2	0	1	5	0	8

TABLE V

Error rate per class when testing the classifier on our own handwriting

True Label	Estimated Labels										Totals
	0	1	2	3	4	5	6	7	8	9	
0	9	0	0	0	0	0	0	1	0	0	10
1	0	10	0	0	0	0	0	0	0	0	10
2	0	0	10	0	0	0	0	0	0	0	10
3	0	0	0	8	0	2	0	0	0	0	10
4	0	0	0	0	9	0	1	0	0	0	10
5	0	0	0	0	0	10	0	0	0	0	10
6	0	0	0	0	0	0	10	0	0	0	10
7	0	2	0	2	0	0	0	6	0	0	10
8	0	0	0	0	0	0	0	0	10	0	10
9	3	0	1	2	0	1	0	0	0	3	10
Total	12	12	11	12	9	13	11	7	10	3	100

TABLE IV

Confusion matrix when testing the classifier on our own handwriting

References

[1] O. Ludwig et al. “Trainable Classifier-Fusion Schemes: An Application To Pedestrian Detection”. In: *12th International IEEE %Conference On Intelligent Transportation Systems* 1 (2009), pp. 432–437.

[2] T.v.d.Laan et al. “Intelligent Multi-camera Video Surveillance”. In: *Christiaan Huygens Magazine* 17.1 (2012), pp. 20–22. URL: `https://ch.tudelft.nl/sites/default/files/j17n1_website_0.pdf`.

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[4] Ferdi van der Heijden. *Classification, parameter estimation and state estimation, an engineering approach using Matlab.*

[5] K Pearson. “On Lines and Planes of Closest Fit to Systems of Points in Space”. In: *Philosophical Magazine* 2.11 (1901), pp. 559–572.

[6] Manabu Sassano. *Classification, parameter estimation and state estimation, an engineering approach using Matlab.* URL: `http://www.aclweb.org/anthology/I/I08/I08-2117.pdf`.