소프트웨어수학

다른 친구들이 입장할 때까지 조금 기다려 주십시오
시간에 맞춰 수업 시작합니다.

다음 사항을 준비해주세요

- 음소거 해주세요 (질문 등 모든 대화는 채팅으로 합니다.)
- 화상(비디오)을 켜주세요
- 채팅창을 열어, 학번과 이름을 남겨주세요(출석)

- **강의 내용을 녹화/녹음하여 사용하는 것은 추후 법적 문제가 발생할 수 있습니다.**

Chapter 1.

The Foundations: Logic and Proofs

Chapter 1, Part I: Propositional Logic

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Presented by Prof. Sang-Woong Lee
Spring 2022

1.1 Propositional Logic

What is logic?

- If Tom is a philosopher, then Tom is poor.
  Tom is a philosopher.
  Therefore, ?

- If K>10, then K>2.
  K>10.
  Therefore, K>2.

If P, then Q. P. Therefore, Q.

(the principle of reasoning for validating these arguments)

- If John is in Europe, then John is not in China.
  John is in Europe.
  Therefore, John is not in China.

Logic (논리, 논리학)

- **Logic: the study of reasoning**
  - Defines a formal language for representing knowledge and for making logical inferences (i.e., the **principle** of **correct reasoning**)
  - focus on
    - method or process by which an argument unfolds
    - (NOT whether any arbitrary statement or series of statements is "true" or accurate)
  - Essential for **mathematical reasoning / in reading and developing proof**

- Example use of logic
  - In mathematics:
    - To prove theorems
  - In computer science:
    - **To prove that programs do what they are supposed to do**

Propositions (명제)

- **Definition:**
  - A proposition is **a statement** that is either **true** or **false** (not both).

Truth value of a proposition:

- **T**: true value
- **F**: false value
- Corresponds to 1 (=true) and 0 (=false) in digital circuits

Examples of propositions

"Elephants are bigger than mice."

Is this a statement?	**yes**
Is this a proposition?	**yes**
What is the truth value of the proposition?	**True (T)**

Examples of propositions

Please do not fall asleep

Is this a statement? **no**

It's a request

Is this a proposition? **no**

Only statements can be propositions.

Examples of propositions

520 < 111

Is this a statement? **yes**

Is this a proposition? **yes**

What is the truth value of the proposition? **false (F)**

Examples of propositions

Today is March 5 and $99 < 5$

Is this a statement? **yes**

Is this a proposition? **yes**

What is the truth value of the proposition? **false (F)**

Examples of propositions

$x > 100$

Is this a statement? **yes**

Is this a proposition? **no**

Its truth value depends on the value of x, but this value is not specified.
We call this type of statement a propositional function or open sentence.

Examples of propositions

x < y if and only if 2y > 2x."

Is this a statement? **yes**

Is this a proposition? **yes**

… because its truth value does not depend on specific values of x and y

What is the truth value of the proposition? **True (T)**

Propositions(3/31)

- Examples of propositions:
  a) The Moon is made of green cheese.
  b) Seoul is the capital of S. Korea.
  c) Toronto is the capital of Canada.
  d) $1 + 0 = 1$
  e) $0 + 0 = 2$
- Examples that are not propositions.
  a) Sit down!
  b) What time is it?
  c) $x + 1 = 2$
  d) $x + y = z$

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Combining Propositions

- **More complex propositional statements** can be build from elementary statements using **logical connectives (logical operators).**

- **Example:**
  - Proposition A: It rains outside
  - Proposition B: We will see a movie
  - A new (combined) proposition:

    If it rains outside **then** we will see a movie

We formalize this by denoting propositions with letters such as p, q, r, s, and introducing several **logical operators**.

Propositional Logic

- Propositional variables : *p, q, r, s* …
  - Variables that represent propositions
  - T: true value, F: false value
- **Compound Propositions**
  - constructed from logical operators (connectives) and other propositions
- **Logical operators** (logical connectives)
  - Negation $\neg$
  - Conjunction $\wedge$
  - Disjunction $\vee$
  - Implication $\rightarrow$ (if-then)
  - Biconditional $\leftrightarrow$ (if and only if)

Logical Operator: Negation

- **Definition: Negation**
  - Let p be a proposition. The statement "It is not the case that p." is another proposition, called the **negation of p**. The *negation* of p is denoted by $\neg p$ $(or\ \overline{p})$ and read as "not p."

- **Examples**
  - If p denotes "The earth is round.", then $\neg p$ denotes "It is not the case that the earth is round," or more simply "The earth is not round."
  - p : It is raining today.
    - $\neg p$: It is not raining today.
  - p : 2 is a prime number.
    - $\neg p$: 2 is not a prime number

Negation (=NOT)

- A **truth table** displays **the relationships between truth values** (T or F) of different propositions

p	$\neg p$
True (T)	False (F)
False (F)	True (T)

Conjunction (=AND)

- **Definition: conjunction**
  - Let p and q be propositions. The proposition "**p and q**" denoted by $p \wedge q$, is true when both p and q are true and is false otherwise. The proposition $p \wedge q$ is called the **conjunction** of p and q.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- **Example**: If p denotes "I am at home." and q denotes "It is raining." then $p \wedge q$ denotes "I am at home and it is raining."

Disjunction (=OR)

- **Definition: disjunction**
  - Let p and q be propositions. The proposition "**p or q**" denoted by $p \vee q$, is false when both p and q are false and is true otherwise. The proposition $p \vee q$ is called the **disjunction** of p and q.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- **Example**: If p denotes "I am at home." and q denotes "It is raining." then $p \vee q$ denotes "I am at home or it is raining."

Conjunction and Disjunction

- **Conjunction and disjunction**
- Four different combinations of values for *p* and *q*

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

- **C.f. –Compound Propositions 를 바라보는 자세**

  $p \wedge q$, $p \vee q$, $p \rightarrow q$ 등의 compound proposition은 (p, q 따로따로 분해해서 보지 말고..) 통으로 된 1개의 proposition으로 바라보고 이해해야

The Connective Or in English

- In English "Or" has two distinct meanings.
  - "Inclusive Or" - In the sentence "Students who have taken CS202 or Math120 may take this class," we assume that students need to have taken one of the prerequisites, but may have taken both. This is the meaning of disjunction. For $p \vee q$ to be true, either one or both of p and q must be true.
  - "Exclusive Or" - When reading the sentence "Soup or salad comes with this entrée," we do not expect to be able to get both soup and salad. (next page…)

Exclusive OR (XOR)

- **Definition:**
  - Let p and q be propositions. The proposition "**p exclusive or q**" denoted by $p \oplus q$, is true when exactly one of p and q if true and is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statement $p \to q$ (or Implication)

- **Definition: Conditional statement / Implication**
  - Let p and q be propositions. The proposition "***p* implies *q***" denoted by $p \to q$, is called **implication**, It is false when p is true q is false and is true otherwise. It is read as "if p, then q"

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

- In $p \to q$, p is the ***hypothesis*** (*antecedent* or *premise*) and q is the ***conclusion*** (or *consequence*).
- **Example**: If p denotes "It is raining" and q denotes "The home team wins." then $p \to q$ denotes "If It is raining, then the home team wins."

Understanding Implication/Conditional Statement

- One way to view the logical conditional is to think of an obligation or contract.
  - "If I am elected, then I will lower taxes."
  - "If you get 100% on the final, then you will get an A."

- If the politician is elected and does not lower taxes, then the voters can say that he or she has broken the campaign pledge. Something similar holds for the professor. This corresponds to the case where p is true and q is false.

Understanding Implication (cont)

- In $p \to q$ there does not need to be any connection between the hypothesis (antecedent) or the conclusion (consequent).
  The "meaning" of $p \to q$ depends only on the truth values of p and q.

p	q	$p \to q$
T	**T**	**T**
T	**F**	**F**
F	T	**T**
F	F	**T**

- Examples
  - if L.A Dodgers win the World Series in 2019 then 2 is a prime.
    -
  - if January has 29 days then 2 * 3 = 8.
    -

Different Ways of Expressing $p \rightarrow q$

- if p, then q
- if p, q
- **q unless $\neg p$**
- q if p
- q whenever p
- q follows from p
- p implies q
- **p only if q**
- q when p
- p is sufficient for q
- q is necessary for p

- a necessary condition for p is q
- a sufficient condition for q is p

- E.g., You get 100% on the final only if you get an A
- ***p* only if *q*:** *p* cannot be true when *q* is not true. So, when "*p* = true & *q* = false", the statement is FALSE

Example 7

Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

Example 8

What is the value of the variable x after the statement

$$\textbf{if } 2 + 2 = 4 \textbf{ then } x := x + 1$$

if $x = 0$ before this statement is encountered? (The symbol $:=$ stands for assignment. The statement $x := x + 1$ means the assignment of the value of $x + 1$ to x.)

Converse, Contrapositive, and Inverse

- From $p \to q$ we can form new conditional statements .
  - $q \to p$ is the **converse** of $p \to q$
  - $\neg q \to \neg p$ is the **contrapositive** of $p \to q$
  - $\neg p \to \neg q$ is the **inverse** of $p \to q$

※ 역(converse), 이(inverse), 대우(transposition)

Example: Find the converse, inverse, and contrapositive of "It's raining is a sufficient condition for my not going to town."

Biconditional

- **Definition:**
  - Let p and q be propositions. The **biconditional** $p \leftrightarrow q$ **(read as "p if and only if q")** is true when p and q have the same truth values and is false otherwise.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- If p denotes "The home team wins." and q denotes "It is raining." then $p \leftrightarrow q$ denotes "The home team wins if and only if it is raining."

Expressing the Biconditional

- Some alternative ways "*p* if and only if *q*" is expressed in English:
  - *p* **is necessary and sufficient for** *q*
  - **if** *p* **then** *q* **, and conversely**
  - *p* **iff** *q*

Summary: Implication (if - then)

- Binary Operator, Symbol: $\rightarrow$

p	q	p→q
true	true	true
true	false	false
false	true	true
false	false	true

Summary: Biconditional (if and only if)

- Binary Operator, Symbol: $\leftrightarrow$

p	q	$p \leftrightarrow q$
true	true	true
true	false	false
false	true	false
false	false	true

Precedence of Logical Operators

※수학: 괄호 → ! → *exponential* → *, / → +, -

Operator	Precedence
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

- *Examples:*

$p \wedge q \vee r$ means $(p \wedge q) \vee r$

$p \vee q \rightarrow \neg r$ is equivalent to $(p \vee q) \rightarrow \neg r$

If the intended meaning is $p \vee (q \rightarrow \neg r)$

then parentheses must be used.

Truth Tables of Compound Propositions

- Construction of a truth table:
- Rows
  - Need a row for every possible combination of values for the atomic propositions.

※atomic ?

- Columns
  - Need a column for the compound proposition (usually at far right)
  - Need a column for the truth value of each expression that occurs in the compound proposition as it is built up.
    - This includes the atomic propositions

Example Truth Table

- Example: Construct a truth table for

$$(p \rightarrow q) \land (\neg p \leftrightarrow q)$$

First step:
define columns & rows

Think about what atomic propositions are in this case.

Example Truth Table

- Example: Construct a truth table for

$$(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

two atomic propositional variables

Columns: simpler if we decompose the sentence to elementary and intermediate propositions

Example Truth Table

- Example: Construct a truth table for

$$(p \rightarrow q) \land (\neg p \leftrightarrow q)$$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p \rightarrow q) \land (\neg p \leftrightarrow q)$
T	T				
T	F				
F	T				
F	F				

Questions: How many rows are there in a truth table with *n* propositional variables?

Solution: 2^n

Example Truth Table

- Example: Construct a truth table for

$$(p \to q) \land (\neg p \leftrightarrow q)$$

p	q	$\neg p$	$p \to q$	$\neg p \leftrightarrow q$	$(p \to q) \land (\neg p \leftrightarrow q)$

Example Truth Table

- Example: Construct a truth table for

$$p \vee q \to \neg r$$

p	q	r			

Example Truth Table

- Example: Construct a truth table for

$$p \vee q \rightarrow \neg r$$

p	q	r	$\neg$r	p $\vee$ q	p $\vee$ q $\rightarrow$ $\neg$r
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

Equivalent Propositions

- Two propositions are ***equivalent*** if they always have the same truth value.
- **Example**: Show using a truth table that the conditional is equivalent to the contrapositive.

Solution:

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Using a Truth Table to Show Non-Equivalence

Example: Show using truth tables that neither the converse nor inverse of an implication are not equivalent to the implication.

Solution:

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$q \rightarrow p$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Computer representation of True and False

- **We need to encode two values True and False:**
  - **Computers represents data and programs using 0s and 1s**
  - Logical truth values – **True** and **False**
  - A bit is sufficient to represent two possible values:
    - **0** (**False**) or **1** (**True**)

- Boolean variable
  - A variable that takes on values **0** or **1**

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Bit and Bitwise Operations

- Bit
  - A symbol with two possible values, 0 and 1

Truth Value	Bit
T	1
F	0

OR → 둘다 0이면 0

AND → 둘다 1이면 1

XOR → 두개가 다르면 1

- Bit string
  - Sequence of zero or more bits
  - Length of a string is the number of bits in the string
- Bitwise OR/AND/XOR

```
01 1011 0110
11 0001 1101
----------------------------
11 1011 1111   bitwise OR
01 0001 0100   bitwise AND
10 1010 1011   bitwise XOR
```

Exercise

둘 중 하나라도 1이면 1　　　　두개가 다르면 1

- Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pairs of bit strings.

둘 다 1이어야 1

  – a) 101 1110,　　010 0001

  – b) 1111 0000,　　1010 1010

  – c) 00 0111 0001,　　10 0100 1000

  – d) 11 1111 1111,　　00 0000 0000

Section Summary

- Propositions
- Connectives
  - Negation
  - Conjunction
  - Disjunction
  - Implication; contrapositive, inverse, converse
  - Biconditional
- Truth Tables

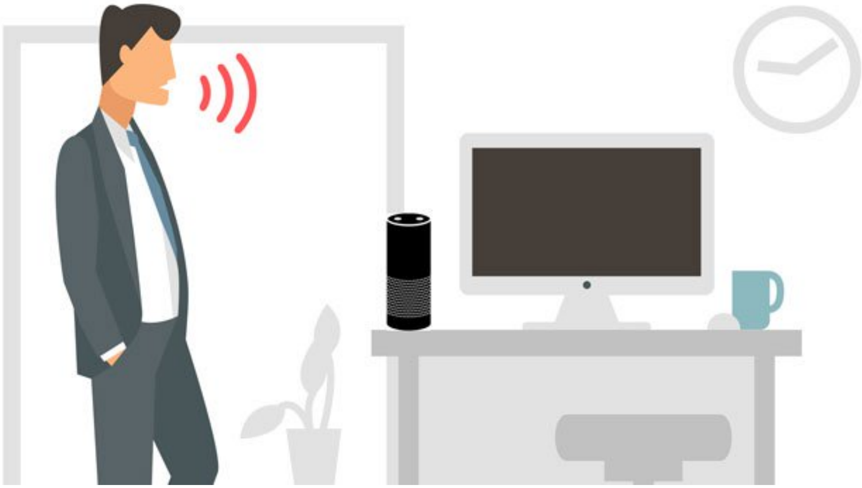
1.2 Applications of Propositional Logic

Applications of Propositional Logic

- **Translation of English sentences**
- **Inference and reasoning:**
  - new true propositions are inferred from existing ones
  - Used in Artificial Intelligence:
    - Rule based (expert) systems
    - Automatic theorem provers
- **Design of logic circuit**

Translating English Sentences

- Steps to convert an English sentence to a statement in propositional logic
  - 1. *find logical connectives* If / or / ,
  - *2. break the sentence into atomic (elementary) propositions*
  - *3. rewrite the sentence in propositional logic*
- "If I go to Harry's or to the country, I will not go shopping."



Translating English Sentences

- Steps to convert an English sentence to a statement in propositional logic
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Translating English Sentences

- Steps to convert an English sentence to a statement in propositional logic
  - 1. *find logical connectives*
  - ***2. break the sentence into atomic (elementary) propositions***
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- "If $\underbrace{\text{I go to Harry's}}_{p}$ or $\underbrace{\text{to the country}}_{q}$, $\underbrace{\text{I will not go shopping.}}_{r}$"
  - *p*: I go to Harry's
  - q: I go to the country.
  - *r*: I will go shopping.

Translating English Sentences

- Steps to convert an English sentence to a statement in propositional logic
  - 1. *find logical connectives*
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- "If $\underbrace{\text{I go to Harry's}}_{p}$ or $\underbrace{\text{to the country}}_{q}$, $\underbrace{\text{I will not go shopping.}}_{r}$"
  - *p*: I go to Harry's
  - q: I go to the country.
  - *r*: I will go shopping.

If *p* or *q* then not *r*.

$$(p \vee q) \rightarrow \neg r$$

$\neg$: not

$\wedge$: and

$\vee$: or

$\rightarrow$

$\leftrightarrow$

Example

Problem: Translate the following sentence into propositional logic:

a $\qquad$ c

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

$\neg f$

One Solution: Let a, c, and f represent respectively "You can access the internet from campus," "You are a computer science major," and "You are a freshman."

$$\cancel{(c \vee f) \to a} \qquad a \to (c \vee \neg f)$$

System Specifications

- System and Software engineers take requirements in English and express them in a precise specification language based on logic.

Example: Express in propositional logic:

"The automated reply cannot be sent when the file system is full"

One possible solution: Let ***p*** denote "The automated reply can be sent" and ***q*** denote "The file system is full."

$$q \rightarrow \neg p$$

Digital Circuits

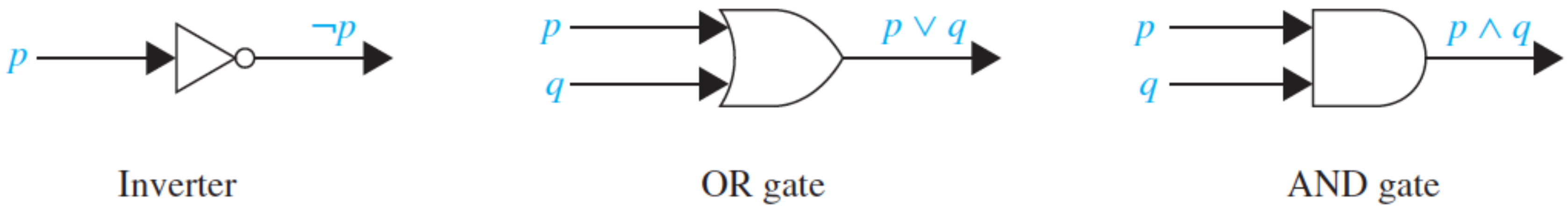


FIGURE 1 **Basic logic gates.**

$(p \wedge \neg q) \vee (\neg r)$

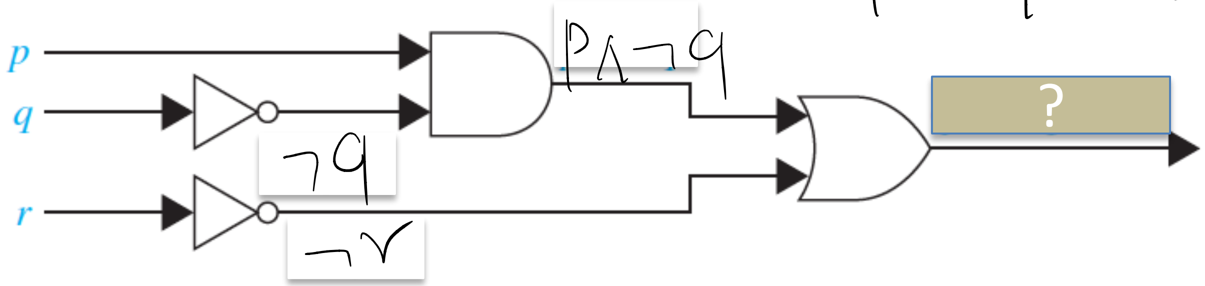


FIGURE 2 **A combinatorial circuit.**

Question

- Question :
  Build a digital circuit that produces the output
  (p∨¬r) ∧ (¬p∨(q∨¬r))
  when given input bits p, q, and r.

역으로 유추할 수도
있어야함!

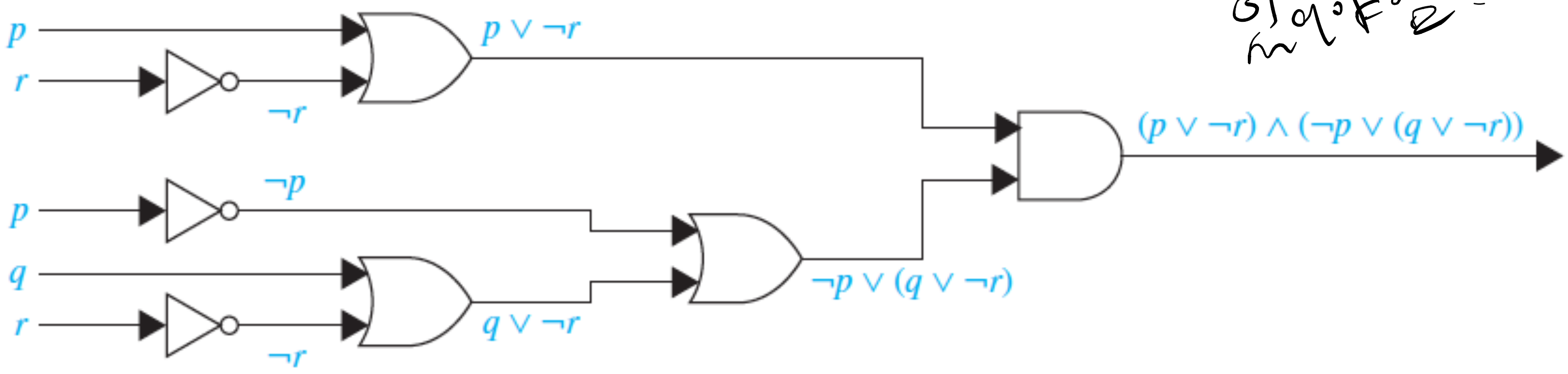


FIGURE 3 The circuit for $(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$.

Applications of Propositional Logic: Summary

- **Translating English to Propositional Logic**
- **System Specifications**
- *Boolean Searching*
- ***Logic Circuits***

1.3 Propositional Equivalences

Tautology, Contradiction, and Contingency

- A *tautology* is a proposition which is always true (no matter what the truth values of the propositional variables that occur in it)
  - Example: $p \vee \neg p$ 항상 참인 명제
- A *contradiction* is a proposition which is always false.
  - Example: $p \wedge \neg p$ 항상 거짓 명제
- A *contingency* is a proposition which is neither a tautology nor a contradiction, such as *p*

P	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Equivalence (Logically Equivalent)

- Two <u>compound</u> propositions ***p*** and ***q*** **are logically equivalent** if $p \leftrightarrow q$ is a tautology.
- We write this as $p \Leftrightarrow q$ or as $p \equiv q$ where p and q are compound propositions.
- Two compound propositions p and q are equivalent if and only if the columns in a truth table giving their truth values agree.
- e.g., This truth table show "$\neg p \vee q$" is equivalent to "$p \rightarrow q$".

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Augustus De Morgan

1806-1871

This truth table shows that De Morgan's Second Law holds.

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Key Logical Equivalences

$\emptyset$ P or T → T P and F → F

- Identity Laws: $p \wedge T \equiv p$, $p \vee F \equiv p$
- Domination Laws: $p \vee T \equiv T$, $p \wedge F \equiv F$
- Idempotent laws: $p \vee p \equiv p$, $p \wedge p \equiv p$
- Double Negation Law: $\neg(\neg p) \equiv p$
- Negation Laws: $p \vee \neg p \equiv T$, $p \wedge \neg p \equiv F$

Key Logical Equivalences (*cont*)

- Commutative Laws: $p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$
- Associative Laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
  $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- Distributive Laws: $(p \vee (q \wedge r)) \equiv (p \vee q) \wedge (p \vee r)$
  $(p \wedge (q \vee r)) \equiv (p \wedge q) \vee (p \wedge r)$

$\rightarrow 3 \times (5+1)$

$= (3\times 5)+(3\times 1)$

- Absorption Laws: $p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$

More Logical Equivalences

$p \to q$ T
$q \to p$ F

P
Q

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Constructing New Logical Equivalences

- We can show that two expressions are logically equivalent by developing a series of logically equivalent statements.
- To prove that $A \equiv B$ we produce a series of equivalences beginning with A and ending with B.

$$\begin{aligned} A &\equiv A_1 \\ &\vdots \\ A_n &\equiv B \end{aligned}$$

- Keep in mind that whenever a proposition (represented by a propositional variable) occurs in the equivalences listed earlier, it may be replaced by an arbitrarily complex compound proposition.

Equivalence Proofs

Example: Show that $\neg(p \vee (\neg p \wedge q))$

is logically equivalent to $\neg p \wedge \neg q$

ㄱㄱ 어렵다!

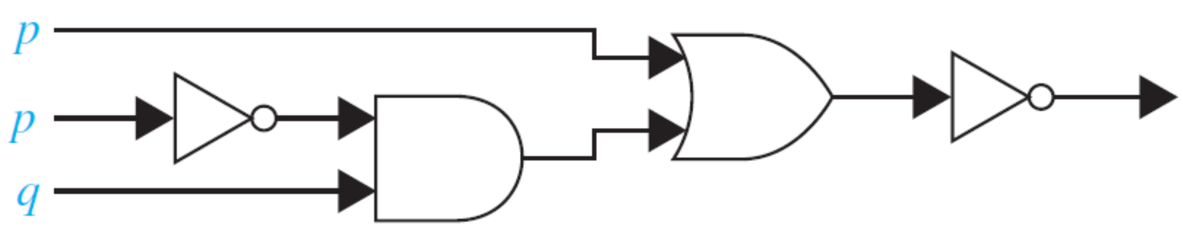
Solution:

$$
\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{by the second De Morgan law} \\
&\equiv \neg p \wedge [\neg(\neg p) \vee \neg q] && \text{by the first De Morgan law} \\
&\equiv \neg p \wedge (p \vee \neg q) && \text{by the double negation law} \\
&\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{by the second distributive law} \\
&\equiv F \vee (\neg p \wedge \neg q) && \text{because } \neg p \wedge p \equiv F \\
&\equiv (\neg p \wedge \neg q) \vee F && \text{by the commutative law} \\
& && \text{for disjunction} \\
&\equiv (\neg p \wedge \neg q) && \text{by the identity law for } \mathbf{F}
\end{aligned}
$$

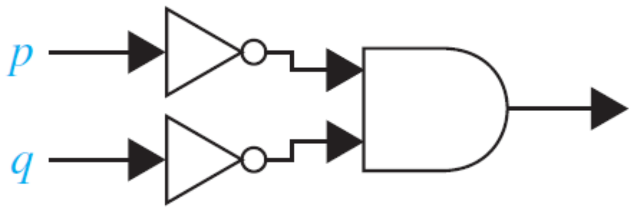
Equivalence Proofs

Example: Show that $\neg(p \vee (\neg p \wedge q))$

is logically equivalent to $\neg p \wedge \neg q$



$\equiv$



Replaced by
a simple and straightforward circuit
with the same function

Equivalence Proofs

Example: Show that $(p \land q) \to (p \lor q)$
is a tautology.

Solution:

$$
\begin{array}{rcll}
(p \land q) \to (p \lor q) & \equiv & \neg(p \land q) \lor (p \lor q) & \text{by truth table for } \to \\
& \equiv & (\neg p \lor \neg q) \lor (p \lor q) & \text{by the first De Morgan law} \\
& \equiv & (\neg p \lor p) \lor (\neg p \lor \neg q) & \text{by associative and} \\
& & & \text{commutative laws} \\
& & & \text{laws for disjunction} \\
& \equiv & T \lor T & \text{by truth tables} \\
& \equiv & T & \text{by the domination law}
\end{array}
$$

Acknowledgement

- Some slides are taken from lecture notes of Prof. Marc Pomplun at UMB.

Propositional Satisfiability

- A compound proposition is ***satisfiable*** if there is an assignment of truth values to its variables that make it true. When no such assignments exist, the compound proposition is *unsatisfiable*.

- A compound proposition is unsatisfiable if and only if its negation is a tautology.

Example

Example: Determine the satisfiability of the following compound propositions:

$$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$$

Solution: Satisfiable. Assign **T** to p, q, and r.

$$(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$$

Solution: Satisfiable. Assign **T** to p and $\boldsymbol{F}$ to q.

$$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$$

Solution: Not satisfiable. Check each possible assignment of truth values to the propositional variables and none will make the proposition true.