Data Structures:
Internal Sorting: Insertion Sort, Selection Sort, Merge Sort, Quick Sort

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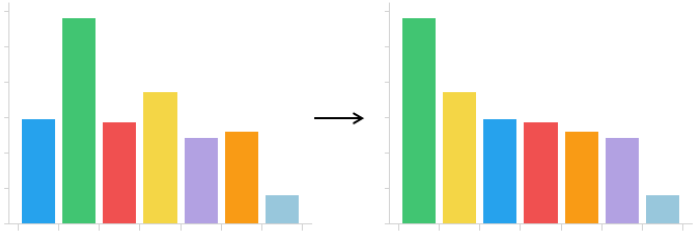
(Slide credits to Won Kim)
Spring 2022

Sorting - Internal

Sorting

- Definition
  - Arranging elements of a list in some order
    - Lexicographical order (symbols, numbers)
    - Ascending order or descending order
    - Composite-key sort
- Why sort?
  - Required end result
    - Easy visual search



  - Efficient intermediate data structure for computing the end result
- In general, entire records are sorted and change positions.

Motivating Example 1

name	age	salary	employer
Kim	25	200	LG
Lee	30	300	IBM
Park	23	250	LG
Cho	40	500	Samsung
…			
Chung	28	900	Samsung

Motivating Example 1 (cont'd)

name	age	salary	employer
Cho	40	500	Samsung
Chung	28	900	Samsung
Kim	25	200	LG
Lee	30	300	IBM
…			
Park	23	250	LG

This is easier to search by name

Motivating Example 2

name	age	salary	employer
Kim	25	200	LG
Lee	30	300	IBM
Park	23	250	LG
Cho	40	500	Samsung
…			
Chung	28	900	Samsung

Who works for Samsung?

Who is the oldest?
highest paid?

Motivating Example 2 (cont'd)

name	age	salary	employer
Chung	28	900	Samsung
Cho	40	500	Samsung
Park	23	250	LG
Kim	25	200	LG
…			
Lee	30	300	IBM

Who works for Samsung?

Who is the oldest?
highest paid?

What the Sorted Results Make Possible

- Speedy sequential search (that can stop in the middle of a search)
- Binary search ($O(\log_2 n)$ performance)
- Fast join of multiple tables (in a relational database)

Join: What is the number of employees of the company where Cho works?

To find the answer, we need to join two tables: employee and company.

employee

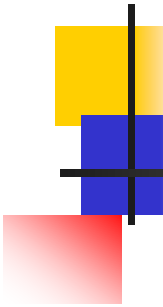
name	age	salary	employer
Kim	25	200	LG
Lee	30	300	IBM
Park	23	250	LG
Cho	40	500	Samsung
…			
Chung	28	900	Samsung

company

company	years	country	employees
IBM	107	USA	450,000
LG	60	Korea	38,000
Samsung	49	250	310,000
SAP	46	500	15,000

Sorting Algorithms

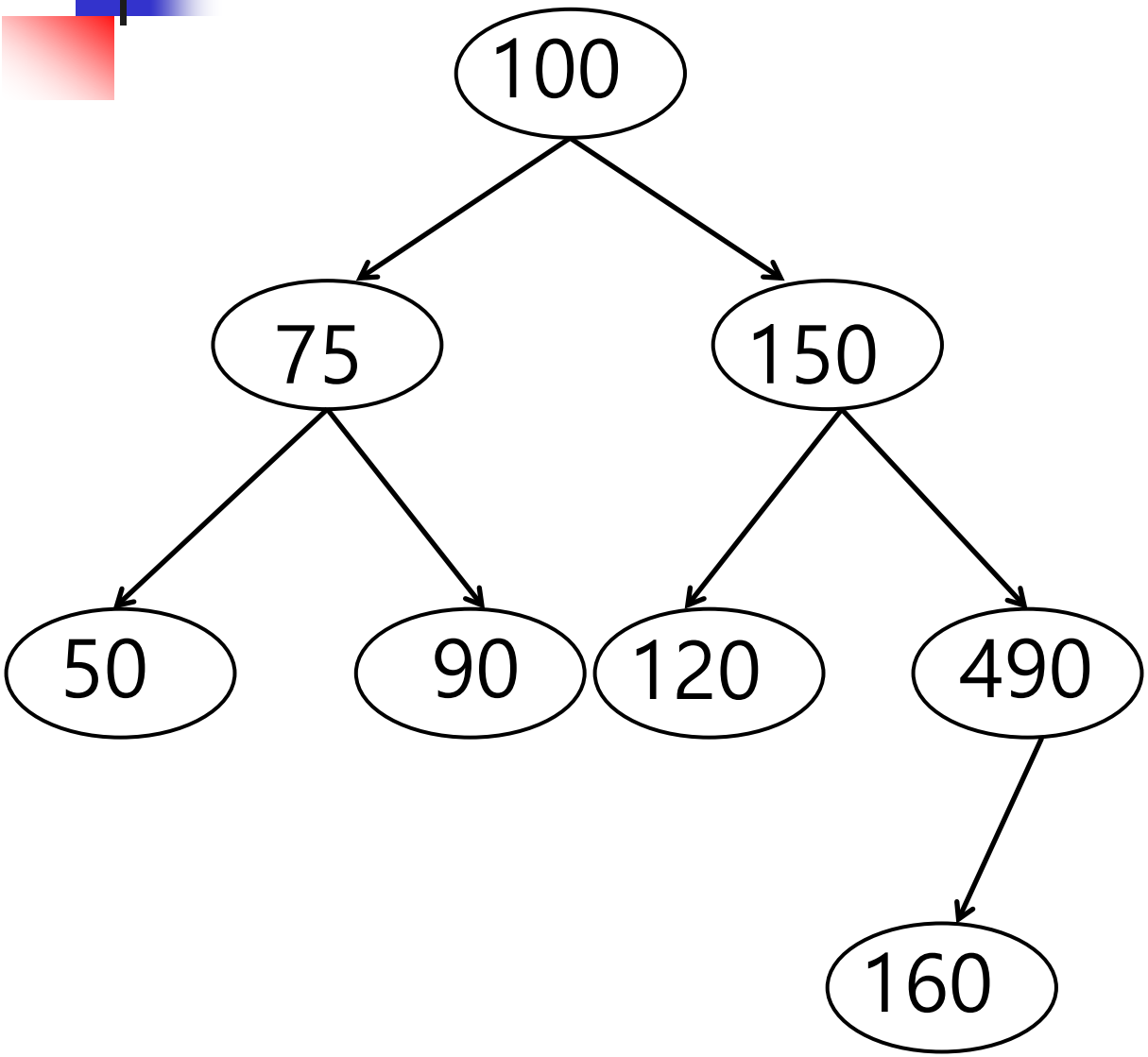
- Internal Sorting
  - List to be sorted fits in main memory.
- External Sorting
  - List to be sorted resides on secondary storage.
- Time Complexity & Space Complexity
- Stable Sorting
  - "equal" elements are ordered in the same order in the sorted list.
  - The relative order of the duplicate elements should not change.
- In-place Sorting
  - A sort algorithm in which the sorted items occupy the same storage as the original ones.
  - These algorithms may use O(n) additional memory for bookkeeping, but at most a constant number of items are kept in auxiliary memory at any time. O(1)



Sorting Algorithms

- Internal Sorting
  - Insertion sort
  - Selection sort
  - (Bubble sort – worst; please self-study it.)
  - Merge sort
  - Quick sort
  - Heap sort
  - Radix sort

Inorder Traversal of a Binary Search Tree: Outputs Keys in Ascending Sort Order



50
75
90
100
120
150
160
490

Insertion Sort

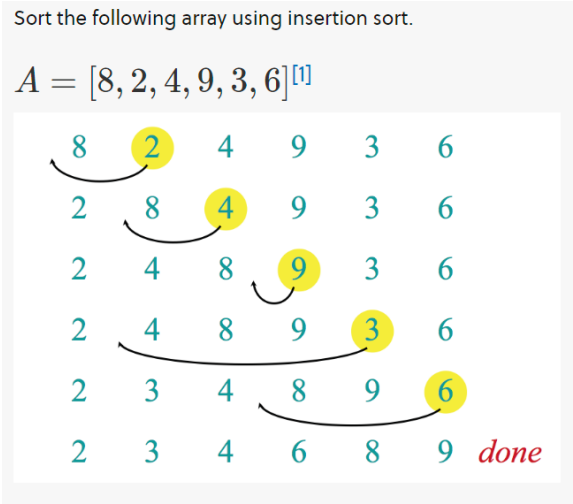
Insertion Sort: Concept

- Start with a sorted list.
- To insert a new key, search the list for a correct position, and insert it there.

insert: 170	(170)
insert: 90	(90, 170)
insert: 2	(2, 90, 170)
insert: 802	(2, 90, 170, 802)
insert: 24	(2, 24, 90, 170, 802)
insert: 45	(2, 24, 45, 90, 170, 802)
insert: 75	(2, 24, 45, 75, 90, 170, 802)
insert: 66	(2, 24, 45, 66, 75, 90, 170, 802)

Insertion Sort: Method

- Unsorted Array → Sorted Array
  - Note the SWAP operation!



- Examples:
  - https://youtu.be/OGzPmgsI-pQ
  - https://www.hackerearth.com/practice/algorithms/sorting/insertion-sort/visualize/

Insertion Sort: Exercise

sort the list:

22, 25, 7, 3, 30, 11, 14, 8

sort the list:

10, 20, 30, 40, 50, 60

sort the list:

60, 50, 40, 30, 20, 10

Insertion Sort: Performance and Qualities

- Average & Worst Case: $O(n^2)$
  - n = the number of keys in the list
  - O(n): time to swap the keys for the correct position for a key on the current sorted list
- Best Case: O(n)
  - If the list is already sorted, the swap becomes O(1)
- Space Complexity: O(1) (in-place sort)
- Simple and good for a short list
- Good when the list is already partially sorted.
- Stable Sorting

10, 10, 10, 20, 30, 40

Insertion Sort: Example code

```c
// C program for insertion sort
#include <math.h>
#include <stdio.h>

/* Function to sort an array using insertion sort*/
void insertionSort(int arr[], int n)
{
    int i, key, j;
    for (i = 1; i < n; i++) {
        key = arr[i];
        j = i - 1;

        /* Move elements of arr[0..i-1], that are
          greater than key, to one position ahead
          of their current position */
        while (j >= 0 && arr[j] > key) {
            arr[j + 1] = arr[j];
            j = j - 1;
        }
        arr[j + 1] = key;
    }
}

// A utility function to print an array of size n
void printArray(int arr[], int n)
{
    int i;
    for (i = 0; i < n; i++)
        printf("%d ", arr[i]);
    printf("\n");
}

/* Driver program to test insertion sort */
int main()
{
    int arr[] = { 12, 11, 13, 5, 6 };
    int n = sizeof(arr) / sizeof(arr[0]);

    insertionSort(arr, n);
    printArray(arr, n);

    return 0;
}
```

https://www.geeksforgeeks.org/insertion-sort/

Selection Sort

Selection Sort

- n-1 passes over a list of n keys
- On the i^{th} pass
  - select the largest key among the first n-i keys, and exchange it with the $n-i+1^{th}$ key
- Reading
  - http://en.wikipedia.org/wiki/Selection_sort

Selection Sort: Example

sort the list:
13, 4, 9, 21, 37, 17, 22, 3, 8

pass 1: 13, 4, 9, 21, 8, 17, 22, 3, 37

pass 2: 13, 4, 9, 21, 8, 17, 3, 22, 37

pass 3: 13, 4, 9, 3, 8, 17, 21, 22, 37

……

pass 8: 3, 4, 8, 9, 13, 17, 21, 22, 37

Selection Sort: Method

- On the i^{th} pass
  - It's fine to select the smallest key.
  - Note how the SWAP operation works.
- Examples:
  - https://www.youtube.com/watch?v=xWBP4lzkoyM
  - https://youtu.be/wnKQsow7ERI
  - https://www.hackerearth.com/practice/algorithms/sorting/insertion-sort/visualize/

Selection Sort: Exercise

sort the list:

22, 25, 7, 3, 30, 11, 14, 8

Selection Sort: Performance

- $O(n^2)$ performance (Best, Average, Worst)
  - n is the number of keys in the list
  - n^2 comparisons
  - generally worse than insertion sort
  - O(n) exchanges
- Space Complexity: O(1) (in-place sort)
- Useful for a small list stored in EEPROM or Flash.
  - “exchanges” require writing to an array.
  - For an array stored in EEPROM or Flash, writing to memory is significantly more expensive than reading.
- Unstable sorting

20, 30, 20, 10, 40, 50

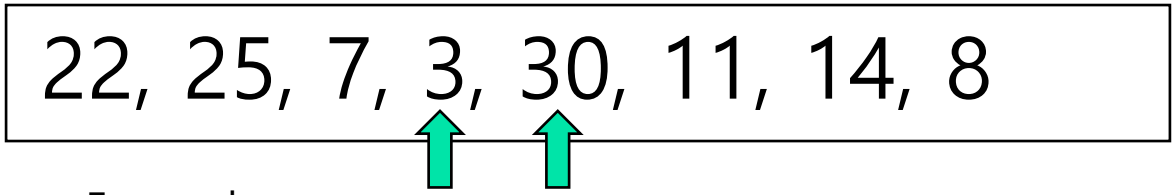
Selection Sort: Example code

```c
// C program for implementation of selection sort
#include <stdio.h>

void swap(int *xp, int *yp)
{
    int temp = *xp;
    *xp = *yp;
    *yp = temp;
}

void selectionSort(int arr[], int n)
{
    int i, j, min_idx;

    // One by one move boundary of unsorted subarray
    for (i = 0; i < n-1; i++)
    {
        // Find the minimum element in unsorted array
        min_idx = i;
        for (j = i+1; j < n; j++)
          if (arr[j] < arr[min_idx])
            min_idx = j;

        // Swap the found minimum element with the first element
        swap(&arr[min_idx], &arr[i]);
    }
}

/* Function to print an array */
void printArray(int arr[], int size)
{
    int i;
    for (i=0; i < size; i++)
        printf("%d ", arr[i]);
    printf("\n");
}

// Driver program to test above functions
int main()
{
    int arr[] = {64, 25, 12, 22, 11};
    int n = sizeof(arr)/sizeof(arr[0]);
    selectionSort(arr, n);
    printf("Sorted array: \n");
    printArray(arr, n);
    return 0;
}
```

https://www.geeksforgeeks.org/selection-sort/

Duplex Selection Sort

- (n-1)/2 passes over a list of n keys
  - Select the largest key and the smallest key, and
  - Exchange the largest key with the last key, and
  - Exchange the smallest key with the first key.



For each pass,
select the maximum & minimum at the same time

- $O(n^2)$ performance
  - n is the number of keys in the list

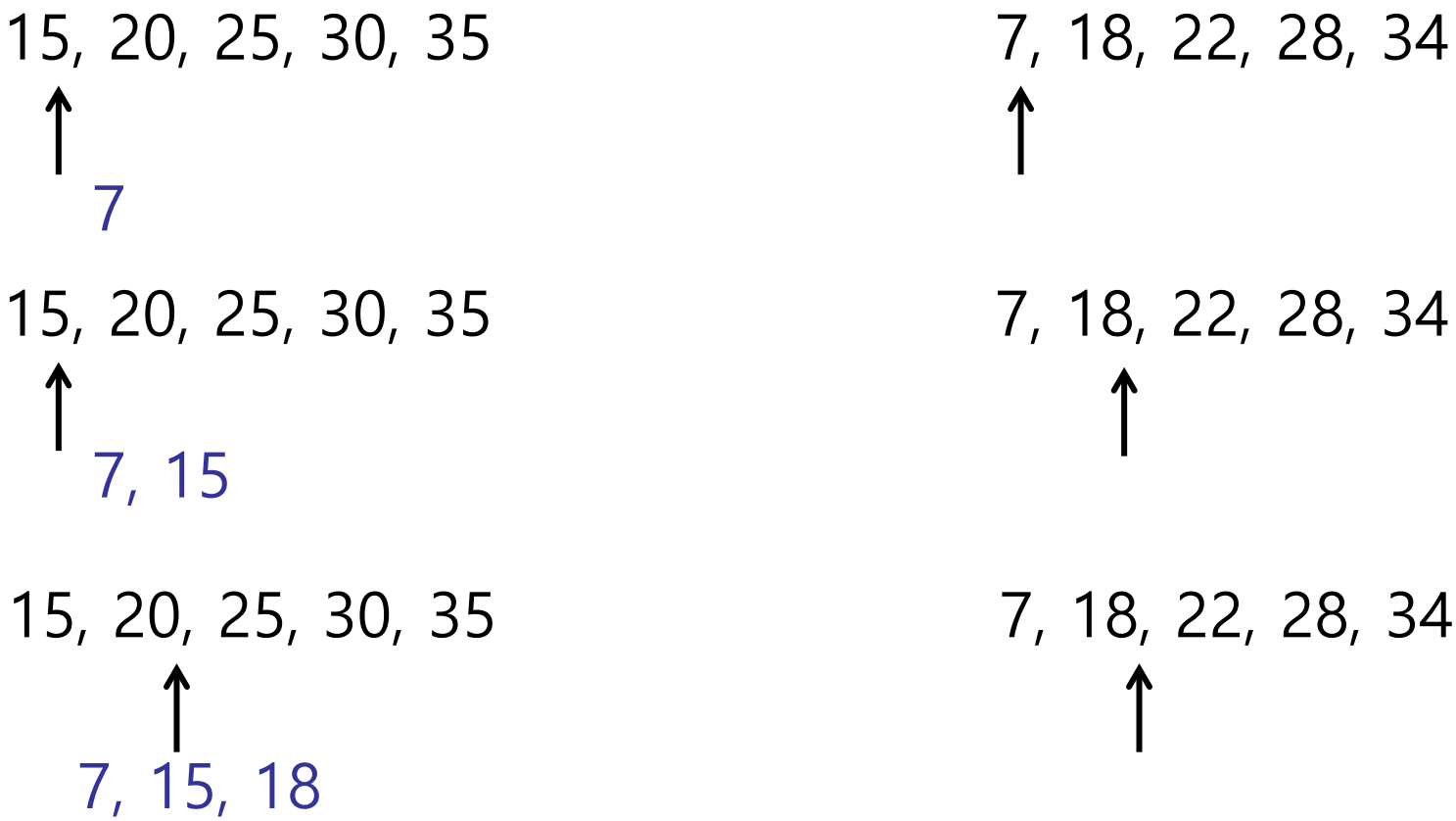
Merge Sort

Merge Sort

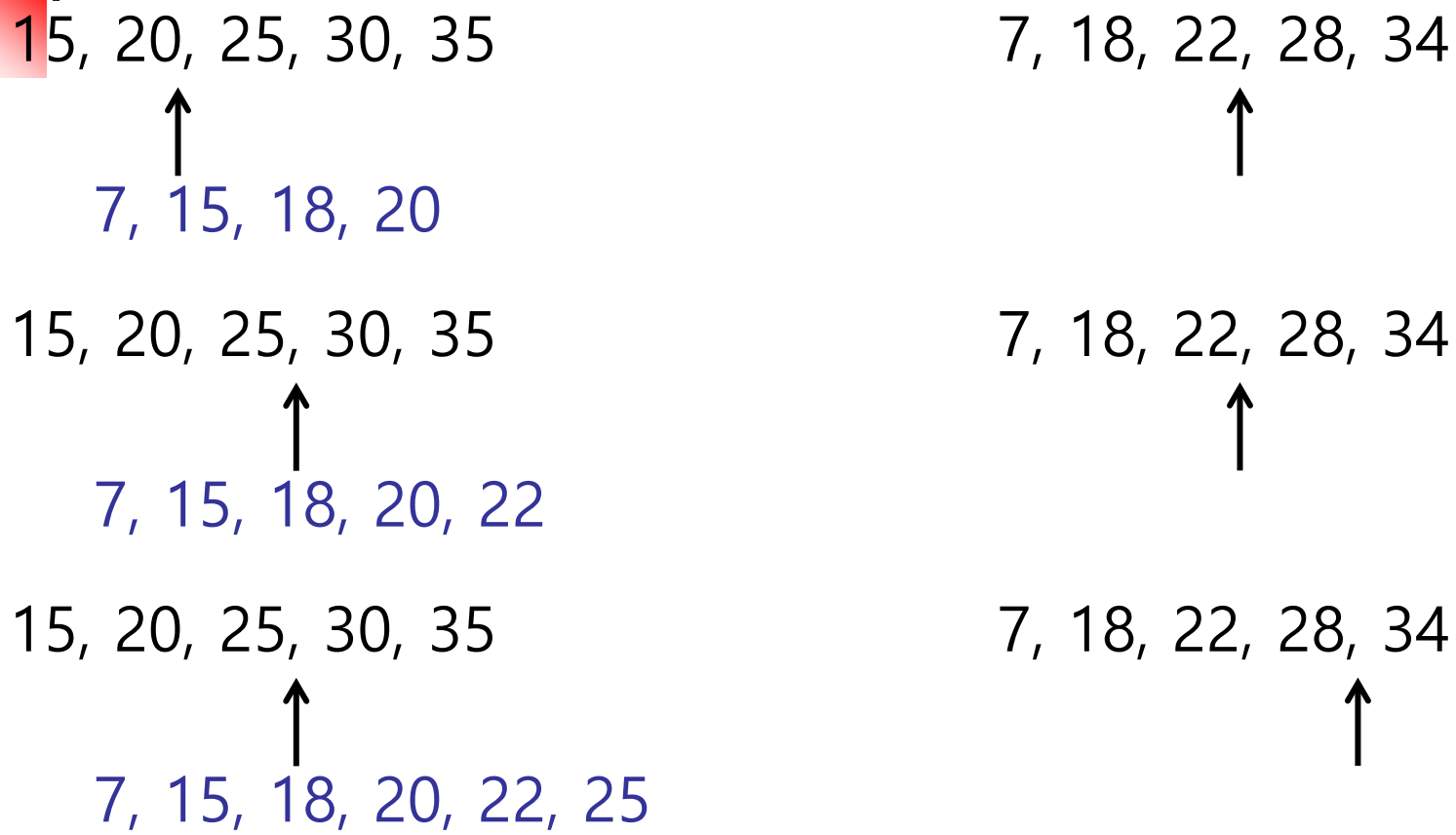
- Divide and Conquer
  - **Divide** the problem into multiple small problems
  - **Conquer** the subproblems
  - **Combine** the solutions of the subproblems
- Split and Merge
  - Split: Divide a list successively into two sorted sub-lists.
  - Merge: Merge the two sub-lists successively into a single sorted list.
- Invented by John von Neumann in 1945
- Supplementary Reading
  - http://en.wikipedia.org/wiki/Merge_sort

Merge Example

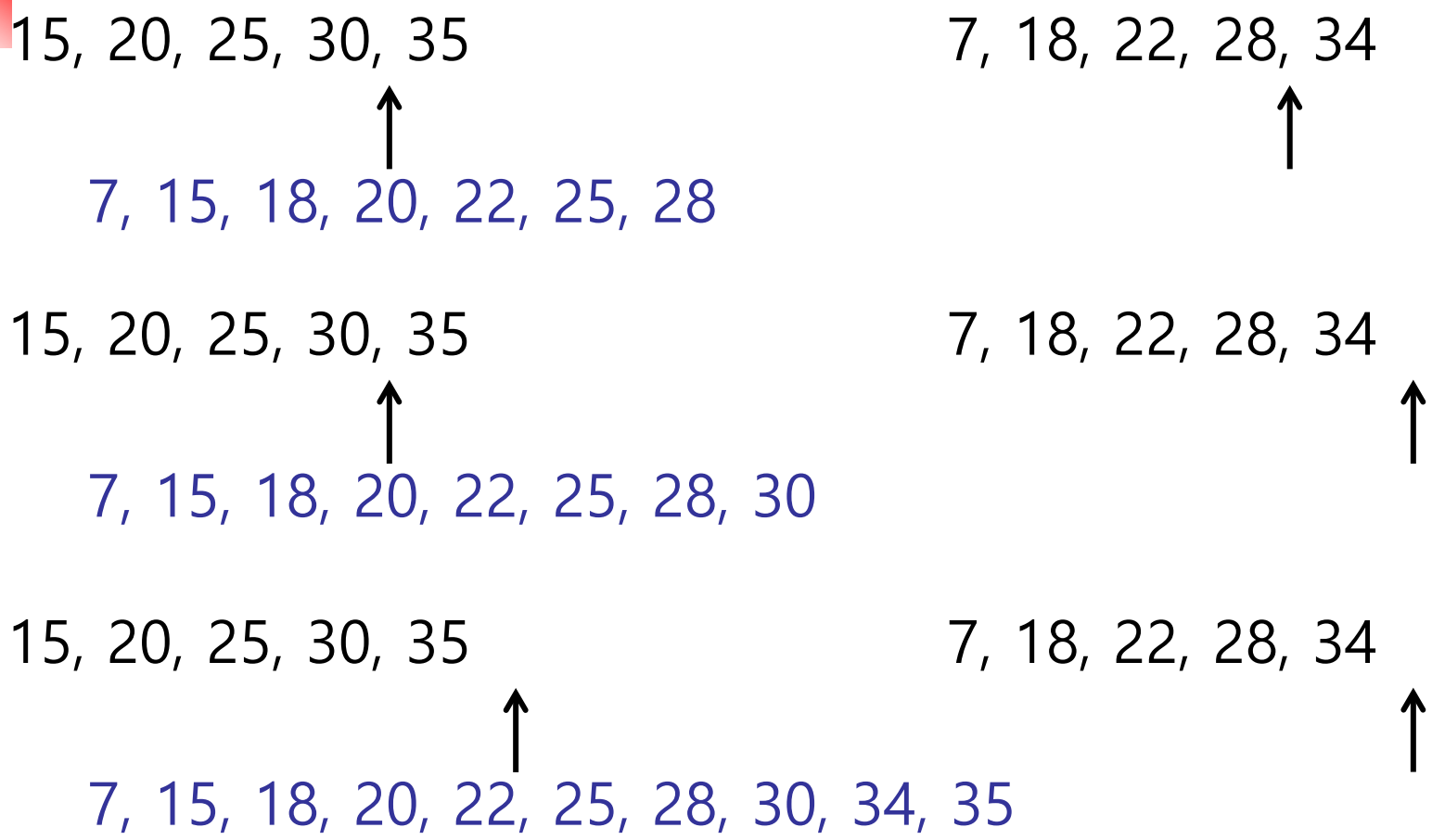
merge two sorted lists:

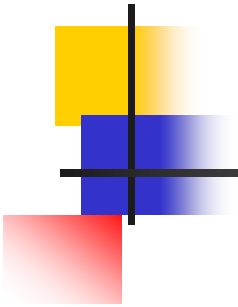


Merge Example (cont'd)



Merge Example (cont'd)





Exercise: Merge

merge the two lists:

35, 20, 25, 40, 3, 9

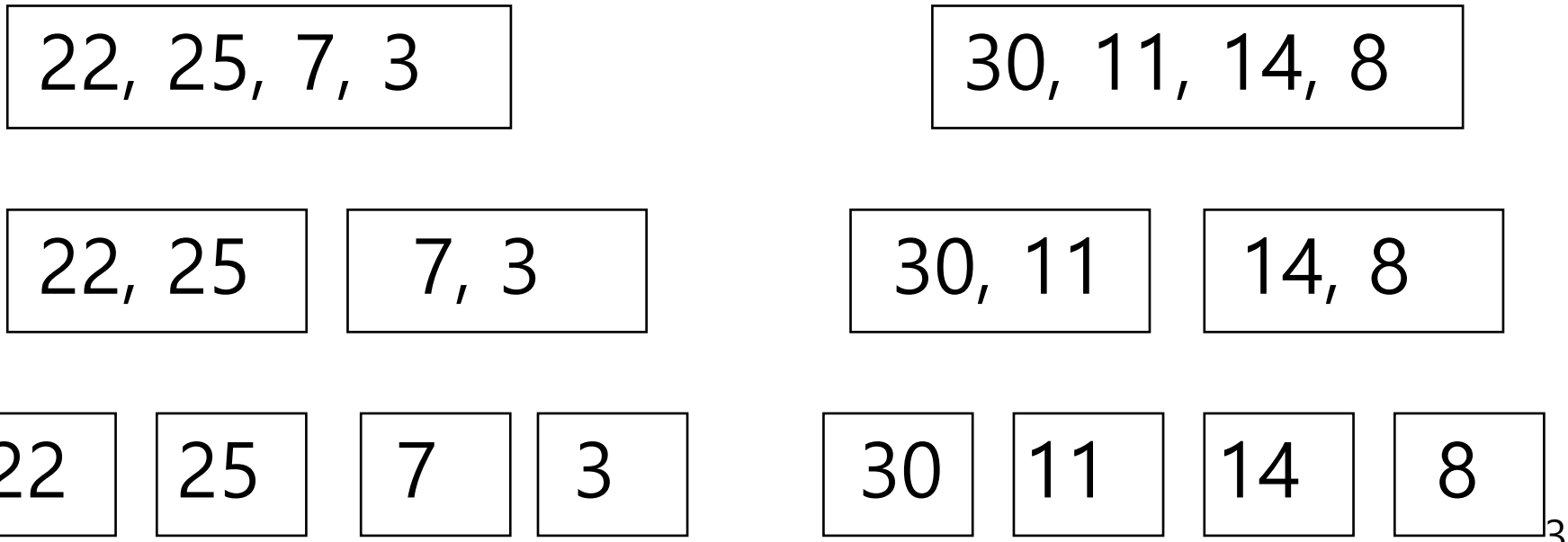
7, 18, 22, 28, 34

Merge Sort

sort the list:

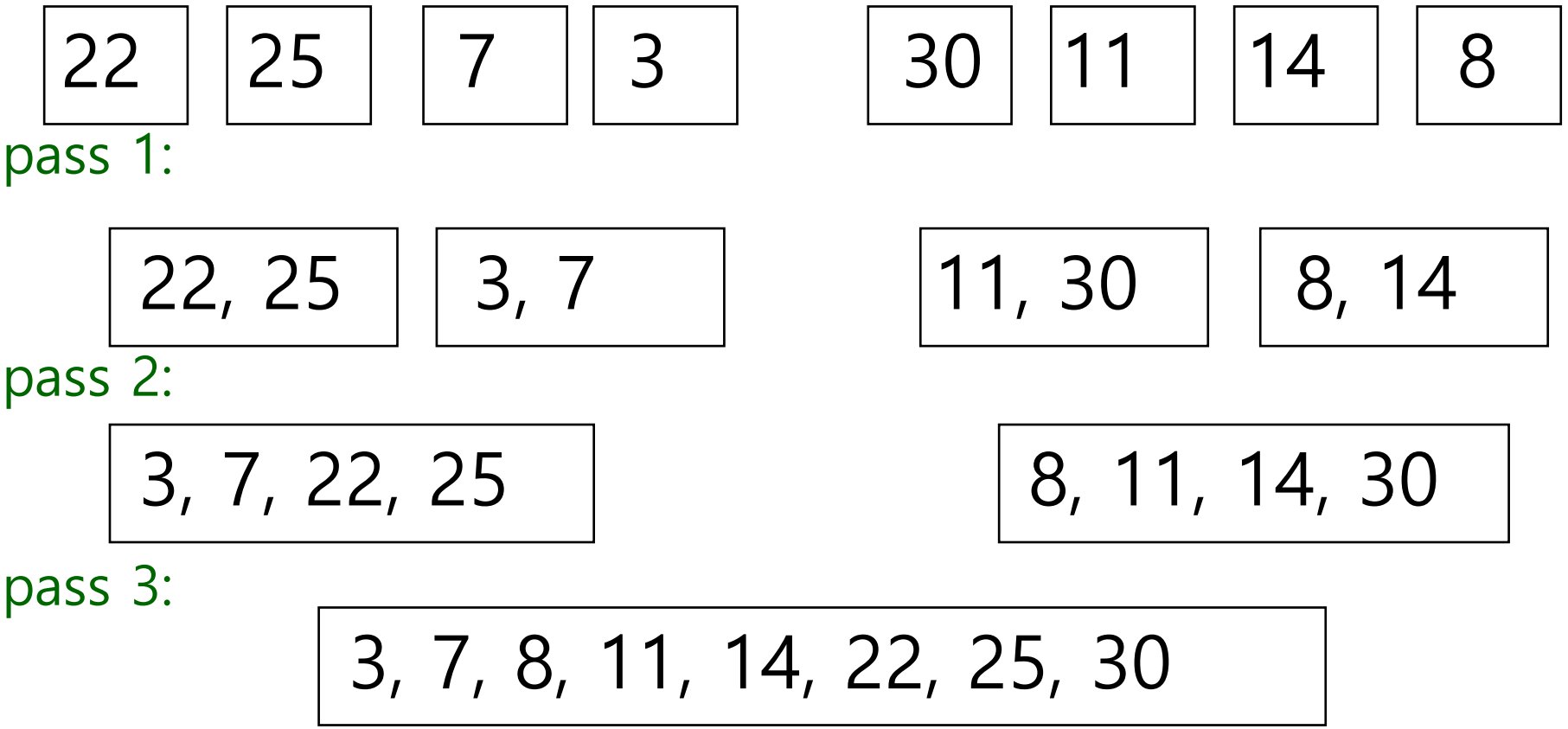
22, 25, 7, 3, 30, 11, 14, 8

splitting phase



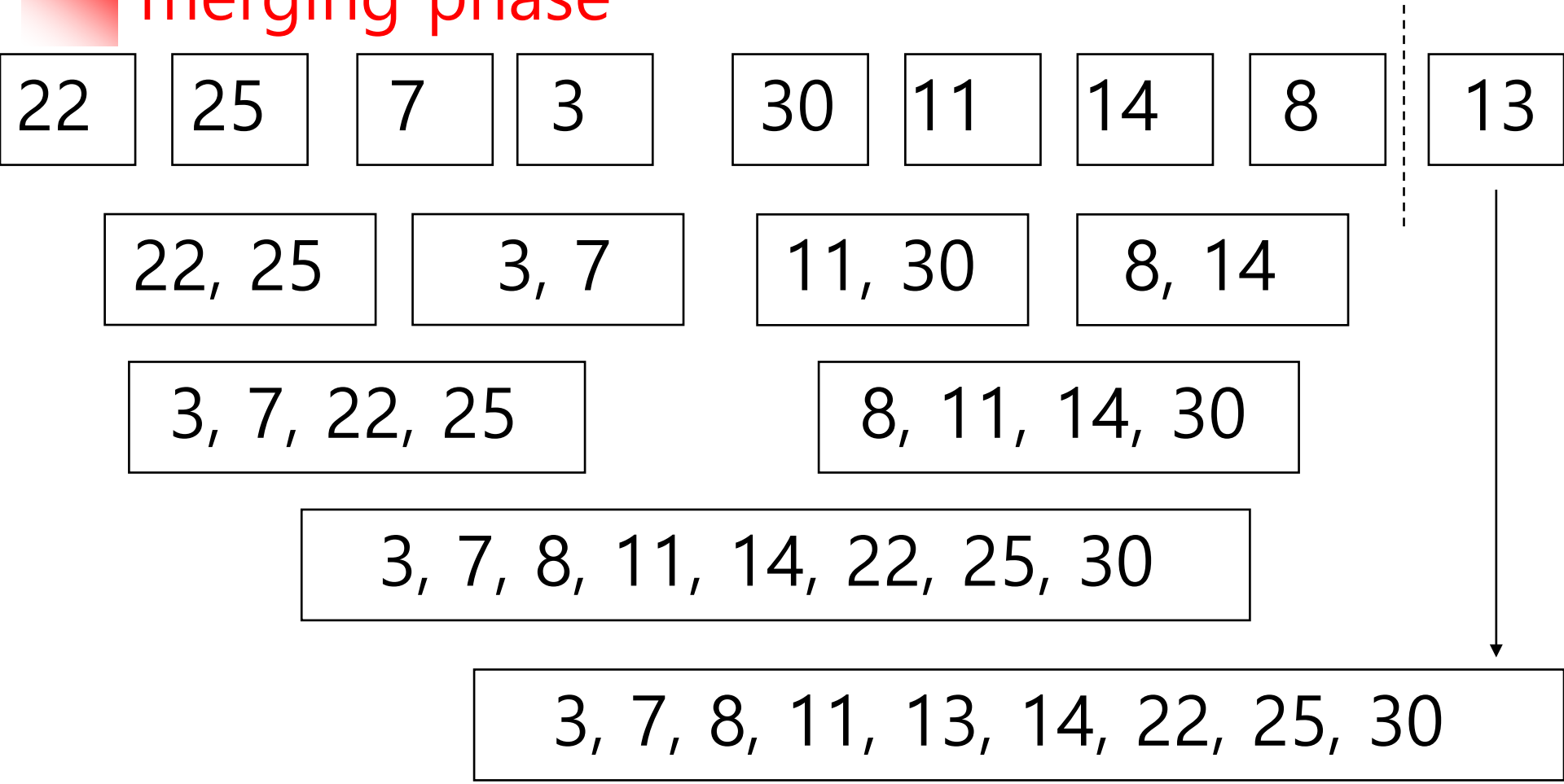
Merge Sort: (cont'd)

merging phase



Merge Sort: Merging an Odd Number of Data

merging phase



Merge Sort: Performance and Qualities

- Merge: O(n)
  - The cursors always move to the right since the sublists are already sorted.
- O(n $\log_2$ n) comparisons and copying of keys
  - n: number of keys in the list
  - $\log_2$ n times Merge (guaranteed) (Best, Average, Worst)
  - best “worst case” sorting
- Space Complexity: O(n) (**Not** in-place sort)
  - The sublists requires equal amount of additional space as the unsorted array.
- Stable Sort
  - Since Merge does not swap for duplicates.
- Amenable to parallel processing
- Amenable to adaptation for external sorting

Merge Sort: Example code

```c
/* C program for Merge Sort */
#include <stdio.h>
#include <stdlib.h>

// Merges two subarrays of arr[].
// First subarray is arr[l..m]
// Second subarray is arr[m+1..r]
void merge(int arr[], int l, int m, int r)
{
    int i, j, k;
    int n1 = m - l + 1;
    int n2 = r - m;

    /* create temp arrays */
    int L[n1], R[n2];

    /* Copy data to temp arrays L[] and R[] */
    for (i = 0; i < n1; i++)
        L[i] = arr[l + i];
    for (j = 0; j < n2; j++)
        R[j] = arr[m + 1 + j];

    /* Merge the temp arrays back into arr[l..r]*/
    i = 0; // Initial index of first subarray
    j = 0; // Initial index of second subarray
    k = l; // Initial index of merged subarray
    while (i < n1 && j < n2) {
        if (L[i] <= R[j]) {
            arr[k] = L[i];
            i++;
        }
        else {
            arr[k] = R[j];
            j++;
        }
        k++;
    }

    /* Copy the remaining elements of L[], if there
    are any */
    while (i < n1) {
        arr[k] = L[i];
        i++;
        k++;
    }

    /* Copy the remaining elements of R[], if there
    are any */
    while (j < n2) {
        arr[k] = R[j];
        j++;
        k++;
    }
}
```

```c
/* l is for left index and r is right index of the
sub-array of arr to be sorted */
void mergeSort(int arr[], int l, int r)
{
    if (l < r) {
        // Same as (l+r)/2, but avoids overflow for
        // large l and h
        int m = l + (r - l) / 2;

        // Sort first and second halves
        mergeSort(arr, l, m);
        mergeSort(arr, m + 1, r);

        merge(arr, l, m, r);
    }
}

/* UTILITY FUNCTIONS */
/* Function to print an array */
void printArray(int A[], int size)
{
    int i;
    for (i = 0; i < size; i++)
        printf("%d ", A[i]);
    printf("\n");
}

/* Driver code */
int main()
{
    int arr[] = { 12, 11, 13, 5, 6, 7 };
    int arr_size = sizeof(arr) / sizeof(arr[0]);

    printf("Given array is \n");
    printArray(arr, arr_size);

    mergeSort(arr, 0, arr_size - 1);

    printf("\nSorted array is \n");
    printArray(arr, arr_size);
    return 0;
}
```

https://www.geeksforgeeks.org/merge-sort/

Performance Comparison

N^2 vs. $N \log_2 N$

N^2

	$n = 10^3$	$n = 10^6$	$n = 10^9$
PC		2.8 hr	317 yr
super com		1 sec	1.7 wk

$N \log_2 N$

	$n = 10^3$	$n = 10^6$	$n = 10^9$
PC		1 sec	18 min
super com			

pc: 10^8 compares/sec
supercom: 10^{12} compares/sec

Quick Sort

Quick Sort

- One of the most widely used sorting algorithms
- Sort by divide and conquer
- Invented by C.A.R. Hoare in 1961
- Types of quick sort
  - Plain quick sort (creates new lists for intermediate results)
  - In-place quick sort (uses the original list)
- Supplementary Reading
  - http://en.wikipedia.org/wiki/Quicksort
  - http://www.cs.purdue.edu/homes/ayg/CS251/slides/chap8b.pdf

Quick Sort

- Select a Pivot.
  - Arbitrary item on the list (first, last, middle,…)
- Partition
  - Move Keys < Pivot into a L-List.
  - Move Keys = Pivot into a P-List.
  - Move Keys > Pivot into a R-List.
- Recurse on the L-List, and R-List, until both lists are sorted.
- Concatenate L-List, P-List, R-List into a new list.

Quick Sort Pseudo Code

```
QUICKSORT(A, p, r)
1  if p < r
2      q = PARTITION(A, p, r)
3      QUICKSORT(A, p, q − 1)
4      QUICKSORT(A, q + 1, r)
```

```
PARTITION(A, p, r)
1  x = A[r]
2  i = p − 1
3  for j = p to r − 1
4      if A[j] ≤ x
5          i = i + 1
6          exchange A[i] with A[j]
7  exchange A[i + 1] with A[r]
8  return i + 1
```

Quick Sort: Example 1

sort the list: (pivot=first element)
70, 90, 12, 80, 24, 45, 75, 66

pass 1: 12, 24, 45, 66, 70, 90, 80, 75

pass 2: 12, 24, 45, 66, 70, 80, 75, 90

pass 3: 12, 24, 45, 66, 70, 75, 80, 90

pass 4: 12, 24, 45, 66, 70, 75, 80, 90

Quick Sort: Exercise

sort the list:(pivot=first element)
17, 90, 2, 80, 14, 13, 75, 16

Quick Sort: Solution

sort the list:
17, 90, 2, 80, 14, 13, 75, 16

pass 1: 2, 14, 13, 16, 17, 90, 80, 75

pass 2: 2, 14, 13, 16, 17, 80, 75, 90

pass 3: 2, 13, 14, 16, 17, 75, 80, 90

Quick Sort: Exercise

sort the list:(pivot=last element)
17, 90, 2, 80, 14, 13, 75, 16

Quick Sort: Example 2 (worst case)

sort the list: (pivot=first element)

2, 24, 45, 66, 75, 90, 170, 802

pass 1: 2, 24, 45, 66, 75, 90, 170, 802

pass 2: 2, 24, 45, 66, 75, 90, 170, 802

pass 3: 2, 24, 45, 66, 75, 90, 170, 802

pass 4: 2, 24, 45, 66, 75, 90, 170, 802

pass 5: 2, 24, 45, 66, 75, 90, 170, 802

pass 6: 2, 24, 45, 66, 75, 90, 170, 802

pass 7: 2, 24, 45, 66, 75, 90, 170, 802

In-Place Quick Sort

- Use the last element in the list as pivot for ascending order sorting.
- In the Partition Step
  - Use 2 cursors (L cursor and R cursor)
    - L cursor scans the list from left to right
    - R cursor scans the list from right to left
  - swap left key > pivot with right key < pivot
  - When the L cursor and R cursor cross, move only one of them.
    - Move the L cursor until it finds a key > pivot or reaches the pivot
    - Swap the last key found with the pivot, if necessary

In-Place Quick Sort: Concepts

sort the list: (pivot=last element)

85, 24, 63, 45, 17, 31, 96, 50

L ⟶ ⟵ R

look for key>pivot look for key<pivot

85, 24, 63, 45, 17, 31, 96, 50

swap two keys so that

key<pivot will go to the left and

key>pivot will go to the right

In-Place Quick Sort Example

sort the list:

85, 24, 63, 45, 17, 31, 96, 50

L ⟶ ⟵ R

pass 1: 85, 24, 63, 45, 17, 31, 96, 50

31, 24, 63, 45, 17, 85, 96, 50

exchange 31 and 85

and continue search

L ⟶ ⟵ R

In-Place Quick Sort Example (cont'd)

sort the list:

85, 24, 63, 45, 17, 31, 96, 50

L ⟶ ⟵ R

pass 1: 85, 24, 63, 45, 17, 31, 96, 50

31, 24, 63, 45, 17, 85, 96, 50

31, 24, 63, 45, 17, 85, 96, 50

31, 24, 17, 45, 63, 85, 96, 50

L ⟶

⟵ R

R stops, but L continues until it finds a key > pivot

The last key found by L is 63.

It is > pivot (50), so swap it with the pivot.

Exercise: In-Place Quick Sort

- Continue and complete the example

  sort the list: (pivot=last element)
  85, 24, 63, 45, 17, 31, 96, 50

pass 1 result	31, 24, 17, 45, 50, 85, 96, 63
pass 2:	31, 24, 17, 45, 50, 63, 96, 85
pass 3:	17, 24, 31, 45, 50, 63, 85, 96
pass 4:	17, 24, 31, 45, 50, 63, 85, 96

Exercise (show all the steps)

sort the list: (pivot=last element)
(* do not split up the P list already formed *)

85, 17, 63, 45, 17, 31, 85, 50

Exercise (show all the steps)

- The first element of the list may be selected as pivot to do descending order sorting.
- In this case, the L cursor and R cursor is reversed in direction.
- ** This is left as exercise.

sort the list: (pivot=first element)

85, 24, 63, 45, 17, 31, 96, 50

Quick Sort: Pivot Selection

- First or Last
- Median of Three
  - Select 3 keys, and select the middle (sized) key.
    - first 3, last 3, random 3
    - (first, last, middle)
    - median of median of three

Quick Sort: Performance and Problems

- Partition O(n)
  - n comparisons & copying of keys on each pass
- O(n $\log_2$ n) (Best, Average)
  - about $\log_2$ n passes
    - if the distribution of the keys is reasonably balanced around the pivot
- Quicksort: (**Not** in-place sort. Use in-place quicksort)
  - O(n) extra storage space for the temporary lists
- Terrible on an already sorted or nearly sorted list
  - worst case: $O(n^2)$
- Randomized quicksort
  - Use quick sort with randomized pivot selection to avoid the worst case performance
- Unstable sort
  - Partition may swap the relative orders according to pivot's position for duplicate elements

Quick Sort: Example code

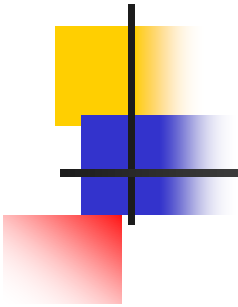
```c
#include <stdio.h>

// function to swap values of two variables
void swap(int *a, int *b) {
  int temp;
  temp = *a;
  *a = *b;
  *b = temp;
}

int partition(int a[], int start, int end) {
  int pivot = a[end];
  int i = start-1;
  int j;

  for(j=start; j<=end-1; j++) {
    if(a[j]<=pivot) {
      i=i+1;
      swap(&a[i], &a[j]);
    }
  }
  swap(&a[i+1], &a[end]);
  return i+1;
}
```

```c
void quicksort(int a[], int start, int end) {
  if(start < end) {
    int q = partition(a, start, end);
    quicksort(a, start, q-1);
    quicksort(a, q+1, end);
  }
}

int main() {
  int a[] = {4, 8, 1, 3, 10, 9, 2, 11, 5, 6};
  quicksort(a, 0, 9);

  //printing array
  int i;
  for(i=0; i<10; i++) {
    printf("%d ",a[i]);
  }
  printf("\n");
  return 0;
}
```

https://www.codesdope.com/course/algorithms-quicksort/
https://www.geeksforgeeks.org/quick-sort/

Quick Sort vs. Merge Sort

	quick sort	merge sort
worst case	$O(n^2)$	$O(n \log_2 n)$
best case	$O(n \log_2 n)$	$O(n \log_2 n)$
average	$O(n \log_2 n)$	$O(n \log_2 n)$
storage	$O(n)$	$O(n)$
stable?	no	yes



End of Lecture