MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) A government issue bonds to support the government spending, in which mainly used to raise money for finance project or daily operation.
   (b) Since the slope of the yield will provide an idea of future interest rate change and economic activity to government, government can predict future change in economic output and growth based on the yield curve.
   (c) Government reduce the money supply by selling bonds, by which money in the market are exchanged to bonds.

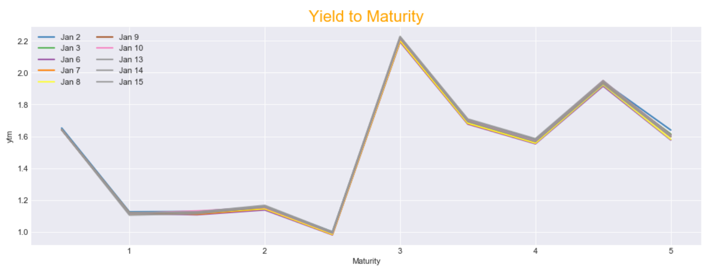
2. My choice of bonds is based on following rules. Firstly, I choose bonds mature in each year's March and September(June for 2022 and 2023, because of lack of September), because it can form basically standard semi-annual periods. Secondly, I choose bonds with similar coupon for calculation accuracy. Thirdly, for the bonds with same maturity date and similar coupon I chose the one with more recent issue date, which will eliminate some historical problem of the bonds.

CDA 2020 1.50 Mar 1	CDA 2021 0.75 Sep 1	CDA 2023 1.75 Mar 1	CDA 19/24 1.50 Sep 1
CDA 2020 0.75 Sep 1	CDA 2022 0.50 Mar 1	CDA 2023 1.50 Jun 1	
CDA 2021 0.75 Mar 1	CDA 2022 2.75 Jun 1	CDA 2024 2.25 Mar 1	

3. The PCA evaluates how data distributed and can be illustrated by covariance matrix. The eigenvectors tell us how the direction of the variance of the data is distributed, and it also implies the yield dynamics. The eigenvalues tell us the magnitude of those associated eigenvectors, which means the magnitude of the directions of the variance. Thus, the eigenvector with largest eigenvalue will imply the direction of the largest variance.

Empirical Questions - 75 points

4.



(a) Because of those semi-annual bonds, I calculate the yield to maturity based on the equation: (results plotted as above)

$$price = \sum_{t=1}^{T} \frac{Coupon}{(1+\frac{ytm}{2})^t} + \frac{FaceValue}{(1+\frac{ytm}{2})^T}$$

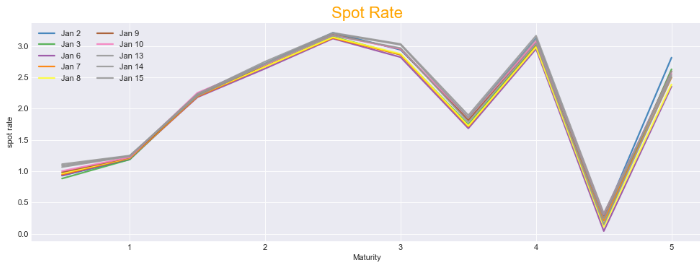
(b) Step 1: For the bond with maturity less than 6 months, we calculate the spot rate by

$$r(T) = -\frac{log(Price/Notional)}{T}$$

Step 2: For the bond with maturity more than 6 months, I use bootstrap to generate the next spot rate by previous, under the equation:

$$P = p_1 \cdot e^{-r(t_1) \cdot t_1} + p_2 \cdot e^{-r(t_2) \cdot t_2}$$

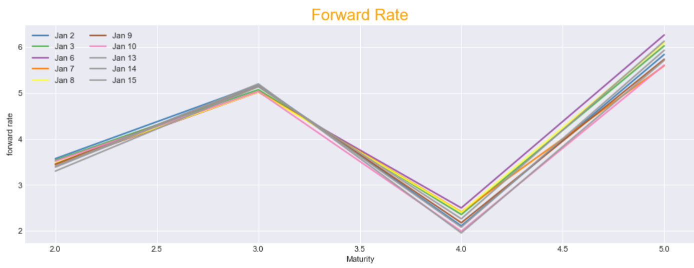
Here, P is the dirty price, which equals to accrued interest plus the clean price; t_1 and t_2 are current timing and the next timing;p_1 and p_2 stand for payment at t_1 and t_2; $r(t_1)$ is the spot rate. Then, I use known information to calculate the unknown spot rate $r(t_2)$ at next timing. Results plotted as following:



(c) Step 1: Select bonds which mature in March, since we only need one-year forward curve
Step 2: calculate the forward rate using following equation with corresponding time to maturity (in year): t_a (longer) and t_b (shorter); and corresponding spot rate: r_a and r_b:

$$\text{Forward Rate} = \frac{(1+r_a)^{t_a}}{(1+r_b)^{t_b}} - 1$$

Results plotted as following:



5. The covariance matrix for the time series of daily log-returns of yields is as following:

$$\begin{bmatrix} 4.20829608e-06 & 8.35094792e-06 & 8.96441258e-06 & 7.04529221e-06 & 5.55865878e-06 \\ 8.35094792e-06 & 6.08789517e-05 & 2.27782359e-05 & 2.97986333e-05 & 2.74060785e-05 \\ 8.96441258e-06 & 2.27782359e-05 & 4.86395639e-05 & 3.71676865e-05 & 3.35212957e-05 \\ 7.04529221e-06 & 2.97986333e-05 & 3.71676865e-05 & 3.41035118e-05 & 3.47306472e-05 \\ 5.55865878e-06 & 2.74060785e-05 & 3.35212957e-05 & 3.47306472e-05 & 5.13832779e-05 \end{bmatrix}$$

The covariance matrix for the time series of daily log-returns of forward rates is as following:

$$\begin{bmatrix} 0.0002627 & -0.00011951 & -0.00063028 & -0.00026769 \\ -0.00011951 & 0.00034788 & -0.00060065 & -0.00027594 \\ -0.00063028 & -0.00060065 & 0.0056215 & 0.00270675 \\ -0.00026769 & -0.00027594 & 0.00270675 & 0.00364266 \end{bmatrix}$$

6. The eigenvalues for the time series of daily log-returns of yields are as following:
1.43175287e-04, 3.59807059e-05, 1.71332901e-05, 2.23233301e-06, 6.91985898e-07

The eigenvectors for the time series of daily log-returns of yields are as following:

$$[-0.10678883, -0.02307529, -0.17997387, -0.93274333, 0.29268236]^T$$
$$[-0.49401893, -0.8474293, -0.06496969, 0.03352302, -0.18017775]^T$$
$$[-0.49993539, 0.43597153, -0.58508853, 0.01300613, -0.46636432]^T$$
$$[-0.47683189, 0.13086032, -0.12287419, 0.32505989, 0.79670881]^T$$
$$[-0.51695008, 0.27227837, 0.77843541, -0.15176627, -0.17214065]^T$$

The largest eigenvalue 1.43175287e-04 counts around 71.9% of sum of eigenvalues, which means that the corresponding eigenvector $[-0.10678883 - 0.02307529 - 0.17997387 - 0.932743330.29268236]^T$ represent the direction of the largest variance.

The eigenvalues for the time series of daily log-returns of forward rates are as following:
7.63116070e-03, 4.37405860e-05, 4.30121818e-04, 1.76971598e-03
The eigenvectors for the time series of daily log-returns of forward rates are as following:

$$[-0.08887154, 0.77652387, 0.61856615, 0.08055085]^T$$
$$[-0.08720643, 0.60877056, -0.78560527, 0.0679544]^T$$
$$[0.81539556, 0.16160553, -0.01335444, -0.55572961]^T$$
$$[0.56535562, -0.01710933, -0.00468335, 0.82465651]^T$$

The largest eigenvalue 7.63116070e-03 counts around 77.3% of sum of eigenvalues, which means that the corresponding eigenvector $[-0.08887154, 0.77652387, 0.61856615, 0.08055085]^T$ represent the direction of the largest variance.

References and GitHub Link to Code

1. GitHub: https://github.com/MinhuiXu/APM466
2. intuition of bonds: https://www.investopedia.com/terms/g/government-bond.asp
3. intuition of yields: https://www.investopedia.com/terms/y/yieldcurve.asp
https://www.investopedia.com/articles/economics/08/monetary-policy-recession.asp
4. intuition of variance matrices: https://www.visiondummy.com/2014/04/geometric-interpretation-covariance-matrix/
5. ituition of PCA: https://towardsdatascience.com/a-one-stop-shop-for-principal-component-analysis-5582fb7e0a9c