Central Connecticut State University

CET346 – Electrical System Analysis

Spring 2023

PROJECT 12

Frequency Response of a Circuit to an Arbitrary Periodic Signal

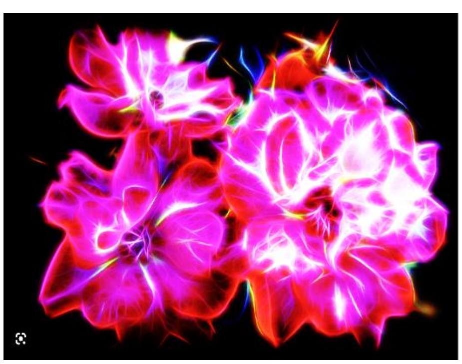
FINAL

To :

Professor : David J. Broderick, Ph.D.

CCSU- CET 346 Spring 2023

Electrical System Analysis



From :

Van Nguyen

CET 346 Spring 2023

May 08-2023

Central Connecticut State University

CET 346 Project

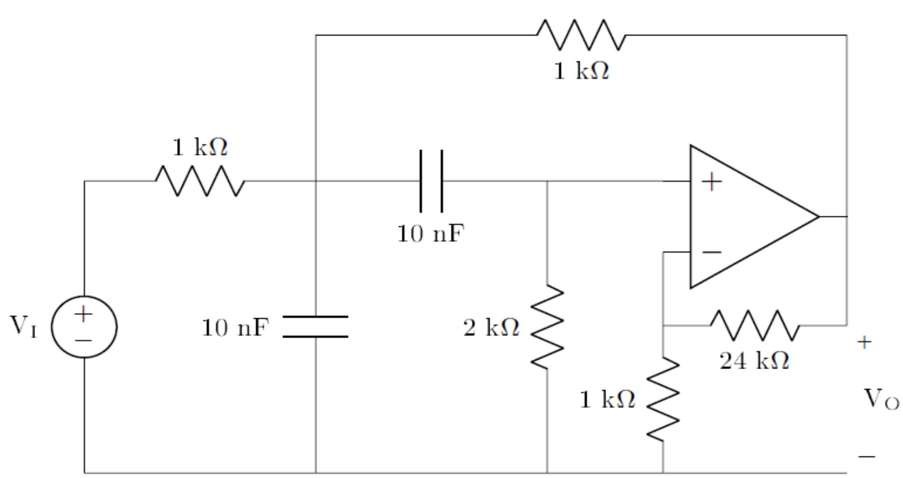
Frequency Response of a Circuit to an Arbitrary Periodic Signal

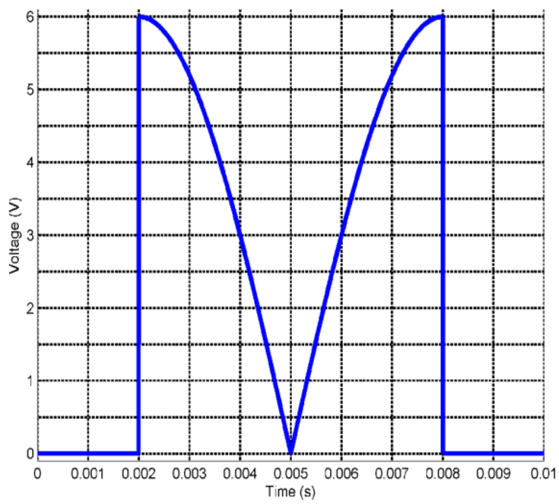
Objectives:

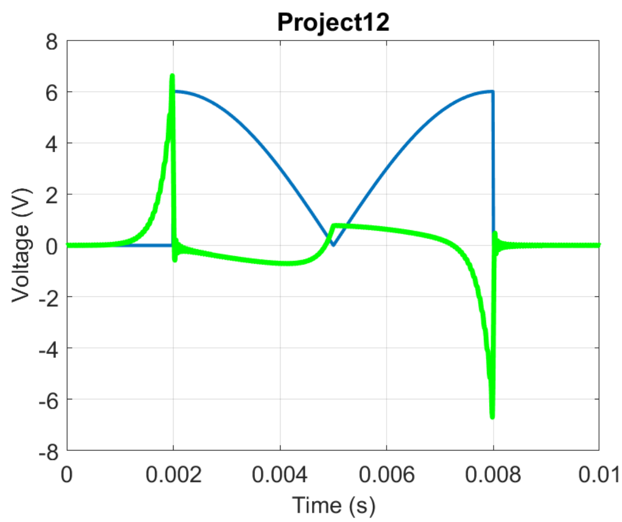
1. Determine the transfer function of a circuit with an operational amplifier
2. Generate a Bode plot for the circuit
3. Represent an arbitrary periodic signal as a function using Heaviside and other functions
4. Use Fourier series to break the signal into a series of sinusoidal functions
5. Perform circuit analysis on the individual sinusoids to predict the output of the circuit

Procedure:

1. Determine the transfer function of the circuit included in this handout. Use of MATLAB is not only permitted, it is encouraged.
2. Generate a Bode plot over a reasonable range of frequencies. (Label all axes, units, and remember to include both plots)
3. Examine the signal included in this handout to be used as the input (VI) to the circuit. Represent the signal as a function of time. Plot your function and compare it to the desired signal.
4. Use Fourier series to determine the coefficients (ak and bk) for the signal. Find coefficients for k=0 to 200. Plot the two coefficients on the same plot against the frequency of each.
5. Estimate the original signal from a summation of sinusoids and plot it with your function found above. Use the same time domain as the one shown in this handout.
6. Use the transfer function you found earlier and/or additional circuit analysis to predict the output of the circuit to the signal provided. Plot the input signal along with your predicted output using the same time domain as the one shown in this handout.







To turn in:

1. A clearly formatted transfer function for the included circuit

2. A bode plot for the included circuit

3. A clearly formatted function representing the included signal

4. A time-domain plot of the included signal generated with your own MATLAB code

5. A plot of Fourier coefficients for frequencies corresponding to k=0 to 200

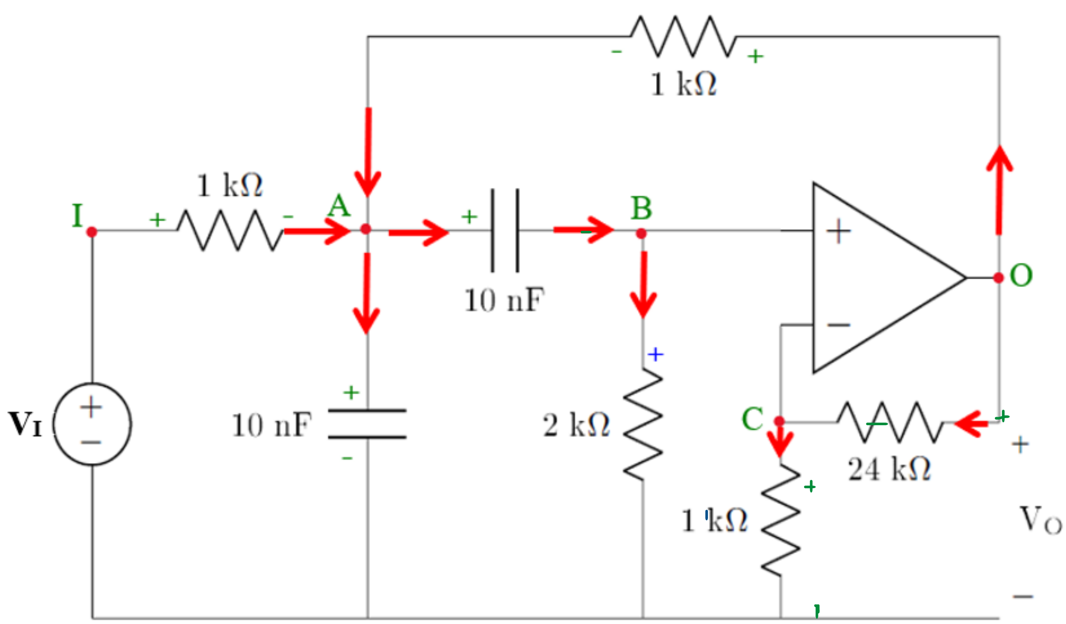
6. Your original function for the included signal plotted with your estimated function using the results of

the Fourier series.

7. Your predicted output of the circuit plotted with the input signal included in this handout

8. Any and all code you used in generating the plots or performing circuit analysis. Code should be clearly formatted and commented appropriately

1) - Find transfer function for the included circuit (H(s))



Nodal Analysis R = 1 kΩ ; C = 10 nF ; $Z_c = \frac{1}{sC}$

KVL node I : $V_i = V_I$

KVL Op-Amp : $V_B = V_C$

KVL node A :

$$\frac{V_I - V_A}{R} + \frac{V_O - V_A}{R} - \frac{V_A - V_B}{Z_c} - \frac{V_A}{Z_C} = 0$$

KVL node B :

$$\frac{V_A - V_B}{Z_c} - \frac{V_B}{2R} = 0$$

KVL node C :

$$\frac{V_0 - V_C}{24R} - \frac{V_C}{R} = 0$$

Using MATLAB Find Transfer Function (FN_01)

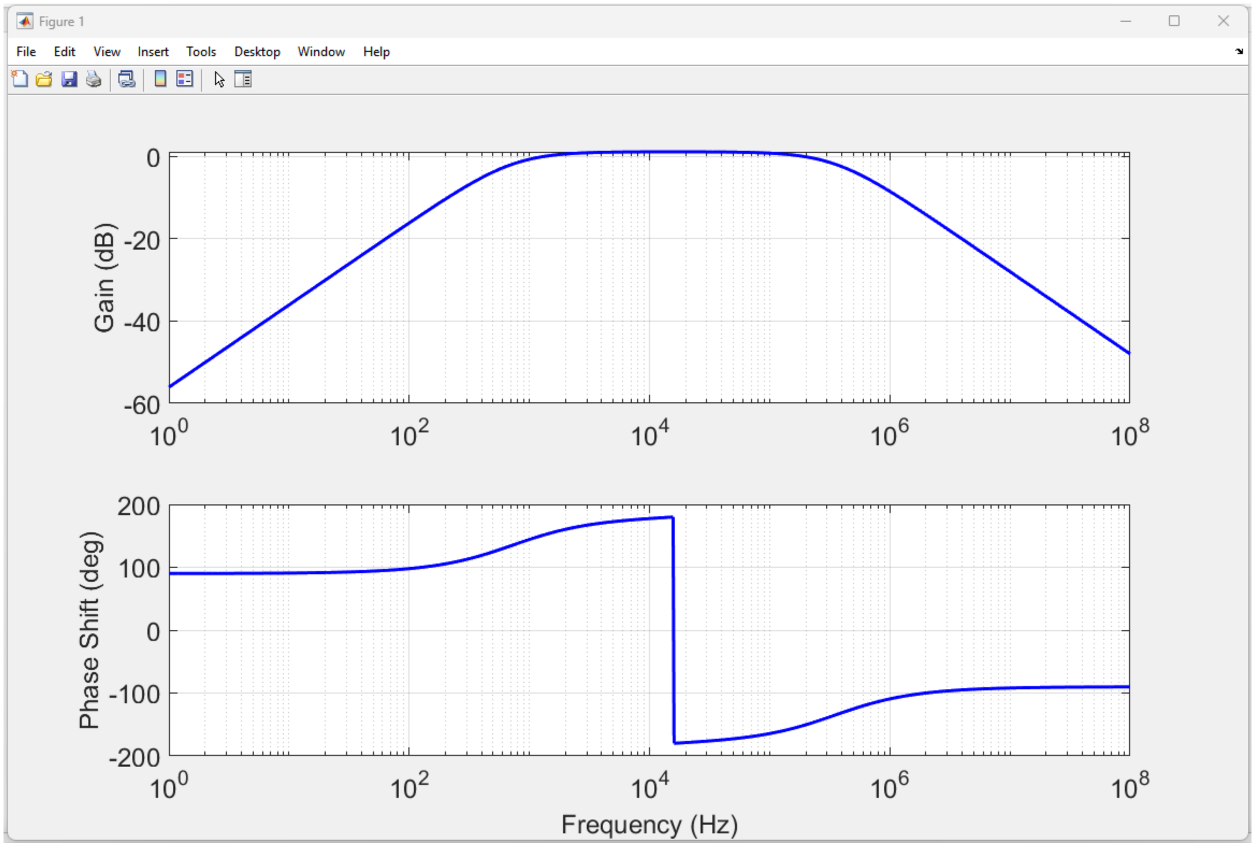
```
clear all
close all
clc
format short Eng
 format compact

syms R C s Vi Va Vb Vc Vo
ZC=inv(s*C);

eqn(1)=Vb==Vc;                                   % KVL Op-Amp

eqn(2)=((Vi-Va)/(R))+((Vo-Va)/(R))-((Va-Vb)/(ZC))-((Va)/(ZC))==0; % KCL A
eqn(3)=((Va-Vb)/(ZC))-((Vb)/(2*R))==0;           % KCL Node B
eqn(4)=((Vo-Vc)/(24*R))-((Vc)/(R))==0;    % KCL Node C
sol=solve(eqn,Va,Vb,Vc,Vo);

H=sol.Vo/Vi;                         % Gain is output over input
H=subs(H,[R C],[1e3 10e-9]);
pretty(simplify(H))                     % Print the transfer function
```

$$H(s) = \frac{2\,500\,000\,s}{s^2 - 2\,200\,000\,s + 10\,000\,000\,000}$$

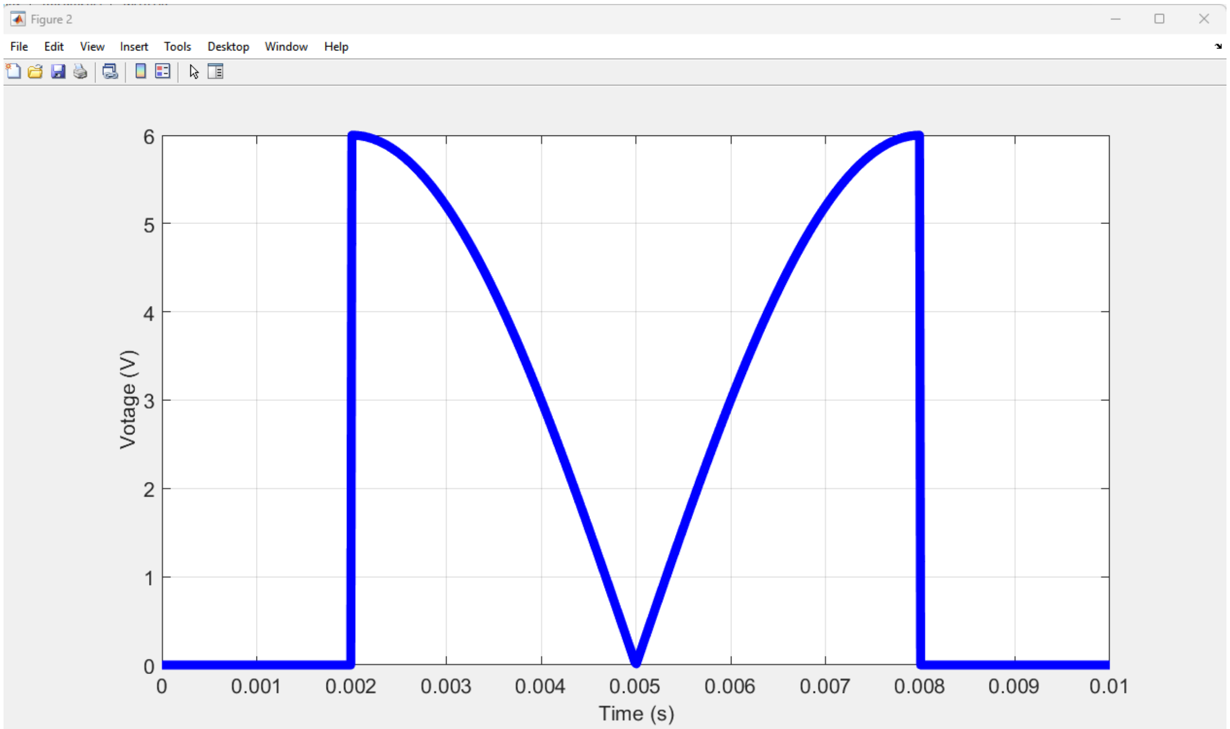
2)- MATLAB plots Bode Plot for the transfer function of the circuit (FN_02)

```
clear all
close all
clc
format short Eng
format compact

syms R C s Vi Va Vb Vc Vo
ZC=inv(s*C);

eqn(1)=Vb==Vc;                                  % KVL Op Amp

eqn(2)=((Vi-Va)/(R))+((Vo-Va)/(R))-((Va-Vb)/(ZC))-((Va)/(ZC))==0; % KCL A
eqn(3)=((Va-Vb)/(ZC))-((Vb)/(2*R))==0;          % KCL Node B
eqn(4)=((Vo-Vc)/(24*R))-((Vc)/(R))==0;          % KCL Node C
sol=solve(eqn,Va,Vb,Vc,Vo);

H=sol.Vo/Vi;                        % Gain is output over input
H=subs(H,[R C],[1e3 10e-9]);
pretty(simplify(H))                 % Print the transfer function

% Bode Plots
f=logspace(0,8,1000);
w=2*pi*f;
H=subs(H,s,j*w);
figure(1)
subplot(2,1,1)
semilogx(f,20*log10(abs(H)),'LineWidth',2,'color','b')
grid on
ylabel('Gain (dB)')

fig=gcf;
set(findall(fig,'-property','FontSize'),'FontSize',18)

subplot(2,1,2)
semilogx(f,angle(H)*(180/pi),'LineWidth',2,'color','b')
grid on

xlabel('Frequency (Hz)')
ylabel('Phase Shift (deg)')

fig=gcf;
set(findall(fig,'-property','FontSize'),'FontSize',18)
```

Second-order Sallen-Key **Band-pass _ Bode plot**



3)- Formatted function representing the signal :



- We have signal input and output as :

6cos(ω_0t - 0.002) * **[u(t-0.002) – u(t-0.008)]**

- We also have the time-domain signal of a sinusoidal circuit can be expressed as:

$$x(t) = \mathbf{A}sin(\omega t + \theta\ )$$

Or $V(t) = \boldsymbol{V_p}cos(\omega t + \theta\ ) =$ 6cos(ωt- 0.002)

Where : A : Amplitude in Voltage = 6 V

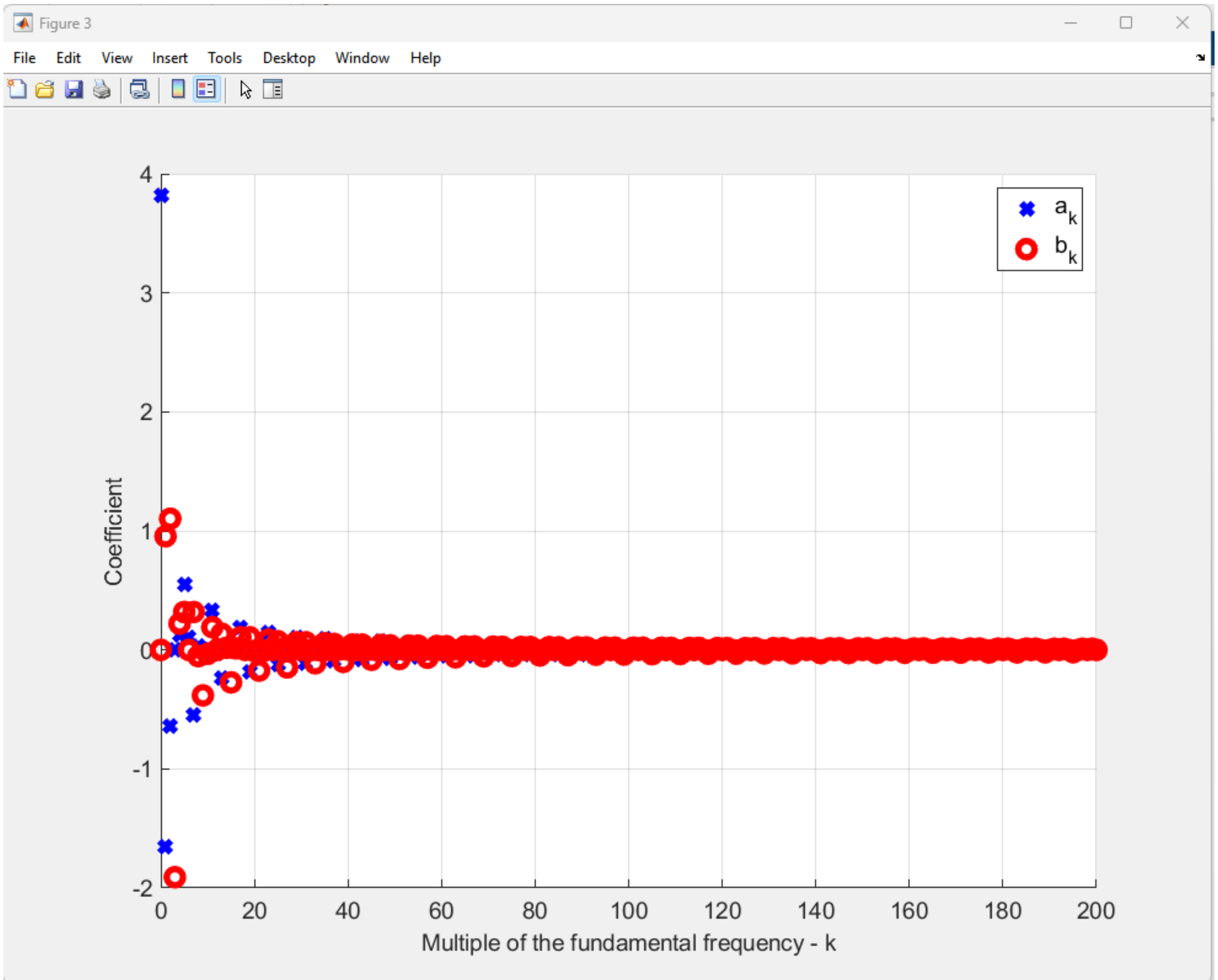
ω : Angular Frequency $= \frac{2\pi}{T_0}$

θ : Phase shift = - 0.002

4)- Using MATLAB draw for A time-domain plot of the signal (FN_03)

```
clear all
close all
clc
format short eng
format compact

T0=0.012;
w0=((2*pi)/T0);     % angular frequency

t=0:T0/1000:T0;
u=@(t) heaviside(t)

Vi=@(t) 6*abs(cos(w0*(t-0.002))).*(u(t-0.002)-u(t-0.008))

figure(1)
plot(t,Vi(t), 'LineWidth',7,'Color','blue')

hold on
grid on

xlabel('Time (s)')
ylabel('Voltage (V)')
ylim([0,6])
xlim([0,0.01])

fig=gcf;
set(findall(fig,'-property','FontSize'),'FontSize',16)
```

5)- Plot Fourier coefficients for frequencies form k = 0 to 200




```
u =

  function_handle with value:

    @(t)heaviside(t)

Vi =

  function_handle with value:

    @(t)6*abs(cos(w0*(t-0.002))).*(u(t-0.002)-u(t-0.008))
```

We have formula for Trigonometric Fourier Series as :

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

$$\mathrm{a_k} = \frac{2}{T_0} = \int_{T_0} x(t)\cos(k\omega_0 t)\,dt$$

$$\mathrm{b_k} = \frac{2}{T_0} = \int_{T_0} x(t)\sin(k\omega_0 t)\,dt$$

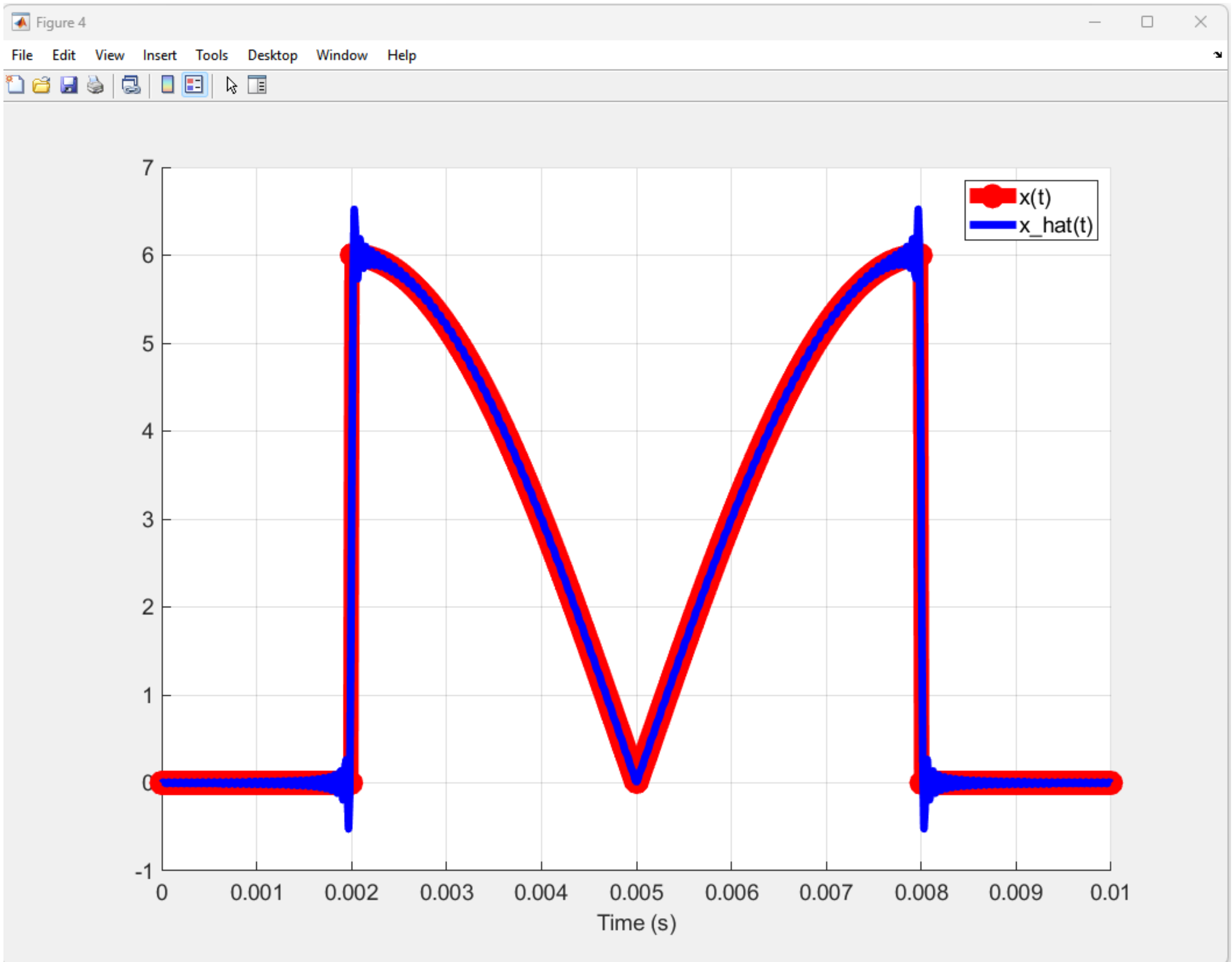
- n = 200
- k = 0 to k = 200
- $T_0 = t_1 - t_0 = 0.012 - 0 = 0.012$
- $\omega_0 = \frac{2\pi}{T_0}$

- $x(t) = Vi = @(t)\ 6 * abs\left(\cos\left(\omega_0 * (t - 0.002)\right)\right) * (u(t - 0.002) - u(t - 0.008))$

MATLAB plot Fourier coefficients of frequencies to k=0 to 200 (FN_04)

```
clear all
close all
clc
format short eng
format compact

% break signal into phasors Using Fourier series
n=200;
t0=0;t1=0.012;
T0=t1-t0;
w0=(2*pi)/T0;
u=@(t) heaviside(t)
Vi=@(t) 6*abs(cos(w0*(t-0.002))).*(u(t-0.002)-u(t-0.008))

for k=0:n;
    integrand=@(t) (Vi(t)).*cos(k.*w0.*t);
    a(k+1)=(2/T0)*integral(integrand,t0,t1);

    integrand=@(t) (Vi(t)).*sin(k.*w0.*t);
    b(k+1)=(2/T0)*integral(integrand,t0,t1);
end

figure(3)
hold on;grid on
semilogy(0:length(a)-1,a,'bx','LineWidth',4,'MarkerSize',10)
semilogy(0:length(a)-1,b,'ro','LineWidth',4,'MarkerSize',10)

xlabel('Multiple of the fundamental frequency - k')
ylabel('Coefficient')
legend('a_k','b_k')

t=t0:T0/1000:t1;
x_hat=(a(1)/2);

for k=1:n;
    x_hat=x_hat+ a(k+1).*cos(k.*w0.*t)+ b(k+1).*sin(k.*w0.*t);
end
set(findall(gcf,'-property','FontSize'),'FontSize',14)
```

6)- Plot original function Signal using the results of the Fourier series.

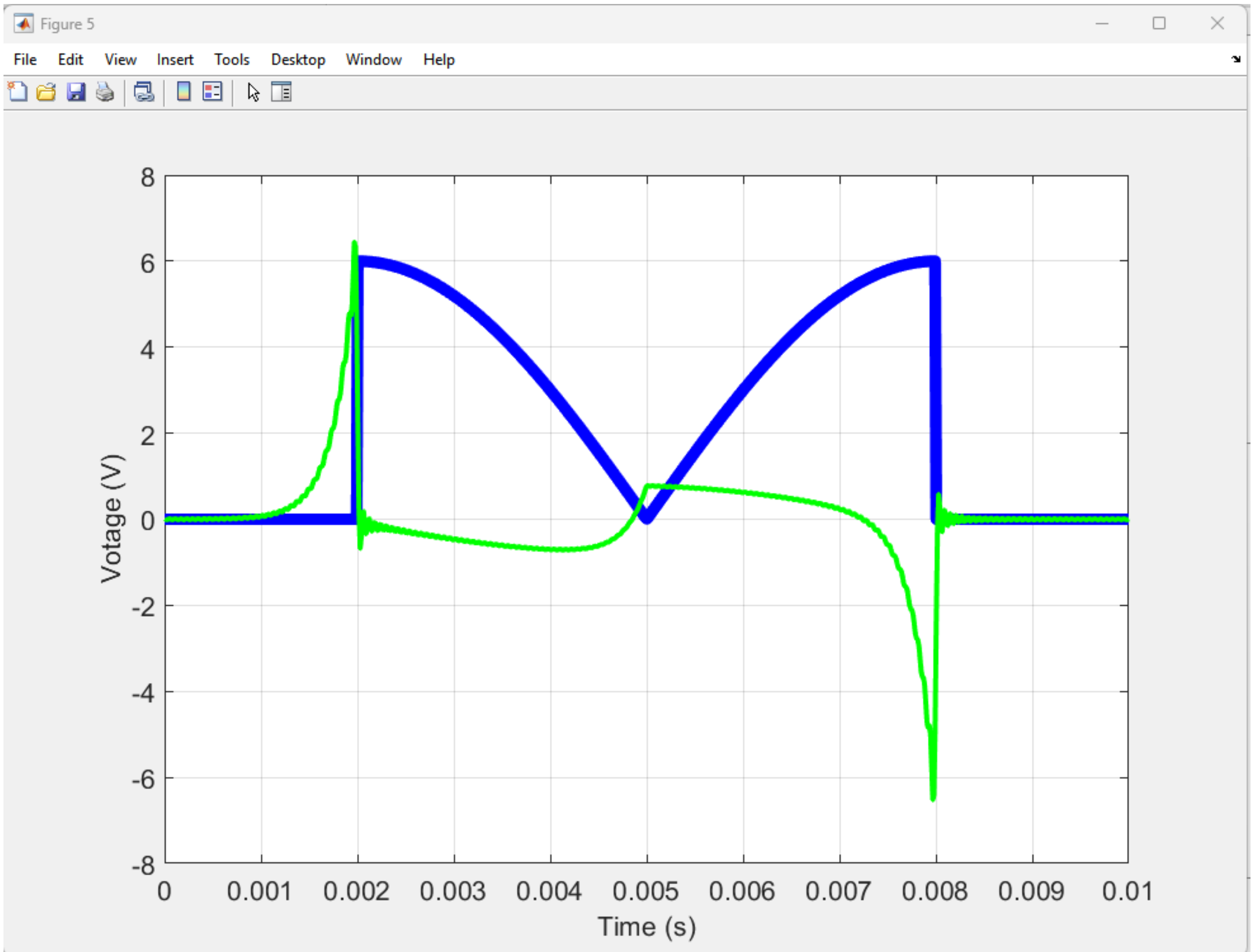



```
u =

  function_handle with value:

    @(t)heaviside(t)

Vi =

  function_handle with value:

    @(t)6*abs(cos(w0*(t-0.002))).*(u(t-0.002)-u(t-0.008))
```

MATLAB Plot original function Signal vs Fourier series (FN_05)

```
clear all
close all
clc
format short eng
format compact

% break signal into phasors Using Fourier series
n=200;
t0=0;t1=0.012;
T0=t1-t0;
w0=(2*pi)/T0;
u=@(t) heaviside(t)
Vi=@(t) 6*abs(cos(w0*(t-0.002))).*(u(t-0.002)-u(t-0.008))
x=@(t) Vi(t);

for k=0:n;
    integrand=@(t) (x(t)).*cos(k.*w0.*t);
    a(k+1)=(2/T0)*integral(integrand,t0,t1);

    integrand=@(t) (x(t)).*sin(k.*w0.*t);
    b(k+1)=(2/T0)*integral(integrand,t0,t1);
end

t=t0:T0/1000:t1;
x_hat=(a(1)/2);
for k=1:n;
    x_hat=x_hat+ a(k+1).*cos(k.*w0.*t)+ b(k+1).*sin(k.*w0.*t);
end

figure(4)
hold on; grid on
plot(t,x(t),'ro-','LineWidth',10)
plot(t,x_hat,'b','LineWidth',5)
xlabel('Time (s)')
xlim([0,0.01])
legend('x(t)','x\_hat(t)')
set(findall(gcf,'-property','FontSize'),'FontSize',14)
```

7)-Plot the Output of the circuit with the input signal handout




```
u =

  function handle with value:

    @(t)heaviside(t)

Vi =

  function handle with value:

    @(t)6*abs(cos(w0*(t-0.002))).*(u(t-0.002)-u(t-0.008))

H =

  function handle with value:

    @(s)(s.*2.5e+6)./(s.*-2.2e+6+s.^2+1.0e+10)
```

Solution :

- The input signals

$$\omega_0 = \frac{2\pi}{T_0} \quad ; \; T_0 = t_1 - t_0 = 0.012 - 0 = 0.012 \; ; \; n = 200$$

$$Vi(t) = \boldsymbol{V_p} cos(\omega t + \theta)$$

u = `@(t)heaviside(t)`

$$\boldsymbol{Vi(t)} = \boldsymbol{6cos(\omega_0}\ \texttt{(t-0.02))*(u(t-0.002)-u(t-0.008))}$$

- The transfer Function

R = 1 kΩ ; C = 10 nF ; $Z_c = \frac{1}{sC}$

$$H(s) = \frac{2\,500\,000\,s}{s^2 - 2\,200\,000\,s + 10\,000\,000\,000}$$

- The Output

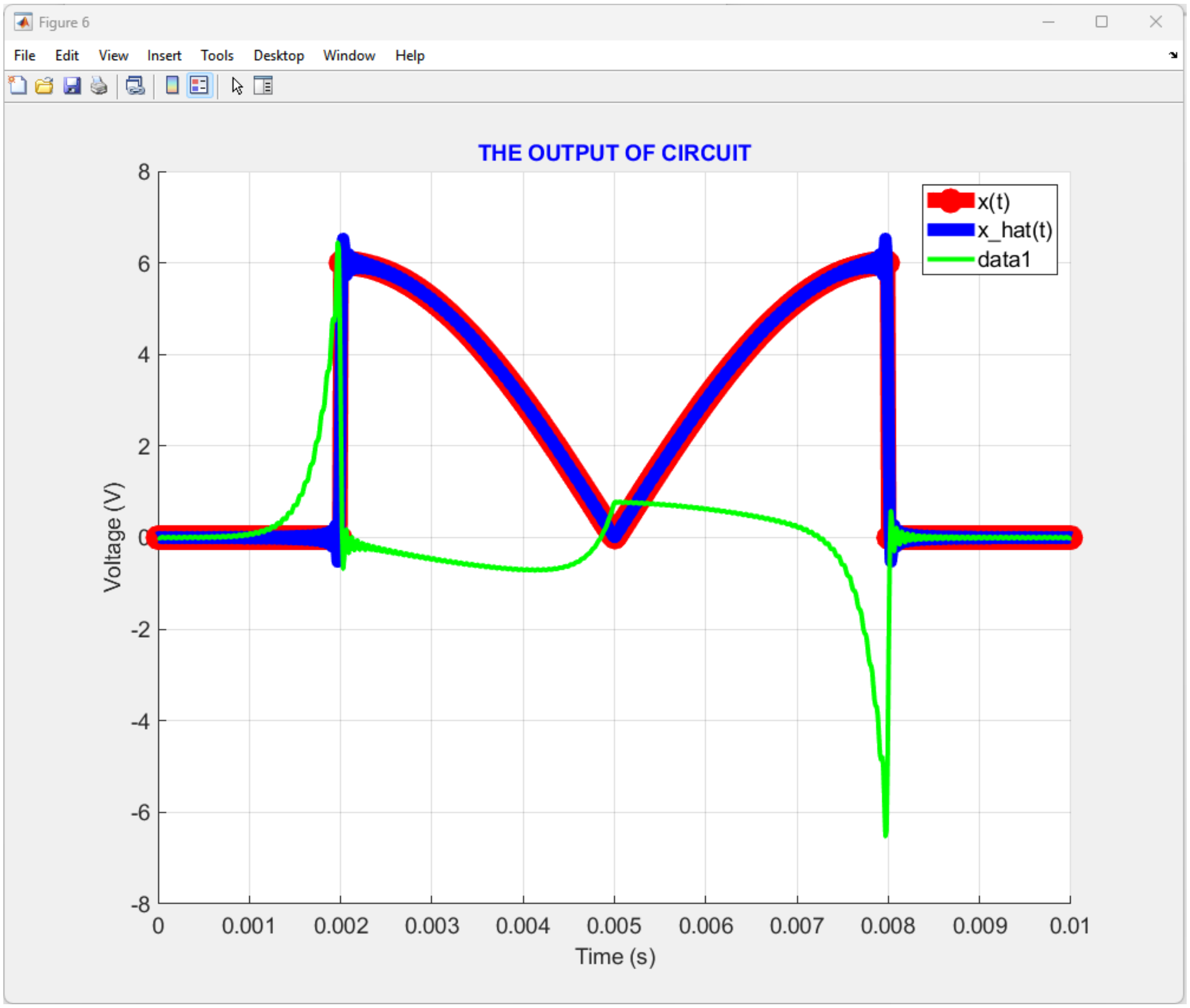
$$V_o = V_i(t)\, H(s)$$

MATLAB plot the Output with the input signal & Transfer Function (FN_06)

```
clear all
close all
clc
format short eng
format compact

% plot signal with heavisides
T0=0.012;
w0=((2*pi)/T0);    % angular frequency

t=0:T0/1000:T0;
u=@(t) heaviside(t)

Vi=@(t) 6*abs(cos(w0*(t-0.002))).*(u(t-0.002)-u(t-0.008))

figure(5)
plot(t,Vi(t), 'LineWidth',7,'Color','blue')

hold on
grid on
ylim([-8,8])
xlim([0,0.01])
xlabel('Time (s)')
ylabel('Votage (V)')
fig=gcf;
set(findall(fig,'-property','FontSize'),'FontSize',16)

% break signal into phasors Using Fourier series

n=200;
t0=0;t1=0.012;
T0=t1-t0;
w0=(2*pi)/T0;
x=@(t) Vi(t);

for k=0:n;
    integrand=@(t) (x(t)).*cos(k.*w0.*t);
    a(k+1)=(2/T0)*integral(integrand,t0,t1);

    integrand=@(t) (x(t)).*sin(k.*w0.*t);
    b(k+1)=(2/T0)*integral(integrand,t0,t1);
end
```

```
t=t0:T0/1000:t1;
x_hat=(a(1)/2);

for k=1:n;
    x_hat=x_hat+ a(k+1).*cos(k.*w0.*t)+ b(k+1).*sin(k.*w0.*t);
end

% find Transfer Function

syms R C s Vi Va Vb Vc Vo

ZC=inv(s*C);

eqn(1)=Vb==Vc;            % KVL Op Amp
eqn(2)=-((Vi-Va)/(R))+((Va-Vo)/(R))+((Va-Vb)/(ZC))+((Va)/(ZC))==0;% KCL A
eqn(3)=((Va-Vb)/(ZC))-((Vb)/(2*R))==0;    % KCL Node B
eqn(4)=-((Vo-Vc)/(24*R))+((Vc)/(R))==0;   % KCL Node C
sol=solve(eqn,Va,Vb,Vc,Vo);

H = sol.Vo/Vi;                 % Gain is output over input

H=simplify(subs(H,[R C],[1e3 10e-9]));
H=matlabFunction(H)

% DC component output

vo_hat=(a(0+1)/2)*(H(j*0)); %DC input times DC gain

for k=1:n
    Vi=a(k+1)-j*b(k+1);
    Vo=Vi*H(j*k*w0);
    vo_hat=vo_hat+abs(Vo).*cos(k.*w0.*t+angle(Vo));
end

plot(t,vo_hat,'g','LineWidth',3)
```

8)- Output Vs Signal input, Transfer function and Fourier coefficients (FN_07)




```
u =

  function_handle with value:

    @(t)heaviside(t)

Vi =

  function_handle with value:

    @(t)6*abs(cos(w0*(t-0.002))).*(u(t-0.002)-u(t-0.008))

H =

  function_handle with value:

    @(s)(s.*2.5e+6)./(s.*-2.2e+6+s.^2+1.0e+10)
```

MATLAB Plots Output vs input, Transfer function & Fourier coefficients

```
clear all
close all
clc
format short eng
format compact

clear all
close all
clc
format short eng
format compact

%% plot signal with heavisides
T0=0.012;
w0=((2*pi)/T0);     % angular frequency

t=0:T0/1000:T0;
u=@(t) heaviside(t)

Vi=@(t) 6*abs(cos(w0*(t-0.002))).*(u(t-0.002)-u(t-0.008))

%% break signal into phasors Using Fourier series

n=200;
t0=0;t1=0.012;
T0=t1-t0;
w0=(2*pi)/T0;
x=@(t) Vi(t);

for k=0:n;
    integrand=@(t) (x(t)).*cos(k.*w0.*t);
    a(k+1)=(2/T0)*integral(integrand,t0,t1);

    integrand=@(t) (x(t)).*sin(k.*w0.*t);
    b(k+1)=(2/T0)*integral(integrand,t0,t1);
end

t=t0:T0/1000:t1;
x_hat=(a(1)/2);
for k=1:n;
    x_hat=x_hat+ a(k+1).*cos(k.*w0.*t)+ b(k+1).*sin(k.*w0.*t);
end
```

```
figure(6)
title('THE OUTPUT OF CIRCUIT','Color','b','FontSize',30)
hold on;grid on
plot(t,x(t),'ro-','LineWidth',10)
plot(t,x_hat,'b','LineWidth',8)
xlabel('Time (s)')
xlim([0,0.01])
ylabel('Voltage (V)')
legend('x(t)','x\_hat(t)')
set(findall(gcf,'-property','FontSize'),'FontSize',14)

% Finding a transfer function

syms R C s Vi Va Vb Vc Vo

ZC=inv(s*C);

eqn(1)=Vb==Vc; % KVL Op Amp
eqn(2)=-((Vi-Va)/(R))+((Va-Vo)/(R))+((Va-Vb)/(ZC))+((Va)/(ZC))==0; % KCL A
eqn(3)=((Va-Vb)/(ZC))-((Vb)/(2*R))==0; % KCL Node B
eqn(4)=-((Vo-Vc)/(24*R))+((Vc)/(R))==0; % KCL Node C
sol=solve(eqn,Va,Vb,Vc,Vo);

H=sol.Vo/Vi;% Gain is output over input

H=simplify(subs(H,[R C],[1e3 10e-9]));
H=matlabFunction(H)

% DC component output
vo_hat=(a(0+1)/2)*(H(j*0)); %DC input times DC gain

for k=1:n
    Vi=a(k+1)-j*b(k+1);
    Vo=Vi*H(j*k*w0);
    vo_hat=vo_hat+abs(Vo).*cos(k.*w0.*t+angle(Vo));
end

plot(t,vo_hat,'g','LineWidth',3)
```