Plans:

1/ Finding the global static feedback law (with T=fixed) is very difficult if not impossible because the solution migh not be time-invariant as Dr. Zeng has found a time-varying state feedback law for the harmonic oscilator aircraft, i.e. u(t) = k(x,t).

Consider a point **x** and its energy optimal controlled trajectory **X**=[x1,....,xn] as well as a second point **y=xi** (i.e **y** is an arbitrary point on the controlled trajectory X) and its energy optimal controlled trajectory **Y**=[y1,....,yn], we can see the optimal control signal and the optimal controlled trajectory for **x** at xi is usually different from those for **y=xi.**

One way to think about this is to think about the amount of effort vs the budget of time. Even if we start at the same location, the difference in time budgets significantly dictates the discrepancy in how much effort should be utilized to reach a common goal.

--> Therefore, a time-energy objective might yield a global feedback law. Specifically, given a same starting location, there should be a unique steering signal and a single resulted controlled trajectory to the target if the objective is to both minimize time and energy. This intuition can be linked to the Bellman principle of optimality and the famous HJB equation.

2/ Then for each point, we have a control signal U=[u1,....,un]. This generates a data set to train a NN whose role is a global controller to stablize a nonlinear control system in a time-energy optimal way.

3/ Additionally, we can explore the robustness of this controller by changing the parameters of the system. Try to give a rigorous analysis of robustness (maybe via gradient/jacobian of the NN). If it is not possible, we ought to qualitatively show the robustness by ploting the tolerant region of parameter variations.

4/ Finally, we can apply this method to the Furuta pendulum (physical platform). To see how robust it is.

Try: swing up the Furuta pendulum from bottom with the time-energy minimum, then take a point on the controlled trajectory, and apply minimum time-energy synthesis to see if it yields the same result. Maybe try this first with the Van-der-pol.

Plans:

1/ Discretize the optimal controlled trajectory to get more training data: divide an i-th interval (t_i) into n smaller intervals (n=mod(t_i/dt)) so that each approximately equals some desired fine step (e.g. dt=0.1ms). This process results in a much more number of training states and inputs. In addtition to getting more training data, this procedure helps to dictate what would be an appropriate time step for the later control implementation of the trained NN static state-feedback controller.

--> Since the minimum value of an t_i interval of all the optimal controlled trajectories is extremely small, i.e., about e^-15, the 0.1ms discretization in task 1 might not be enough. Therefore, some constraints on the minimum interval need to be imposed during the optimal trajectories synthesis.

2/ Provided with the training data from task 1, we now train a NN to map state x_i to the corresponding optimal control input u_i.

3/ Validation: Plot an optimal controlled trajectory from the synthesis against the resulted controlled trajectory from the feedback implementation of the trained NN. Hopefully, they are vey similar.