Drone Path Planning: From Mixed-Integer Program to QUBO

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July 3, 2025

1 The Original Constrained Problem

The process begins with the Mixed-Integer Quadratic Program (MIQP) for simultaneous path and morphing planning, as defined in the source document. The goal is to find an optimal path for a tensegrity drone, which involves minimising a cost function subject to several operational constraints.

1.1 Objective Function

The primary objective is to minimise the squared Euclidean distance from the start and end points, while also promoting a smooth path by penalising large distances between consecutive steps.

$$\underset{\mathbf{x}_k, c_{t,k}, s_{q,k}}{\text{minimise}} \quad ||\mathbf{x}_1 - \mathbf{x}_{\text{start}}||^2 + ||\mathbf{x}_K - \mathbf{x}_{\text{goal}}||^2 + w \sum_{k=1}^{K-1} ||\mathbf{x}_{k+1} - \mathbf{x}_k||^2 \tag{1}$$

1.2 Constraints

The optimisation is subject to the following constraints for all relevant indices i, k, t, q:

1. **Region Fit (Inequality):** The drone's structure (represented by its nodes) must be contained within a selected safe convex region at each step k. This is managed using the big-M relaxation method.

$$A_t(P_i \mathbf{r}^q + \mathbf{x}_k) \leq \mathbf{b}_t + (1 - c_{t,k})M \cdot \mathbf{1}_t + (1 - s_{q,k})M \cdot \mathbf{1}_t \tag{2}$$

2. **Region Selection (Equality):** Exactly one convex region must be chosen at each step k.

$$\sum_{t=1}^{T} c_{t,k} = 1 \tag{3}$$

3. **Configuration Selection (Equality):** Exactly one drone configuration must be chosen at each step k.

$$\sum_{q=1}^{Q} s_{q,k} = 1 \tag{4}$$

4. **Binary Variables:** The selection variables must be binary.

$$c_{t,k}, s_{q,k} \in \{0, 1\} \tag{5}$$

2 Conversion to an Unconstrained Function

To prepare the problem for a QUBO solver, we convert the constrained problem into an unconstrained one. This is achieved by moving the operational constraints into the objective function as quadratic penalty terms. The binary constraint (5) is not formulated as a penalty, as it will be handled by the inherent structure of the final QUBO model.

2.1 Steps to Achieve the Unconstrained Function

The conversion follows a systematic process:

1. **Formulate Quadratic Penalties:** Each of the operational constraints is converted into a penalty term. The equality constraints (3) and (4) are squared directly. For the inequality constraint (2), the slack is added for each iteration of the summation.

$$P_1 = \lambda_1 \left(\sum_{i,k,t,q} A_t(P_i \mathbf{r}^q + \mathbf{x}_k) - \mathbf{b}_t - (1 - c_{t,k})M \cdot \mathbf{1}_t - (1 - s_{q,k})M \cdot \mathbf{1}_t + y_{i,k,t,q} \right)^2 \tag{6}$$

$$P_2 = \lambda_2 \sum_k \left(\sum_{t=1}^{T} c_{t,k} - 1 \right)^2 \tag{7}$$

$$P_3 = \lambda_3 \sum_k \left(\sum_{q=1}^{Q} s_{q,k} - 1 \right)^2 \tag{8}$$

2. **Combine Objective and Penalties:** The final unconstrained objective function, $U(\mathbf{x}, c, s, y)$, is the sum of the original objective function (1) and the three penalty terms: $U = \text{Objective} + P_1 + P_2 + P_3$.

2.2 The Full Unconstrained Objective Function

Combining these steps yields the complete unconstrained function. Note the revised structure for the first penalty term, where the summation now occurs inside the square.

$$\begin{aligned} U(\mathbf{x}, c, s, y) = & \left(||\mathbf{x}_1 - \mathbf{x}_{\text{start}}||^2 + ||\mathbf{x}_K - \mathbf{x}_{\text{goal}}||^2 + w \sum_{k=1}^{K-1} ||\mathbf{x}_{k+1} - \mathbf{x}_k||^2 \right) \\ & + \lambda_1 \left(\sum_{i,k,t,q} A_t(P_i \mathbf{r}^q + \mathbf{x}_k) - \mathbf{b}_t - (1 - c_{t,k})M \cdot \mathbf{1}_t - (1 - s_{q,k})M \cdot \mathbf{1}_t + y_{i,k,t,q} \right)^2 \\ & + \lambda_2 \sum_k \left(\sum_{t=1}^{T} c_{t,k} - 1 \right)^2 \\ & + \lambda_3 \sum_k \left(\sum_{q=1}^{Q} s_{q,k} - 1 \right)^2 \end{aligned} \tag{9}$$

3 Discretisation and Expansion into a Binary Polynomial

The next stage is to convert the unconstrained function U into a pure quadratic polynomial where every variable is binary. This involves discretising the non-binary variables (x_k, y) and then expanding all terms.

3.1 Discretisation of Continuous and Integer Variables

A QUBO model requires all variables to be binary. We use a binary expansion to represent the continuous position variables and the integer slack variable. For any integer variable v, its binary expansion is:

$$v = \sum_{p=0}^{P-1} 2^p b_p \tag{10}$$

where b_p are new binary variables and P is the number of bits determining the precision. This simplified formula assumes that the problem space is appropriately scaled and offset to a non-negative integer range during implementation.

3.2 Expansion of the Path Objective

The path objective consists of three parts: a term for the starting position, a term for the goal position, and a term for the smoothness of the path between steps.

$$\text{Objective} = ||\mathbf{x}_1 - \mathbf{x}_{\text{start}}||^2 + ||\mathbf{x}_K - \mathbf{x}_{\text{goal}}||^2 + w \sum_{k=1}^{K-1} ||\mathbf{x}_{k+1} - \mathbf{x}_k||^2 \tag{11}$$

3.2.1 Binarisation of Position Variables

First, we replace each continuous position coordinate, $x_{k,d}$, with its binary expansion. We use the variable h for these binaries to avoid confusion with the vector $\mathbf{b}_t$.

$$x_{k,d} = \sum_{p=0}^{P-1} 2^p h_{k,d,p} \tag{12}$$

The constant vectors $\mathbf{x}_{\text{start}}$ and $\mathbf{x}_{\text{goal}}$ are known values and do not need to be binarised.

3.2.2 Expansion of the Start and Goal Terms

Let's expand the first term, $||\mathbf{x}_1 - \mathbf{x}_{\text{start}}||^2$. This is equivalent to $\sum_d (x_{1,d} - x_{\text{start},d})^2$.

$$(x_{1,d} - x_{\text{start},d})^2 = x_{1,d}^2 - 2x_{\text{start},d}x_{1,d} + x_{\text{start},d}^2$$

Now, we substitute the binary expansion for $x_{1,d}$:

$$= \left(\sum_{p=0}^{P-1} 2^p h_{1,d,p}\right)^2 - 2x_{\text{start},d}\left(\sum_{p=0}^{P-1} 2^p h_{1,d,p}\right) + x_{\text{start},d}^2$$

Expanding the squared summation using the identity $h^2 = h$ gives:

$$\left(\sum_{p=0}^{P-1} 2^p h_{1,d,p}\right)^2 = \sum_{p=0}^{P-1} (2^p)^2 h_{1,d,p} + \sum_{p \neq m} 2 \cdot 2^p \cdot 2^m h_{1,d,p} h_{1,d,m}$$

So, the full expansion for the start term for a single dimension d is:

$$\left(\sum_{p=0}^{P-1} 4^p h_{1,d,p} + \sum_{p \neq m} 2^{p+m+1} h_{1,d,p} h_{1,d,m}\right) - \left(\sum_{p=0}^{P-1} (2x_{\text{start},d} \cdot 2^p) h_{1,d,p}\right) + x_{\text{start},d}^2$$

The expansion for the goal term $||\mathbf{x}_K - \mathbf{x}_{\text{goal}}||^2$ follows the exact same structure.

3.2.3 Expansion of the Path Smoothness Term

Now let's expand the main summation term, $w \sum_{k=1}^{K-1} ||\mathbf{x}_{k+1} - \mathbf{x}_k||^2$. We focus on a single component for a given k and d: $w(x_{k+1,d} - x_{k,d})^2$.

$$\begin{aligned} w(x_{k+1,d} - x_{k,d})^2 &= w\left(\left(\sum_{p=0}^{P-1} 2^p h_{k+1,d,p}\right) - \left(\sum_{m=0}^{P-1} 2^m h_{k,d,m}\right)\right)^2 \\ &= w\left[\left(\sum_p 2^p h_{k+1,d,p}\right)^2 - 2\left(\sum_p 2^p h_{k+1,d,p}\right)\left(\sum_m 2^m h_{k,d,m}\right) + \left(\sum_m 2^m h_{k,d,m}\right)^2\right] \end{aligned}$$

Using the expansion for the squared summations from the previous section, the final binarised form for this component is:

$$= w\left[\left(\sum_p 4^p h_{k+1,d,p} + \sum_{p\neq p'} 2^{p+p'+1} h_{k+1,d,p} h_{k+1,d,p'}\right) - \left(\sum_{p,m} 2^{p+m+1} h_{k+1,d,p} h_{k,d,m}\right) + \left(\sum_m 4^m h_{k,d,m} + \sum_{m\neq m'} 2^{m+m'+1} h_{k,d,m} h_{k,d,m'}\right)\right]$$

The complete path objective is the sum of these expanded polynomials over all dimensions d and steps k.

3.2.4 Simplified Final Form

To simplify the final expression, we can represent each of the fully expanded parts of the path objective with a new letter. Let Φ (Phi) represent these binarised quadratic polynomials.

Let Φ_{start} be the full binarised expansion of $||\mathbf{x}_1 - \mathbf{x}_{\text{start}}||^2$:

$$\Phi_{\text{start}} = \sum_d \left[\left(\sum_{p=0}^{P-1} 4^p h_{1,d,p} + \sum_{p\neq m} 2^{p+m+1} h_{1,d,p} h_{1,d,m}\right) - \left(\sum_{p=0}^{P-1} (2x_{\text{start},d} \cdot 2^p) h_{1,d,p}\right) + x_{\text{start},d}^2\right]$$

Let Φ_{goal} be the full binarised expansion of $||\mathbf{x}_K - \mathbf{x}_{\text{goal}}||^2$:

$$\Phi_{\text{goal}} = \sum_d \left[\left(\sum_{p=0}^{P-1} 4^p h_{K,d,p} + \sum_{p\neq m} 2^{p+m+1} h_{K,d,p} h_{K,d,m}\right) - \left(\sum_{p=0}^{P-1} (2x_{\text{goal},d} \cdot 2^p) h_{K,d,p}\right) + x_{\text{goal},d}^2\right]$$

Let $\Phi_{\text{smooth}}(k,d)$ be the binarised expansion for a single component of the smoothness term:

$$\Phi_{\text{smooth}}(k,d) = w\left[\left(\sum_p 4^p h_{k+1,d,p} + \dots\right) - \left(\sum_{p,m} 2^{p+m+1} h_{k+1,d,p} h_{k,d,m}\right) + \left(\sum_m 4^m h_{k,d,m} + \dots\right)\right]$$

The complete binarised path objective, which we can call $\text{Objective}_{\text{binary}}$, is then:

$$\text{Objective}_{\text{binary}} = \Phi_{\text{start}} + \Phi_{\text{goal}} + \sum_{k=1}^{K-1} \sum_{d=1}^{D} \Phi_{\text{smooth}}(k,d)$$

3.3 Expansion of the P_1 Penalty Term

The penalty term P_1 is a sum of squared penalties, where each individual scalar constraint is penalised independently. Let u be the index for the planes defining the polytope $\mathcal{A}_t$. For each combination of (i,k,t,q,u), we introduce a unique slack variable $y_{i,k,t,q,u}$.

The penalty term is then the sum of the squares of all these individual constraint equalities:

$$P_1 = \lambda_1 \sum_{i,k,t,q,u} \left((A_t(P_i \mathbf{r}^q + \mathbf{x}_k))_u - (\mathbf{b}_t)_u - (1-c_{t,k})M - (1-s_{q,k})M + y_{i,k,t,q,u}\right)^2 \tag{13}$$

For clarity, let's define the expression inside the parentheses for a single term as $\mathcal{L}_{i,k,t,q,u}$. Our goal is to expand the sum of squares, $\sum(\mathcal{L}_{i,k,t,q,u})^2$.

3.3.1 Binarising Each Component of $\mathcal{L}_{i,k,t,q,u}$

First, we must binarise every non-binary variable within a single term $\mathcal{L}_{i,k,t,q,u}$. The variables $c_{t,k}$ and $s_{q,k}$ are already binary.

- **Position Variables ($\mathbf{x}_k$):** Each coordinate $x_{k,d}$ is replaced by its binary expansion. We use the variable h for these binaries to avoid confusion with the vector $\mathbf{b}_t$.

$$x_{k,d} = \sum_{p=0}^{P-1} 2^p h_{k,d,p} \tag{14}$$

- **Slack Variable ($y_{i,k,t,q,u}$):** Each individual slack variable is replaced by its own unique binary expansion:

$$y_{i,k,t,q,u} = \sum_{m=0}^{P_s-1} 2^m z_{i,k,t,q,u,m} \tag{15}$$

Now, we substitute these into a single term $\mathcal{L}_{i,k,t,q,u}$. The u-th component of the vector $A_t\mathbf{x}_k$ is $(A_t\mathbf{x}_k)_u = \sum_d A_{t,u,d}x_{k,d}$, where $A_{t,u,d}$ is the element of the matrix A_t at row u and column d. After binarisation, this becomes:

$$\sum_{d=1}^{D} A_{t,u,d} \left(\sum_{p=0}^{P-1} 2^p h_{k,d,p} \right) \tag{16}$$

3.3.2 The Binarised Expression for a Single Term $\mathcal{L}_{i,k,t,q,u}$

By substituting the binary expansions, each term $\mathcal{L}_{i,k,t,q,u}$ becomes a linear combination of binary variables.

$$\mathcal{L}_{i,k,t,q,u} = \left(\sum_{d=1}^{D} \sum_{p=0}^{P-1} A_{t,u,d} 2^p h_{k,d,p} \right) + Mc_{t,k} + Ms_{q,k} + \left(\sum_{m=0}^{P_s-1} 2^m z_{i,k,t,q,u,m} \right) + G_{i,t,q,u}$$

Where $G_{i,t,q,u}$ is the constant part for that specific term: $G_{i,t,q,u} = (A_tP_i\mathbf{r}^q)_u - (b_t)_u - 2M$. Note that M is a single large constant value.

3.3.3 Expanding the Square of a Single Term

The final step is to expand the square of the linear polynomial $(\mathcal{L}_{i,k,t,q,u})^2$. This expansion reveals all the linear and quadratic terms that will contribute to the final QUBO matrix from this single penalty. For simplicity, let's denote the four variable parts as H, C, S, Z and the constant as G.

$$(\mathcal{L}_{i,k,t,q,u})^2 = (H + C + S + Z + G)^2$$

Where each variable part corresponds to a specific addend:

$$\mathcal{L}_{i,k,t,q,u} = \underbrace{\left(\sum_{d=1}^{D} \sum_{p=0}^{P-1} A_{t,u,d} 2^p h_{k,d,p} \right)}_{H} + \underbrace{Mc_{t,k}}_{C} + \underbrace{Ms_{q,k}}_{S} + \underbrace{\left(\sum_{m=0}^{P_s-1} 2^m z_{i,k,t,q,u,m} \right)}_{Z} + \underbrace{G_{i,t,q,u}}_{G}$$

The full expansion is:

$$\begin{aligned}
(\mathcal{L}_{i,k,t,q,u})^2 = {} & H^2 + C^2 + S^2 + Z^2 + G^2 \\
& + 2HC + 2HS + 2HZ + 2HG \\
& + 2CS + 2CZ + 2CG \\
& + 2SZ + 2SG \\
& + 2ZG
\end{aligned}$$

3.3.4 Defining the Full Expansion of a Single Term

The expansion contains squared terms for each part and cross-product terms between all parts.

3.3.5 Squared Terms

These terms produce linear and quadratic interactions within the same variable type.

$$
\begin{aligned}
H^2 &= \left(\sum_{d,p} A_{t,u,d} 2^p h_{k,d,p}\right)^2 = \sum_{d,p} (A_{t,u,d} 2^p)^2 h_{k,d,p} + \sum_{d,p \neq d',p'} 2(A_{t,u,d} 2^p)(A_{t,u,d'} 2^{p'}) h_{k,d,p} h_{k,d',p'} \\
C^2 &= (M c_{t,k})^2 = M^2 c_{t,k} \\
S^2 &= (M s_{q,k})^2 = M^2 s_{q,k} \\
Z^2 &= \left(\sum_{m} 2^m z_{i,k,t,q,u,m}\right)^2 = \sum_{m} (2^m)^2 z_{i,k,t,q,u,m} + \sum_{m \neq m'} 2(2^m)(2^{m'}) z_{i,k,t,q,u,m} z_{i,k,t,q,u,m'} \\
G^2 &= (G_{i,t,q,u})^2 \quad \text{(This is a constant and can be ignored)}
\end{aligned}
$$

3.3.6 Cross-Product Terms

These terms produce the quadratic interactions between different types of variables.

$$
\begin{aligned}
2HC &= 2\left(\sum_{d,p} A_{t,u,d} 2^p h_{k,d,p}\right)(M c_{t,k}) \\
2HS &= 2\left(\sum_{d,p} A_{t,u,d} 2^p h_{k,d,p}\right)(M s_{q,k}) \\
2HZ &= 2\left(\sum_{d,p} A_{t,u,d} 2^p h_{k,d,p}\right)\left(\sum_{m} 2^m z_{i,k,t,q,u,m}\right) \\
2CS &= 2(M c_{t,k})(M s_{q,k}) = 2M^2 c_{t,k} s_{q,k} \\
2CZ &= 2(M c_{t,k})\left(\sum_{m} 2^m z_{i,k,t,q,u,m}\right) \\
2SZ &= 2(M s_{q,k})\left(\sum_{m} 2^m z_{i,k,t,q,u,m}\right)
\end{aligned}
$$

3.3.7 Linear Terms from the Constant

These terms are formed by the cross-product of each variable part with the constant G.

$$
\begin{aligned}
2HG &= 2G_{i,t,q,u}\left(\sum_{d,p} A_{t,u,d} 2^p h_{k,d,p}\right) \\
2CG &= 2G_{i,t,q,u}(M c_{t,k}) \\
2SG &= 2G_{i,t,q,u}(M s_{q,k}) \\
2ZG &= 2G_{i,t,q,u}\left(\sum_{m} 2^m z_{i,k,t,q,u,m}\right)
\end{aligned}
$$

3.4 Expansion of Equality Penalties (P_2 and P_3)

The equality penalty terms already consist of binary variables and can be expanded directly. For a single step k, the expansion of the P_2 penalty is:

$$\left(\sum_{t=1}^{T} c_{t,k} - 1\right)^2 = \sum_{t \neq j} c_{t,k}c_{j,k} - \sum_{t=1}^{T} c_{t,k} + 1$$

Similarly, the expansion of the P_3 penalty is:

$$\left(\sum_{q=1}^{Q} s_{q,k} - 1\right)^2 = \sum_{q \neq j} s_{q,k}s_{j,k} - \sum_{q=1}^{Q} s_{q,k} + 1$$

3.5 The Complete Binarised Objective Function

The full expression for U_{binary} is the sum of the binarised path objective and all binarised penalty terms:

$$
\begin{aligned}
U_{\text{binary}} = \; & \text{\% Path Objective Terms} \\
& \sum_d \left[\left(\sum_p 4^p h_{1,d,p} + \sum_{p \neq m} 2^{p+m+1} h_{1,d,p} h_{1,d,m} \right) - \left(\sum_p (2x_{\text{start},d} \cdot 2^p) h_{1,d,p} \right) + x_{\text{start},d}^2 \right] \\
& + \sum_d \left[\left(\sum_p 4^p h_{K,d,p} + \sum_{p \neq m} 2^{p+m+1} h_{K,d,p} h_{K,d,m} \right) - \left(\sum_p (2x_{\text{goal},d} \cdot 2^p) h_{K,d,p} \right) + x_{\text{goal},d}^2 \right] \\
& + w \sum_{k=1}^{K-1} \sum_d \left[\left(\sum_p 4^p h_{k+1,d,p} + \sum_{p \neq p'} 2^{p+p'+1} h_{k+1,d,p} h_{k+1,d,p'} \right) \right. \\
& \quad \left. - \left(\sum_{p,m} 2^{p+m+1} h_{k+1,d,p} h_{k,d,m} \right) + \left(\sum_m 4^m h_{k,d,m} + \sum_{m \neq m'} 2^{m+m'+1} h_{k,d,m} h_{k,d,m'} \right) \right] \\
& \text{\% Inequality Penalty Term (P1)} \\
& + \lambda_1 \sum_{i,k,t,q,u} \Bigg[\\
& \text{\% — Squared Terms —} \\
& \quad \left(\sum_{d,p} (A_{t,u,d} 2^p)^2 h_{k,d,p} + \sum_{d,p \neq d',p'} 2(A_{t,u,d} 2^p)(A_{t,u,d'} 2^{p'}) h_{k,d,p} h_{k,d',p'} \right) \quad \text{\% } H^2 \\
& \quad + M^2 c_{t,k} \quad \text{\% } C^2 \\
& \quad + M^2 s_{q,k} \quad \text{\% } S^2 \\
& \quad + \left(\sum_m 4^m z_{i,k,t,q,u,m} + \sum_{m \neq m'} 2^{m+m'+1} z_{i,k,t,q,u,m} z_{i,k,t,q,u,m'} \right) \quad \text{\% } Z^2 \\
& \quad + G_{i,t,q,u}^2 \quad \text{\% } G^2 \\
& \text{\% — Cross-Product Terms —} \\
& \quad + 2M c_{t,k} \left(\sum_{d,p} A_{t,u,d} 2^p h_{k,d,p} \right) \quad \text{\% 2HC} \\
& \quad + 2M s_{q,k} \left(\sum_{d,p} A_{t,u,d} 2^p h_{k,d,p} \right) \quad \text{\% 2HS} \\
& \quad + 2 \left(\sum_{d,p} A_{t,u,d} 2^p h_{k,d,p} \right) \left(\sum_m 2^m z_{i,k,t,q,u,m} \right) \quad \text{\% 2HZ} \\
& \quad + 2M^2 c_{t,k} s_{q,k} \quad \text{\% 2CS} \\
& \quad + 2M c_{t,k} \left(\sum_m 2^m z_{i,k,t,q,u,m} \right) \quad \text{\% 2CZ} \\
& \quad + 2M s_{q,k} \left(\sum_m 2^m z_{i,k,t,q,u,m} \right) \quad \text{\% 2SZ} \\
& \text{\% — Linear Terms from Constant G —} \\
& \quad + 2G_{i,t,q,u} \left(\sum_{d,p} A_{t,u,d} 2^p h_{k,d,p} \right) \quad \text{\% 2HG} \\
& \quad + 2G_{i,t,q,u} M c_{t,k} \quad \text{\% 2CG} \\
& \quad + 2G_{i,t,q,u} M s_{q,k} \quad \text{\% 2SG} \\
& \quad + 2G_{i,t,q,u} \left(\sum_m 2^m z_{i,k,t,q,u,m} \right) \quad \text{\% 2ZG} \\
& \quad \Bigg] \\
& \text{\% Equality Penalty Terms (P2 and P3)} \\
& + \lambda_2 \sum_k \left(\sum_{t \neq j} c_{t,k} c_{j,k} - \sum_{t=1}^{T} c_{t,k} + 1 \right) \\
& + \lambda_3 \sum_k \left(\sum_{q \neq j} s_{q,k} s_{j,k} - \sum_{q=1}^{Q} s_{q,k} + 1 \right)
\end{aligned}
$$

3.5.1 Summary using Compact Notation

To clarify the overall structure, we can represent the complete binarised objective function using the simplified terms we developed. Let Φ represent the binarised quadratic polynomials for the path objective, and let $\mathcal{L}_{i,k,t,q,u}$ represent the binarised linear polynomial for a single inequality constraint.

The final QUBO objective function can then be written more compactly as:

$$
\begin{aligned}
U_{\text{binary}} = \quad & \left(\Phi_{\text{start}} + \Phi_{\text{goal}} + \sum_{k=1}^{K-1} \sum_{d=1}^{D} \Phi_{\text{smooth}}(k,d) \right) \quad \text{\% Path Objective Binary} \\
& + \lambda_1 \sum_{i,k,t,q,u} (\mathcal{L}_{i,k,t,q,u})^2 \quad \text{\% Inequality Penalty } (P_1) \text{ Binary} \\
& + \lambda_2 \sum_{k} \left(\sum_{t \neq j} c_{t,k} c_{j,k} - \sum_{t=1}^{T} c_{t,k} + 1 \right) \quad \text{\% Equality Penalty } (P_2) \text{ Binary} \\
& + \lambda_3 \sum_{k} \left(\sum_{q \neq j} s_{q,k} s_{j,k} - \sum_{q=1}^{Q} s_{q,k} + 1 \right) \quad \text{\% Equality Penalty } (P_3) \text{ Binary}
\end{aligned}
$$

Or:

$$
U_{\text{binary}} = \text{Objective}_{\text{binary}} + P_{1\text{Binary}} + P_{2\text{Binary}} + P_{3\text{Binary}} \tag{17}
$$