Mathematical Induction

Prove $1 + 2 + ... + n = \frac{n(n+1)}{2}$.

$n = 1$	$1 = \frac{n(n+1)}{2}$
$n = 2$	$1 + 2 = \frac{2(2+1)}{2}$
$n = 3$	$1 + 2 + 3 = \frac{3(3+1)}{2}$
$n = 4$	$1 + 2 + 3 + 4 = \frac{4(4+1)}{2}$

Principal of Mathematical Induction

Take a statement $P(n)$ (can evaluate as true or false)

Base Case

Prove $P(1)$

Induction

Assume $P(n)$ is true. Prove $P(n+1)$ is true.

Then you can conclude $P(n)$ is true for all $n \geq 1$.

Solution

Prove $1 + 2 + ... + n = \frac{n(n+1)}{2}$.

Solution (by Induction)

BASE CASE: $n = 1$

LHS: 1

RHS: $\frac{1(2)}{2} = 1$

Assume for some $n \geq 1$, $1 + 2 + ... + n = \frac{n(n+1)}{2}$. We need to show (and you should write this down):

$$1 + 2 + ... + n + (n+1) = \frac{(n+1)(n+2)}{2}$$

Let's start with the LHS We see the inductive hypothesis in the statement I'm evalulating.

$$[1 + 2 + ... + n] + (n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{n(n+1) + 2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$$

And we're done.

Polya's Conjecture

Even type means an even # of prime factors. **Odd** type means odd # of prime factors.

E	1
O	2
O	3
E	4=2 x 2
O	5
E	6=3 x 2
O	7
O	8=2 x 2 x 2
E	9 = 3 x 3

$E(n) = k \leq n$ of even type

$O(n) = k < n$ of odd type

For $n \geq 2$, $O(n) \geq E(n)$

(1919) Polya $n \leq 1500$

$E(6) = 3 \leq O(6) = 3$ $E(7) = 3 < O(7) = 4$ $E(8) = 3 < O(8) = 5$

Computer checked $E(n) \leq O(n)$ for all $n \leq 1,000,000$

Lehman showed $E(n) = O(n) + 1$ for $n = 906,180,359$.

So you want to show your proof. Even if you brute force, it isn't enough.