Pigeonhole Principle

"combinatorics is an honest subject. You can count balls in boxes and either you have the right number or you haven't" - Gian-Carlo ROTA

Combinatorics is a concrete subject: you can do it with your hands. Construct an image, draw an example.

Pigeonhole Principle

k balls into n boxes and $k > n$

$\rightarrow$ at least 1 box has 2 balls

When we have a statement that is *painfully* obvious, we will use proof by contradiction.

Proof

Suppose not ($k > n$ s.t. no box has at least 2 balls *i.e. each box has* $\leq$ *1 ball*). That means the number of balls k must be at most the number of boxes n (i.e. $k \leq n$). Which is a contradiciton.

Example

Among any 7 people living in the contiguous U.S., at least 2 people live within 1500 miles of one another.

Ask: which are the balls? and which are the boxes?

- You can think of "objects" as balls and "properties" as boxes.

So the balls are *people* and the boxes are *anything that has the property of being close together* This means I need 6 (or less) "boxes". Draw a picture: (pigeonholeex1.png)