

Algorithm Design and Implementation

Principle of Algorithms XII

NP Problem I

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Poly-Time Reductions

Algorithm design patterns.

- Greedy.
- Divide and conquer.
- Dynamic programming.
- Duality.
- Reductions.
- Local search.
- Randomization.

Algorithm design antipatterns.

- | | |
|-----------------------|--|
| • NP-completeness. | $O(n^k)$ algorithm unlikely. |
| • PSPACE-completeness | $O(n^k)$ certification algorithm unlikely. |
| • Undecidability | No algorithm possible. |

Classify problems according to computational requirements

Q. Which problems will we be able to solve in practice?

A **working definition**. Those with poly-time algorithms.



von Neumann
(1953)



Nash
(1955)



Gödel
(1956)



Cobham
(1964)



Edmonds
(1965)



Rabin
(1966)

Theory. Definition is broad and robust.

- Turing machine, word RAM, uniform circuits, ...

Practice. Poly-time algorithms scale to huge problems.

Classify problems according to computational requirements

Q. Which problems will we be able to solve in practice?

A [working definition](#). Those with poly-time algorithms.

yes	probably no
shortest path	longest path
min cut	max cut
2-satisfiability	3-satisfiability
planar 4-colorability	planar 3-colorability
bipartite vertex cover	vertex cover
2D-matching	3D-matching
primality testing	factoring
linear programming	integer linear programming

Classify problems

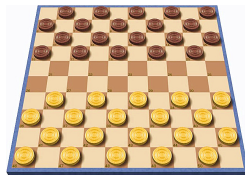
Requirement. Classify problems according to those that can be solved in polynomial time and those that cannot.

Provably requires exponential time.

- Given a constant-size program, does it halt in at most k steps?
- Given a board position in an n -by- n generalization of checkers, can black guarantee a win?



Alan designed the perfect computer



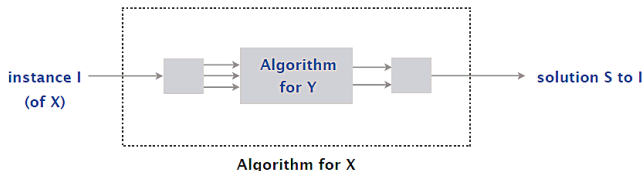
Frustrating news. Huge number of fundamental problems have defied classification for decades.

Poly-time reductions

Suppose we could solve problem Y in polynomial time. What else could we solve in polynomial time?

Reduction. Problem X polynomial-time (Cook) reduces to problem Y if arbitrary instances of problem X can be solved using:

- Polynomial number of standard computational steps, plus
- Polynomial number of calls to oracle that solves problem Y .



Poly-time reductions

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- Polynomial number of standard computational steps, plus
- Polynomial number of calls to oracle that solves problem Y .

Notation. $X \leq_P Y$.

Note. We pay for time to write down instances of Y sent to oracle \Rightarrow instances of Y must be of polynomial size.

Novice mistake. Confusing $X \leq_P Y$ with $Y \leq_P X$.

Quiz 1

Suppose that $X \leq_P Y$. Which of the following can we infer?

- A. If X can be solved in polynomial time, then so can Y .
- B. X can be solved in poly time iff Y can be solved in poly time.
- C. If X cannot be solved in polynomial time, then neither can Y .
- D. If Y cannot be solved in polynomial time, then neither can X .

Which of the following poly-time reductions are known?

- A. $\text{FIND-MAX-FLOW} \leq_P \text{FIND-MIN-CUT}$.
- B. $\text{FIND-MIN-CUT} \leq_P \text{FIND-MAX-FLOW}$.
- C. Both A and B.
- D. Neither A nor B.

Poly-time reductions

Design algorithms. If $X \leq_P Y$ and Y can be solved in polynomial time, then X can be solved in polynomial time.

Establish intractability. If $X \leq_P Y$ and X cannot be solved in polynomial time, then Y cannot be solved in polynomial time.

Establish equivalence. If both $X \leq_P Y$ and $Y \leq_P X$, we use notation $X \equiv_P Y$. In this case, X can be solved in polynomial time iff Y can be.

Bottom line. Reductions classify problems according to **relative** difficulty.

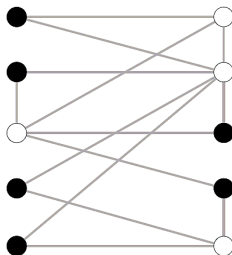
Packing and Covering Problems

Independent set

Independent Set. Given a graph $G = (V, E)$ and an integer k , is there a subset of k (or more) vertices such that no two are adjacent?

Example. Is there an independent set of size ≥ 6 ?

Example. Is there an independent set of size ≥ 7 ?

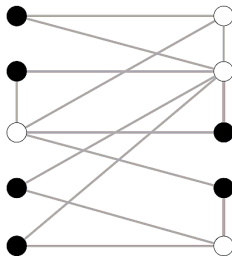


Vertex cover

Vertex Cover. Given a graph $G = (V, E)$ and an integer k , is there a subset of k (or fewer) vertices such that each edge is incident to at least one vertex in the subset?

Example. Is there a vertex cover of size ≤ 4 ?

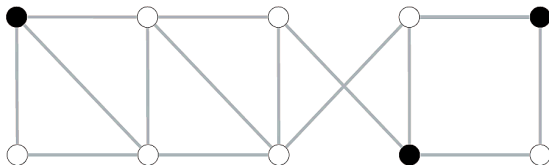
Example. Is there a vertex cover of size ≤ 3 ?



Quiz 3

Consider the following graph G . Which are true?

- A. The white vertices are a vertex cover of size 7.
- B. The black vertices are an independent set of size 3.
- C. Both A and B.
- D. Neither A nor B.

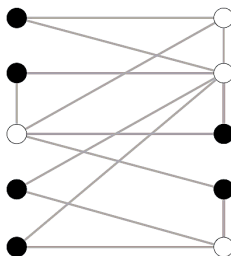


Vertex cover and independent set reduce to one another

Theorem

Independent Set \equiv_P Vertex Cover.

Proof. We show S is an independent set of size k iff $V - S$ is a vertex cover of size $n - k$.



Vertex cover and independent set reduce to one another

Theorem

Independent Set \equiv_P Vertex Cover.

Proof. We show S is an independent set of size k iff $V - S$ is a vertex cover of size $n - k$.

\Rightarrow

- Let S be any independent set of size k .
- $V - S$ is of size $n - k$.
- Consider an arbitrary edge $(u, v) \in E$.
- - S independent \Rightarrow either $u \notin S$, or $v \notin S$, or both.
 - \Rightarrow either $u \in V - S$, or $v \in V - S$, or both.
- Thus, $V - S$ covers (u, v) .

Vertex cover and independent set reduce to one another

Theorem

Independent Set \equiv_P Vertex Cover.

Proof. We show S is an independent set of size k iff $V - S$ is a vertex cover of size $n - k$.



- Let $V - S$ be any independent set of size $n - k$.
- S is of size k .
- Consider an arbitrary edge $(u, v) \in E$.
-

$V - S$ is a vertex cover \Rightarrow either $u \in V - S$, or $v \in V - S$, or both.
 \Rightarrow either $u \notin S$, or $v \notin S$, or both.

- Thus, S is an independent set.

Set Cover. Given a set U of elements, a collection S of subsets of U , and an integer k , are there $\leq k$ of these subsets whose union is equal to U ?

Sample application.

- m available pieces of software.
- Set U of n capabilities that we would like our system to have.
- The i^{th} piece of software provides the set $S_i \subseteq U$ of capabilities.
- **Goal:** achieve all n capabilities using fewest pieces of software.

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

$$S_a = \{3, 7\}$$

$$S_b = \{2, 4\}$$

$$S_c = \{3, 4, 5, 6\}$$

$$S_d = \{5\}$$

$$S_e = \{1\}$$

$$S_f = \{1, 2, 6, 7\}$$

$$k = 2$$

a set cover instance

Quiz 4

Given the universe $U = \{1, 2, 3, 4, 5, 6, 7\}$ and the following sets, which is the minimum size of a set cover?

- A. 1
- B. 2
- C. 3
- D. None of the above.

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

$$S_a = \{1, 4, 6\}$$

$$S_b = \{1, 6, 7\}$$

$$S_c = \{1, 2, 3, 6\}$$

$$S_d = \{1, 3, 5, 7\}$$

$$S_e = \{2, 6, 7\}$$

$$S_f = \{3, 4, 5\}$$

Vertex cover reduces to set cover

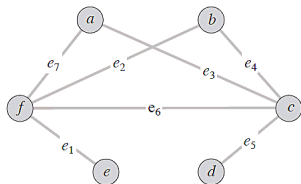
Theorem

Vertex Cover \leq_P *Set Cover*.

Proof. Given a *Vertex Cover* instance $G = (V, E)$ and k , we construct a *Set Cover* instance (U, S, k) that has a set cover of size k iff G has a vertex cover of size k .

Construction.

- Universe $U = E$.
- Include one subset for each node
 $v \in V : S_v = \{e \in E : e \text{ incident to } v\}$.



vertex cover instance

$(k = 2)$

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

$$S_a = \{3, 7\}$$

$$S_b = \{2, 4\}$$

$$S_c = \{3, 4, 5, 6\}$$

$$S_d = \{5\}$$

$$S_e = \{1\}$$

$$S_f = \{1, 2, 6, 7\}$$

set cover instance

$(k = 2)$

Vertex cover reduces to set cover

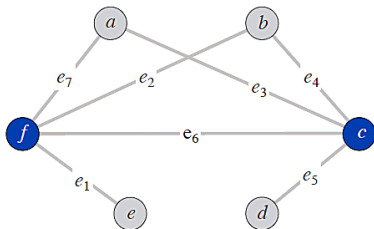
Lemma

$G = (V, E)$ contains a vertex cover of size k iff (U, S, k) contains a set cover of size k .

Proof. \Rightarrow

Let $X \subseteq V$ be a vertex cover of size k in G .

- Then $Y = \{S_v : v \in X\}$ is a set cover of size k .



vertex cover instance
($k = 2$)

$U = \{1, 2, 3, 4, 5, 6, 7\}$

$S_a = \{3, 7\}$

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$S_c = \{3, 4, 5, 6\}$

$S_d = \{5\}$

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$S_f = \{1, 2, 6, 7\}$

set cover instance
($k = 2$)

Vertex cover reduces to set cover

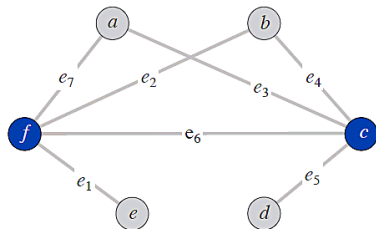
Lemma

$G = (V, E)$ contains a vertex cover of size k iff (U, S, k) contains a set cover of size k .

Proof. \Leftarrow

Let $Y \subseteq S$ be a set cover of size k in (U, S, k) .

- Then $X = \{v : S_v \in Y\}$ is a vertex cover of size k in G .



vertex cover instance
($k = 2$)

$U = \{1, 2, 3, 4, 5, 6, 7\}$

$S_a = \{3, 7\}$

$S_b = \{2, 4\}$

$S_c = \{3, 4, 5, 6\}$

$S_d = \{5\}$

$S_e = \{1\}$

$S_f = \{1, 2, 6, 7\}$

set cover instance
($k = 2$)

Constraint Satisfaction Problems

Literal. A Boolean variable or its negation: x_i or \bar{x}_i .

Clause. A disjunction of literals: $C_j = x_1 \vee \bar{x}_2 \vee x_3$

Conjunctive normal form (CNF): $\Phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

SAT. Given a CNF formula Φ , does it have a satisfying truth assignment?

3-SAT. SAT where each clause contains exactly 3 literals (and each literal corresponds to a different variable).

$$\Phi = (\bar{x}_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_4)$$

yes instance: $x_1 = \text{true}$, $x_2 = \text{true}$, $x_3 = \text{false}$ $x_4 = \text{false}$

Key application. Electronic design automation (EDA).

Satisfiability is hard

Scientific hypothesis. There does not exist a poly-time algorithm for 3-SAT.

P vs. NP This hypothesis is equivalent to $P \neq NP$ conjecture.



Donald J. Trump ✓

@realDonaldTrump

Following



Computer Scientists have so much funding and time and can't even figure out the boolean satisfiability problem. SAT!

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6:31 AM - 17 Apr 2017

↩ 20K

↻ 17K

♥ 50K

3-satisfiability reduces to independent set

Theorem

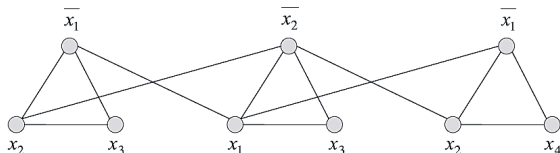
$3\text{-SAT} \leq_P \text{Independent Set}$.

Proof.

Given an instance Φ of 3-SAT , we construct an instance (G, k) of Independent Set that has an independent set of size $k = |\Phi|$ iff Φ is satisfiable.

Construction.

- G contains 3 nodes for each clause, one for each literal.
- Connect 3 literals in a clause in a triangle.
- Connect literal to each of its negations.



$$\Phi = (\overline{x_1} \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee x_4)$$

3-satisfiability reduces to independent set

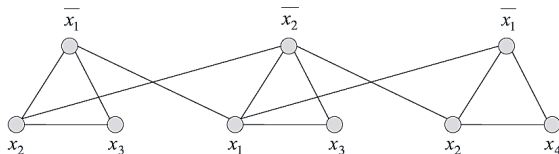
Theorem

$3\text{-SAT} \leq_P \text{Independent Set}$.

Proof.

⇒ Consider any satisfying assignment for Φ .

- Select one true literal from each clause/triangle.
- This is an independent set of size $k = |\Phi|$.



$$\Phi = (\overline{x_1} \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee x_4)$$

3-satisfiability reduces to independent set

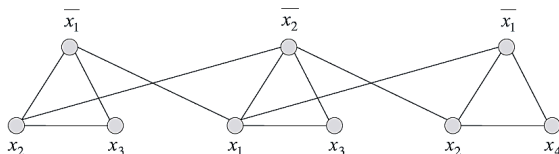
Theorem

$3\text{-SAT} \leq_P \text{Independent Set}$.

Proof.

⇐ Let S be independent set of size k .

- S must contain exactly one node in each triangle.
- Set these literals to **true** and remaining literals consistently.
- All clauses in Φ are satisfied.



$$\Phi = (\overline{x_1} \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee x_4)$$

Basic reduction strategies.

- Simple equivalence: Independent Set \equiv_P Vertex Cover
- Special case to general case: Vertex Cover \leq_P Set Cover.
- Encoding with gadgets: 3-SAT \leq_P Independent Set.

Transitivity. If $X \leq_P Y$ and $Y \leq_P Z$, then $X \leq_P Z$.

Proof idea. Compose the two algorithms.

Example. 3-SAT \leq_P Independent Set \leq_P Vertex Cover \leq_P Set Cover.

Decision, search and optimization problems

Decision problem. Does there **exist** a vertex cover of size $\leq k$?

Search problem. **Find** a vertex cover of size $\leq k$.

Optimization problem. **Find** a vertex cover of **minimum** size.

Goal. Show that all three problems poly-time reduce to one another.

Search problems VS. Decision problems

Vertex cover. Does there exist a vertex cover of size $\leq k$?

Find vertex cover. Find a vertex cover of size $\leq k$.

Theorem. Vertex cover \equiv_P Find vertex cover.

Proof.

\leq_P . Decision problem is a special case of search problem.

\geq_P . To find a vertex cover of size $\leq k$:

- Determine if there exists a vertex cover of size $\leq k$.
- Find a vertex v such that $G - \{v\}$ has a vertex cover of size $\leq k - 1$.
(any vertex in any vertex cover of size $\leq k$ will have this property)
- Include v in the vertex cover.
- Recursively find a vertex cover of size $\leq k - 1$ in $G - \{v\}$.

Optimization problems VS. Search problems VS. Decision problems

Find vertex cover. Find a vertex cover of size $\leq k$.

Find min vertex cover. Find a vertex cover of minimum size.

Theorem. Find vertex cover \equiv_P Find min vertex cover.

Proof.

\leq_P . Search problem is a special case of optimization problem.

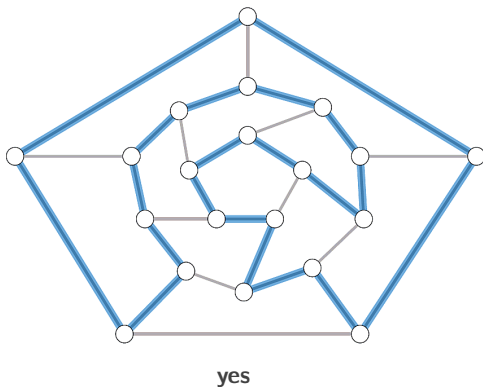
\geq_P . To find vertex cover of minimum size:

- Binary search (or linear search) for size k^* of min vertex cover.
- Solve search problem for given k^* .

Sequencing Problems

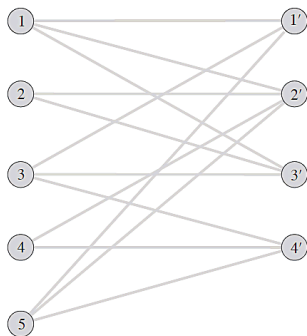
Hamilton cycle

Hamilton cycle. Given an undirected graph $G = (V, E)$, does there exist a cycle Γ that visits every node exactly once?



Hamilton cycle

Hamilton cycle. Given an undirected graph $G = (V, E)$, does there exist a cycle Γ that visits every node exactly once?



no

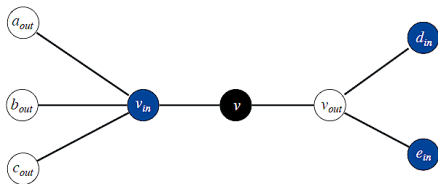
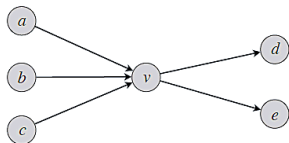
Directed Hamilton cycle reduces to Hamilton cycle

Directed Hamilton cycle. Given a directed graph $G = (V, E)$, does there exist a directed cycle Γ that visits every node exactly once?

Theorem

Directed Hamilton cycle \leq_P Hamilton cycle.

Proof. Given a directed graph $G = (V, E)$, construct a graph G' with $3n$ nodes.



Directed Hamilton cycle reduces to Hamilton cycle

Lemma

G has a directed Hamilton cycle iff G' has a Hamilton cycle.

Proof.



- Suppose G has a directed Hamilton cycle Γ .
- Then G' has an undirected Hamilton cycle (same order).



- Suppose G' has an undirected Hamilton cycle Γ' .
- Γ' must visit nodes in G' using one of following two orders:
... , black, white, blue, black, white, blue, black, white, blue, ...
... , black, blue, white, black, blue, white, black, blue, white, ...
- Black nodes in Γ' comprise either a directed Hamilton cycle Γ in G , or reverse of one.

3-satisfiability reduces to directed Hamilton cycle

Theorem

$3\text{-SAT} \leq_P \text{Directed Hamilton cycle}$.

Proof.

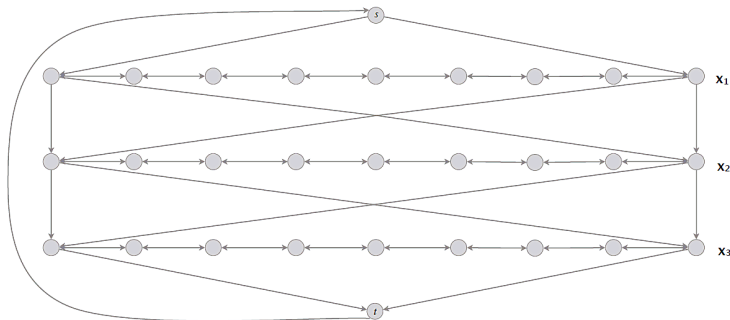
Given an instance Φ of 3-SAT, we construct an instance G of Directed Hamilton cycle that has a Hamilton cycle iff Φ is satisfiable.

Construction overview. Let n denote the number of variables in Φ . We will construct a graph G that has 2^n Hamilton cycles, with each cycle corresponding to one of the 2^n possible truth assignments.

3-satisfiability reduces to directed Hamilton cycle

Construction. Given 3-SAT instance Φ with n variables x_i and k clauses.

- Construct G to have 2^n Hamilton cycles.
- **Intuition:** traverse path i from left to right \Leftrightarrow set variables $x_i = \text{true}$



Quiz 5

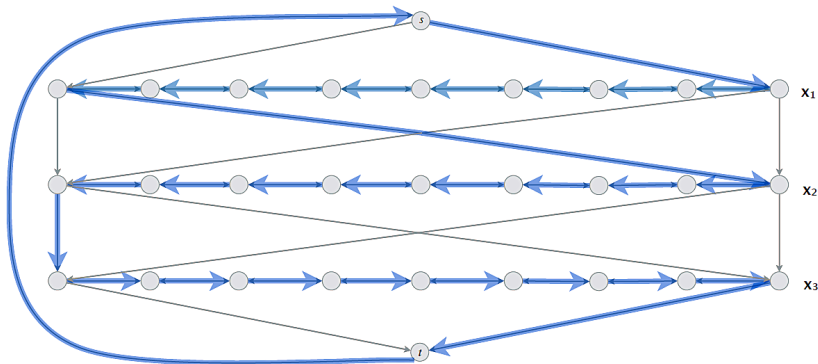
Which is truth assignment corresponding to Hamilton cycle below?

A. $x_1 = \text{true}, x_2 = \text{true}, x_3 = \text{true}$

C. $x_1 = \text{false}, x_2 = \text{false}, x_3 = \text{true}$

B. $x_1 = \text{true}, x_2 = \text{true}, x_3 = \text{false}$

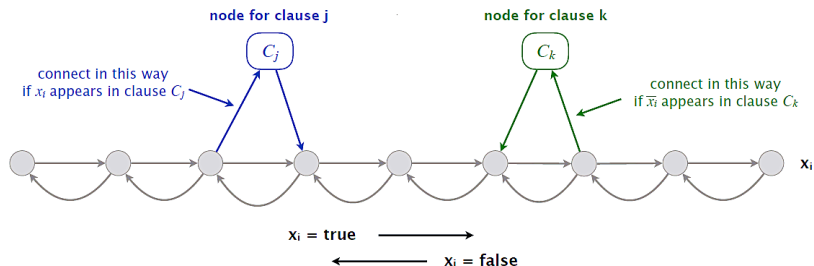
D. $x_1 = \text{false}, x_2 = \text{false}, x_3 = \text{false}$



3-satisfiability reduces to directed Hamilton cycle

Construction. Given 3-SAT instance Φ with n variables x_i and k clauses.

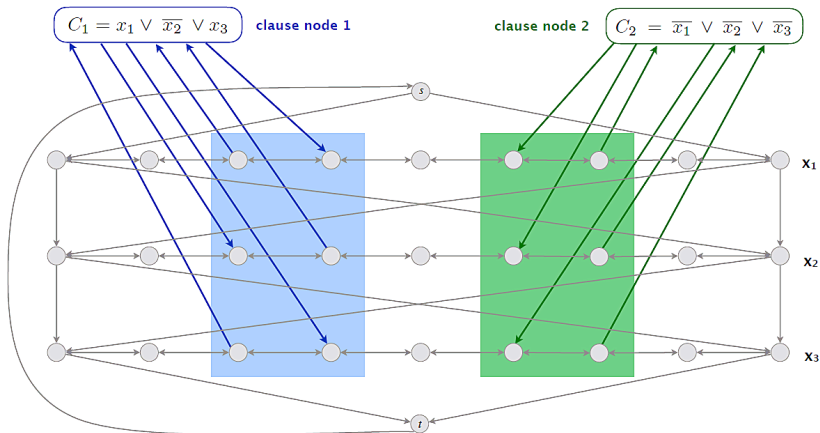
- For each clause: add a node and 2 edges per literal.



3-satisfiability reduces to directed Hamilton cycle

Construction. Given 3-SAT instance Φ with n variables x_i and k clauses.

- For each clause: add a node and 2 edges per literal.



3-satisfiability reduces to directed Hamilton cycle

Lemma

Φ is satisfiable iff G has a Hamilton cycle.

Proof.

\Rightarrow

- Suppose 3-SAT instance Φ has satisfying assignment x^* .
- Then, define Hamilton cycle Γ in G as follows:
 - if $x_i^* = \text{true}$, traverse row i from left to right.
 - if $x_i^* = \text{false}$, traverse row i from right to left.
 - for each clause C_j , there will be at least one row i in which we are going in "correct" direction to splice clause node C_j into cycle (and we splice in C_j exactly once)

3-satisfiability reduces to directed Hamilton cycle

Lemma

Φ is satisfiable iff G has a Hamilton cycle.

Proof.

\Leftarrow

- Suppose G has a Hamilton cycle Γ .
- If Γ enters clause node C_j , it must depart on mate edge.
 - nodes immediately before and after C_j are connected by an edge $e \in E$.
 - removing C_j from cycle, and replacing it with edge e yields Hamilton cycle on $G - \{C_j\}$.
- Continuing in this way, we are left with a Hamilton cycle Γ' in $G - \{C_1, C_2, \dots, C_k\}$.
- Set $x_i^* = \text{true}$ if Γ' traverses row i left-to-right; otherwise, set $x_i^* = \text{false}$.
- traversed in “correct” direction, and each clause is satisfied.

Graph Coloring

Home reading!

Numerical Problems

Subset sum. Given n natural numbers w_1, \dots, w_n and an integer W , is there a subset that adds up to exactly W ?

Example. $\{ 215, 215, 275, 275, 355, 355, 420, 420, 580, 580, 655, 655 \}$,
 $W = 1505$.

Yes. $215 + 355 + 355 + 580 = 1505$.

Remark. With arithmetic problems, input integers are encoded in binary. Poly-time reduction must be polynomial in **binary** encoding.

Theorem

$3\text{-SAT} \leq_P \text{Subset sum}$.

Proof. Given an instance Φ of 3-SAT , we construct an instance of Subset sum that has solution iff Φ is satisfiable.

3-satisfiability reduces to subset sum

Construction. Given 3-SAT instance Φ with n variables and k clauses, form $2n + 2k$ decimal integers, each having $n + k$ digits:

- Include one digit for each variable x_i and one digit for each clause C_j .
- Include two numbers for each variable x_i .
- Include two numbers for each clause C_j .
- Sum of each x_i digit is 1;
sum of each C_j digit is 4.

Key property. No carries possible \Rightarrow
each digit yields one equation.

$C_1 =$	$\neg x_1$	\vee	x_2	\vee	x_3
$C_2 =$	x_1	\vee	$\neg x_2$	\vee	x_3
$C_3 =$	$\neg x_1$	\vee	$\neg x_2$	\vee	$\neg x_3$

3-SAT instance

dummies to get clause
columns to sum to 4

	x_1	x_2	x_3	C_1	C_2	C_3	
x_1	1	0	0	0	1	0	100,010
$\neg x_1$	1	0	0	1	0	1	100,101
x_2	0	1	0	1	0	0	10,100
$\neg x_2$	0	1	0	0	1	1	10,011
x_3	0	0	1	1	1	0	1,110
$\neg x_3$	0	0	1	0	0	1	1,001
{	0	0	0	1	0	0	100
	0	0	0	2	0	0	200
	0	0	0	0	1	0	10
	0	0	0	0	2	0	20
	0	0	0	0	0	1	1
	0	0	0	0	0	2	2
W	1	1	1	4	4	4	111,444

Subset sum instance

3-satisfiability reduces to subset sum

Lemma

Φ is satisfiable iff there exists a subset that sums to W .

Proof. \Rightarrow Suppose 3-SAT instance Φ has satisfying assignment x^* .

- If $x_i^* = \text{true}$, select integer in row x_i ;
otherwise, select integer in row $\neg x_i$.

- Each x_i digit sums to 1.
- Since Φ is satisfiable, each C_j digit sums to at least 1 from x_i and $\neg x_i$ rows.
- Select dummy integers to make C_j digits sum to 4.

$$\begin{array}{l} C_1 = \neg x_1 \vee x_2 \vee x_3 \\ C_2 = x_1 \vee \neg x_2 \vee x_3 \\ C_3 = \neg x_1 \vee \neg x_2 \vee \neg x_3 \end{array}$$

3-SAT instance

dummies to get clause columns to sum to 4

	x_1	x_2	x_3	C_1	C_2	C_3	
x_1	1	0	0	0	1	0	100,010
$\neg x_1$	1	0	0	1	0	1	100,101
x_2	0	1	0	1	0	0	10,100
$\neg x_2$	0	1	0	0	1	1	10,011
x_3	0	0	1	1	1	0	1,110
$\neg x_3$	0	0	1	0	0	1	1,001
	0	0	0	1	0	0	100
	0	0	0	2	0	0	200
	0	0	0	0	1	0	10
	0	0	0	0	2	0	20
	0	0	0	0	0	1	1
	0	0	0	0	0	2	2
W	1	1	1	4	4	4	111,444

Subset sum instance

3-satisfiability reduces to subset sum

Lemma

Φ is satisfiable iff there exists a subset that sums to W .

Proof. \Leftarrow Suppose there exists a subset S^* that sums to W .

- Digit x_i forces subset S^* to select either row x_i or row $\neg x_i$ (but not both).
- If row x_i selected, assign $x_i^* = \text{true}$; otherwise, assign $x_i^* = \text{false}$.

Digit C_j forces subset S^* to select at least one literal in clause.

$$\begin{aligned} C_1 &= \neg x_1 \vee x_2 \vee x_3 \\ C_2 &= x_1 \vee \neg x_2 \vee x_3 \\ C_3 &= \neg x_1 \vee \neg x_2 \vee \neg x_3 \end{aligned}$$

3-SAT instance

dummies to get clause columns to sum to 4

	x_1	x_2	x_3	C_1	C_2	C_3	
x_1	1	0	0	0	1	0	100,010
$\neg x_1$	1	0	0	1	0	1	100,101
x_2	0	1	0	1	0	0	10,100
$\neg x_2$	0	1	0	0	1	1	10,011
x_3	0	0	1	1	1	0	1,110
$\neg x_3$	0	0	1	0	0	1	1,001
	0	0	0	1	0	0	100
	0	0	0	2	0	0	200
	0	0	0	0	1	0	10
	0	0	0	0	2	0	20
	0	0	0	0	0	1	1
	0	0	0	0	0	2	2
W	1	1	1	4	4	4	111,444

Subset sum instance

Subset sum reduces to knapsack

Subset sum. Given n natural numbers w_1, \dots, w_n and an integer W , is there a subset that adds up to exactly W ?

Knapsack. Given a set of items X , weights $u_i \geq 0$, values $v_i \geq 0$, a weight limit U , and a target value V , is there a subset $S \subseteq X$ such that:

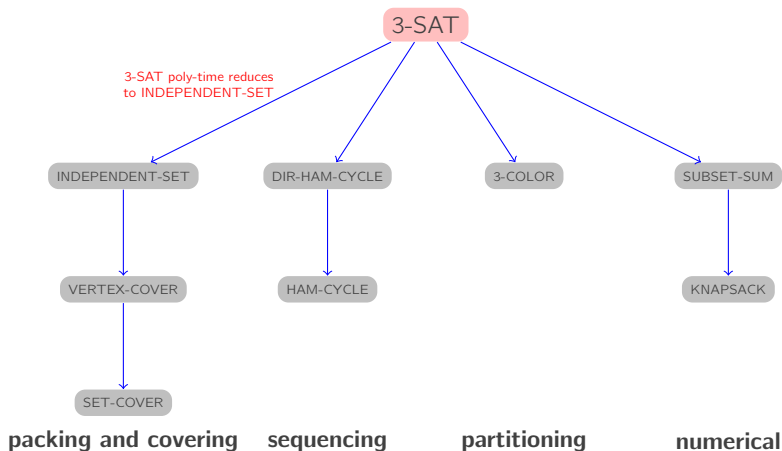
$$\sum_{i \in S} u_i \leq U, \quad \sum_{i \in S} v_i \geq V$$

Recall. $O(nU)$ dynamic programming algorithm for knapsack.

Challenge. Prove subset sum \leq_P Knapsack.

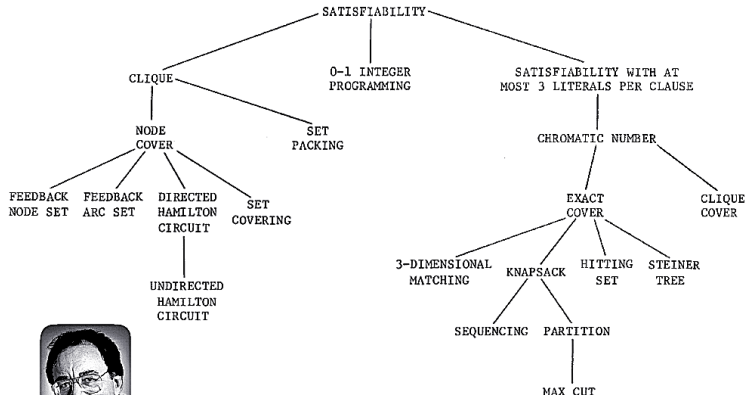
Poly-time reductions

constraint satisfaction



Karp's 20 poly-time reductions from satisfiability

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Dick Karp (1972)
1985 Turing Award

FIGURE 1 - Complete Problems

RICHARD M. KARP