Homework1

Due date: Sept 30, 2020

1. Consider a linear regression on data set $D=\{(x_1, t_1), (x_2, t_2),\ldots,(x_N, t_N)\}$ in which each data point is associated with a weighting factor $r_n > 0$, so that the sum-of-squares error function becomes:

$$L(W|D) = \frac{1}{2}\sum_{n=1}^{N} r_n\{t_n - W^T\varphi(x_n)\}^2$$

where $\varphi(x_n)$ is the basic function which transforms data into a computation-friendly shape and **W** is the model parameter to be estimated. Find an expression for the solution **W*** that minimizes this error function. (Hint: consider the **matrix form** of the objective function)

2. Suppose it is known that email arrival at a mail server behaves as a Poisson process with email arrival events occurring independently at a fixed average rate $\lambda > 0$. Then the time between successive events, denoted by the nonnegative real-valued variable x, follows an exponential distribution with the following probability density function:

$$p(x)=\lambda e^{-\lambda x},$$

where $x \geqslant 0$.

(a) Given a set of independent and identically distributed (i.i.d.) observations $\chi = \{x^{(\ell)}\}_{\ell=1}^{N}$ for the time intervals between successive events, write down an expression for the likelihood function of λ given χ.

(b) Derive the maximum likelihood estimate of λ given χ.

(c) Suppose λ is a random variable with prior probability density $p(\lambda)$ of the following form:

$$p(\lambda) \propto \lambda^{\alpha-1}e^{-\lambda\beta},$$

where α and β are two positive parameters. Show that the posterior density $p(\lambda|\chi)$ has the same form as the prior density $p(\lambda)$ but with different value for α and β. What are the new parameter values?