



# Minizinc Handbook

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MiniZinc is a free and open-source constraint modeling language. You can use MiniZinc to model constraint satisfaction and optimization problems in a high-level, solver-independent way, taking advantage of a large library of pre-defined constraints.

This handbook consists of four parts: [Section 1](#) covers installation and basic steps; [Section 2](#) is a tutorial-style introduction into modelling with MiniZinc; [Section 3](#) is a user manual for the individual tools in the MiniZinc tool chain; and [Section 4](#) is a reference to the language.

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## **Part 1**

# **Overview**



# CHAPTER 1.1

---

## Introduction

---

MiniZinc is a language for specifying constrained optimization and decision problems over integers and real numbers. A MiniZinc model does not dictate *how* to solve the problem - the MiniZinc compiler can translate it into different forms suitable for a wide range of *solvers*, such as Constraint Programming (CP), Mixed Integer Linear Programming (MIP) or Boolean Satisfiability (SAT) solvers.

The MiniZinc language lets users write models in a way that is close to a mathematical formulation of the problem, using familiar notation such as existential and universal quantifiers, sums over index sets, or logical connectives like implications and if-then-else statements. Furthermore, MiniZinc supports defining predicates and functions that let users structure their models (similar to procedures and functions in regular programming languages).

MiniZinc models are usually *parametric*, i.e., they describe a whole *class* of problems rather than an individual problem instance. That way, a model of, say, a vehicle routing problem could be reused to generate weekly plans, by instantiating it with the updated customer demands for the upcoming week.

MiniZinc is designed to interface easily to different backend solvers. It does this by transforming an input MiniZinc model and data file into a FlatZinc model. FlatZinc models consist of variable declarations and constraint definitions as well as a definition of the objective function if the problem is an optimization problem. The translation from MiniZinc to FlatZinc makes use of a library of function and predicate definitions for the particular target solver, which allows the MiniZinc compiler to produce specialised FlatZinc that only contains the types of variables and constraints that are supported by the target. In particular, MiniZinc allows the specification of

*global constraints* by *decomposition*. Furthermore, *annotations* of the model let the user fine tune the behaviour of the solver, independent of the declarative meaning of the model.

### 1.1.1 Structure

This documentation consists of four parts. *The first part* (page 5) includes this introduction and then describes how to download and install MiniZinc and how to make your first steps using the MiniZinc IDE and the command line tools. *The second part* (page 25) is a tutorial introduction to modelling with MiniZinc, from basic syntax and simple modelling techniques to more advanced topics. It also explains how MiniZinc is compiled to FlatZinc. *The third part* (page 149) is a user manual for the tools that make up the MiniZinc tool chain. Finally, *The fourth part* (page 199) contains the reference documentation for MiniZinc, including a definition of the MiniZinc language, documentation on how to interface a solver to FlatZinc, and an annotated list of all predicates and functions in the MiniZinc standard library.

### 1.1.2 How to Read This

If you are new to MiniZinc, follow the installation instructions and the introduction to the IDE and then work your way through the tutorial. Most example code can be downloaded, but it is sometimes more useful to type it in yourself to get the language into your muscle memory! If you need help, visit the MiniZinc web site at <http://www.minizinc.org> where you find a discussion forum.

Some of the code examples are shown in boxes like the one below. If a code box has a heading, it usually lists the name of a file that can be downloaded from <http://minizinc.org/doc-latest/en/downloads/index.html>.

**Listing 1.1.1: A code example (`dummy.mzn`)**

```
% Just an example
var int: x;
solve satisfy;
```

Throughout the documentation, some concepts are defined slightly more formally in special sections like this one.

**More details**

These sections can be skipped over if you just want to work through the tutorial for the first time, but they contain important information for any serious MiniZinc user!

Finally, if you find a mistake in this documentation, please report it through our GitHub issue tracker.



# CHAPTER 1.2

---

## Installation

---

A complete installation of the MiniZinc system comprises the MiniZinc *compiler tool chain*, one or more *solvers*, and (optionally) the *MiniZinc IDE*. We provide fully self-contained binary packages for all major operating systems that contain all of these components. Alternatively, it is possible to compile all components from source code.

### 1.2.1 Binary Packages

The easiest way to get a full, working MiniZinc system is to use the **bundled binary packages**, available from <http://www.minizinc.org/software.html>.

The bundled binary packages contain the compiler and IDE, as well as the following solvers: Gecode, Chuffed, COIN-OR CBC, and a Gurobi interface (the Gurobi library itself is not included). For backwards compatibility with older versions of MiniZinc, the packages also contain the now deprecated G12 suite of solvers (G12 fd, G12 lazy, G12 MIP).

#### 1.2.1.1 Microsoft Windows

To install the bundled binary packages, simply download the installer, double-click to execute it, and follow the prompts. **Note:** you should select the 64 bit version of the installer if your Windows is a 64 bit operating system, otherwise pick the 32 bit version.

After installation is complete, you can find the MiniZinc IDE installed as a Windows application. The file extensions .mzn, .dzn and .fzn are linked to the IDE, so double-clicking any MiniZinc file should open it in the IDE.

If you want to use MiniZinc from a command prompt, you need to add the installation directory to the PATH environment variable. In a Windows command prompt you could use the following command:

```
C:\>setx PATH "%PATH%;C:\Program Files\MiniZinc 2.2.2 (bundled)\"
```

### 1.2.1.2 Linux

The MiniZinc bundled binary distribution for Linux is provided as an archive that contains everything that is needed to run MiniZinc. It was compiled on a Ubuntu 16.04 LTS system, but it bundles all required libraries except for the system C and C++ libraries (so it should be compatible with any Linux distribution that uses the same C and C++ libraries as Ubuntu 16.04). **Note:** you should select the 64 bit version of the installer if your Linux is a 64 bit operating system, otherwise pick the 32 bit version.

After downloading, uncompress the archive, for example in your home directory or any other location where you want to install it:

```
$ tar xf MiniZincIDE-2.2.2-bundle-linux-x86_64.tgz
```

This will unpack MiniZinc into a directory that is called the same as the archive file (without the .tgz). You can run the MiniZinc IDE or any of the command line tools directly from that directory, or add it to your PATH environment variable for easier access. **Note:** the MiniZinc IDE needs to be started using the MiniZincIDE.sh script, which sets up a number of paths that are required by the IDE.

### 1.2.1.3 Apple macOS

The macOS bundled binary distribution works with any version of OS X starting from 10.9. After downloading the disk image (.dmg) file, double click it if it doesn't open automatically. You will see an icon for the MiniZinc IDE that you can drag into your Applications folder (or anywhere else you want to install MiniZinc).

In order to use the MiniZinc tools from a terminal, you need to add the path to the MiniZinc installation to the PATH environment variable. If you installed the MiniZinc IDE in the standard Applications folder, the following command will add the correct path:

```
$ export PATH=/Applications/MiniZincIDE.app/Contents/Resources:$PATH
```

## 1.2.2 Compilation from Source Code

All components of MiniZinc are free and open source software, and compilation should be straightforward if you have all the necessary build tools installed. However, third-party components, in particular the different solvers, may be more difficult to install correctly, and we cannot provide any support for these components.

The source code for MiniZinc can be downloaded from its GitHub repository at <https://github.com/MiniZinc/libminizinc>. The source code for the MiniZinc IDE is available from <https://github.com/MiniZinc/MiniZincIDE>.

You will also need to install additional solvers to use with MiniZinc. To get started, try Gecode (<http://www.gecode.org>) or Chuffed (<https://github.com/chuffed/chuffed>). We don't cover installation instructions for these solvers here.

### 1.2.2.1 Microsoft Windows

Required development tools:

- CMake, version 3.0.0 or later (<http://cmake.org>)
- Microsoft Visual C++ 2013 or later (e.g. the Community Edition available from <https://www.visualstudio.com/de/downloads/>)
- Optional, only needed for MiniZinc IDE: Qt toolkit, version 5.4 or later (<http://qt.io>)

**Compiling MiniZinc:** Unpack the source code (or clone the git repository). Open a command prompt and change into the source code directory. The following sequence of commands will build a 64 bit version of the MiniZinc compiler tool chain (you may need to adapt the `cmake` command to fit your version of Visual Studio):

```
mkdir build
cd build
cmake -G"Visual Studio 14 2015 Win64" -DCMAKE_INSTALL_PREFIX="C:/Program Files/MiniZinc" ..
cmake --build . --config Release --target install
```

This will install MiniZinc in the usual Program Files location. You can change where it gets installed by modifying the CMAKE\_INSTALL\_PREFIX.

**Compiling the MiniZinc IDE:** Unpack the source code (or clone the git repository). Open a Visual Studio command prompt that matches the version of the Qt libraries installed on your system. Change into the source code directory for the MiniZinc IDE. Then use the following commands to compile:

```
mkdir build  
cd build  
qmake ../MiniZincIDE  
nmake
```

### 1.2.2.2 Linux

Required development tools:

- CMake, version 3.0.0 or later
- A recent C++ compiler (g++ or clang)
- Optional, only needed for MiniZinc IDE: Qt toolkit, version 5.4 or later (<http://qt.io>)

**Compiling MiniZinc:** Unpack the source code (or clone the git repository). Open a terminal and change into the source code directory. The following sequence of commands will build the MiniZinc compiler tool chain:

```
mkdir build  
cd build  
cmake -DCMAKE_BUILD_TYPE=Release ..  
cmake --build .
```

**Compiling the MiniZinc IDE:** Unpack the source code (or clone the git repository). Open a terminal and change into the source code directory for the MiniZinc IDE. Then use the following commands to compile:

```
mkdir build  
cd build  
qmake ../MiniZincIDE  
make
```

### 1.2.2.3 Apple macOS

Required development tools:

- CMake, version 3.0.0 or later (from <http://cmake.org> or e.g. through homebrew)
- The Xcode developer tools
- Optional, only needed for MiniZinc IDE: Qt toolkit, version 5.4 or later (<http://qt.io>)

**Compiling MiniZinc:** Unpack the source code (or clone the git repository). Open a terminal and change into the source code directory. The following sequence of commands will build the MiniZinc compiler tool chain:

```
mkdir build
cd build
cmake -DCMAKE_BUILD_TYPE=Release ..
cmake --build .
```

**Compiling the MiniZinc IDE:** Unpack the source code (or clone the git repository). Open a terminal and change into the source code directory for the MiniZinc IDE. Then use the following commands to compile:

```
mkdir build
cd build
qmake ../MiniZincIDE
make
```

## 1.2.3 Adding Third-party Solvers

Third party solvers for MiniZinc typically consist of two parts: a solver *executable*, and a solver-specific MiniZinc *library*. MiniZinc must be aware of the location of both the executable and the library in order to compile and run a model with that solver. Each solver therefore needs to provide a *configuration file* in a location where the MiniZinc toolchain can find it.

The easiest way to add a solver to the MiniZinc system is via the MiniZinc IDE. This is explained in Section 3.2.5.2. You can also add configuration files manually, as explained in Section 4.3.5.



# CHAPTER 1.3

---

## First steps with MiniZinc

---

We recommend using the bundled binary distribution of MiniZinc introduced in [Section 1.2](#). It contains the MiniZinc IDE, the MiniZinc compiler, and several pre-configured solvers so you can get started straight away.

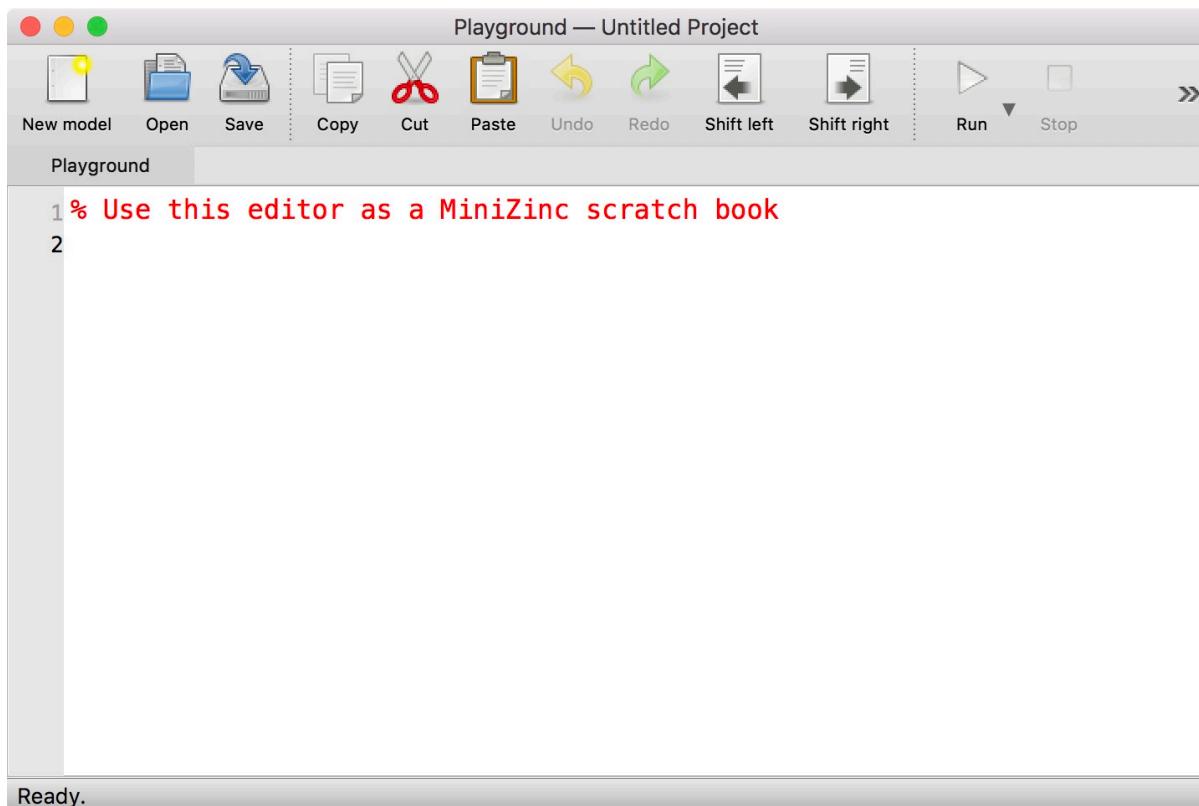
This section introduces the MiniZinc IDE and the command line tool `minizinc` using some very basic examples. This should be enough to get you started with the MiniZinc tutorial in [Section 2](#) (in fact, you only need to be able to use one of the two, for instance just stick to the IDE if you are not comfortable with command line tools, or just use the `minizinc` command if you suffer from fear of mice).

### 1.3.1 The MiniZinc IDE

The MiniZinc IDE provides a simple interface to most of MiniZinc’s functionality. It lets you edit model and data files, solve them with any of the solvers supported by MiniZinc, run debugging and profiling tools, and submit solutions to online courses (such as the MiniZinc Coursera courses).

When you open the MiniZinc IDE for the first time, it will ask you whether you want to be notified when an update is available. If you installed the IDE from sources, it may next ask you to locate your installation of the MiniZinc compiler. Please refer to [Section 3.2.5](#) for more details on this.

The IDE will then greet you with the *MiniZinc Playground*, a window that will look like this:

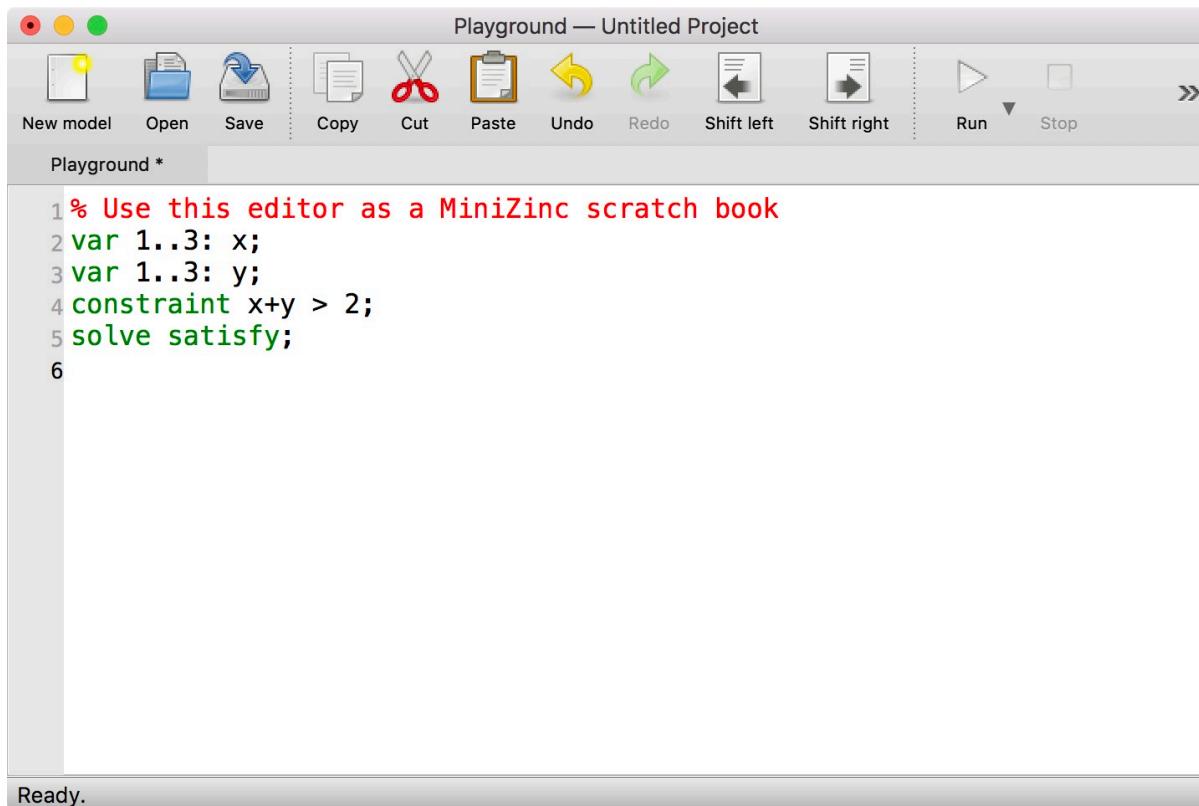


The screenshot shows the MiniZinc IDE interface. At the top is a toolbar with standard file operations (New model, Open, Save, Copy, Cut, Paste, Undo, Redo) and navigation (Shift left, Shift right, Run, Stop). Below the toolbar is a tab bar with 'Playground' selected. The main area contains the following text:

```
1 % Use this editor as a MiniZinc scratch book
2
```

In the bottom status bar, it says 'Ready.'

You can start writing your first MiniZinc model! Let's try something very simple:



The screenshot shows the MiniZinc IDE interface. At the top is a toolbar with standard file operations (New model, Open, Save, Copy, Cut, Paste, Undo, Redo) and navigation (Shift left, Shift right, Run, Stop). Below the toolbar is a tab bar with 'Playground \*' selected. The main area contains the following text:

```
1 % Use this editor as a MiniZinc scratch book
2 var 1..3: x;
3 var 1..3: y;
4 constraint x+y > 2;
5 solve satisfy;
6
```

In the bottom status bar, it says 'Ready.'

In order to solve the model, you click on the *Run* button in the toolbar, or use the keyboard shortcut *Ctrl+R* (or *command+R* on macOS):

The screenshot shows the MiniZinc IDE interface. The top bar displays the title "Playground — Untitled Project". Below the title is a toolbar with icons for New model, Open, Save, Copy, Cut, Paste, Undo, Redo, Shift left, Shift right, Run, and Stop. The main editor window titled "Playground \*" contains the following MiniZinc code:

```

1 % Use this editor as a MiniZinc scratch book
2 var 1..3: x;
3 var 1..3: y;
4 constraint x+y > 2;
5 solve satisfy;
6

```

The "Output" window below the editor shows the results of running the model:

```

Running untitled_model.mzn
x = 2;
y = 1;
-----
Finished in 240msec

```

At the bottom, a status bar indicates "Ready." on the left and "240msec" on the right.

As you can see, an output window pops up that displays a solution to the problem you entered. Let us now try a model that requires some additional data.

The screenshot shows the MiniZinc IDE interface. The top bar displays the title "Playground — Untitled Project". Below the title is a toolbar with icons for New model, Open, Save, Copy, Cut, Paste, Undo, Redo, Shift left, Shift right, Run, and Stop. The main editor window titled "Playground \*" contains the following MiniZinc code:

```

1 % Use this editor as a MiniZinc scratch book
2 int: n;
3 var 1..n: x;
4 var 1..n: y;
5 constraint x+y > n;
6 solve satisfy;
7

```

The "Output" window below the editor shows the results of running the model:

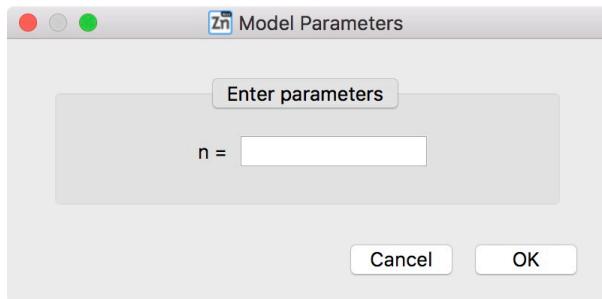
```

Running untitled_model.mzn
x = 2;
y = 1;
-----
Finished in 240msec

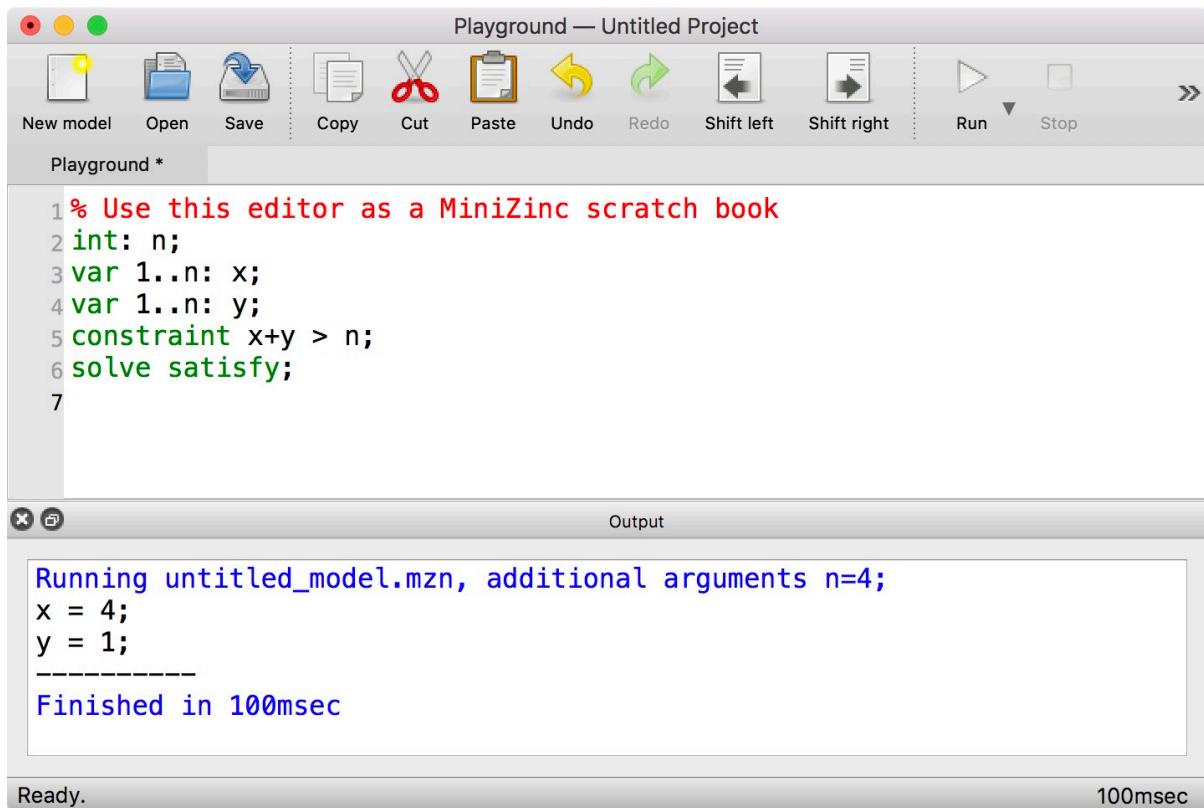
```

At the bottom, a status bar indicates "Ready." on the left and "240msec" on the right.

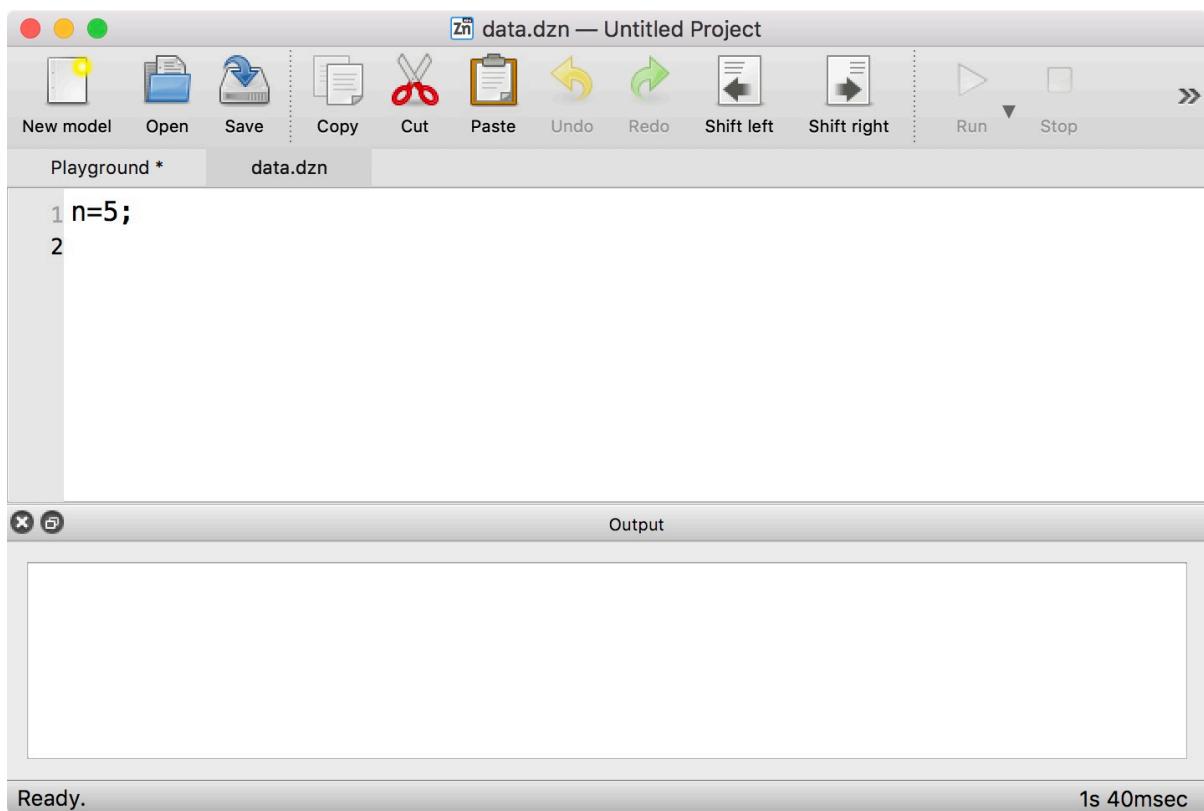
When you run this model, the IDE will ask you to enter a value for the parameter  $n$ :



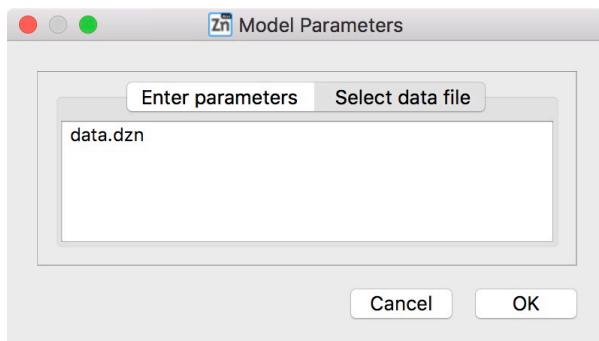
After entering, for example, the value 4 and clicking *Ok*, the solver will execute the model for  $n=4$ :



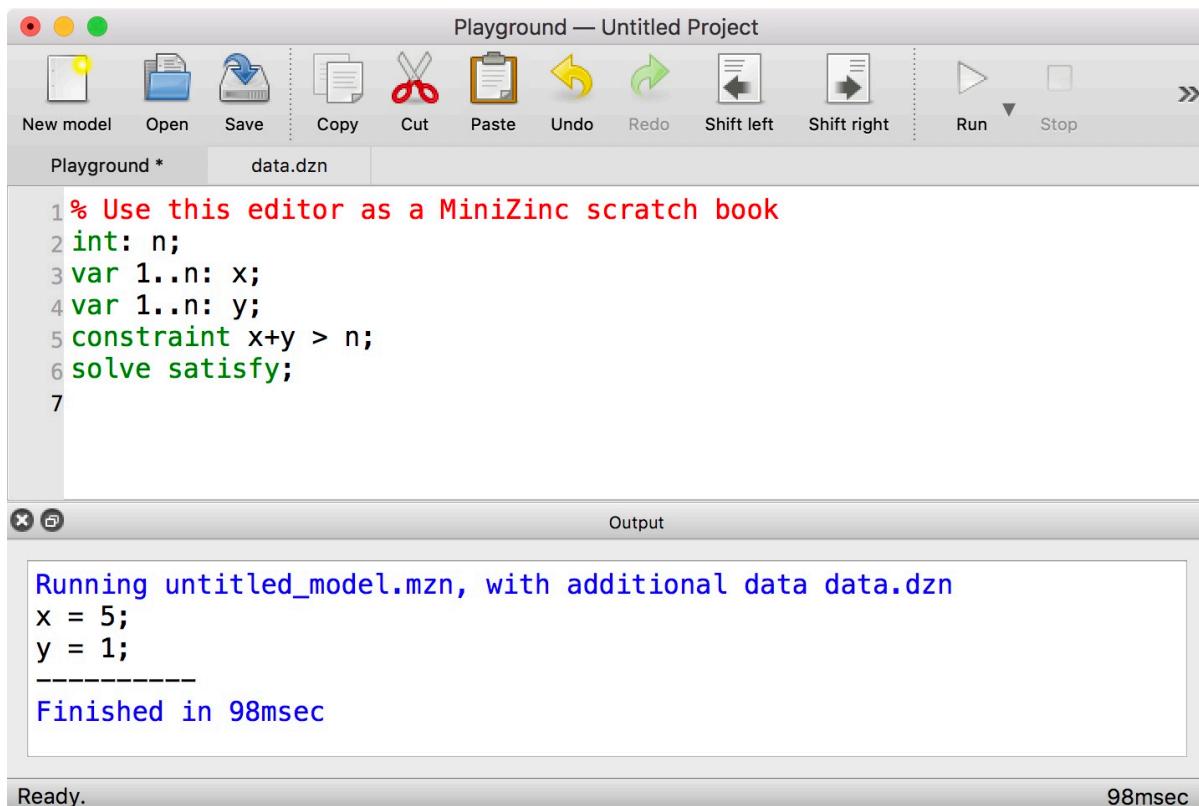
Alternatively, data can also come from a file. Let's create a new file with the data and save it as `data.dzn`:



When you now go back to the *Playground* tab and click *Run*, the IDE will give you the option to select a data file:



Click on the `data.dzn` entry, then on *Ok*, and the model will be run with the given data file:



Of course you can save your model to a file, and load it from a file, and the editor supports the usual functionality.

If you want to know more about the MiniZinc IDE, continue reading from [Section 3.2](#).

### 1.3.2 The MiniZinc command line tool

The MiniZinc command line tool, `minizinc`, combines the functionality of the MiniZinc compiler, different solver interfaces, and the MiniZinc output processor. After installing MiniZinc from the bundled binary distribution, you may have to set up your PATH in order to use the command line tool (see [Section 1.2](#)).

Let's assume we have a file `model.mzn` with the following contents:

```
var 1..3: x;
var 1..3: y;
constraint x+y > 3;
solve satisfy;
```

You can simply invoke `minizinc` on that file to solve the model and produce some output:

```
$ minizinc model.mzn
x = 3;
y = 1;
-----
$
```

If you have a model that requires a data file (like the one we used in the IDE example above), you pass both files to `minizinc`:

```
$ minizinc model.mzn data.dzn
x = 5;
y = 1;
-----
$
```

The `minizinc` tool supports numerous command line options. One of the most useful options is `-a`, which switches between *one solution* mode and *all solutions* mode. For example, for the first model above, it would result in the following output:

```
$ minizinc -a model.mzn
x = 3;
y = 1;
-----
x = 2;
y = 2;
-----
x = 3;
y = 2;
-----
x = 1;
y = 3;
-----
x = 2;
y = 3;
-----
x = 3;
y = 3;
-----
```

```
=====
```

```
$
```

To learn more about the `minizinc` command, explore the output of `minizinc --help` or continue reading in [Section 3.1](#).

## **Part 2**

# **Minizinc 指南**



# CHAPTER 2.1

## MiniZinc 基本模型

在此节中，我们利用两个简单的例子来介绍一个 MiniZinc 模型的基本结构。

### 2.1.1 第一个实例



Fig. 2.1.1: 澳大利亚各州

作为我们的第一个例子，假设我们要去给 Fig. 2.1.1 中的澳大利亚地图涂色。它包含了七个不同的州和地区，而每一块都要被涂一个颜色来保证相邻的区域有不同的颜色。

Listing 2.1.1: 一个用来给澳大利亚的州和地区涂色的 MiniZinc 模型 aust.mzn

```
% 用 nc 个颜色来涂澳大利亚
int: nc = 3;

var 1..nc: wa;  var 1..nc: nt;  var 1..nc: sa;  var 1..nc: q;
var 1..nc: nsw;  var 1..nc: v;  var 1..nc: t;

constraint wa != nt;
constraint wa != sa;
constraint nt != sa;
constraint nt != q;
constraint sa != q;
constraint sa != nsw;
constraint sa != v;
constraint q != nsw;
constraint nsw != v;
solve satisfy;

output ["wa=\\"(wa)\\"t nt=\\"(nt)\\"t sa=\\"(sa)\\"n",
       "q=\\"(q)\\"t nsw=\\"(nsw)\\"t v=\\"(v)\\"n",
       "t=", show(t), "\\"n"];
```

我们可以很容易的用 MiniZinc 给此问题建模。此模型在 Listing 2.1.1 中给出。

模型中的第一行是注释。注释开始于 % 来表明此行剩下的部分是注释。MiniZinc 同时也含有 C 语言风格的由 /\* 开始和 \*/ 结束的块注释。

模型中的下一部分声明了模型中的变量。此行

```
int: nc = 3;
```

定义了一个问题的参数来代表可用的颜色个数。在很多编程语言中，参数和变量是类似的。它们必须被声明并且指定一个类型 type。在此例中，类型是 int。通过赋值，它们被设了值。MiniZinc 允许变量声明时被赋值（就像上面那一行）或者单独给出一个赋值语句。因此下面的表示跟上面的只一行表示是相等的

```
int: nc;
nc = 3;
```

和很多编程语言中的变量不一样的是，这里的参数只可以被赋 唯一 的值。如果一个参数出现在了多于一个的赋值中，就会出现错误。

基本的 参数类型包括 整型 (`int`)，浮点型 (`float`)，布尔型 (`bool`) 以及 字符串型 (`string`)。同时也支持数组和集合。

MiniZinc 模型同时也可能包含另一种类型的变量 决策变量 。决策变量是数学的或者逻辑的变量。和一般的编程语言中的参数和变量不同，建模者不需要给决策变量一个值。而是在开始时，一个决策变量的值是不知道的。只有当 MiniZinc 模型被执行的时候，求解系统才来决定决策变量是否可以被赋值从而满足模型中的约束。若满足，则被赋值。

在我们的模型例子中，我们给每一个区域一个 决策变量 `wa`, `nt`, `sa`, `q`, `nsw`, `v` 和 `t`。它们代表了会被用来填充区域的（未知）颜色。

对于每一个决策变量，我们需要给出变量可能的取值集合。这个被称为变量的 定义域 。定义域部分可以在 变量声明 的时候同时给出，这时决策变量的 类型 就会从定义域中的数值的类型推断出。

MiniZinc 中的决策变量的类型可以为布尔型，整型，浮点型或者集合。同时也可是元素为决策变量的数组。在我们的 MiniZinc 模型例子中，我们使用整型去给不用的颜色建模。通过使用 `var` 声明，我们的每一个决策变量被声明为定义域为一个整数类型的范围表示 `1..nc`，来表明集合  $\{1, 2, \dots, nc\}$ 。所有数值的类型为整型，所以模型中的所有的变量是整型决策变量。

## 标识符

用来命名参数和变量的标识符是一列由大小写字母，数字以及下划线 `_` 字符组成的字符串。它们必须始于一个字母字符。因此 `myName_2` 是一个有效的标识符。MiniZinc (和 Zinc) 的 关键字 不允许被用为标识符名字。它们在 *Identifiers* (page 204) 中被列出。所有的 MiniZinc 操作符 都不能被用做标识符名字。它们在 *Operators* (page 220) 中被列出。

MiniZinc 仔细地区别了以下两种模型变量：参数和决策变量。利用决策变量创建的表达式类型比利用参数可以创建的表达式类型更局限。但是，在任何可以用决策变量的地方，同类型的参数变量也可以被应用。

## 整型变量声明

一个 整型参数变量可以被声明为以下两种方式：

```
int : <变量名>
<l> .. <u> : <变量名>
```

<l> 和 <u> 是固定的整型表达式。

一个整型决策变量被声明为以下两种方式：

```
var int : <变量名>
var <l>..<u> : <变量名>
```

<l> 和 <u> 是固定的整型表达式。

参数和决策变量形式上的区别在于对变量的 实例化。变量的实例化和类型的结合被叫为 **类型-实例化**。既然你已经开始使用 MiniZinc，毫无疑问的你会看到很多 **类型-实例化** 的错误例子。

模型的下一部分是 **约束**。它们详细说明了决策变量想要组成一个模型的有效解必须要满足的布尔型表达式。在这个例子中我们有一些决策变量之间的不等式。它们规定如果两个区域是相邻的，则它们必须有不同的颜色。

## 关系操作符

MiniZinc 提供了以下关系操作符 关系操作符：

相等 (= or ==), 不等 (!=), 小于 (<), 大于 (>), 小于等于 (<=), and 和大于等于 (>=).

模型中的下一行：

```
solve satisfy;
```

表明了它是什么类型的问题。在这个例子中，它是一个 满足问题：我们希望给决策变量找到一个值使得约束被满足，但具体是哪一个值却没有所谓。

模型的最后一个部分是 **输出** 语句。它告诉 MiniZinc 当模型被运行并且找到一个解 解的时候，要输出什么。

## 输出和字符串

一个输出语句跟着一串字符。它们通常或者是写在双引号之间的字符串常量，字符串常量并且对特殊字符用类似 C 语言的标记法，或者是 `show(e)` 格式的表达式，其中 `e` 是 MiniZinc 表达式。例子中的 `\n` 代表换行符，`\t` 代表制表符。

数字的 `show` 有各种不同方式的表示：`show_int(n,X)` 在至少  $|n|$  个字符里输出整型 `X` 的值，若  $n > 0$  则右对齐，否则则左对齐；`show_float(n,d,X)` 在至少  $|n|$  个字符里输出浮点型 `X` 的值，若  $n > 0$  则右对齐，否则则左对齐，并且小数点后有  $d$  个字符。

字符串常量必须在同一行中。长的字符串常量可以利用字符串连接符 `++` 来分成几行。例如，字符串常量

```
"Invalid datafile: Amount of flour is non-negative"
```

和字符串常量表达式

```
"Invalid datafile: " ++
  "Amount of flour is non-negative"
```

是相等的。

MiniZinc 支持内插字符串 内插字符串。表达式可以直接插入字符串常量中。`"\(\text{e})"` 形式的子字符串会被替代为 `show(e)`。例如，`"t=\(t)\n"` 产生和 `"t=" ++ show(t) ++ "\n"` 一样的字符串。

一个模型可以包含多个输出语句。在这种情况下，所有输出会根据它们在模型中出现的顺序连接。

我们可以通过点击 MiniZinc IDE 中的 *Run* 按钮，或者输入

```
$ minizinc --solver Gecode aust.mzn
```

来评估我们的模型。其中 `aust.mzn` 是包含我们的 MiniZinc 模型的文件名字。我们必须使用文件扩展名 `.mzn` 来表明一个 MiniZinc 模型。带有 `--solver Gecode` 选项的命令 `minizinc` 使用 Gecode 有限域求解器去评估我们的模型。如果你使用的是 MiniZinc 二进制发布，这个求解器实际上是预设的，所以你也可以运行 `minizinc aust.mzn`。

当我们运行上面的命令后，我们得到如下的结果：

```
wa=2    nt=3    sa=1
q=2    nsw=3    v=2
t=1
-----
```

10 个破折号 ----- 这行是自动被 MiniZinc 输出的，用来表明一个解已经被找到。

## 2.1.2 算术优化实例

我们的第二个例子来自于要为了本地的校园游乐园烤一些蛋糕的需求。我们知道如何制作两种蛋糕。footnote{警告：请不要在家里使用这些配方制作} 一个香蕉蛋糕的制作需要 250 克自发酵的面粉，2 个捣碎的香蕉，75 克糖和 100 克黄油。一个巧克力蛋糕的制作需要 200 克自发酵的面粉，75 克可可粉，150 克糖和 150 克黄油。一个巧克力蛋糕可以卖 \$4.50，一个香蕉蛋糕可以卖 \$4.00。我们一共有 4 千克的自发酵面粉，6 个香蕉，2 千克的糖，500 克的黄油和 500 克的可可粉。问题是每一种类型的蛋糕，我们需要烤多少来得到最大的利润。一个可能的 MiniZinc 模型在 Listing 2.1.2 中给出。

Listing 2.1.2: 决定为了校园游乐园要烤多少香蕉和巧克力蛋糕的模型。(cakes.mzn)

```
% 为校园游乐园做蛋糕

var 0..100: b; % 香蕉蛋糕的个数
var 0..100: c; % 巧克力蛋糕的个数

% 面粉
constraint 250*b + 200*c <= 4000;
% 香蕉
constraint 2*b <= 6;
% 糖
constraint 75*b + 150*c <= 2000;
% 黄油
constraint 100*b + 150*c <= 500;
% 可可粉
constraint 75*c <= 500;

% 最大化我们的利润
solve maximize 400*b + 450*c;

output ["no. of banana cakes = \$(b)\n",
        "no. of chocolate cakes = \$(c)\n"];
```

第一个新特征是 *arithmetic expressions* 的使用。

## 整数算术操作符

MiniZinc 提供了标准的整数算术操作符。加 (+), 减 (-), 乘 (\*), 整数除 (div) 和整数模 (mod). 同时也提供了一元操作符 + 和 -。

整数模被定义为输出和被除数  $a$  一样正负的  $a \bmod b$  值。整数除被定义为使得  $a = b(a \text{ div } b) + (a \bmod b)$  值。

MiniZinc 提供了标准的整数函数用来取绝对值 (abs) 和幂函数 (pow). 例如 `abs(-4)` 和 `pow(2, 5)` 分别求得数值 4 和 32。

算术常量的语法是相当标准的。整数常量可以是十进制, 十六进制或者八进制。例如 0, 5, 123, `0x1b7`, `0o777`。

例子中的第二个新特征是优化。这行

```
solve maximize 400 * b + 450 * c;
```

指出我们想找一个可以使 `solve` 语句中的表达式（我们叫做 **目标**）最大化的解。这个目标可以为任何类型的算术表达式。我们可以把关键字 `maximize` 换为 `minimize` 来表明一个最小化问题。

当我们运行上面这个模型时，我们得到以下的结果：

```
no. of banana cakes = 2
no. of chocolate cakes = 2
-----
=====
```

一旦系统证明了一个解是最优解，这一行 ===== 在最优化问题中会自动被输出。

## 2.1.3 数据文件和谓词

此模型的一个缺点是如果下一次我们希望解决一个相似的问题，即我们需要为学校烤蛋糕（这是经常发生的），我们需要改变模型中的约束来表明食品间拥有的原料数量。如果我们想重新利用此模型，我们最好使得每个原料的数量作为模型的参数，然后在模型的最上层设置它们的值。

更好的办法是在一个单独的 **数据文件** 中设置这些参数的值。MiniZinc（就像很多其他的建模语言一样）允许使用数据文件来设置在原始模型中声明的参数的值。通过运行不同的数据文件，使得同样的模型可以很容易地和不同的数据一起使用。

数据文件的文件扩展名必须是 `.dzn`，来表明它是一个 MiniZinc 数据文件。一个模型可以被任何一个数据文件运行（但是每个变量/参数在每个文件中只能被赋一个值）

Listing 2.1.3: 独立于数据的用来决定为了校园游乐会要烤多少香蕉和巧克力蛋糕的模型。  
(cakes2.mzn)

```
% 为校园游乐会做蛋糕 (和数据文件一起)

int: flour; % 拥有的面粉克数
int: banana; % 拥有的香蕉个数
int: sugar; % 拥有的糖克数
int: butter; % 拥有的黄油克数
int: cocoa; % 拥有的可可粉克数

constraint assert(flour >= 0, "Invalid datafile: " ++
                  "Amount of flour should be non-negative");
constraint assert(banana >= 0, "Invalid datafile: " ++
                  "Amount of banana should be non-negative");
constraint assert(sugar >= 0, "Invalid datafile: " ++
                  "Amount of sugar should be non-negative");
constraint assert(butter >= 0, "Invalid datafile: " ++
                  "Amount of butter should be non-negative");
constraint assert(cocoa >= 0, "Invalid datafile: " ++
                  "Amount of cocoa should be non-negative");

var 0..100: b; % 香蕉蛋糕的个数
var 0..100: c; % 巧克力蛋糕的个数

% 面粉
constraint 250*b + 200*c <= flour;
% 香蕉
constraint 2*b <= banana;
% 糖
constraint 75*b + 150*c <= sugar;
% 黄油
constraint 100*b + 150*c <= butter;
% 可可粉
constraint 75*c <= cocoa;

% 最大化我们的利润
solve maximize 400*b + 450*c;
```

```
output ["no. of banana cakes = \$(b)\n",
        "no. of chocolate cakes = \$(c)\n"];
```

我们的新模型在 Listing 2.1.3 中给出。我们可以用下面的命令来运行

数据文件 `pantry.dzn` 在 Listing 2.1.4 中给出。我们得到和 `cakes.mzn` 同样的结果。运行下面的命令

```
$ minizinc cakes2.mzn pantry2.dzn
```

利用另外一个 Listing 2.1.5 中定义的数据集，我们得到如下结果

```
no. of banana cakes = 3
no. of chocolate cakes = 8
-----
=====
```

如果我们从 `cakes.mzn` 中去掉输出语句，Minizinc 会使用默认的输出。这种情况下得到的输出是

```
b = 3;
c = 8;
-----
=====
```

## 默认输出

一个没有输出语句的 MiniZinc 模型会给每一个决策变量以及它的值一个输出行，除非决策变量已经在声明的时候被赋了一个表达式。注意观察此输出是如何呈现一个正确的数据文件格式的。

### Listing 2.1.4: `cakes2.mzn` 的数据文件例子 (`pantry.dzn`)

```
flour = 4000;
banana = 6;
sugar = 2000;
butter = 500;
cocoa = 500;
```

## Listing 2.1.5: cakes2.mzn 的数据文件例子 (pantry2.dzn)

```
flour = 8000;
banana = 11;
sugar = 3000;
butter = 1500;
cocoa = 800;
```

通过使用 命令行标示 `-D string`，小的数据文件可以被直接输入而不是必须要创建一个 `.dzn` 文件，其中 `string` 是数据文件里面的内容。

```
$ minizinc cakes2.mzn -D \
"flour=4000;banana=6;sugar=2000;butter=500;cocoa=500;"
```

会给出和

```
$ minizinc cakes2.mzn pantry.dzn
```

一模一样的结果。

数据文件只能包含给模型中的决策变量和参数赋值的语句。

防御性编程建议我们应该检查数据文件中的数值是否合理。在我们的例子中，检查所有原料的份量是否是非负的并且若不正确则产生一个运行错误，这是明智的。MiniZinc 提供了一个内置的布尔型操作符 `断言` 用来检查参数值。格式是 `assert(B,S)`。布尔型表达式 `B` 被检测。若它是假的，运行中断。此时字符串表达式 `S` 作为错误信息被输出。如果我们想当面粉的份量是负值的时候去检测出并且产生合适的错误信息，我们可以直接加入下面的一行

```
constraint assert(flour >= 0, "Amount of flour is non-negative");
```

到我们的模型中。注意 `断言` 表达式是一个布尔型表达式，所以它被看做是一种类型的约束。我们可以加入类似的行来检测其他原料的份量是否是非负值。

## 2.1.4 实数求解

MiniZinc also supports “real number” constraint solving using floating point variables and constraints. Consider a problem of taking out a short loan for one year to be repaid in 4 quarterly

instalments. A model for this is shown in Listing 2.1.6. It uses a simple interest calculation to calculate the balance after each quarter.

通过使用浮点数求解, MiniZinc 也支持“实数”约束求解。考虑一个要在 4 季度分期偿还的一年短期贷款问题。此问题的一个模型在 Listing 2.1.6 中给出。它使用了一个简单的计算每季度结束后所欠款的利息计算方式。

Listing 2.1.6: 确定一年借款每季度还款关系的模型 (loan.mzn)

```
% 变量
var float: R;           % 季度还款
var float: P;           % 初始借贷本金
var 0.0 .. 10.0: I;     % 利率

% 中间变量
var float: B1; % 一个季度后的欠款
var float: B2; % 两个季度后的欠款
var float: B3; % 三个季度后的欠款
var float: B4; % 最后欠款

constraint B1 = P * (1.0 + I) - R;
constraint B2 = B1 * (1.0 + I) - R;
constraint B3 = B2 * (1.0 + I) - R;
constraint B4 = B3 * (1.0 + I) - R;

solve satisfy;

output [
  "Borrowing ", show_float(0, 2, P), " at ", show(I*100.0),
  "% interest, and repaying ", show_float(0, 2, R),
  "\nper quarter for 1 year leaves ", show_float(0, 2, B4), " owing\n"
];
```

注意我们声明了一个浮点型变量 `f`, 它和整型变量很类似。只是这里我们使用关键字 `float` 而不是 `int`。

我们可以用同样的模型来回答一系列不同的问题。第一个问题是: 如果我以利息 4% 借款 \$1000 并且每季度还款 \$260, 我最终还欠款多少? 这个问题在数据文件 `loan1.dzn` 中被编码。

由于我们希望用实数求解, 我们需要使用一个可以支持这种问题类型的求解器。Gecode(Minizinc 绑定二进制发布预设的求解器) 支持浮点型变量, 一个混合整数线性求解

器可能更加适合这种类型的问题。MiniZinc 发布包含了这样的一个求解器。我们可以通过从求解器 IDE 菜单 (*Run* 按钮下面的三角形) 选择 OSICBC 来使用, 或者在命令行中运行命令 `minizinc --solver osicbc`:

```
$ minizinc --solver osicbc loan.mzn loan1.dzn
```

输出是

```
Borrowing 1000.00 at 4.0% interest, and repaying 260.00
per quarter for 1 year leaves 65.78 owing
-----
```

第二个问题是如果我希望用 4% 的利息来借款 \$1000 并且在最后的时候一点都不欠款, 我需要在每季度还款多少? 这个问题在数据文件 `loan2.dzn` 中被编码。运行命令

```
$ minizinc --solver osicbc loan.mzn loan2.dzn
```

后的输出是

```
Borrowing 1000.00 at 4.0% interest, and repaying 275.49
per quarter for 1 year leaves 0.00 owing
-----
```

第三个问题是如果我可以每个季度返还 \$250, 我可以用 4% 的利息来借款多少并且在最后的时候一点都不欠款? 这个问题在数据文件 `loan3.dzn` 中被编码。运行命令

```
$ minizinc --solver osicbc loan.mzn loan3.dzn
```

后的输出是

```
Borrowing 907.47 at 4.0% interest, and repaying 250.00
per quarter for 1 year leaves 0.00 owing
-----
```

#### Listing 2.1.7: `loan.mzn` 的数据文件例子 (`loan1.dzn`)

```
I = 0.04;
P = 1000.0;
```

```
R = 260.0;
```

#### Listing 2.1.8: loan.mzn 的数据文件例子 (loan2.dzn)

```
I = 0.04;
P = 1000.0;
B4 = 0.0;
```

#### Listing 2.1.9: loan.mzn 的数据文件例子 (loan3.dzn)

```
I = 0.04;
R = 250.0;
B4 = 0.0;
```

### 浮点算术操作符

MiniZinc 提供了标准的浮点算术操作符: 加 (+), 减 (-), 乘 (\*) 和浮点除 (/)。同时也提供了一元操作符 + 和 -。

MiniZinc 不会自动地强制转换整数为浮点数。内建函数 `int2float` 被用来达到此目的。注意强制转换的一个后果是表达式 `a / b` 总是被认为是一个浮点除。如果你需要一个整数除, 请确定使用 `div` 操作符。

MiniZinc 同时也包含浮点型函数来计算: 绝对值 (`abs`), 平方根 (`sqrt`), 自然对数 (`ln`), 底数为 2 的对数 (`log2`), 底数为 10 的对数 (`log10`), `$e$` 的幂 (`exp`), 正弦 (`sin`), 余弦 (`cos`), 正切 (`tan`), 反正弦 (`asin`), 反余弦 (`acos`), 反正切 (`atan`), 双曲正弦 (`sinh`), 双曲余弦 (`cosh`), 双曲正切 (`tanh`), 双曲反正弦 (`asinh`), 双曲反余弦 (`acosh`), 双曲反正切 (`atanh`), 和唯一的二元函数次方 (`pow`), 其余的都是一元函数。

算术常量的语法是相当标准的。浮点数常量的例子有 `1.05`, `1.3e-5` 和 `1.3E+5`。

## 2.1.5 模型的基本结构

我们现在可以去总结 MiniZinc 模型的基本结构了。它由多个项组成, 每一个在其最后都有一个分号 ;。项可以按照任何顺序出现。例如, 标识符在被使用之前不需要被声明。

有八种类型的项 items 。

- 引用项允许另外一个文件的内容被插入模型中。它们有以下形式:

```
include <文件名>;
```

其中 <文件名> 是一个字符串常量。它们使得大的模型可以被分为小的子模型以及包含库文件中定义的约束。我们会在 Listing 2.2.4 中看到一个例子。

- 变量声明声明新的变量。这种变量是全局变量，可以在模型中的任何地方被提到。变量有两种。在模型中被赋一个固定值的参数变量以及只有在模型被求解的时候才会被赋值的决策变量。我们称参数是 固定的，决策变量是 不固定的。变量可以选择性地被赋一个值来作为声明的一部分。形式是：

```
<类型-实例化 表达式>: <变量> [ = ] <表达式>;
```

<类型-实例化 表达式> 给了变量的类型和实例化。这些是 MiniZinc 比较复杂的其中一面。用 `par` 来实例化声明参数，用 `var` 来实例化声明决策变量。如果没有明确的实例化声明，则变量是一个参数。类型可以为基类型，一个 整数或者浮点数范围，或者一个数组或集合。基类型有 `float`, `int`, `string`, `bool`, `ann`。其中只有 `float`, `int` and `bool` 可以被决策变量使用。基类型 `ann` 是一个注解 – 我们会在 [搜索 \(page 97\)](#) 中讨论注解。整数范围表达式可以被用来代替类型 `int`. 类似的，浮点数范围表达式可以被用来代替类型 `float`。这些通常被用来定义一个整型决策变量的定义域，但也可以被用来限制一个整型参数的范围。变量声明的另外一个用处是定义 枚举类型，—我们在 [枚举类型 \(page 56\)](#) 中讨论。

- 赋值项给一个变量赋一个值。它们有以下形式：

```
<变量> = <表达式>;
```

数值可以被赋给决策变量。在这种情况下，赋值相当于加入 `constraint <变量> = <表达式>;`

- 约束项是模型的核心。它们有以下形式：

```
constraint <布尔型表达式>;
```

我们已经看到了使用算术比较的简单约束以及内建函数 `assert` 操作符。在下一节我们会看到更加复杂的约束例子。

- 求解项详细说明了到底要找哪种类型的解。正如我们看到的，它们有以下三种形式：

```
solve satisfy;
solve maximize <算术表达式>;
solve minimize <算术表达式>;
```

一个模型必须有且只有一个求解项。

- 输出项用来恰当的呈现模型运行后的结果。它们有下面的形式：

```
output [ <字符串表达式>, ..., <字符串表达式> ];
```

如果没有输出项， MiniZinc 会默认输出所有没有被以赋值项的形式赋值的决策变量值。

- 枚举类型声明. 我们会在[数组和集合](#) (page 41) 和[枚举类型](#) (page 56) 中讨论。
- 谓词函数和测试项被用来定义新的约束, 函数和布尔测试。我们会在[谓词和函数](#) (page 71) 中讨论。
- 注解项用来定义一个新的注解。我们会在[搜索](#) (page 97) 中讨论。



# CHAPTER 2.2

---

## 更多复杂模型

---

在上一节中，我们介绍了 MiniZinc 模型的基本结构。在这一节中，我们介绍数组和集合数据结构，枚举类型，以及更加复杂的约束。

### 2.2.1 数组和集合

在绝大多数情况下，我们都是有兴趣建一个约束和变量的个数依赖于输入数据的模型。为了达到此目的，我们通常会使用 数组。

考虑一个关于金属矩形板温度的简单有限元素模型。通过把矩形板在 2 维的矩阵上分成有限个的元素，我们近似计算矩形板上的温度。一个模型在 Listing 2.2.1 中给出。它声明了有限元素模型的宽  $w$  和高  $h$ 。

声明

```
set of int: HEIGHT = 0..h;
set of int: CHEIGHT = 1..h-1;
set of int: WIDTH = 0..w;
set of int: CWIDTH = 1..w-1;
array[HEIGHT,WIDTH] of var float: t; % 在点 (i,j) 处的温度
```

声明了四个固定的整型集合来描述有限元素模型的尺寸：HEIGHT 是整个模型的整体高度，而 CHEIGHT 是省略了顶部和底部的中心高度，WIDTH 是模型的整体宽度，而 CWIDTH 是省略了左侧

和右侧的中心宽度。最后，声明了一个浮点型变量组成的行编号从 0 到  $w$ ，列编号从 0 到  $h$  的二维数组  $t$  用来表示金属板上每一点的温度。我们可以用表达式  $t[i, j]$  来得到数组中第  $i^{th}$  行和第  $j^{th}$  列的元素。

拉普拉斯方程规定当金属板达到一个稳定状态时，每一个内部点的温度是它的正交相邻点的平均值。约束

```
constraint forall(i in CHEIGHT, j in CWIDTH)(
    4.0*t[i,j] = t[i-1,j] + t[i,j-1] + t[i+1,j] + t[i,j+1]);
```

保证了每一个内部点  $(i, j)$  是它的四个正交相邻点的平均值。约束

```
% 边约束
constraint forall(i in CHEIGHT)(t[i,0] = left);
constraint forall(i in CHEIGHT)(t[i,w] = right);
constraint forall(j in CWIDTH)(t[0,j] = top);
constraint forall(j in CWIDTH)(t[h,j] = bottom);
```

限制了每一个边的温度必须是相等的，并且给了这些温度名字：left，right，top 和 bottom。而约束

```
% 角约束
constraint t[0,0]=0.0;
constraint t[0,w]=0.0;
constraint t[h,0]=0.0;
constraint t[h,w]=0.0;
```

确保了角的温度（这些是不相干的）被设置为 0.0。我们可以用 Listing 2.2.1 中给出的模型来决定一个被分成  $5 \times 5$  个元素的金属板的温度。其中左右下侧的温度为 0，上侧的温度为 100。

Listing 2.2.1: 决定稳定状态温度的有限元平板模型 (laplace.mzn).

```
int: w = 4;
int: h = 4;

% arraydec
set of int: HEIGHT = 0..h;
set of int: CHEIGHT = 1..h-1;
set of int: WIDTH = 0..w;
set of int: CWIDTH = 1..w-1;
```

```

array[HEIGHT,WIDTH] of var float: t; % 在点 (i,j) 处的温度
var float: left;    % 左侧温度
var float: right;   % 右侧温度
var float: top;     % 顶部温度
var float: bottom;  % 底部温度

% 拉普拉斯方程：每一个内部点温度是它相邻点的平均值
constraint forall(i in CHEIGHT, j in CWIDTH)(
    4.0*t[i,j] = t[i-1,j] + t[i,j-1] + t[i+1,j] + t[i,j+1]);
}

% sides
% 边约束
constraint forall(i in CHEIGHT)(t[i,0] = left);
constraint forall(i in CHEIGHT)(t[i,w] = right);
constraint forall(j in CWIDTH)(t[0,j] = top);
constraint forall(j in CWIDTH)(t[h,j] = bottom);

% 角约束
constraint t[0,0]=0.0;
constraint t[0,w]=0.0;
constraint t[h,0]=0.0;
constraint t[h,w]=0.0;
left = 0.0;
right = 0.0;
top = 100.0;
bottom = 0.0;

solve satisfy;

output [ show_float(6, 2, t[i,j]) ++
    if j == h then "\n" else " " endif |
    i in HEIGHT, j in WIDTH
];

```

运行命令

```
$ minizinc --solver osicbc laplace.mzn
```

得到输出

```

0.00 100.00 100.00 100.00 0.00
0.00 42.86 52.68 42.86 0.00
0.00 18.75 25.00 18.75 0.00
0.00 7.14 9.82 7.14 0.00
0.00 0.00 0.00 0.00 0.00
-----

```

## 集合

集合变量用以下方式声明

```
set of <类型-实例化> : <变量名> ;
```

整型，枚举型（参见后面），浮点型和布尔型集合都可以定义。决策变量集合只可以是类型为整型或者枚举型的变量集合。集合常量有以下形式

```
{ <表达式-1>, ..., <表达式-n> }
```

或者是以下形式的整型，枚举型或浮点型 范围表达式

```
<表达式-1> .. <表达式-2>
```

标准的集合操作符有：元素属于 (`in`)，(非严格的) 集合包含 (`subset`)，(非严格的) 超集关系 (`superset`)，并集 (`union`)，交集 (`intersect`)，集合差运算 (`diff`)，集合对称差 (`symdiff`) 和集合元素的个数 (`card`)。

我们已经看到集合变量和集合常量（包含范围）可以被用来作为变量声明时的隐式类型。在这种情况下变量拥有集合元素中的类型并且被隐式地约束为集合中的一个成员。

我们的烤蛋糕问题是一个非常简单的批量生产计划问题例子。在这类问题中，我们希望去决定每种类型的产品要制造多少来最大化利润。同时制造一个产品会消耗不同数量固定的资源。我们可以扩展 Listing 2.1.3 中的 MiniZinc 模型为一个不限制资源和产品类型的模型去处理这种类型的问题。这个模型在 Listing 2.2.2 中给出。一个（烤蛋糕问题的）数据文件例子在 Listing 2.2.3 中给出。

Listing 2.2.2: 简单批量生产计划模型 (`simple-prod-planning.mzn`).

```
% 要制造的产品
enum Products;
% 每种产品的单位利润
array[Products] of int: profit;
% 用到的资源
```

```

enum Resources;
% 每种资源可获得的数量
array[Resources] of int: capacity;

% 制造一个单位的产品需要的资源单位量
array[Products, Resources] of int: consumption;
constraint assert(forall (r in Resources, p in Products)
  (consumption[p,r] >= 0), "Error: negative consumption");

% 产品数量的界
int: mproducts = max (p in Products)
  (min (r in Resources where consumption[p,r] > 0)
  (capacity[r] div consumption[p,r]));

% 变量: 每种产品我们需要制造多少
array[Products] of var 0..mproducts: produce;
array[Resources] of var 0..max(capacity): used;

% 产量不可以使用超过可获得的资源量:
constraint forall (r in Resources) (
  used[r] = sum (p in Products)(consumption[p, r] * produce[p])
);
constraint forall (r in Resources) (
  used[r] <= capacity[r]
);

% 最大化利润
solve maximize sum (p in Products) (profit[p]*produce[p]);

output [
  "\(p) = \$(produce[p]);\n" | p in Products ] ++
  [ "\(r) = \$(used[r]);\n" | r in Resources ];

```

Listing 2.2.3: 简单批量生产计划模型的数据文件例子  
(`simple-prod-planning-data.dzn`).

```

% 简单批量生产计划模型的数据文件
Products = { BananaCake, ChocolateCake };
profit = [400, 450]; % 以分为单位

```

```
Resources = { Flour, Banana, Sugar, Butter, Cocoa };
capacity = [4000, 6, 2000, 500, 500];

consumption= [| 250, 2, 75, 100, 0,
               | 200, 0, 150, 150, 75 |];
```

这个模型的新特征是只用枚举类型 枚举类型。这使得我们可以把资源和产品的选择作为模型的参数。模型的第一个项

```
enum Products;
```

声明 `Products` 为 未知的产品集合。

## 枚举类型

枚举类型，我们称为 `enums`，用以下方式声明

```
enum <变量名> ;
```

一个枚举类型用以下赋值的方式定义

```
enum <变量名> = { <变量名-1>, ..., <变量名-n> } ;
```

其中 `<变量名-1>`, ..., `<变量名-n>` 是名为 `<变量名>` 的枚举类型中的元素。通过这个定义，枚举类型中的每个元素也被有效地声明为这个类型的一个新的常量。声明和定义可以像往常一样结合为一行。

第二个项声明了一个整型数组：

```
array[Products] of int: profit;
```

`profit` 数组的下标集合是 `Products`。理想情况下，这种声明方式表明只有集合 `Products` 中的元素才能被用来做数组的下标。

有  $n$  个元素组成的枚举类型中的元素的行为方式和整数  $1 \dots n$  的行为方式很像。它们可以被比较，它们可以按照它们出现在枚举类型定义中的顺序被排序，它们可以遍历，它们可以作为数组的下标，实际上，它们可以出现在一个整数可以出现的任何地方。

在数据文件例子中，我们用一列整数来初始化数组

```
Products = { BananaCake, ChocolateCake };
profit = [400, 450];
```

意思是香蕉蛋糕的利润是 400，而巧克力蛋糕的利润是 450。在内部，BananaCake 会被看成是像整数 1 一样，而 ChocolateCake 会被看成像整数 2 一样。MiniZinc 虽然不提供明确的列表类型，但用 `1..n` 为下标集合的一维数组表现起来就像列表。我们有时候也会称它们为列表 lists。

根据同样的方法，接下来的两项中我们声明了一个资源集合 Resources，一个表明每种资源可获得量的数组 capacity。

更有趣的是项

```
array[Products, Resources] of int: consumption;
```

声明了一个二维数组 consumption。consumption[p,r] 的值是制造一单位的产品 p 所需要的资源 r 的数量。其中第一个下标是行下标，而第二个下标是列下标。

数据文件包含了一个二维数组的初始化例子：

```
consumption= [| 250, 2, 75, 100, 0,
               | 200, 0, 150, 150, 75 |];
```

注意分隔符 | 是怎样被用来分隔行的。

## 数组

因此, MiniZinc 提供一维和多维数组。它们用以下类型来声明:

```
array [ <下标集合-1>, ..., <下标集合-n> ] of <类型-实例化>
```

MiniZinc 要求数组声明要给出每一维的下标集合。下标集合或者是一个整型范围, 一个被初始化为整型范围的集合变量, 或者是一个枚举类型。数组可以是所有的基类型: 整型, 枚举型, 布尔型, 浮点型或者字符串型。这些可以是固定的或者不固定的, 除了字符串型, 它只可以是参数。数组也可以作用于集合但是不可以作用于数组。

一维数组常量有以下格式

```
[ <表达式-1>, ..., <表达式-n> ]
```

而二维数组常量有以下格式

```
[| <表达式-1-1>, ..., <表达式-1-n> |
 ...
 | <表达式-m-1>, ..., <表达式-m-n> |]
```

其中这个数组有  $m$  行  $n$  列。

内建函数 `array1d`, `array2d` 等家族可以被用来从一个列表 (或者更准确的说是一个一维数组) 去实例化任何维度的数组。调用

```
array<n>d(<下标集合-1>, ..., <下标集合-n>, <列表>)
```

返回一个  $n$  维的数组, 它的下标集合在前  $n$  个参数给出, 最后一个参数包含了数组的元素。例如 `array2d(1..3, 1..2, [1, 2, 3, 4, 5, 6])` 和 `[|1, 2 |3, 4 |5, 6|]` 是相等的。

数组元素按照通常的方式获取: `a[i, j]` 给出第  $i^{th}$  行第  $j^{th}$  列的元素。

串联操作符 `++` 可以被用来串联两个一维的数组。结果得到一个列表, 即一个元素从 1 索引的一维数组。例如 `[4000, 6] ++ [2000, 500, 500]` 求得 `[4000, 6, 2000, 500, 500]`。内建函数 `length` 返回一维数组的长度。

模型的下一项定义了参数 `mproducts`。它被设为可以生产出的任何类型产品的数量上限。这个确实是一个复杂的内嵌数组推导式和聚合操作符例子。在我们试图理解这些项和剩下的模型之前, 我们应该先介绍一下它们。

首先, MiniZinc 提供了在很多函数式编程语言都提供的列表推导式。例如, 列表推导式 `[i + j | i, j in 1..3 where j < i]` 算得 `[1 + 2, 1 + 3, 2 + 3]` 等同于 `[3, 4, 5]`。`[3, 4, 5]` 只是一个下标集合为 `1..3` 的数组。

MiniZinc 同时也提供了集合推导式, 它有类似的语法: 例如

`{i + j | i, j in 1..3 where j < i}` 计算得到集合 {3, 4, 5}。

## 列表和集合推导式

列表推导式的一般格式是

```
[ <表达式> | <生成元表达式> ]
```

<表达式> 指明了如何从 <生成元表达式> 产生的元素输出列表中创建元素。生成元 <generator-exp> 由逗号分开的一列生成元表达式组成，选择性地跟着一个布尔型表达式。两种格式是

```
<生成元>, ..., <生成元>
<生成元>, ..., <生成元> where <布尔表达式>
```

第二种格式中的可选择的 <布尔型表达式> 被用作生成元表达式的过滤器：只有满足布尔型表达式的输出列表中的元素才被用来构建元素。生成元 <generator> 有以下格式

```
<标识符>, ..., <标识符> in <数组表达式>
```

每一个标识符是一个迭代器，轮流从数值表达式中取值，最后一个标识符变化的最迅速。列表推导式的生成元和 <布尔型表达式> 通常不会涉及决策变量。如果它们确实涉及了决策变量，那么产生的列表是一列 `var opt <T>`，其中 \$T\$ 是 <表达式> 的类型。更多细节，请参考 [选项类型 \(page 93\)](#) 中有关选项类型 option types 的论述。

集合推导式几乎和列表推导式一样：唯一的不同是这里使用 { 和 } 括住表达式而不是 [ 和 ]。集合推导式生成的元素必须是固定的 fixed，即不能是决策变量。类似的，集合推导式的生成元和可选择的 <布尔型表达式> 必须是固定的。

第二， MiniZinc 提供了一系列的可以把一维数组的元素聚合起来的内建函数。它们中最有用的可能是 `:mzn:forall`。它接收一个布尔型表达式数组（即，约束），返回单个布尔型表达式，它是对数组中的布尔型表达式的逻辑合取。

例如，以下表达式

```
forall( [a[i] != a[j] | i,j in 1..3 where i < j])
```

其中 `a` 是一个下标集合为 1..3 的算术数组。它约束了 `a` 中的元素是互相不相同的。列表推导式计算得到 `[ a[1] != a[2], a[1] != a[3], a[2] != a[3] ]`，所以 `forall` 函数返回逻辑合取 `a[1] != a[2] /\ a[1] != a[3] /\ a[2] != a[3]`。

## 聚合函数

算术数组的 聚合函数 有: `sum` 把元素加起来, `product` 把元素乘起来, 和 `min` 跟 `max` 各自返回数组中的最小和最大元素。当作用于一个空的数组时, `min` 和 `max` 返回一个运行错误, `sum` 返回 0, `product` 返回 1。

MiniZinc 为数组提供了包含有布尔型表达式的四个聚合函数。正如我们看到的, 它们中的第一个是 `forall`, 它返回一个等于多个约束的逻辑合取的单个约束。第二个函数, `exists`, 返回多个约束的逻辑析取。因此 `forall` 强制数组中的所有约束都满足, 而 `exists` 确保至少有一个约束满足。第三个函数, `xorall` 确保奇数个约束满足。第四个函数, `iffall` 确保偶数个约束满足。

第三个, 也是难点的最后一个部分是当使用数组推导式时, MiniZinc 允许使用一个特别的聚合函数的语法。建模者不仅仅可以用

```
forall( [a[i] != a[j] | i,j in 1..3 where i < j])
```

也可以用一个更加数学的表示

```
forall (i,j in 1..3 where i < j) (a[i] != a[j])
```

两种表达方式是完全相等的: 建模者可以自由使用任何一个他们认为更自然的表达方式。

## 生成表达式

一个 生成表达式 有以下格式

```
<聚合函数> (<生成元表达式>) (<表达式>)
```

圆括号内的 `<生成元表达式>` 以及构造表达式 `<表达式>` 是非选择性的: 它们必须存在。它等同于

```
<聚合函数> ([<表达式> | <生成元表达式> ] )
```

`<聚合函数>` aggregation function 可以是 MiniZinc 的任何由单个数组作为其参数的函数。

接下来我们就来了解 Listing 2.2.2 中的简单批量生产计划模型剩余的部分。现在请暂时忽略定义 `mproducts` 的这部分。接下来的项:

```
array[Products] of var 0..mproducts: produce;
```

定义了一个一维的决策变量数组 `produce`。`produce[p]` 的值代表了最优解中产品 `p` 的数量。下一项

```
array[Resources] of var 0..max(capacity): used;
```

定义了一个辅助变量集合来记录每一种资源的使用量。下面的两个约束

```
constraint forall (r in Resources)
    (used[r] = sum (p in Products) (consumption[p, r] * produce[p]));
constraint forall (r in Resources)(used[r] <= capacity[r]);
```

使用 `used[r]` 计算资源 `r` 的总体消耗以及保证它是少于可获得的资源 `r` 的量。最后，项

```
solve maximize sum (p in Products) (profit[p]*produce[p]);
```

表明这是一个最大化问题以及最大化的目标是全部利润。

现在我们回到 `mproducts` 的定义。对每个产品 `p`，表达式

```
(min (r in Resources where consumption[p,r] > 0)
      (capacity[r] div consumption[p,r]))
```

决定了在考虑了每种资源 `r` 的数量以及制造产品 `p` 需要的 `r` 量的情况下，`p` 可以生产的最大量。注意过滤器 `where consumption[p,r] > 0` 的使用保证了只有此产品需要的资源才会考虑，因此避免了出现除数为零的错误。所以，完整的表达式

```
int: mproducts = max (p in Products)
    (min (r in Resources where consumption[p,r] > 0)
          (capacity[r] div consumption[p,r]));
```

计算 任何 产品可以被制造的最大量，因此它可以被作为 `produce` 中的决策变量定义域的上限。

最后，注意输出项是比较复杂的，并且使用了列表推导式 列表推导式去创建一个易于理解的输出。运行

```
$ minizinc --solver gecode simple-prod-planning.mzn simple-prod-planning-
→data.dzn
```

输出得到如下结果

```
BananaCake = 2;
```

```
ChocolateCake = 2;
```

```
Flour = 900;
```

```
Banana = 4;
```

```
Sugar = 450;
```

```
Butter = 500;
```

```
Cocoa = 150;
```

```
-----
```

```
=====
```

## 2.2.2 全局约束

MiniZinc 包含了一个全局约束的库，这些全局约束也可以被用来定义模型。一个例子是 `alldifferent` 约束，它要求所有参数中的变量都必须是互相不相等的。

Listing 2.2.4: SEND+MORE=MONEY 算式谜题模型 (`send-more-money.mzn`)

```
include "alldifferent.mzn";

var 1..9: S;
var 0..9: E;
var 0..9: N;
var 0..9: D;
var 1..9: M;
var 0..9: O;
var 0..9: R;
var 0..9: Y;

constraint      1000 * S + 100 * E + 10 * N + D
               + 1000 * M + 100 * O + 10 * R + E
               = 10000 * M + 1000 * O + 100 * N + 10 * E + Y;

constraint alldifferent([S,E,N,D,M,O,R,Y]);

solve satisfy;

output ["  \$(S)\$(E)\$(N)\$(D)\n",
```

```
" + \$(M)\$(O)\$(R)\$(E)\n",
" = \$(M)\$(O)\$(N)\$(E)\$(Y)\n"];
```

SEND+MORE=MONEY 问题要求给每一个字母赋不同的数值使得此算术约束满足。 Listing 2.2.4 中的模型使用 `alldifferent([S,E,N,D,M,O,R,Y])` 约束表达式来保证每个字母有不同的数字值。在使用了引用项

```
include "alldifferent.mzn";
```

后，此全局约束 `alldifferent` 可以在模型中使用。我们可以用以下代替此行

```
include "globals.mzn";
```

它包含了所有的全局约束。

一系列所有在 MiniZinc 中定义了的全局约束都被包含在了发布的文档中。对一些重要的全局约束的描述，请参见全局约束 (page 71)。

### 2.2.3 条件表达式

MiniZinc 提供了一个条件表达式 `if-then-else-endif`。它的一个使用例子如下

```
int: r = if y != 0 then x div y else 0 endif;
```

若 `y` 不是零，则 `r` 设为 `x` 除以 `y`，否则则设为零。

#### 条件表达式

条件表达式的格式是

```
if <布尔型表达式> then <表达式-1> else <表达式-2> endif
```

它是一个真表达式而不是一个控制流语句，所以它可以被用于其他表达式中。如果 `<布尔型表达式>` 是真，则它取值 `<表达式-1>`，否则则是 `<表达式-2>`。条件表达式的类型是 `<表达式-1>` 和 `<表达式-2>` 的类型，而它们俩必须有相同的类型。

如果 `<布尔型表达式>` 包含决策变量，则表达式的类型-实例化是 `var <T>`，其中 `<T>` 是 `<表达式-1>` 和 `<表达式-2>` 的类型，就算是在两个表达式都已经固定了的情况下也是如此。

## Listing 2.2.5: 广义数独问题的模型 (sudoku.mzn)

```

include "alldifferent.mzn";

int: S;
int: N = S * S;
int: digs = ceil(log(10.0,int2float(N))); % 输出的数字

set of int: PuzzleRange = 1..N;
set of int: SubSquareRange = 1..S;

array[1..N,1..N] of 0..N: start; %% 板初始 0 = 空
array[1..N,1..N] of var PuzzleRange: puzzle;

% 填充初始板
constraint forall(i,j in PuzzleRange)(
    if start[i,j] > 0 then puzzle[i,j] = start[i,j] else true endif);

% 每行中取值各不相同
constraint forall (i in PuzzleRange) (
    alldifferent( [ puzzle[i,j] | j in PuzzleRange ] ) );

% 每列中取值各不相同
constraint forall (j in PuzzleRange) (
    alldifferent( [ puzzle[i,j] | i in PuzzleRange ] ) );

% 每个子方格块中取值各不相同
constraint
    forall (a, o in SubSquareRange)(
        alldifferent( [ puzzle[(a-1)*S + a1, (o-1)*S + o1] |
                        a1, o1 in SubSquareRange ] ) );

solve satisfy;

output  [ show_int(digs,puzzle[i,j]) ++ " " ++
          if j mod S == 0 then "\n" else "" endif ++
          if j == N then
              if i != N then

```

```

    if i mod S == 0 then "\n\n" else "\n" endif
else "" endif else "" endif
| i,j in PuzzleRange ] ++ ["\n"];

```

Listing 2.2.6: 广义数独问题的数据文件例子 (sudoku.dzn)

```

S=3;
start=[|
0, 0, 0, 0, 0, 0, 0, 0, 0|
0, 6, 8, 4, 0, 1, 0, 7, 0|
0, 0, 0, 0, 8, 5, 0, 3, 0|
0, 2, 6, 8, 0, 9, 0, 4, 0|
0, 0, 7, 0, 0, 0, 9, 0, 0|
0, 5, 0, 1, 0, 6, 3, 2, 0|
0, 4, 0, 6, 1, 0, 0, 0, 0|
0, 3, 0, 2, 0, 7, 6, 9, 0|
0, 0, 0, 0, 0, 0, 0, 0, 0|];

```

6	8	4		1		7		
			8	5		3		
2	6	8		9		4		
	7				9			
5		1		6	3	2		
4		6	1					
3		2		7	6	9		

Fig. 2.2.1: sudoku.dzn 代表的问题。

在创建复杂模型或者复杂输出时，条件表达式是非常有用的。我们来看下 Listing 2.2.5 中的数独问题模型。板的初始位置在参数 `start` 中给出，其中 0 代表了一个空的板位置。通过使用以下条件表达式

```

constraint forall(i,j in PuzzleRange)(
  if start[i,j] > 0 then puzzle[i,j] = start[i,j] else true endif );

```

它被转换为对决策变量 `puzzle` 的约束。

在定义复杂输出时，条件表达式也很有用。`output` 在数独模型 Listing 2.2.5 中，表达式

```
if j mod S == 0 then " " else "" endif
```

在大小为  $S$  的组群之间插入了一个额外的空格。输出表达式同时也使用条件表达式来在每  $S$  行后面加入一个空白行。这样得到的输出有很高的可读性。

剩下的约束保证了每行中，每列中以及每  $S \times S$  子方格块中的值都是互相不相同的。

通过使用标示 `-a` 或 `--all-solutions`，我们可以用 MiniZinc 求解得到一个满足问题 ([solve satisfy](#)) 的所有解。运行

```
$ minizinc --all-solutions sudoku.mzn sudoku.dzn
```

得到

```
5 9 3 7 6 2 8 1 4
2 6 8 4 3 1 5 7 9
7 1 4 9 8 5 2 3 6

3 2 6 8 5 9 1 4 7
1 8 7 3 2 4 9 6 5
4 5 9 1 7 6 3 2 8

9 4 2 6 1 8 7 5 3
8 3 5 2 4 7 6 9 1
6 7 1 5 9 3 4 8 2
-----
=====
```

当系统输出完所有可能的解之后，此行 ===== 被输出。在这里则表明了此问题只有一个解。

## 2.2.4 枚举类型

枚举类型允许我们根据一个或者是数据中的一部分，或者在模型中被命名的对象集合来创建模型。这样一来，模型就更容易被理解和调试。我们之前已经简单介绍了枚举类型或者 `enums`。在这一小分段，我们会探索如何可以全面地使用它们，并且给出一些处理枚举类型的内建函数。

让我们重新回顾一下 [MiniZinc 基本模型](#) (page 25) 中的给澳大利亚涂色问题。

Listing 2.2.7: 使用枚举类型的澳大利亚涂色模型 (aust-enum.mzn).

```
enum Color;
var Color: wa;
var Color: nt;
var Color: sa;
var Color: q;
var Color: nsw;
var Color: v;
var Color: t;
constraint wa != nt /\ wa != sa /\ nt != sa /\ nt != q /\ sa != q;
constraint sa != nsw /\ sa != v /\ q != nsw /\ nsw != v;
solve satisfy;
```

Listing 2.2.7 中的模型声明了一个枚举类型 `Color`，而它必须在数据文件中被定义。每一个州变量被声明为从此枚举类型中取一个值。使用以下方式运行这个程序

```
$ minizinc -D"Color = { red, yellow, blue };" aust-enum.mzn
```

可能会得到输出

```
wa = yellow;
nt = blue;
sa = red;
q = yellow;
nsw = blue;
v = yellow;
t = red;
```

## 枚举类型变量声明

一个枚举类型参数变量被声明为以下两种方式：

```
<枚举名> : <变量名>
<l>..<u> : <变量名>
```

其中 `<枚举名>` 是枚举类型的名字，`<l>` 和 `<u>` 是此枚举类型的固定枚举类型表达式。

枚举类型一个重要的行为是，当它们出现的位置所期望的是整数时，它们会自动地强制转换为

整数。这样一来，这就允许我们使用定义在整数上的全局变量，例如

```
global_cardinality_low_up([wa,nt,sa,q,nsw,v,t],
    [red,yellow,blue],[2,2,2],[2,2,3]);
```

要求每种颜色至少有两个州涂上并且有三个州被涂了蓝色。

## 枚举类型操作符

有一系列关于枚举类型的内部操作符：

- `enum_next(X,x)`: 返回枚举类型 `X` 中 `x` 后的下一个值。这是一个部份函数，如果 `x` 是枚举类型 `X` 最后一个值，则函数会返回 `⊥` 令包含这个表达式的布尔表达式返回 `false`。
- `enum_prev(X,x)` `enum_prev(X,x)`: 返回枚举类型 `X` 中 `x` 的上一个值。`enum_prev` 同样是一个部份函数。
- `to_enum(X,i)`: 映射一个整型表达式 `i` 到一个在 `X` 的枚举类型值，或者如果 `i` 是小于等于 0 或大于 `:mzn:X` 中元素的个数，则返回`:math:bot`。

注意，一些标准函数也是可以应用于枚举类型上

- `card(X)`: 返回枚举类型 `X` 的势。
- `min(X)`: 返回枚举类型 `X` 中最小的元素。
- `max(X)`: 返回枚举类型 `X` 中最大的元素。

## 2.2.5 复杂约束

约束是 MiniZinc 模型的核心。我们已经看到了简单关系表达式，但是约束其实是比这更加强大的。一个约束可以是任何布尔型表达式。想象一个包含两个时间上不能重叠的任务的调度问题。如果 `s1` 和 `s2` 是相对应的起始时间，`d1` 和 `d2` 是相对应的持续时间，我们可以表达约束为：

```
constraint s1 + d1 <= s2  \vee s2 + d2 <= s1;
```

来保证任务之间互相不会重叠。

## 布尔型

MiniZinc 中的布尔型表达式可以按照标准的数学语法来书写。布尔常量是 **真** 或 **假**，布尔型操作符有合取，即，与 (`\wedge`)，析取，即，或 (`\vee`)，必要条件蕴含 (`<-`)，充分条件蕴含 (`->`)，充分必要蕴含 (`<->`) 以及非 (`not`)。内建函数 `bool2int` 强制转换布尔型为整型：如果参数为真，它返回 1，否则返回 0。

Listing 2.2.8: 车间作业调度问题模型 (jobshop.mzn).

```

enum JOB;
enum TASK;

TASK: last = max(TASK);
array [JOB,TASK] of int: d;                                % 任务持续时间
int: total = sum(i in JOB, j in TASK)(d[i,j]);% 总持续时间
int: digs = ceil(log(10.0,int2float(total))); % 输出的数值
array [JOB,TASK] of var 0..total: s;                      % 起始时间
var 0..total: end;                                         % 总结束时间

constraint %% 保证任务按照顺序出现
  forall(i in JOB) (
    forall(j in TASK where j < last)
      (s[i,j] + d[i,j] <= s[i,enum_next(TASK,j)]) /\ 
      s[i,last] + d[i,last] <= end
  );
  );

constraint %% 保证任务之间没有重叠
  forall(j in TASK) (
    forall(i,k in JOB where i < k) (
      s[i,j] + d[i,j] <= s[k,j] \/
      s[k,j] + d[k,j] <= s[i,j]
    )
  );
  );

solve minimize end;

output ["end = \n"] ++
  [ show_int(digs,s[i,j]) ++ " " ++
    if j == last then "\n" else "" endif |
    i in JOB, j in TASK ];

```

Listing 2.2.9: 车间作业调度问题数据 (jdata.dzn).

```

JOB = anon_enum(5);
TASK = anon_enum(5);

```

```
d = [| 1, 4, 5, 3, 6
      | 3, 2, 7, 1, 2
      | 4, 4, 4, 4, 4
      | 1, 1, 1, 6, 8
      | 7, 3, 2, 2, 1 |];
```

Listing 2.2.8 中的车间作业调度模型给出了一个使用析取建模功能的现实例子。车间作业调度问题中，我们有一个作业集合，每一个包含一系列的在不同机器上的任务：任务  $[i, j]$  是在第  $i^{th}$  个作业中运行在第  $j^{th}$  个机器上的任务。每列任务必须按照顺序完成，并且运行在同一个机器上的任何两个任务在时间上都不能重叠。就算是对这个问题的小的实例找最优解都会是很有挑战性的。

命令

```
$ minizinc --all-solutions jobshop.mzn jdata.dzn
```

求解了一个小的车间作业调度问题，并且显示了优化问题在 `all-solutions` 下的表现。在这里，求解器只有当找到一个更好的解时才会输出它，而不是输出所有的可能最优解。这个命令下的（部分）输出是：

```
end = 39
5 9 13 22 30
6 13 18 25 36
0 4 8 12 16
4 8 12 16 22
9 16 25 27 38
-----
end = 37
4 8 12 17 20
5 13 18 26 34
0 4 8 12 16
8 12 17 20 26
9 16 25 27 36
-----
end = 34
0 1 5 10 13
6 10 15 23 31
2 6 11 19 27
1 5 10 13 19
```

```
9 16 22 24 33
```

```
-----
```

```
end = 30
```

```
5 9 13 18 21
```

```
6 13 18 25 27
```

```
1 5 9 13 17
```

```
0 1 2 3 9
```

```
9 16 25 27 29
```

```
-----
```

```
=====
```

表明一个结束时间为 30 的最优解终于被找到，并且被证明为是最优的。通过加一个约束 `end = 30`，并且把求解项改为 `solve satisfy`，然后运行

```
$ minizinc --all-solutions jobshop.mzn jobshop.dzn
```

我们可以得到所有的 最优解。这个问题有 3,444,375 个最优解。

Listing 2.2.10: 稳定婚姻问题模型 (stable-marriage.mzn).

```
int: n;

enum Men = anon_enum(n);
enum Women = anon_enum(n);

array[Women, Men] of int: rankWomen;
array[Men, Women] of int: rankMen;

array[Men] of var Women: wife;
array[Women] of var Men: husband;

% assignment
constraint forall (m in Men) (husband[wife[m]] = m);
constraint forall (w in Women) (wife[husband[w]] = w);
% ranking
constraint forall (m in Men, o in Women) (
    rankMen[m, o] < rankMen[m, wife[m]] ->
    rankWomen[o, husband[o]] < rankWomen[o, m] );
```

```

constraint forall (w in Women, o in Men) (
    rankWomen[w,o] < rankWomen[w,husband[w]] ->
    rankMen[o,wife[o]] < rankMen[o,w] );
solve satisfy;

output ["wives= \$(wife)\nhusbands= \$(husband)\n"];

```

Listing 2.2.11: 稳定婚姻问题模型的数据文件例子。(stable-marriage.dzn).

```

n = 5;
rankWomen =
[| 1, 2, 4, 3, 5,
 | 3, 5, 1, 2, 4,
 | 5, 4, 2, 1, 3,
 | 1, 3, 5, 4, 2,
 | 4, 2, 3, 5, 1 |];

rankMen =
[| 5, 1, 2, 4, 3,
 | 4, 1, 3, 2, 5,
 | 5, 3, 2, 4, 1,
 | 1, 5, 4, 3, 2,
 | 4, 3, 2, 1, 5 |];

```

MiniZinc 中的另外一个强大的建模特征是决策变量可以被用来访问数组 array access。作为一个例子，考虑（老式的）稳定婚姻问题。我们有  $n$  个（直）女以及  $n$  个（直）男。每一个男士有一个女士排行榜，女士也是。我们想给每一个女士/男士找一个丈夫/妻子来使得所有的婚姻按以下意义上来说都是 稳定的：

- 每当  $m$  喜欢另外一个女士  $o$  多过他的妻子  $w$  时， $o$  喜欢她的丈夫多过  $m$ ，以及
- 每当  $w$  喜欢另外一个男士  $o$  多过她的丈夫  $m$  时， $o$  喜欢他的妻子多过  $w$ 。

这个问题可以很优雅地在 MiniZinc 中建模。模型和数据例子在 Listing 2.2.10 和 Listing 2.2.11 中分别被给出。

模型中的前三项声明了男士/女士的数量以及男士和女士的集合。在这里我们介绍 匿名枚举类型 的使用。Men 和 Women 都是大小为  $n$  的集合，但是我们不希望把它们混合到一起，所以我们使用了一个匿名枚举类型。这就允许 MiniZinc 检测到使用 Men 为 Women 或者反之的建模错误。

矩阵 rankWomen 和 rankMen 分别给出了男士们的女士排行以及女士们的男士排行。因此，项

`rankWomen[w,m]` 给出了女士 `texttt{w}` 的关于男士 `texttt{m}` 的排行。在排行中的数目越小，此男士或者女士被选择的倾向越大。

有两个决策变量的数组：`wife` 和 `husband`。这两个分别代表了每个男士的妻子和每个女士的丈夫。

前两个约束

```
constraint forall (m in Men) (husband[wife[m]] = m);
constraint forall (w in Women) (wife[husband[w]] = w);
```

确保了丈夫和妻子的分配是一致的：`w` 是 `m` 的妻子蕴含了 `m` 是 `w` 的丈夫，反之亦然。注意在 `husband[wife[m]]` 中，下标表达式 `wife[m]` 是一个决策变量，而不是一个参数。

接下来的两个约束是稳定条件的直接编码：

```
constraint forall (m in Men, o in Women) (
    rankMen[m,o] < rankMen[m,wife[m]] ->
    rankWomen[o,husband[o]] < rankWomen[o,m] );
constraint forall (w in Women, o in Men) (
    rankWomen[w,o] < rankWomen[w,husband[w]] ->
    rankMen[o,wife[o]] < rankMen[o,w] );
```

在有了用决策变量作为数组的下标和用标准的布尔型连接符构建约束的功能后，稳定婚姻问题的自然建模才变得可行。敏锐的读者可能会在这时产生疑问，如果数组下标变量取了一个超出数组下标集合的值，会产生什么情况。MiniZinc 把这种情况看做失败：一个数组访问 `a[e]` 在其周围最近的布尔型语境中隐含地加入了约束 `e in index_set(a)`，其中 `index_set(a)` 给出了 `a` 的下标集合。

## 匿名枚举类型

一个匿名枚举类型表达式有格式 `anon_enum(<n>)`，其中 `<n>` 是一个固定的整型表达式，它定义了枚举类型的大小。

除了其中的元素没有名字，匿名枚举类型和其他的枚举类型一样。当被输出时，它们根据枚举类型的名字被给定独有的名字。

例如，如下的变量声明

```
array[1..2] of int: a = [2,3];
var 0..2: x;
```

```
var 2..3: y;
```

约束 `a[x] = y` 会在  $x = 1 \wedge y = 2$  和  $x = 2 \wedge y = 3$  时得到满足。约束 `not a[x] = y` 会在  $x = 0 \wedge y = 2$ ,  $x = 0 \wedge y = 3$ ,  $x = 1 \wedge y = 3$  和  $x = 2 \wedge y = 2$  时得到满足。

当参数无效访问数组时，正式的 MiniZinc 语义会把此情况看成失败来确保参数和决策变量的处理方式是一致的，但是会发出警告，因为这种情况下几乎总是会有错误出现。

Listing 2.2.12: 魔术串问题模型 (`magic-series.mzn`).

```
int: n;
array[0..n-1] of var 0..n: s;

constraint forall(i in 0..n-1) (
    s[i] = (sum(j in 0..n-1)(bool2int(s[j]=i)));
);

solve satisfy;

output [ "s = \\"(s);\n" ] ;
```

强制转换函数 `bool2int` 可以被任何布尔型表达式调用。这就使得 MiniZinc 建模者可以使用所谓的高价约束。举个简单的例子，请看 魔术串问题：找到一列数字  $s = [s_0, \dots, s_{n-1}]$  使得： $s_i$  是数字  $i$  出现在  $s$  的次数。一个解的例子是  $s = [1, 2, 1, 0]$ 。

这个问题的一个 MiniZinc 模型在 Listing 2.2.12 中给出。Listing 2.2.12 的使用使得我们可以把函数 `s[j]=i` 满足的次数加起来。运行命令

```
$ minizinc --all-solutions magic-series.mzn -D "n=4;"
```

得到输出

```
s = [1, 2, 1, 0];
-----
s = [2, 0, 2, 0];
-----
=====
```

确切地显示出这个问题的两个解。

注意当有需要的时候，MiniZinc 会自动地强制转换布尔型为整型以及整型为浮点型。我们可以

把 Listing 2.2.12 中的约束项替换为

```
constraint forall(i in 0..n-1) (
    s[i] = (sum(j in 0..n-1)(s[j]=i)));
```

由于 MiniZinc 系统实际上会自动地加入缺失的 `bool2int`，布尔型表达式 `s[j] = i` 会被自动地强制转换为整型，所以会得到同样的结果。

### 强制转换

MiniZinc 中，通过使用函数 `bool2int`，我们可以把一个布尔型数值 `emph{强制转换}` 为一个整型数值。同样地，通过使用函数 `int2float`，我们也可以把一个整型数值强制转换为一个浮点型数值。被强制转换的数值的实例化和原数值一样。例如，`par bool` 被强制转换为 `par int`，而 `var bool` 被强制转换为 `var int`。

通过适当地在模型中加入 `bool2int` 和 `int2float`，MiniZinc 会自动地强制转换布尔型表达式为整型表达式，以及整型表达式为浮点型表达式。注意通过两步转换，它也会强制转换布尔型为浮点型。

## 2.2.6 集合约束

MiniZinc 另外一个强大的建模特征是它允许包含整数的集合是决策变量：这表示当模型被评估时，求解器会查找哪些元素在集合中。

举个简单的例子，*0/1 背包问题*。这个问题是背包问题的局限版本，即我们或者选择把物品放入背包或者不放。每一个物品有一个重量和一个利润，在受限制于背包不能太满的条件下，我们想找到选取哪些物品会得到最大化利润。

很自然地，我们在 MiniZinc 中使用单个的决策变量来建模：`var set of ITEM: knapsack` 其中 `ITEM` 是可放置的物品集合。如果数组 `weight[i]` 和 `profit[i]` 分别给出物品 `i` 的重量和利润，以及背包可以装载的最大重量是 `capacity`，则一个自然的模型在 Listing 2.2.13 中给出。

Listing 2.2.13: 0/1 背包问题模型 (`knapsack.mzn`).

```
enum ITEM;
int: capacity;

array[ITEM] of int: profits;
array[ITEM] of int: weights;

var set of ITEM: knapsack;
```

```

constraint sum (i in knapsack) (weights[i]) <= capacity;

solve maximize sum (i in knapsack) (profits[i]) ;

output ["knapsack = \$(knapsack)\n"];

```

注意，关键字 `var` 出现在 `set` 声明之前，表明这个集合本身是决策变量。这就和一个 `var` 关键字描述其中元素而不是数组自身的数组形成对比，因为此时数组的基本结构，即它的下标集合，是固定了的。

Listing 2.2.14: 高尔夫联谊问题模型 (`social-golfers.mzn`).

```

include "partition_set.mzn";

int: weeks;      set of int: WEEK = 1..weeks;
int: groups;     set of int: GROUP = 1..groups;
int: size;       set of int: SIZE = 1..size;
int: ngolfers = groups*size;
set of int: GOLFER = 1..ngolfers;

array[WEEK, GROUP] of var set of GOLFER: Sched;

% constraints
constraint
    forall (i in 1..weeks-1) (
        Sched[i,1] < Sched[i+1,1]
    ) /\%
    forall (i in WEEK, j in GROUP) (
        card(Sched[i,j]) = size
        /\ forall (k in j+1..groups) (
            %           Sched[i,j] < Sched[i,k]
            %           /\
            Sched[i,j] intersect Sched[i,k] = {}
        )
    ) /\
    forall (i in WEEK) (
        partition_set([Sched[i,j] | j in GROUP], GOLFER)
        % /\ forall (j in 1..groups-1) (
        %         Sched[i,j] < Sched[i,j+1]
    )

```

```

%           )
) /\

forall (i in 1..weeks-1, j in i+1..weeks) (
    forall (x,y in GROUP) (
        card(Sched[i,x] intersect Sched[j,y]) <= 1
    )
);

% symmetry
constraint

% Fix the first week %
forall (i in GROUP, j in SIZE) (
    ((i-1)*size + j) in Sched[1,i]
) /\

% Fix first group of second week %
forall (i in SIZE) (
    ((i-1)*size + 1) in Sched[2,1]
) /\

% Fix first 'size' players
forall (w in 2..weeks, p in SIZE) (
    p in Sched[w,p]
);

solve satisfy;

output [ show(Sched[i,j]) ++ " " ++
    if j == groups then "\n" else "" endif |
    i in WEEK, j in GROUP ];

```

我们来看一个更复杂的关于集合约束的例子， Listing 2.2.14 中给出的高尔夫联谊问题。这个问题的目的是给 `groups` × `size` 个高尔夫手在 `weeks` 时间内安排一个高尔夫联赛。每一周我们需要安排 `groups` 个大小为 `size` 的不同的组。任何一对高尔夫手都不能一起出现于两个组中进行比赛。

模型中的变量是第  $i^{th}$  周第  $j^{th}$  组的高尔夫手:`mzn:Sched[i,j]` 组成的集合。

11-32 行中的约束首先对每一周的第一个集合进行一个排序来去除掉周之间可以互相调换的对称。然后它对每一周内的集合进行了一个排序，同时使得每一个集合的势为 `size`。接下来通过使用全局约束 `partition_set`，确保了每一周都是对高尔夫手集合的一个划分。最后一个约束确保了任何两个高尔夫手都不会一起在两个组内比赛（因为任何两个组的交集的势最多都是 1）。

我们也有

在 34-46 行中，我们也给出了去对称初始化约束：第一周被固定为所有的高尔夫手都按顺序排列；第二周的第一组被规定为是由第一周的前几组的第一个选手组成；最后，对于剩下的周，模型规定第一个 size 内的高尔夫手们出现在他们相对应的组数中。

运行命令

```
$ minizinc social-golfers.mzn social-golfers.dzn
```

其中数据文件定义了一个周数为 4，大小为 3，组数为 4 的问题，得到如下结果

```
1..3 4..6 7..9 10..12
{ 1, 4, 7 } { 2, 5, 10 } { 3, 9, 11 } { 6, 8, 12 }
{ 1, 5, 8 } { 2, 6, 11 } { 3, 7, 12 } { 4, 9, 10 }
{ 1, 6, 9 } { 2, 4, 12 } { 3, 8, 10 } { 5, 7, 11 }
-----
```

注意范围集合是如何以范围格式输出的。

## 2.2.7 汇总

我们以一个可以阐释这一章介绍的大部分特征的复杂例子来结束这一节，包括枚举类型，复杂约束，全局约束以及复杂输出。

Listing 2.2.15: 使用枚举类型规划婚礼座位 (wedding.mzn).

```
enum Guests = { bride, groom, bestman, bridesmaid, bob, carol,
    ted, alice, ron, rona, ed, clara};
set of int: Seats = 1..12;
set of int: Hatreds = 1..5;
array[Hatreds] of Guests: h1 = [groom, carol, ed, bride, ted];
array[Hatreds] of Guests: h2 = [clara, bestman, ted, alice, ron];
set of Guests: Males = {groom, bestman, bob, ted, ron,ed};
set of Guests: Females = {bride,bridesmaid,carol,alice,rona,clara};

array[Guests] of var Seats: pos; % 客人的座位
array[Hatreds] of var Seats: p1; % 互相憎恶的客人 1 的座位
array[Hatreds] of var Seats: p2; % 互相憎恶的客人 2 的座位
```

```

array[Hatreds] of var 0..1: sameside; % 互相憎恶的客人是否坐在同一边
array[Hatreds] of var Seats: cost; % 互相憎恶的客人的距离

include "alldifferent.mzn";
constraint alldifferent(pos);
constraint forall(g in Males)( pos[g] mod 2 == 1 );
constraint forall(g in Females)( pos[g] mod 2 == 0 );
constraint not (pos[ed] in {1,6,7,12});
constraint abs(pos[bride] - pos[groom]) <= 1 /\ 
    (pos[bride] <= 6 <-> pos[groom] <= 6);
constraint forall(h in Hatreds)(
    p1[h] = pos[h1[h]] /\ 
    p2[h] = pos[h2[h]] /\ 
    sameside[h] = bool2int(p1[h] <= 6 <-> p2[h] <= 6) /\ 
    cost[h] = sameside[h] * abs(p1[h] - p2[h]) + 
    (1 - sameside[h]) * (abs(13 - p1[h] - p2[h]) + 1));
    
solve maximize sum(h in Hatreds)(cost[h]);
    
output [ show(g)++" " | s in Seats, g in Guests where fix(pos[g]) == s]
++ ["\n"];

```

Listing 2.2.15 中的模型安排婚礼桌上的座位。这个桌子有 12 个编码的顺序排列的座位，每边有 6 个。男士必须坐奇数号码的座位，女士坐偶数。Ed 由于恐惧症不能坐在桌子的边缘，新郎和新娘必须坐在彼此旁边。我们的目的是最大化已知的互相憎恶的人之间的距离。如果在同一边，座位之间的距离是座位号码之间的差，否则则是和其对面座位的距离 + 1。

注意在输出语句中我们观察每个座位 `s` 来找一个客人 `g` 分配给此座位。我们利用内建函数 `fix`，它检查一个决策变量是否是固定的以及输出它的固定值，否则的话中断。在输出语句中使用此函数总是安全的，因为当输出语句被运行的时候，所有的决策变量都应该是固定了的。

运行

```
$ minizinc wedding.mzn
```

得到输出

```
ted bride groom rona ed carol ron alice bob bridesmaid bestman clara
-----
```

```
=====
```

最终得到的座位安排在 Fig. 2.2.2 中给出。其中连线表示互相憎恶，总的距离是 22.

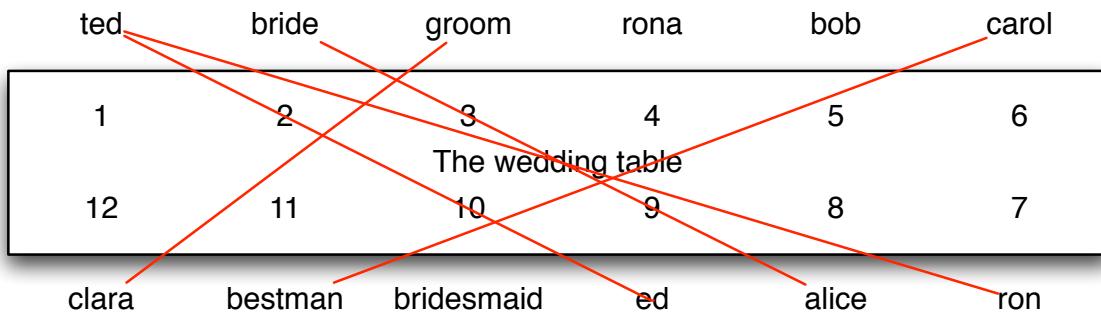


Fig. 2.2.2: 婚礼桌上座位的安排

## 固定

输出项中，内建函数 `fix` 检查一个决策变量的值是否固定，然后把决策变量的实例化强制转换为参数。

# CHAPTER 2.3

---

## 谓词和函数

---

MiniZinc 中的谓词允许我们用简洁的方法来表达模型中的复杂约束。MiniZinc 中的谓词利用预先定义好的全局约束建模，同时也让建模者获取以及定义新的复杂约束。MiniZinc 中的函数用来捕捉模型中的共同结构。实际上，一个谓词就是一个输出类型为 `var bool` 的函数。

### 2.3.1 全局约束

MiniZinc 中定义了很多可以在建模中使用的全局约束。由于全局约束的列表一直在慢慢增加，最终确定的列表可以在发布的文档中找到。下面我们讨论一些最重要的全局约束。

#### 2.3.1.1 Alldifferent

约束 `alldifferent` 的输入为一个变量数组，它约束了这些变量取不同的值。`alldifferent` 的使用有以下格式

```
alldifferent(array[int] of var int: x)
```

即，参数是一个整型变量数组。

`alldifferent` 是约束规划中被最多研究以及使用的全局约束之一。它被用来定义分配子问题，人们也给出了 `alldifferent` 的高效全局传播器。`send-more-money.mzn` (Listing 2.2.4) 和 `sudoku.mzn` (Listing 2.2.5) 是使用 `alldifferent` 的模型例子。

### 2.3.1.2 Cumulative

约束 `cumulative` 被用来描述资源累积使用情况。

```
cumulative(array[int] of var int: s, array[int] of var int: d,
           array[int] of var int: r, var int: b)
```

规定对于一个起始时间为 `s`，持续时间为 `d` 以及资源需求量为 `r` 的任务集合，在任何时间对资源的需求量都不能超过一个全局资源量界限 `b`。

Listing 2.3.1: 使用 `cumulative` 来建模搬运家具问题的模型 (`moving.mzn`).

```
include "cumulative.mzn";

enum OBJECTS;
array[OBJECTS] of int: duration; % 移动的持续时间
array[OBJECTS] of int: handlers; % 需要的搬运工的数量
array[OBJECTS] of int: trolleys; % 需要的手推车的数量

int: available_handlers;
int: available_trolleys;
int: available_time;

array[OBJECTS] of var 0..available_time: start;
var 0..available_time: end;

constraint cumulative(start, duration, handlers, available_handlers);
constraint cumulative(start, duration, trolleys, available_trolleys);

constraint forall(o in OBJECTS)(start[o] +duration[o] <= end);

solve minimize end;

output [ "start = \nstart\nend = \nend\n" ];
```

Listing 2.3.2: 使用 `cumulative` 来建模搬运家具问题的数据 (`moving.dzn`).

```
OBJECTS = { piano, fridge, doublebed, singlebed,
           wardrobe, chair1, chair2, table };
```

```

duration = [60, 45, 30, 30, 20, 15, 15, 15];
handlers = [3, 2, 2, 1, 2, 1, 1, 2];
trolleys = [2, 1, 2, 2, 2, 0, 0, 1];

available_time = 180;
available_handlers = 4;
available_trolleys = 3;

```

[Listing 2.3.1](#) 中的模型为搬运家具规划一个行程表使得每一份家具在搬运的过程中都有足够的搬运工和足够的手推车可以使用。允许的时间，可以使用的搬运工以及手推车被给出，每个物体的搬运持续时间，需要的搬运工和手推车的数量等数据也被给出。使用 [Listing 2.3.2](#) 中的数据，命令

```
$ minizinc moving.mzn moving.dzn
```

可能会得到如下输出

```

start = [0, 60, 60, 90, 120, 0, 15, 105]
end = 140
-----
=====
```

[Fig. 2.3.1](#) and [Fig. 2.3.2](#) 给出了这个解中搬运时每个时间点所需要的搬运工和手推车。

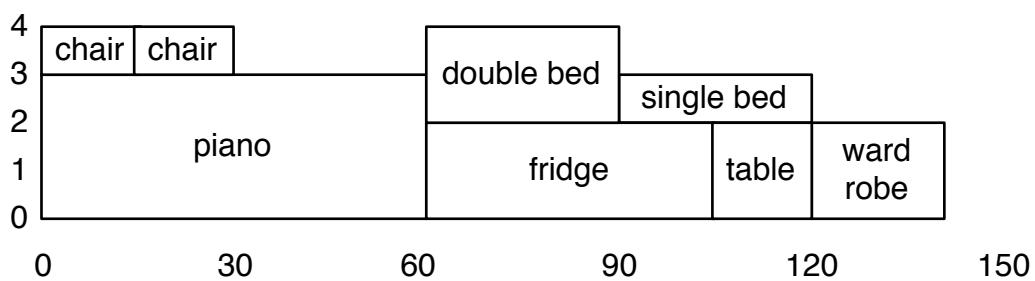


Fig. 2.3.1: 搬运时搬运工使用量直方图

### 2.3.1.3 Table

约束 `table` 强制变量元组从一个元组集合中取值。由于 MiniZinc 中没有元组，我们用数组来描述它。根据元组是布尔型还是整型，`table` 的使用有以下两种格式

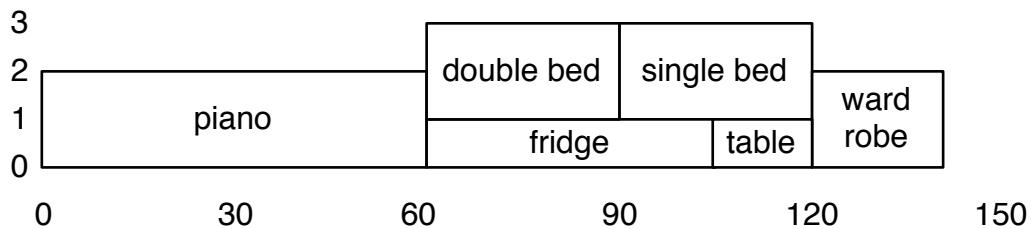


Fig. 2.3.2: 搬运时手推车使用量直方图

```
table(array[int] of var bool: x, array[int, int] of bool: t)
table(array[int] of var int: x, array[int, int] of int: t)
```

强制约束了  $x \in t$ ，其中  $x$  和  $t$  中的每一行是元组， $t$  是一个元组集合。

Listing 2.3.3: 使用 table 约束来建模食物规划问题的模型 (meal.mzn).

```
% 规划均衡的膳食
include "table.mzn";
int: min_energy;
int: min_protein;
int: max_salt;
int: max_fat;
set of FOOD: desserts;
set of FOOD: mains;
set of FOOD: sides;
enum FEATURE = { name, energy, protein, salt, fat, cost};
enum FOOD;
array[FOOD,FEATURE] of int: dd; % 食物数据库

array[FEATURE] of var int: main;
array[FEATURE] of var int: side;
array[FEATURE] of var int: dessert;
var int: budget;

constraint main[name] in mains;
constraint side[name] in sides;
constraint dessert[name] in desserts;
constraint table(main, dd);
constraint table(side, dd);
```

```

constraint table(dessert, dd);
constraint main[energy] + side[energy] + dessert[energy] >= min_energy;
constraint main[protein]+side[protein]+dessert[protein] >= min_protein;
constraint main[salt] + side[salt] + dessert[salt] <= max_salt;
constraint main[fat] + side[fat] + dessert[fat] <= max_fat;
constraint budget = main[cost] + side[cost] + dessert[cost];

solve minimize budget;

output ["main = ",show(to_enum(FOOD,main[name])),",
        ", side = ",show(to_enum(FOOD,side[name])),",
        ", dessert = ",show(to_enum(FOOD,dessert[name]))],
        ", cost = ",show(budget), "\n"];

```

Listing 2.3.4: 定义 table 的食物规划的数据 (meal.dzn).

```

FOODS = { icecream, banana, chocolatecake, lasagna,
          steak, rice, chips, brocolli, beans} ;

dd = [| icecream,      1200,   50,   10,  120,   400      % 冰淇淋
       | banana,        800,  120,    5,   20,   120      % 香蕉
       | chocolatecake, 2500,   400,   20,  100,   600      % 巧克力蛋糕
       | lasagna,       3000,   200,  100,  250,   450      % 千层面
       | steak,         1800,   800,   50,  100,  1200      % 牛排
       | rice,          1200,   50,    5,   20,   100      % 米饭
       | chips,         2000,   50,  200,  200,   250      % 薯条
       | brocolli,      700,   100,   10,   10,   125      % 花椰菜
       | beans,         1900,   250,   60,   90,   150 |]; % 黄豆

min_energy = 3300;
min_protein = 500;
max_salt = 180;
max_fat = 320;
desserts = { icecream, banana, chocolatecake };
mains = { lasagna, steak, rice };
sides = { chips, brocolli, beans };

```

Listing 2.3.3 中的模型寻找均衡的膳食。每一个食物项都有一个名字（用整数表示），卡路里数，蛋白质克数，盐毫克数，脂肪克数以及单位为分的价钱。这些个项之间的关系用一个 `table` 约

束来描述。模型寻找拥有最小花费，最少卡路里数 `min_energy`，最少蛋白质量 `min_protein`，最大盐分 `max_salt` 以及脂肪 `max_fat` 的膳食。

### 2.3.1.4 Regular

约束 `regular` 用来约束一系列的变量取有限自动机定义的值。`regular` 的使用有以下方式

```
regular(array[int] of var int: x, int: Q, int: S,
        array[int,int] of int: d, int: q0, set of int: F)
```

它约束了 `x` 中的一列值（它们必须是在范围 `range 1..S` 内）被一个有 `Q` 个状态，输入为 `1..S`，转换函数为 `d` ( $<1..Q, 1..S>$  映射到  $0..Q$ )，初始状态为 `q0`（必须在 `1..Q` 中）和接受状态为 `F``（必须在 `:mzn:`1..Q` 中）的 DFA 接受。状态 0 被保留为总是会失败的状态。

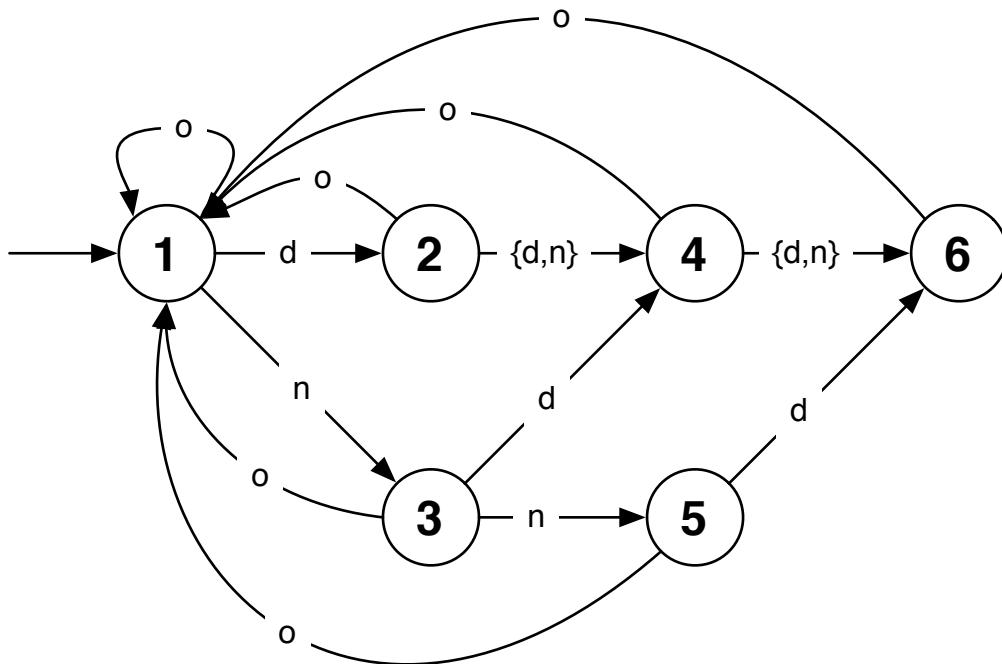


Fig. 2.3.3: 判定正确排班的 DFA。

我们来看下护士排班问题。每一个护士每一天被安排为以下其中一种：(d) 白班 (n) 夜班或者 (o) 休息。每四天，护士必须有至少一天的休息。每个护士都不可以被安排为连续三天夜班。这个问题可以使用 Fig. 2.3.3 中的不完全 DFA 来表示。我们可以把这个 DFA 表示为初始状态是 1，结束状态是 1..6，转换函数为

	d	n	o
1	2	3	1
2	4	4	1
3	4	5	1
4	6	6	1
5	6	0	1
6	0	0	1

注意状态表中的状态 0 代表一个错误状态。Listing 2.3.5 中给出的模型为 `num_nurses` 个护士 `num_days` 天寻找一个排班，其中我们要求白天有 `req_day` 个护士值班，晚上有 `req_night` 个护士值班，以及每个护士至少有 `min_night` 个夜班。

Listing 2.3.5: 使用 `regular` 约束来建模的护士排班问题模型 (`nurse.mzn`)

```
% 简单护士排班问题
include "regular.mzn";
enum NURSE;
enum DAY;
int: req_day;
int: req_night;
int: min_night;

enum SHIFT = { d, n, o };
int: S = card(SHIFT);

int: Q = 6; int: q0 = 1; set of int: STATE = 1..Q;
array[STATE,SHIFT] of int: t =
[| 2, 3, 1    % 状态 1
  | 4, 4, 1    % 状态 2
  | 4, 5, 1    % 状态 3
  | 6, 6, 1    % 状态 4
  | 6, 0, 1    % 状态 5
  | 0, 0, 1 |]; % 状态 6

array[NURSE,DAY] of var SHIFT: roster;

constraint forall(j in DAY)(
    sum(i in NURSE)(roster[i,j] == d) == req_day /\
    sum(i in NURSE)(roster[i,j] == n) == req_night /\
    sum(i in NURSE)(roster[i,j] == o) == min_night
)
```

```

    );
constraint forall(i in NURSE)(
    regular([roster[i,j] | j in DAY], Q, S, t, q0, STATE) /\ 
    sum(j in DAY)(roster[i,j] == n) >= min_night
);

solve satisfy;

output [ show(roster[i,j]) ++ if j==card(DAY) then "\n" else " " endif
| i in NURSE, j in DAY ];

```

运行命令

```
$ minizinc nurse.mzn nurse.dzn
```

找到一个给 7 个护士 10 天的排班，要求白天有 3 个人值班，夜晚有 2 个人值班，以及每个护士最少有 2 个夜班。一个可能的结果是

```

o d n n o n n d o o
d o n d o d n n o n
o d d o d o d n n o
d d d o n n d o n n
d o d n n o d o d d
n n o d d d o d d d
n n o d d d o d d d
-----

```

另外一种 regular 约束是 `regular_nfa`。它使用 NFA（没有 `\epsilon` 弧）来定义 regular 表达式。此约束有以下格式

```

regular_nfa(array[int] of var int: x, int: Q, int: S,
            array[int,int] of set of int: d, int: q0, set of int: F)

```

它约束了数组 `x` 中的数值序列（必须在范围 `1..S` 中）被含有 `Q` 个状态，输入为 `1..S`，转换函数为 `d`（映射 `<1..Q, 1..S>` 到 `1..Q` 的子集），初始状态为 `q0`（必须在范围 `1..Q` 中）以及接受状态为 `F`（必须在范围 `1..Q` 中）的 NFA 接受。在这里，我们没必要再给出失败状态 0，因为转换函数可以映射到一个状态的空集。

## 2.3.2 定义谓词

MiniZinc 的其中一个最强大的建模特征是建模者可以定义他们自己的高级约束。这就使得他们可以对模型进行抽象化和模块化。也允许了在不同的模型之间重新利用约束以及促使了用来定义标准约束和类型的特殊库应用的发展。

Listing 2.3.6: 使用谓词的车间作业调度问题模型 (jobshop2.mzn)

```

int: jobs;                                     % 作业的数量
set of int: JOB = 1..jobs;
int: tasks;                                    % 每个作业的任务数量
set of int: TASK = 1..tasks;
array [JOB,TASK] of int: d;                   % 任务持续时间
int: total = sum(i in JOB, j in TASK)(d[i,j]); % 总持续时间
int: digs = ceil(log(10.0,total));             % 输出的数值
array [JOB,TASK] of var 0..total: s;          % 起始时间
var 0..total: end;                            % 总结束时间

% nooverlap
predicate no_overlap(var int:s1, int:d1, var int:s2, int:d2) =
    s1 + d1 <= s2 \vee s2 + d2 <= s1;

constraint %% 保证任务按照顺序出现
forall(i in JOB) (
    forall(j in 1..tasks-1)
        (s[i,j] + d[i,j] <= s[i,j+1]) /\ 
        s[i,tasks] + d[i,tasks] <= end
);

constraint %% 保证任务之间没有重叠
forall(j in TASK) (
    forall(i,k in JOB where i < k) (
        no_overlap(s[i,j], d[i,j], s[k,j], d[k,j])
    )
);

solve minimize end;

output ["end = \n(end)\n"] ++

```

```
[ show_int(digs,s[i,j]) ++ " " ++
  if j == tasks then "\n" else "" endif |
  i in JOB, j in TASK];
```

我们用一个简单的例子开始，回顾下前面章节中的车间作业调度问题。这个模型在 Listing 2.3.6 中给出。我们感兴趣的项是 **谓词** 项：

```
predicate no_overlap(var int:s1, int:d1, var int:s2, int:d2) =
  s1 + d1 <= s2 \/ s2 + d2 <= s1;
```

它定义了一个新的约束用来约束起始时间为 `s1`，持续时间为 `d1` 的任务不能和起始时间为 `s2`，持续时间为 `d2` 的任务重叠。它可以在模型的任何（包含决策变量的）布尔型表达式可以出现的地方使用。

和谓词一样，建模者也可以定义只涉及到参数的新的约束。和谓词不一样的是，它们可以被用在条件表达式的测试中。它们被关键字 `test` 定义。例如

```
test even(int:x) = x mod 2 = 0;
```

## 谓词定义

使用以下形式的语句，我们可以定义谓词

```
predicate <谓词名> (<参数定义>, ..., <参数定义>) = <布尔表达式>
```

<谓词名> 必须是一个合法的 MiniZinc 标识符，每个 <参数定义> 都是一个合法的 MiniZinc 类型 type 声明。

参数定义的一个松弛是数组的索引类型可以是 没有限制地写为 `int`。

类似的，使用以下形式的语句，我们定义测试

```
test <谓词名> (<参数定义>, ..., <参数定义>) = <布尔表达式>
```

其中的 <布尔表达式> 必须是固定的。

另外我们介绍一个谓词中使用到的 `assert` 命令的新形式。

```
assert (<布尔表达式>, <字符串表达式>, <表达式>)
```

`assert` 表达式的类型和最后一个参数的类型一样。`assert` 表达式检测第一个参数是否为假，如果是则输出第二个参数字符串。如果第一个参数是真，则输出第三个参数。

注意 `assert` 表达式中的第三个参数是延迟的，即如果第一个参数是假，它就不会被评估。所以

它可以被用来检查

```
predicate lookup(array[int] of var int:x, int: i, var int: y) =
    assert(i in index_set(x), "index out of range in lookup"
        y = x[i]
    );
```

此代码在  $i$  超出数组  $x$  的范围时不会计算  $x[i]$ 。

### 2.3.3 定义函数

MiniZinc 中的函数和谓词一样定义，但是它有一个更一般的返回类型。

下面的函数定义了一个数独矩阵中的第  $a^{th}$  个子方块的第  $a1^{th}$  行。

```
function int: posn(int: a, int: a1) = (a-1) * S + a1;
```

有了这个定义之后，我们可以把 Listing 2.2.5 中的数独问题的最后一个约束替换为

```
constraint forall(a, o in SubSquareRange)(
    alldifferent([ puzzle [ posn(a,a0), posn(o,o1) ] |
        a1,o1 in SubSquareRange ] ) );
```

函数对于描述模型中经常用到的复杂表达式非常有用。例如，想象下在  $n \times n$  的方格的不同位置上放置数字 1 到  $n$  使得任何两个数字  $i$  和  $j$  之间的曼哈顿距离比这两个数字其中最大的值减一还要大。我们的目的是最小化数组对之间的总的曼哈顿距离。曼哈顿距离函数可以表达为：

```
function var int: manhattan(var int: x1, var int: y1,
    var int: x2, var int: y2) =
    abs(x1 - x2) + abs(y1 - y2);
```

完整的模型在 Listing 2.3.7 中给出。

Listing 2.3.7: 阐释如何使用函数的数字放置问题模型 (`manhattan.mzn`).

```
int: n;
set of int: NUM = 1..n;
```

```

array[NUM] of var NUM: x;
array[NUM] of var NUM: y;
array[NUM,NUM] of var 0..2*n-2: dist =
    array2d(NUM,NUM,[
        if i < j then manhattan(x[i],y[i],x[j],y[j]) else 0 endif
        | i,j in NUM ]);
    ];

% manf
function var int: manhattan(var int: x1, var int: y1,
                           var int: x2, var int: y2) =
    abs(x1 - x2) + abs(y1 - y2);

constraint forall(i,j in NUM where i < j)
    (dist[i,j] >= max(i,j)-1);

var int: obj = sum(i,j in NUM where i < j)(dist[i,j]);
solve minimize obj;

% 简单地显示结果
include "alldifferent_except_0.mzn";
array[NUM,NUM] of var 0..n: grid;
constraint forall(i in NUM)(grid[x[i],y[i]] = i);
constraint alldifferent_except_0([grid[i,j] | i,j in NUM]);

output ["obj = \obj;\n"] ++
    [ if fix(grid[i,j]) > 0 then show(grid[i,j]) else "." endif
    ++ if j = n then "\n" else "" endif
    | i,j in NUM ];

```

## 函数定义

函数用以下格式的语句定义

```

function <返回类型> : <函数名> ( <参数定义>, ..., <参数定义> ) =
    ↳<表达式>

```

<函数名>必须是一个合法的 MiniZinc 标识符。每一个<参数定义>是一个合法的 MiniZinc 类型声明。<返回类型>是函数的返回类型，它必须是<表达式>的类型。参数和谓词定义中的参数有一样的限制。

MiniZinc 中的函数可以有任何返回类型，而不只是固定的返回类型。在定义和记录多次出现在模型中的复杂表达式时，函数是非常有用的。

### 2.3.4 反射函数

为了方便写出一般性的测试和谓词，各种反射函数会返回数组的下标集合，var 集合的定义域以及决策变量范围的信息。关于下标集合的有以下反射函数 `index_set(<1-D array>)`, `index_set_1of2(<2-D array>)`, `index_set_2of2(<2-D array>)`, 以及关于更高维数组的反射函数。

车间作业问题的一个更好的模型是把所有的对于同一个机器上的不重叠约束结合为一个单个的析取约束。这个方法的一个优点是虽然我们只是初始地把它建模成一个 `non-overlap` 约束的连接，但是如果下层的求解器对于解决析取约束有一个更好的方法，在对我们的模型最小改变的情况下，我们可以直接使用它。这个模型在 Listing 2.3.8 中给出。

Listing 2.3.8: 使用 `disjunctive` 谓词的车间作业调度问题模型 (`jobshop3.mzn`).

```

include "disjunctive.mzn";

int: jobs;                                     % 作业的数量
set of int: JOB = 1..jobs;
int: tasks;                                     % 每个作业的任务数量
set of int: TASK = 1..tasks;
array [JOB,TASK] of int: d;                   % 任务持续时间
int: total = sum(i in JOB, j in TASK)(d[i,j]); % 总持续时间
int: digs = ceil(log(10.0,total));             % 输出的数值
array [JOB,TASK] of var 0..total: s;          % 起始时间
var 0..total: end;                            % 总结束时间

constraint % 保证任务按照顺序出现
forall(i in JOB) (
    forall(j in 1..tasks-1)
        (s[i,j] + d[i,j] <= s[i,j+1]) /\ 
        s[i,tasks] + d[i,tasks] <= end
);

constraint % 保证任务之间没有重叠
forall(j in TASK) (
    disjunctive([s[i,j] | i in JOB], [d[i,j] | i in JOB])
)

```

```

);
solve minimize end;

output ["end = \n(end)\n"] ++
[ show_int(digs,s[i,j]) ++ " " ++
  if j == tasks then "\n" else "" endif |
  i in JOB, j in TASK ];

```

约束 `disjunctive` 获取每个任务的开始时间数组以及它们的持续时间数组，确保每次只有一个任务是被激活的。我们定义析取约束为一个有以下特征的 谓词

```
predicate disjunctive(array[int] of var int:s, array[int] of int:d);
```

在 Listing 2.3.8 中，我们可以用这个析取约束定义任务之间不重叠。我们假设 `disjunctive` 谓词的定义已经在模型中引用的文件 `disjunctive.mzn` 中给出。

如果下层的系统直接支持 `disjunctive`，则会在它的全局目录下包含一个 `disjunctive.mzn` 文件（拥有上述特征定义内容）。如果我们使用的系统不直接支持析取，通过创建文件 `disjunctive.mzn`，我们可以给出我们自己的定义。最简单的实现是单单使用上面定义的 `no_overlap` 谓词。一个更好的实现是利用全局约束 `cumulative`，假如下层求解器支持它的话。Listing 2.3.9 给出了一个 `disjunctive` 的实现。注意我们使用 `index_set` 反射函数来 (a) 检查 `disjunctive` 的参数是有意义的，以及 (b) 构建 `cumulative` 的合适大小的资源利用数组。另外注意这里我们使用了 `assert` 的三元组版本。

Listing 2.3.9: 使用 cumulative 来定义一个 disjunctive 谓词 (disjunctive.mzn).

```

include "cumulative.mzn";

predicate disjunctive(array[int] of var int:s, array[int] of int:d) =
    assert(index_set(s) == index_set(d), "disjunctive: " ++
        "first and second arguments must have the same index set",
        cumulative(s, d, [ 1 | i in index_set(s) ], 1)
    );

```

## 2.3.5 局部变量

在谓词，函数或者测试中，引进 **局部变量** 总是非常有用的。表达式 `let` 允许你去这样做。它可以被用来引进决策变量 决策变量和 参数，但是参数必须被初始化。例如：

```

var s..e: x;
let {int: l = s div 2; int: u = e div 2; var l .. u: y;} in x = 2*y

```

引进了参数 `l` 和 `u` 以及变量 `y`。`let` 表达式虽然在 谓词，函数和测试定义中最有用，它也可以被用在其他的表达式中。例如，来消除共同的子表达式：

```

constraint let { var int: s = x1 + x2 + x3 + x4 } in
    l <= s /\ s <= u;

```

局部变量可以被用在任何地方，在简化复杂表达式时也很有用。通过使用局部变量来定义目标 objective 函数而不是显式地加入很多个变量， Listing 2.3.10 给出了稳定婚姻模型的一个改进版本。

Listing 2.3.10: 使用局部变量来定义一个复杂的目标函数 (wedding2.mzn).

```

enum Guests = { bride, groom, bestman, bridesmaid, bob, carol,
    ted, alice, ron, rona, ed, clara};
set of int: Seats = 1..12;
set of int: Hatreds = 1..5;
array[Hatreds] of Guests: h1 = [groom, carol, ed, bride, ted];
array[Hatreds] of Guests: h2 = [clara, bestman, ted, alice, ron];
set of Guests: Males = {groom, bestman, bob, ted, ron, ed};

```

```

set of Guests: Females = {bride,bridesmaid,carol,alice,rona,clara};

array[Guests] of var Seats: pos; % 客人的座位

include "alldifferent.mzn";
constraint alldifferent(pos);

constraint forall(g in Males)( pos[g] mod 2 == 1 );
constraint forall(g in Females)( pos[g] mod 2 == 0 );

constraint not (pos[ed] in {1,6,7,12});
constraint abs(pos[bride] - pos[groom]) <= 1 /\
    (pos[bride] <= 6 <-> pos[groom] <= 6);

solve maximize sum(h in Hatreds)(
    let { var Seats: p1 = pos[h1[h]];
          var Seats: p2 = pos[h2[h]];
          var 0..1: same = bool2int(p1 <= 6 <-> p2 <= 6); } in
    same * abs(p1 - p2) + (1-same) * (abs(13 - p1 - p2) + 1));

output [ show(g)++" " | s in Seats, g in Guests where fix(pos[g]) == s]
++ ["\n"];

```

## 2.3.6 语境

有一个局限，即含有决策变量并且在声明时没有初始化的谓词和函数不可以被用在一个否定语境下。下面例子是非法的

```

predicate even(var int:x) =
    let { var int: y } in x = 2 * y;

constraint not even(z);

```

原因是求解器只解决存在约束的问题。如果我们在否定语境下引入了一个局部变量，则此变量是普遍地量化了，因此超出下层求解器的解决范围。例如， $\neg \text{even}(z)$  等价于  $\neg \exists y. z = 2y$ ，然后等价于  $\forall y. z \neq 2y$ 。

如果局部变量被赋了值，则它们可以被用在否定语境中。下面的例子是合法的

```

predicate even(var int:x) =
    let { var int: y = x div 2; } in x = 2 * y;

constraint not even(z);

```

注意，现在 `even` 的意思是正确的，因为如果  $x$  是偶数，则  $x = 2 * (x \text{ div } 2)$ 。由于 `math:y` 被  $z$  功能性定义了， $\neg \text{even}(z)$  等价于  $\neg \exists y. y = z \text{ div } 2 \wedge z = 2y$ ，同时等价于  $\exists y. y = z \text{ div } 2 \wedge \neg z \neq 2y$ 。

MiniZinc 中的任意表达式都出现在以下四种语境中的一种中：根，肯定，否定，或者混合。非布尔型表达式的语境直接地为包含其最近的布尔型表达式的语境。唯一的例外是目标表达式出现在一个根语境下（由于它没有包含其的布尔型表达式）。

为了方便定义语境，我们把蕴含表达式  $e_1 \rightarrow e_2$  等价地写为 `not e1 \vee e2`， $e_1 \leftarrow e_2$  等价地写为  $e1 \vee \text{not } e2$ 。

一个布尔型表达式的语境可能有：

**根** 根语境是任何作为 `constraint` 的参数或者作为一个赋值项 `assignment` 出现的表达式  $e$  的语境，或者作为一个出现在根语境中的  $e_1 \wedge e_2$  的子表达式  $e_1$  或  $e_2$  的语境。

根语境下的布尔型表达式必须在问题的任何模型中都满足。

**肯定** 肯定语境是任何作为一个出现在根语境或者肯定语境中的  $e_1 \vee e_2$  的子表达式  $e_1$  或  $e_2$  的语境，或者是作为一个出现在肯定语境中的  $e_1 \wedge e_2$  的子表达式  $e_1$  或  $e_2$  的语境，或者是作为一个出现在否定语境中的 `:mzn:` `not e` 的子表达式  $e$  的语境。

肯定语境下的布尔型表达式不是必须要在模型中满足，但是满足它们会增加包含其的约束被满足的可能性。对于一个肯定语境下的表达式，从包含其的根语境到此表达式有偶数个否定。

**否定** 否定语境是任何作为一个出现在根语境或者否定语境中的  $e_1 \vee e_2$  或  $e_1 \wedge e_2$  的子表达式  $e_1$  或  $e_2$ ，或者是作为一个出现在肯定语境中的 `not e` 的子表达式  $e$  的语境。

否定语境下的布尔型表达式不是必须要满足，但是让它们成假会增加包含其的约束被满足的可能性。对于一个否定语境下的表达式，从包含其的根语境到此表达式有奇数个否定。

**混合** 混合语境是任何作为一个出现在 `e1 <-> e2`, `e1 = e2` 或者 `bool2int(e)` 中的子表达式  $e_1$  或  $e_2$  的语境。

混合语境下的表达式实际上既是肯定也是否定的。通过以下可以看出：`e1 <-> e2` 等价于  $(e_1 \wedge e_2) \vee (\text{not } e_1 \wedge \text{not } e_2)$  以及 `x = bool2int(e)` 等价于  $(e \wedge x=1) \vee (\text{not } e \wedge x=0)$ 。

观察以下代码段

```
constraint x > 0 /\ (i <= 4 -> x + bool2int(x > i) = 5);
```

其中 `x > 0` 在根语境中, `i <= 4` 在否定语境中, `x + bool2int(x > i) = 5` 在肯定语境中, `x > i` 在混合语境中。

## 2.3.7 局部约束

Let 表达式也可以被用来引入局部约束, 通常用来约束局部变量的行为。例如, 考虑只利用乘法来定义开根号函数:

```
function var float: mysqrt(var float:x) =
    let { var float: y;
          constraint y >= 0;
          constraint x = y * y; } in y;
```

局部约束确保了 `y` 取正确的值; 而此值则会被函数返回。

局部约束可以在 let 表达式中使用, 尽管最普遍的应用是在定义函数时。

### Let 表达式

局部变量可以在任何以下格式的 `let` 表达式 中引入:

```
let { <声明>; ... <声明>; } in <表达式>
```

声明 `<dec>` 可以是决策变量或者参数 (此时必须被初始化) 或者约束项的声明。任何声明都不能在一个新的声明变量还没有引进时使用它。

注意局部变量和约束不可以出现在测试中。局部变量不可以出现在 否定或者 混合语境下的谓词和函数中, 除非这个变量是用表达式定义的。

## 2.3.8 定义域反射函数

其他重要的反射函数有允许我们对变量定义域进行访问的函数。表达式 `lb(x)` 返回一个小于等于 `x` 在一个问题的解中可能取的值的数值。通常它会是 `x` 声明的下限。如果 `x` 被声明为一个非有限类型, 例如, 只是 `var int`, 则它是错误的。类似地, 表达式 `dom(x)` 返回一个 `$x$` 在问题的任何解中的可能值的 (非严格) 超集。再次, 它通常是声明的值, 如果它不是被声明为有限则会出现错误。

Listing 2.3.11: 使用反射谓词 (`reflection.mzn`).

```
var -10..10: x;
constraint x in 0..4;
int: y = lb(x);
set of int: D = dom(x);
solve satisfy;
output ["y = ", show(y), "\nD = ", show(D), "\n"];
```

例如, Listing 2.3.11 中的模型或者输出

```
y = -10
D = -10..10
-----
```

或

```
y = 0
D = {0, 1, 2, 3, 4}
-----
```

或任何满足  $-10 \leq y \leq 0$  和  $\{0, \dots, 4\} \subseteq D \subseteq \{-10, \dots, 10\}$  的答案。

变量定义域反射表达式应该以在任何安全近似下都正确的的方式使用。但是注意这个是没有被检查的! 例如加入额外的代码

```
var -10..10: z;
constraint z <= y;
```

不是一个定义域信息的正确应用。因为使用更紧密 (正确的) 近似会比使用更弱的初始近似产生更多的解。

## 定义域反射

我们有查询包含变量的表达式的可能值的反射函数：

- `dom(<表达式>)` 返回 `<表达式>` 所有可能值的安全近似。
- `lb(<表达式>)` 返回 `<表达式>` 下限值的安全近似。
- `ub(<表达式>)` 返回 `<表达式>` 上限值的安全近似。

`lb` 和 `ub` 的表达式必须是 `int`, `bool`, `float` 或者 `set of int` 类型。`dom` 中表达式的类型不能是 `float`。如果 `<表达式>` 中的一个变量有一个非有限声明类型（例如，`var int` 或 `var float` 类型），则会出现一个错误。

我们也有直接作用于表达式数组的版本（有类似的限制）：

- `dom_array(<数组表达式>)`: 返回数组中出现的表达式的所有可能值的并集的一个安全近似。
- `lb_array(<数组表达式>)` 返回数组中出现的所有表达式的下限的安全近似。
- `ub_array(<数组表达式>)` 返回数组中出现的所有表达式的上限的安全近似。

谓词，局部变量和定义域反射的结合使得复杂全局约束通过分解定义变为可能。利用 Listing 2.3.12 中的代码，我们可以定义 `cumulative` 约束的根据时间的分解。

Listing 2.3.12: 利用分解来定义一个 谓词 (`cumulative.mzn`).

```
%-----%
% 需要给出一个任务集合，其中起始时间为's'，
% 持续时间'd' 以及资源需求量'r'，
% 任何时候需求量都不能超过
% 一个全局资源界限'b'。
% 假设：
% - forall i, d[i] >= 0 and r[i] >= 0
%-----%
predicate cumulative(array[int] of var int: s,
                      array[int] of var int: d,
                      array[int] of var int: r, var int: b) =
  assert(index_set(s) == index_set(d) /\
         index_set(s) == index_set(r),
        "cumulative: the array arguments must have identical index sets",
        assert(lb_array(d) >= 0 /\ lb_array(r) >= 0,
        "cumulative: durations and resource usages must be non-negative",
        let {
          set of int: times =
            lb_array(s) ..
            max([ ub(s[i]) + ub(d[i]) | i in index_set(s) ])
```

```

    }
    in
    forall( t in times ) (
        b >= sum( i in index_set(s) ) (
            bool2int( s[i] <= t /\ t < s[i] + d[i] ) * r[i]
        )
    )
);

```

这个分解利用 `lb` 和 `ub` 来决定任务可以执行的时间范围集合。接下来，它对 `times` 中的每个时间 `times` 都断言在此时间 `t` 激活的所有任务所需要的资源量总和小于界限 `b`。

### 2.3.9 作用域

MiniZinc 中声明的作用域值得我们简单地介绍下。MiniZinc 只有一个作用域，所以出现在声明中的所有变量都可以在模型中的每个表达式中可见。用以下几个方式，MiniZinc 引进局部作用域变量：

- 推导式表达式中的 迭代器
- 使用 `let` 表达式
- 谓词和函数中的 参数

任何局部作用域变量都会覆盖同名称的外部作用域变量。

Listing 2.3.13: 阐述变量作用域的模型 (scope.mzn).

```

int: x = 3;
int: y = 4;
predicate smallx(var int:y) = -x <= y /\ y <= x;
predicate p(int: u, var bool: y) =
    exists(x in 1..u)(y \ / smallx(x));
constraint p(x, false);
solve satisfy;

```

例如，在 Listing 2.3.13 中给出的模型中， $-x \leq y$  中的 `x` 是全局 `x`，`smallx(x)` 中的 `x` 是迭代器 `x in 1..u`，而析取中的 `y` 是谓词的第二个参数。



# CHAPTER 2.4

## 选项类型

选项类型是一个强大的抽象，使得简洁建模成为可能。一个选项类型决策变量代表了一个有其他可能  $\top$  的变量，在 MiniZinc 中表达为  $\text{<外层表达式>} \text{? } \text{<内层表达式>}$ ，代表了这个变量是 缺失的。选项类型变量在建模一个包含在其他变量没做决定之前不会有意义的变量的问题时是很有用的。

### 2.4.1 声明和使用选项类型

#### 选项类型变量

一个选项类型变量被声明为：

```
var opt <类型> : <变量名>;
```

其中 **<类型>** 是 `int` , `float` 或 `bool` 中的一个，或者是一个固定范围的表达式。选项类型变量可以是参数，但是这个不常用。

一个选项类型变量可以有附加值  $\text{? } \text{<表达式>}$  表明它是 缺失的。

三个内建函数被提供给选项类型变量：`absent(v)` 只有在选项类型变量  $v$  取值  $\text{? } \text{<表达式>}$  时返回 `true` , `occurs(v)` 只有在选项类型变量  $v$  不 取值  $\text{? } \text{<表达式>}$  时返回 `true` , 以及  $\text{? } \text{<表达式>}$  返回  $v$  的正常值或者当它取值  $\text{? } \text{<表达式>}$  时返回失败。

选项类型最常被用到的地方是调度中的可选择任务。在灵活的车间作业调度问题中，我们有  $n$  个在  $k$  个机器上执行的任务，其中完成每个机器上每个任务的时间可能是不一样的。我们的目标是最小化所有任务的总完成时间。一个使用选项类型来描述问题的模型在 Listing 2.4.1 中给

出。在建模这个问题的时候，我们使用  $n \times k$  个可选择的任务来代表每个机器上每个任务的可能性。我们使用 `alternative` 全局约束来要求任务的起始时间和它的持续时间跨越了组成它的可选择任务的时间，同时要求只有一个会实际运行。我们使用 `disjunctive` 全局变量在每个机器上最多有一个任务在运行，这里我们延伸到可选择的任务。最后我们约束任何时候最多有  $\$k$  个任务在运行，利用一个在实际（不是可选择的）任务上作用的冗余约束。

Listing 2.4.1: 使用选项类型的灵活车间作业调度模型 (`flexible-js.mzn`).

```

int: horizon;                                % 时间范围
set of int: Time = 0..horizon;
enum Task;
enum Machine;

array[Task,Machine] of int: d; % 每个机器上的持续时间
int: maxd = max([d[t,m] | t in Task, m in Machine]);
int: mind = min([d[t,m] | t in Task, m in Machine]);

array[Task] of var Time: S;                  % 起始时间
array[Task] of var mind..maxd: D;           % 持续时间
array[Task,Machine] of var opt Time: O; % 可选择的任务起始

constraint forall(t in Task)(alternative(S[t],D[t],
                                         [0[t,m]|m in Machine],[d[t,m]|m in Machine]));
constraint forall(m in Machine)
    (disjunctive([0[t,m]|t in Task],[d[t,m]|t in Task]));
constraint cumulative(S,D,[1|i in Task],k);

solve minimize max(t in Task)(S[t] + D[t]);

```

## 2.4.2 隐藏选项类型

当列表推导式是从在变量集合迭代上创建而来，或者 `where` 从句中的表达式还没有固定时，选项类型变量会隐式地出现。

例如，模型片段

```

var set of 1..n: x;
constraint sum(i in x)(i) <= limit;

```

是以下的语法糖

```
var set of 1..n: x;
constraint sum(i in 1..n)(if i in x then i else <> endif) <= limit;
```

内建函数 `sum` 实际上在一列类型-实例化 `var opt int` 上操作。由于 `<>` 在 `+` 中表现为标识 0，我们会得到期望的结果。

类似地，模型片段

```
array[1..n] of var int: x;
constraint forall(i in 1..n where x[i] >= 0)(x[i] <= limit);
```

是以下的语法糖

```
array[1..n] of var int: x;
constraint forall(i in 1..n)(if x[i] >= 0 then x[i] <= limit else <> endif);
```

同样地，函数 `forall` 实际上在一列类型-实例化 `var opt bool` 上操作。由于 `<>` 在 `\wedge` 上表现为标识 `true`，我们可以得到期望的结果。

尽管我们已经很小心了，隐式的使用可能会导致意外的行为。观察

```
var set of 1..9: x;
constraint card(x) <= 4;
constraint length([ i | i in x]) > 5;
solve satisfy;
```

它本应该是一个不可满足的问题。它返回  $x = \{1, 2, 3, 4\}$  作为一个解例子。这个是正确的因为第二个约束等于

```
constraint length([ if i in x then i else <> endif | i in 1..9 ]) > 5;
```

而可选择整数列表的长度总会是 9，所以这个约束总是会满足。

我们可以通过不在变量集合上创建迭代或者使用不固定的 `where` 从句来避免隐式的选项类型。例如，上面的两个例子可以不使用选项类型重写为

```
var set of 1..n: x;
constraint sum(i in 1..n)(bool2int(i in x)*i) <= limit;
```

和

```
array[1..n] of var int: x;  
constraint forall(i in 1..n)(x[i] >= 0 -> x[i] <= limit);
```

# CHAPTER 2.5

---

## 搜索

---

MiniZinc 默认没有我们想如何搜索解的声明。这就把搜索全部都留给下层的求解器了。但是有些时候，尤其是对组合整数问题，我们或许想规定搜索应该如何去进行。这就需要我们和求解器沟通出一个搜索策略 search。注意，搜索策略 不真的是模型的一部分。实际上，我们不要求每个求解器把所有可能的求解策略都实现了。MiniZinc 通过使用 *annotations* 来用一个稳定的方法跟约束求解器沟通额外的信息。

### 2.5.1 有限域搜索

利用有限域求解器搜索涉及到检查变量剩余的可能值以及选择进一步地约束一些变量。搜索则会加一个新的约束来限制这个变量的剩余值（实际上猜测解可能存在哪里），然后使用传播来确定其他的值是否可能存在于解中。为了确保完全性，搜索会保留另外一个选择，而它是新约束的否定。搜索会当有限域求解器发现所有的约束都被满足，此时一个解已经被找到，或者有约束不被满足时停止。当不可满足出现的时候，搜索必须换另外一个不同的选择集合继续下去。通常有限域求解器使用深度优先搜索，它会撤销最后一个做的选择然后尝试做一个新的选择。

Listing 2.5.1: n 皇后问题模型 (`nqueens.mzn`).

```
int: n;  
array [1..n] of var 1..n: q; % i 列的皇后在行 q[i]  
  
include "alldifferent.mzn";
```

```

constraint alldifferent(q); % 不同行
constraint alldifferent([ q[i] + i | i in 1..n]); % 不同对角线
constraint alldifferent([ q[i] - i | i in 1..n]); % 上 + 下

% 搜索
solve :: int_search(q, first_fail, indomain_min, complete)
    satisfy;
output [ if fix(q[j]) == i then "Q" else "." endif ++
        if j == n then "\n" else "" endif | i,j in 1..n]

```

有限域问题的一个简单例子是  $n$  皇后问题，它要求我们放置  $n$  个皇后在  $n \times n$  棋盘上使得任何两个都不会互相攻击。变量  $q[i]$  记录了在  $i$  列的皇后放置在哪一行上。[Fig. 2.5.1](#) 的左边给出了  $n = 9$  的典型（部分）搜索树。我们首选设置  $q[1] = 1$ ，这样就可以从其他变量的定义域里面移除一些数值，例如  $q[2]$  不能取值 1 或者 2。我们接下来设置  $q[2] = 3$ ，然后进一步地从其他变量的定义域里面移除一些数值。我们设置  $q[3] = 5$ （它最早可能的值）。在这三个决策后，棋盘的状态显示为 [Fig. 2.5.2](#)。其中皇后表示已经固定的皇后位置。星星表示此处放置的皇后会攻击到已经放置的皇后，所以我们不能在此处放置皇后。

一个搜索策略决定要做哪一个选择。我们目前所做的决定都按照一个简单的策略：选择第一个还没有固定的变量，尝试设置它为它的最小可能值。按照这个策略，下一个决策应该是  $q[4] = 7$ 。变量选择的另外一个策略是选择现在可能值集合 定义域 最小的变量。按照这个所谓 最先失败 变量选择策略，下一个决策应该是  $q[6] = 4$ 。如果我们做了这个决策，则初始的传播会去除掉 [Fig. 2.5.3](#) 中显示的额外的值。但是它使得  $q[8]$  只剩余有一个值。所以  $q[8] = 7$  被执行。但是这又使得  $q[7]$  和  $q[9]$  也只剩余一个值 2。因此这个时候有个约束一定会被违反。我们检测到了不满足性，求解器必须回溯取消最后一个决策  $q[6] = 4$  并且加入它的否定  $q[6] \neq 4$ （引导我们到了 [Fig. 2.5.1](#) 中树的状态 (c)），即强制使  $q[6] = 8$ 。这使得有些值从定义域中被去除。我们接下来继续重新启用搜索策略来决定该怎么做。

很多有限域搜索被定义为这种方式：选择一个变量来进一步约束，然后选择如何进一步地约束它。

## 2.5.2 搜索注解

MiniZinc 中的搜索注解注明了为了找到一个问题的解应如何去搜索。注解附在求解项，在关键字 `solve` 之后。搜索注解

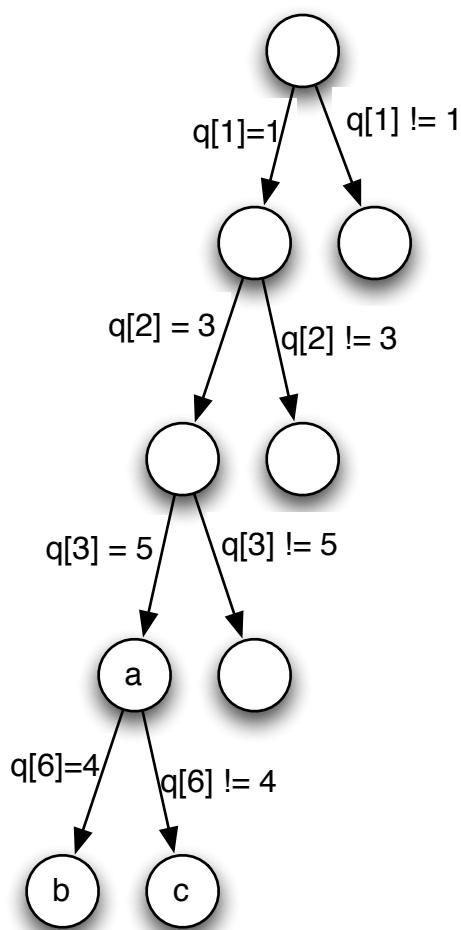


Fig. 2.5.1: 9 皇后问题的部分搜索树

	q[1]	q[2]	q[3]	q[4]	q[5]	q[6]	q[7]	q[8]	q[9]
1									
2									
3									
4									
5									
6									
7									
8									
9									

Fig. 2.5.2: 在加入  $q[1] = 1, q[2] = 4, q[3] = 5$  的状态

	q[1]	q[2]	q[3]	q[4]	q[5]	q[6]	q[7]	q[8]	q[9]
1									
2									
3									
4									
5									
6									
7									
8									
9									

Fig. 2.5.3: 在进一步加入  $q[6] = 4$  后的初始传播

```
solve :: int_search(q, first_fail, indomain_min, complete)
    satisfy;
```

出现在求解项中。注解使用连接符 `::` 附为模型的一部分。这个搜索注解意思是我们应该按照从整型变量数组 `q` 中选择拥有最小现行定义域的变量（这个是 `first_fail` 规则），然后尝试设置其为它的最小可能值（`indomain_min` 值选择），纵观整个搜索树来搜索（`complete` 搜索）。

## 基本搜索注解

我们有三个基本搜索注解，相对应于不同的基本搜索类型：

- `int_search( <变量>, <变量选择>, <约束选择>, <策略> )` 其中 `<变量>` 是一个 `var int` 类型的一维数组，`<变量选择>` 是一个接下来会讨论的变量选择注解，`<约束选择>` 是一个接下来会讨论的如何约束一个变量的选择，`<策略>` 是一个搜索策略，我们暂时假设为 `complete`
- `bool_search( <变量>, <变量选择>, <约束选择>, <策略> )` 其中 `<变量>` 是一个 `var bool` 类型的一维数组，剩余的和上面一样。
- `set_search( <变量>, <变量选择>, <约束选择>, <策略> )` 其中 `<变量>` 是一个 `var set of int` 类型的一维数组，剩余的和上面一样。
- `float_search( <变量>, <精度>, <变量选择>, <约束选择>, <策略> )` 其中 `<变量>` 是一个一维 `var float` 数组，`<precision>` 是一个固定的用于表示  $\epsilon$  浮点数，其中两个数之差低于这个浮点数时被认为相等。剩余的和上面一样。

变量选择注解的例子有：

- `input_order`: 从数组中按照顺序选择
- `first_fail`: 选择拥有最小定义域大小的变量，以及
- `smallest`: 选择拥有最小值的变量。

约束一个变量的方式有：

- `indomain_min`: 赋最小的定义域内的值给变量，
- `indomain_median`: 赋定义域内的中间值给变量，
- `indomain_random`: 从定义域中取一个随机的值赋给变量，以及
- `indomain_split` 把变量定义域一分为二然后去除掉上半部分。

对于完全搜索，`<策略>` 基本都是 `complete`。关于一份完整的变量和约束选择注解，请参看 MiniZinc 参考文档中的 FlatZinc 说明书。

利用搜索构造注解，我们可以创建更加复杂的搜索策略。目前我们只有一个这样的注解。

```
seq_search([ <搜索注解>, ... , <搜索注解> ])
```

顺序搜索构造首先执行对列表中的第一个注解所指定的变量的搜索，当这个注解中的所有的变量都固定后，它执行第二个搜索注解，等等。直到所有的搜索注解都完成。

我们来看一下 Listing 2.3.8 中给出的车间作业调度模型。我们可以替换求解项为

```
solve :: seq_search([
    int_search(s, smallest, indomain_min, complete),
    int_search([end], input_order, indomain_min, complete)])
minimize end
```

通过选择可以最早开始的作业并设置其为 `s`，起始时间被设置为 `s`。当所有的起始时间都设置完后，终止时间 `end` 或许还没有固定。因此我们设置其为它的最小可能取值。

### 2.5.3 注解

在 MiniZinc 中，注解是第一类对象。我们可以在模型中声明新的注解，以及声明和赋值给注解变量。

#### 注解

注解有一个类型 `ann`。你可以声明一个注解参数 `parameter`（拥有可选择的赋值）：

```
ann : <标识符>;
ann : <标识符> = <注解表达式> ;
```

对注解变量赋值和对其他参数赋值一样操作。

表达式，变量声明，和 `solve` 项都可以通过使用 `::` 操作符来成为注解。

使用注解项 `annotation`，我们可以声明一个新的注解 `annotation` 项：

```
annotation <注解名> ( <参数定义>, ..., <参数定义> ) ;
```

Listing 2.5.2: n 皇后问题的注解模型 (`nqueens-ann.mzn`).

```
annotation bitdomain(int:nwords);

include "alldifferent.mzn";

int: n;
array [1..n] of var 1..n: q :: bitdomain(n div 32);

constraint alldifferent(q) :: domain;
constraint alldifferent([ q[i] + i | i in 1..n]) :: domain;
```

```

constraint alldifferent([ q[i] - i | i in 1..n]) :: domain;

ann: search_ann;

solve :: search_ann satisfy;

output [ if fix(q[j]) == i then "Q" else "." endif ++
        if j == n then "\n" else "" endif | i,j in 1..n]

```

Listing 2.5.2 中的程序阐述了注解声明，注解和注解变量的使用。我们声明一个新的注解 `bitdomain`，意思是来告诉求解器变量定义域应该通过大小为 `nwords` 的比特数组来表示。注解附注在变量 `q` 的声明之后。每一个 `alldifferent` 约束都被注解为内部注解 `domain`，而它指导求解器去使用 `alldifferent` 的定义域传播版本（如果有的话）。一个注解变量 `search_ann` 被声明和使用来定义搜索策略。我们可以在一个单独的数据文件中来给出搜索策略的值。

搜索注解的例子或许有以下几种（我们假设每一行都在一个单独的数据文件中）

```

search_ann = int_search(q, input_order, indomain_min, complete);
search_ann = int_search(q, input_order, indomain_median, complete);
search_ann = int_search(q, first_fail, indomain_min, complete);
search_ann = int_search(q, first_fail, indomain_median, complete);

```

第一个只是按顺序来选择皇后然后设置其为最小值。第二个按顺序来选择皇后，但是设置中间值给它。第三个选择定义域大小最小的皇后，然后设置最小值给它。最后一个策略选择定义域大小最小的皇后，设置中间值给它。

不同的搜索策略对于能多容易找到解有显著的差异。下面的表格给出了一个简单的关于使用 4 种不同的搜索策略找到 `n` 皇后问题的第一个解所要做的决策个数（其中—表示超过 100,000 个决策）。很明显地看到，合适的搜索策略会产生显著的提高。

<code>n</code>	<code>input-min</code>	<code>input-median</code>	<code>ff-min</code>	<code>ff-median</code>
10	28	15	16	20
15	248	34	23	15
20	37330	97	114	43
25	7271	846	2637	80
30	—	385	1095	639
35	—	4831	—	240
40	—	—	—	236

# CHAPTER 2.6

## Minizinc 中的有效建模实践

对同一个问题，几乎总是存在多种方式来建模。其中一些产生的模型可以很有效地求解，另外一些则不是。通常情况下，我们很难提前判断哪个模型是对解决一个特定的问题最有效的。事实上，这或许十分依赖于我们使用的底层求解器。在这一章中，我们专注于建模实践，来避免产生模型的过程和产生的模型低效。

### 2.6.1 变量界限

有限域传播器，是 Minizinc 所针对的求解器中的核心类型。在当其所涉及到的变量的界限越紧凑时，此传播器越有效。它也会当问题含有会取很大整型数值的子表达式时表现得很差，因为它们可能会隐式地限制整型变量的大小。

Listing 2.6.1: 没有无界整数的模型 (grocery.mzn).

```
var int: item1;
var int: item2;
var int: item3;
var int: item4;

constraint item1 + item2 + item3 + item4 == 711;
constraint item1 * item2 * item3 * item4 == 711 * 100 * 100 * 100;
```

```

constraint      0 < item1 /\ item1 <= item2
               /\ item2 <= item3 /\ item3 <= item4;

solve satisfy;

output ["{", show(item1), ", ", show(item2), ", ", show(item3), ", ",
        show(item4), "}\n"];

```

在 Listing 2.6.1 中的杂货店问题要找寻 4 个物品使得它们的价格加起来有 7.11 元并且乘起来也有 7.11 元。变量被声明为无界限。运行

```
$ minizinc --solver g12fd grocery.mzn
```

得到

```

=====UNSATISFIABLE=====
% /tmp/mznfile85EWzj.fzn:11: warning: model inconsistency detected before
→ search.

```

这是因为乘法中的中间表达式的类型也会是 `var int`，也会被求解器给一个默认的界限  $-1,000,000 \dots 1,000,000$ 。但是这个范围太小了以至于不能承载住中间表达式所可能取的值。

更改模型使得初始变量都被声明为拥有更紧致的界限

```

var 1..711: item1;
var 1..711: item2;
var 1..711: item3;
var 1..711: item4;

```

我们得到一个更好的模型，因为现在 MiniZinc 可以推断出中间表达式的界限，并且使用此界限而不是默认的界限。在做此更改后，求解模型我们得到

```
{120,125,150,316}
```

注意，就算是改善的模型也可能对于某些求解器来说会很难解决。运行

```
$ minizinc --solver g12lazy grocery.mzn
```

不能得到任何结果，因为求解器给中间产生的变量创建了巨大的表示。

### 给变量加界限

在模型中要尽量使用有界限的变量。当使用 `let` 声明来引进新的变量时，始终尽量给它们定义正确的和紧凑的界限。这会使得你的模型更有效率，避免出现意外溢出的可能性。一个例外是当你引进一个新的变量然后立刻定义它等于一个表达式，通常 MiniZinc 都可以从此表达式推断出此变量有效的界限。

## 2.6.2 有效的生成元

想象下我们想要计算在一个图中出现的三角形的个数 ( $K_3$  子图)。假设此图由一个邻接矩阵定义：如果点  $i$  和  $j$  邻接，则  $\text{adj}[i, j]$  为真。我们或许可以写成

```
int: count = sum ([ 1 | i,j,k in NODES where i < j /\ j < k
                  /\ adj[i,j] /\ adj[i,k] /\ adj[j,k]]);
```

这当然是对的，但是它检查了所有点可能组成的三元组。如果此图是稀疏的，在意识到一旦我们选择了  $i$  和  $j$ ，就可以进行一些测试之后，我们可以做得更好。

```
int: count = sum ([ 1 | i,j in NODES where i < j /\ adj[i,j],
                    k in NODES where j < k /\ adj[i,k] /\ adj[j,k]]);
```

你可以使用内建 `trace` 函数来帮助决定在生成元内发生了什么。

### 追踪

函数 `trace(s,e)` 在对表达式 `e` 求值并返回它的值之前就输出字符串 `s`。它可以在任何情境下使用。

例如，我们可以查看在两种计算方式下的内部循环中分别进行了多少次测试。

```
int:count=sum([ 1 | i,j,k in NODES where
                trace("+" , i<j /\ j<k /\ adj[i,j] /\ adj[i,k] /\ adj[j,k]) ]);
adj = [| false, true, true, false
        | true, false, true, false
        | true, true, false, true
        | false, false, true, false |];
constraint trace("\n",true);
solve satisfy;
```

得到输出：

```
+++++
-----
```

表示内部循环进行了 64 次，而

```
int: count = sum ([ 1 | i,j in NODES where i < j /\ adj[i,j],
                    k in NODES where trace("++", j < k /\ adj[i,k] /\_
                    adj[j,k])]);
```

得到输出

```
+++++
-----
```

表示内部循环进行了 16 次。

注意你可以在 `trace` 中使用单独的字符串来帮助你理解模型创建过程中发生了什么。

```
int: count = sum( i,j in NODES where i < j /\ adj[i,j])(_
    sum([trace("(++show(i)++","++show(j)++","++show(k)++)",1) |_
        k in NODES where j < k /\ adj[i,k] /\ adj[j,k]]));
```

会输出在计算过程中找到的每个三角形。得到输出

```
(1,2,3)
-----
```

我们要承认这里我们有一点点作弊：在某些情况下，MiniZinc 编译器实际上会把 `where` 从句中的参数自动重新排序，所以他们会尽快地被计算。在这种情况下，加入 `trace` 函数实际上阻止了这种优化。一般来说，通过分离 `where` 从句把它们摆到尽量接近生成元，这其实是一个很好的主意来帮助编译器运作正常。

### 2.6.3 冗余约束

模型的形式会影响约束求解器求解它的效率。在很多情况下加入冗余约束，即被现有的模型逻辑上隐含的约束，可能会让求解器在更早时候产生更多可用的信息从而提高找寻解的搜索效率。

回顾下第复杂约束 (page 58) 节中的魔术串问题。

运行 `n = 16` 时的模型：

```
$ minizinc --all-solutions --statistics magic-series.mzn -D "n=16;"
```

`n = 16`

```
s = [12, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0];
-----
=====
```

统计数据显示需要有 89 个失败。

我们可以在模型中加入冗余约束。由于序列中的每个数字是用来计算某一个数字出现的次数，我们知道它们的和肯定是 `n`。类似地，由于这个序列是魔术的，我们知道 `s[i] * i` 的和肯定也是 `n`。使用如下方式把这些约束加入我们的模型 Listing 2.6.2.

Listing 2.6.2: 使用冗余约束求解魔术串问题模型 (`magic-series2.mzn`).

```
int: n;
array[0..n-1] of var 0..n: s;

constraint forall(i in 0..n-1) (
    s[i] = (sum(j in 0..n-1)(bool2int(s[j]=i)));
% redundant
constraint sum(i in 0..n-1)(s[i]) = n;
constraint sum(i in 0..n-1)(s[i] * i) = n;
solve satisfy;

output [ "s = ", show(s), ";" ] ;
```

像之前那样求解同一个问题

```
$ minizinc --all-solutions --statistics magic-series2.mzn -D "n=16;"
```

产生了同样的输出。但是统计显示只搜索了 13 个决策点。这些冗余约束使得求解器更早地去剪枝。

## 2.6.4 模型选择

在 MiniZinc 中有很多方式去给同一个问题建模，尽管其中有些模型或许会比另外的一些模型更自然。不同的模型或许会产生不同的求解效率。更糟糕的是，在不同的求解后端中，不同的模型或许会更好或更差。但是，我们还是可以给出一些关于普遍情况下产生更好的模型的指导：

### 模型之间的选择

一个好的模型倾向于有以下特征

- 更少量的变量，或者至少是更少量的没有被其他变量功能上定义的变量。
- 更小的变量定义域范围
- 模型的约束定义更简洁或者直接
- 尽可能地使用全局约束

实际情况中，所有这些都需要通过检查这个模型的搜索到底多有效率来断定模型好坏。通常除了用实验之外，我们很难判断搜索是否高效。

观察如下问题，我们要找寻 1 到  $n$  这  $n$  个数字的排列，使得相邻数字的差值也形成一个 1 到  $n$  的排列。Listing 2.6.3 中给出了一个用直观的方式来建模此问题的模型。注意变量  $u$  被变量  $x$  功能性定义。所以最差情况下的搜索空间是  $n^n$ 。

Listing 2.6.3: 对于 CSplib prob007 所有间隔系列问题的模型 (allinterval.mzn).

```
include "alldifferent.mzn";

int: n;

array[1..n] of var 1..n: x;          % 数字的序列
array[1..n-1] of var 1..n-1: u;      % 差的序列

constraint alldifferent(x);
constraint alldifferent(u);
constraint forall(i in 1..n-1)(u[i] = abs(x[i+1] - x[i]));

solve :: int_search(x, first_fail, indomain_min, complete)
        satisfy;
output ["x = ", show(x), "\n"];
```

在这个模型中，数组  $x$  代表  $n$  个数字的排序。约束自然地可用 `alldifferent` 来表示。

求解模型

```
$ minizinc --solver g12fd --all-solutions --statistics allinterval.mzn -D
→"n=10;"
```

在 84598 个决策点和 3 秒的时间内找到了所有的解。

另外一个模型是使用数组  $y$ ，其中  $y[i]$  代表数字  $i$  在序列中的位置。我们同时也使用变量  $v$  来建模表示差的位置。 $v[i]$  表示了绝对值差  $i$  在序列出现的位置。如果  $y[i]$  和  $y[j]$  差别为一，其中  $j > i$ ，则代表了它们的位置是相邻的。所以  $v[j-i]$  被约束为两个位置中最早的那个。我们可以给这个模型加入两个冗余约束：由于我们知道差值  $n-1$  肯定会产生，我们就可以推断出 1 和  $n$  的位置必须是相邻的  $\text{abs}(y[1] - y[n]) = 1$ 。同时也告诉我们差值  $n-1$  的位置就是在  $y[1]$  和  $y[n]$  中的最早的那个位置，即  $v[n-1] = \min(y[1], y[n])$ 。有了这些之后，我们可以建模此问题为:[numref:ex-allint2](#)。输出语句从位置数组  $y$  里重现了原本的序列  $x$ 。

Listing 2.6.4: CSplib 中全区间序列问题 prob007 的一个逆向模型。(allinterval2.mzn).

```
include "alldifferent.mzn";

int: n;

array[1..n] of var 1..n: y; % 每个数值的位置
array[1..n-1] of var 1..n-1: v; % 差值 i 的位置

constraint alldifferent(y);
constraint alldifferent(v);
constraint forall(i,j in 1..n where i < j)(
    (y[i] - y[j] = 1 -> v[j-i] = y[j]) /\ 
    (y[j] - y[i] = 1 -> v[j-i] = y[i])
);

constraint abs(y[1] - y[n]) = 1 /\ v[n-1] = min(y[1], y[n]);

solve :: int_search(y, first_fail, indomain_min, complete)
    satisfy;

output [ "x = [", ] ++
    [ show(i) ++
        if j == n then "]\\n;" else ", " endif
        | j in 1..n, i in 1..n where j == fix(y[i]) ];
```

逆向模型跟初始模型有同样的变量和定义域大小。但是相对于给变量  $x$  和  $u$  的关系建模，逆向模型使用了一个更加非直接的方式来给变量  $y$  和  $v$  的关系建模。所以我们或许期望初始模型更

好些。

命令

```
$ minizinc --solver g12fd --all-solutions --statistics allinterval2.mzn -D
→ "n=10;"
```

在 75536 个决策点和 18 秒内找到了所有的解。有趣的是，尽管这个模型不是简洁的，在变量  $y$  上搜索比在变量  $x$  上搜索更加有效率。简洁的缺乏意味着尽管搜索需要更少的决策点，但是在时间上实质上会更慢。

## 2.6.5 多重建模和连通

当我们对同一个问题有两个模型时，由于每个模型可以给求解器不同的信息，通过把两个模型中的变量系到一起从而同时使用两个模型或许对我们是有帮助的。

Listing 2.6.5: CSplib 中全区间序列问题 prob007 的一个双重模型。(allinterval3.mzn).

```
include "inverse.mzn";

int: n;

array[1..n] of var 1..n: x; % 数值的序列
array[1..n-1] of var 1..n-1: u; % 差值的序列

constraint forall(i in 1..n-1)(u[i] = abs(x[i+1] - x[i]));

array[1..n] of var 1..n: y; % 每个数值的位置
array[1..n-1] of var 1..n-1: v; % 差值的位置

constraint inverse(x,y);
constraint inverse(u,v);

constraint abs(y[1] - y[n]) = 1 /\ v[n-1] = min(y[1], y[n]);

solve :: int_search(y, first_fail, indomain_min, complete)
      satisfy;

output ["x = ", show(x), "\n"];
```

[Listing 2.6.5](#) 给出了一个结合 `allinterval.mzn` 和 `allinterval2.mzn` 特征的双重模型。模型的开始来自于 `allinterval.mzn`。我们接着介绍了来自于 `allinterval2.mzn` 中的变量 `y` 和 `v`。我们使用全局约束 `inverse` 来把变量绑到一起: `inverse(x,y)` 约束 `y` 为 `x` 的逆向函数 (反之亦然), 即,  $x[i] = j \Leftrightarrow y[j] = i$ 。[Listing 2.6.6](#) 中给出了它的一个定义。这个模型没有包含把变量 `y` 和 `v` 关联起来的约束, 它们是冗余的 (实际上是传播冗余)。所以它们不会给基于传播的求解器多余的信息。`alldifferent` 也不见了。原因是它们被逆向约束变得冗余了 (传播冗余)。唯一的约束是关于变量 `x` 和 `u` 和关系的约束以及 `y` 和 `v` 的冗余约束。

[Listing 2.6.6](#): 全局约束 `inverse` 的一个定义 (`inverse.mzn`).

```
predicate inverse(array[int] of var int: f,
                 array[int] of var int: invf) =
    forall(j in index_set(invf))(invf[j] in index_set(f)) /\ 
    forall(i in index_set(f))(
        f[i] in index_set(invf) /\ 
        forall(j in index_set(invf))(j == f[i] <-> i == invf[j])
    );

```

双重模型的一个优点是我们可以有更多的定义不同搜索策略的视角。运行双重模型,

```
$ minizinc --solver g12fd --all-solutions --statistics allinterval3.mzn -D
→ "n=10;"
```

注意它使用逆向模型的搜索策略, 标记变量 `y`, 在 1714 决策点和 0.5 秒内找到了所有的解。注意标记变量 `x` 来运行同样的模型, 需要 13142 个决策点和 1.5 秒。

## 2.6.6 对称

对称在约束满足和优化问题中是常见的。我们可以再次通过 [Listing 2.5.1](#) 来看一下这个问题。在 [Fig. 2.6.1](#) 棋盘的左上方展示了一个 8 皇后问题的解 (标记为 “original”)。剩下的棋盘展示了 7 个解的对称版本: 旋转 90 度, 180 度和 270 度, 还有垂直翻转。

如果我们想要穷举 8 皇后问题的所有解, 很明显我们需要通过穷举彼此之间不对称的解为求解器省下一些工作, 然后生成这些的对称版本。这是我们想要在约束模型中摆脱对称的一个理由。另外一个更重要的理由是, 我们的求解器也可能会探索非解状态的对称版本!

举个例子, 一个典型的约束求解器可能会尝试把第 1 列皇后放在第 1 行上 (这是可以的), 然后尝试把第 2 列的皇后放到第 3 行上。这在第一眼看是没有违反任何约束的。然而, 这种设置不能被完成成为一个解 (求解器会在一些搜索之后发现这一点)。[Fig. 2.6.2](#) 在左上方的棋盘展示了这种设置。现在没有任何东西会阻止求解器去尝试, 比如, 在 [Fig. 2.6.2](#) 最低一行的左边第二种设置。

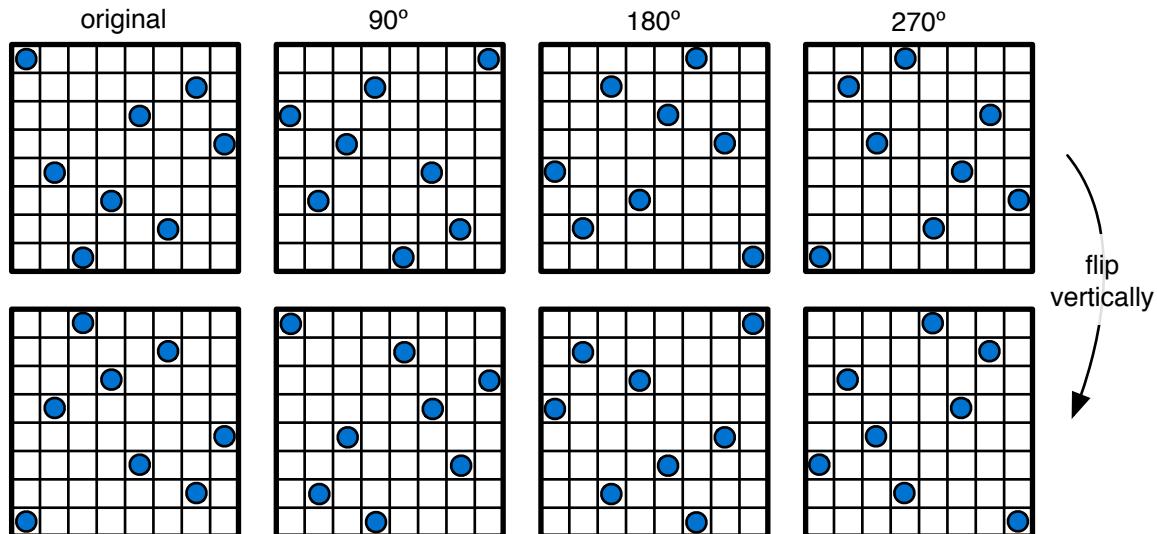


Fig. 2.6.1: 8 皇后问题解的对称变化

其中第 1 列的皇后仍然在第 1 行, 而第 3 列的皇后放在第 2 行。于是, 即便是搜索一个解, 求解器可能需要探索很多它已经看到并证明不可满足状态的对称状态!

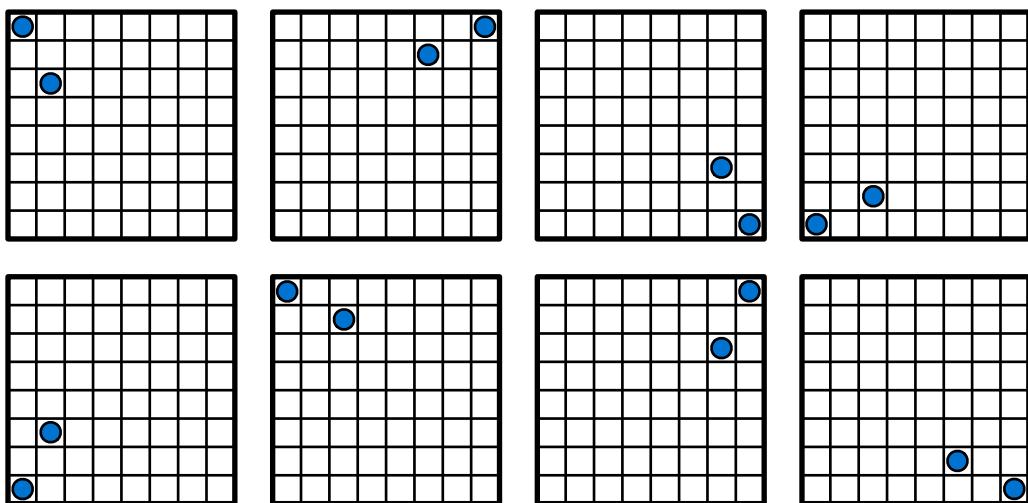


Fig. 2.6.2: 8 皇后问题不可满足约束的部分解的对称版本

### 2.6.6.1 静态的对称性破缺

解决对称问题的建模技巧叫做 **对称性破缺**, 在它的最简单形式里, 需要在模型中加入约束使一个(不完整的)赋值的所有对称变换去除掉而保留一个。这些约束称作 **静态的对称性破缺约束**。

对称性破缺背后基本的想法是加入 **顺序**。举个例子, 我们可以通过简单地加入约束使第一列的皇后必须在棋盘的上半部分, 从而去除掉所有棋盘垂直翻转的。

```
constraint q[1] <= n div 2;
```

请相信以上约束会去除掉在 Fig. 2.6.1 中所有对称变换的一半。为了去除所有对称, 我们需要更多工作。

当我们把所有对称都表示成数组变量的排列, 一组字典顺序约束可以用于破坏所有对称。举个例子, 如果数组变量名为  $x$ , 而翻转数组是这个问题的一种对称, 那么以下约束可以破坏那种对称:

```
constraint lex_lesseq(x, reverse(x));
```

那么二维数组又怎么样呢? 字典顺序同样适用, 我们只需要把数组转换成一维的。举个例子, 下面的约束破坏了沿着其中一个对角线翻转数组的对称性(注意到第二个生成式里对换的数组下标):

```
array[1..n,1..n] of var int: x;
constraint lex_lesseq([x[i,j] | i,j in 1..n], [x[j,i] | i,j in 1..n]);
```

字典排序约束的好处在于我们可以加入多个(同时破坏几个对称性), 而不需要它们互相干扰, 只要我们保持第一个参数中的顺序一致即可。

对于  $n$  皇后问题, 很不幸的是这个技巧不能马上适用, 因为又一些对称不能被描述成数组  $q$  的排列。克服这个问题的技巧是把  $n$  皇后问题表示成布尔变量。对于每个棋盘的每个格子, 布尔变量表示是否有一个皇后在上面。现在所有的对称性都可以表示成这个数组的排列。因为主要的  $n$  皇后问题的主要约束在整型数组  $q$  里面更加容易表达, 我们只需要两个模型都用起来, 然后在它们之间加入连通约束。正如[多重建模和连通](#)(page 112)中解释的一样。

加入布尔变量, 连通约束和对称性破缺约束的完整模型展示在 Listing 2.6.7 里面。我们可以做一些小实验来检查它是否成功的破坏所有对称性。尝试用不断增加的  $n$  运行模型, 比如从 1 到 10, 数一下解的个数(比如, 使用 Gecode 求解器的  $-s$  标志, 或者选择 IDE 中“Print all solutions”和“Statistics for solving”)。你应该可以获得以下数列的解: 1, 0, 0, 1, 2, 1, 6, 12, 46, 92。你可以搜索 *On-Line Encyclopedia of Integer Sequences* (<http://oeis.org>) 来校验这个序列。

Listing 2.6.7:  $n$  皇后问题对称性破缺的部分模型 (full model: `nqueens_sym.mzn`).

```
% 映射每一个位置 i,j 到一个布尔变量上来表示在 i,j 上是否有一个皇后
array[1..n,1..n] of var bool: qb;
```

```
% 连通约束
constraint forall (i,j in 1..n) ( qb[i,j] <-> (q[i]=j) );
```

```
% 字典排序对称性破缺
```

```
constraint
```

```

lex_lesseq(array1d(qb), [ qb[j,i] | i,j in 1..n ])
/\ lex_lesseq(array1d(qb), [ qb[i,j] | i in reverse(1..n), j in 1..n ])
/\ lex_lesseq(array1d(qb), [ qb[j,i] | i in 1..n, j in reverse(1..n) ])
/\ lex_lesseq(array1d(qb), [ qb[i,j] | i in 1..n, j in reverse(1..n) ])
/\ lex_lesseq(array1d(qb), [ qb[j,i] | i in reverse(1..n), j in 1..n ])
/\ lex_lesseq(array1d(qb), [ qb[i,j] | i,j in reverse(1..n) ])
/\ lex_lesseq(array1d(qb), [ qb[j,i] | i,j in reverse(1..n) ])
;

```

### 2.6.6.2 其他对称的例子

许多其他问题都有内在的对称性，破坏这些对称性常常会令求解表现不一样。以下是一些常见的例子：

- 装箱问题：当尝试把物品装入箱子时，任意两个有相同容量的箱子都是对称的
- 涂色问题：当尝试为一个图涂色使得所有相邻的节点都有不同的颜色时，我们通常用整型变量对颜色建模。但是，对颜色的任意排列都是一种合法的涂色方案。
- 车辆路线问题：如果任务是给顾客分配一些车辆，任何两辆有相同容量的车可能是对称的（这跟装箱问题是相似的）
- 排班/时间表问题：两个有能力的职员可能是可以相互交换的，就像两个有相同容量或者设备的房间一样

# CHAPTER 2.7

## 在 MiniZinc 中对布尔可满足性问题建模

MiniZinc 可以被用来给布尔可满足性问题建模，这种问题的变量被限制为是布尔型 (`bool`)。MiniZinc 可以使用有效率的布尔可满足性求解器来有效地解决求得的模型。

### 2.7.1 整型建模

很多时候，尽管我们想要使用一个布尔可满足性求解器，我们可能也需要给问题的整数部分建模。有三种通用的方式使用布尔型变量对定义域为范围  $0 \dots m$  内的整型变量  $I$  建模，其中  $m = 2^k - 1$ 。

- 二元： $I$  被表示为  $k$  个二元变量  $i_0, \dots, i_{k-1}$ ，其中  $I = 2^{k-1}i_{k-1} + 2^{k-2}i_{k-2} + \dots + 2i_1 + i_0$ 。在 MiniZinc 中，这可表示为

```
array[0..k-1]  of var bool: i;  
var 0..pow(2,k)-1: I = sum(j in 0..k-1)(bool2int(i[j])*pow(2,j));
```

- 一元： $I$  被表示为  $m$  个二元变量  $i_1, \dots, i_m$  且  $i = \sum_{j=1}^m \text{bool2int}(i_j)$ 。由于在一元表示中有大量的冗余表示，我们通常要求  $i_j \rightarrow i_{j-1}, 1 < j \leq m$ 。在 MiniZinc 中，这可表示为

```
array[1..m]  of var bool: i;  
constraint forall(j in 2..m)(i[j] -> i[j-1]);  
var 0..m: I = sum(j in 1..m)(bool2int(i[j]));
```

- 值: 其中  $I$  被表示为  $m + 1$  个二元变量  $i_0, \dots, i_m$ ，其中  $i = k \Leftrightarrow i_k$  并且  $i_0, \dots, i_m$  中最多有一个为真。在 MiniZinc 中，这可表示为

```
array[0..m] of var bool: i;
constraint sum(j in 0..m)(bool2int(i[j])) == 1;
var 0..m: I;
constraint foall(j in 0..m)(I == j <-> i[j]);
```

每种表示都有其优点和缺点。这取决于模型中需要对整数做什么样的操作，而这些操作在哪一种表示上更为方便。

## 2.7.2 非等式建模

接下来，让我们考虑如何为一个拉丁方问题建模。一个拉丁方问题是在  $n \times n$  个网格上放置  $1..n$  之间的数值使得每个数在每行每列都仅出现一次。图 Listing 2.7.1 中给出了拉丁方问题的的一个整数模型。

Listing 2.7.1: 拉丁方问题的整数模型 (`latin.mzn`).

```
int: n; % 拉丁方的大小
array[1..n,1..n] of var 1..n: a;

include "alldifferent.mzn";
constraint forall(i in 1..n)(
    alldifferent(j in 1..n)(a[i,j]) /\ 
    alldifferent(j in 1..n)(a[j,i])
);
solve satisfy;
output [ show(a[i,j]) ++ if j == n then "\n" else " " endif |
    i in 1..n, j in 1..n];
```

整型变量直接的唯一的约束实际上是非等式，而它在约束 `alldifferent` 中被编码。数值表示是表达非等式的最佳方式。图 Listing 2.7.2 给出了一个关于拉丁方问题的只含有布尔型变量的模型。注意每个整型数组元素  $a[i, j]$  被替换为一个布尔型数组。我们使用谓词 `exactlyone` 来约束每个数值在每行每列都仅出现一次，也用来约束有且仅有一个布尔型变量对应于整型数组元素  $a[i, j]$  为真。

Listing 2.7.2: 拉丁方问题的布尔型模型 (latinbool.mzn).

```

int: n; % 拉丁方的大小
array[1..n,1..n] of var bool: a;

predicate atmostone(array[int] of var bool:x) =
    forall(i,j in index_set(x) where i < j)(
        (not x[i] \wedge not x[j]));
predicate exactlyone(array[int] of var bool:x) =
    atmostone(x) /\ exists(x);

constraint forall(i,j in 1..n)(
    exactlyone(k in 1..n)(a[i,j,k]) /\ 
    exactlyone(k in 1..n)(a[i,k,j]) /\ 
    exactlyone(k in 1..n)(a[k,i,j])
);

solve satisfy;
output [ if fix(a[i,j,k]) then
    show(k) ++ if j == n then "\n" else " " endif
    else "" endif | i,j,k in 1..n ];

```

### 2.7.3 势约束建模

让我们来看下如何对点灯游戏建模。这个游戏由一个矩形网格组成，每个网格为空白或者被填充。每个被填充的方格可能包含 1 到 4 之间的数字，或者没有数字。我们的目标是放置灯泡在空白网格使得

- 每个空白的网格是“被照亮的”，也就是说它可以透过一行没有被打断的空白网格看到光亮。
- 任何两个灯泡都不能看到彼此。
- 一个有数值的填充的网格相邻的灯泡个数必须等于这个网格中的数值。

Fig. 2.7.1 给出了点灯游戏的一个例子以及 Fig. 2.7.2 给出了它的解。

这个问题很自然地可以使用布尔型变量建模。布尔型变量用来决定哪一个网格包含有一个点灯以及哪一个没有。同时我们也有一些作用于填充的网格上的整数算术运算要考虑。

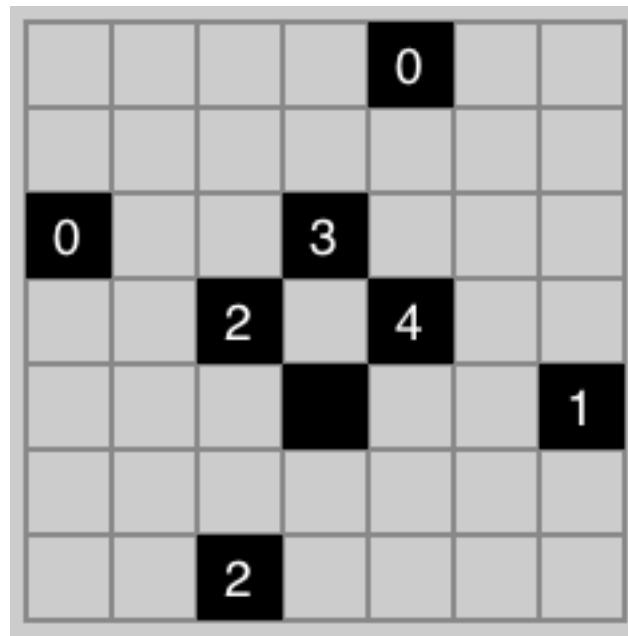


Fig. 2.7.1: 点灯游戏的一个例子展示

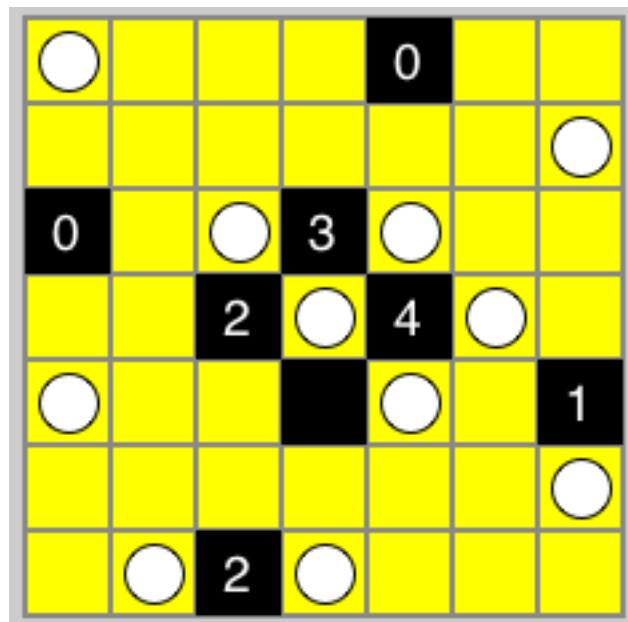


Fig. 2.7.2: 点灯游戏的完整的解

Listing 2.7.3: 点灯游戏的 SAT 模型 (lightup.mzn).

```

int: h; set of int: H = 1..h; % 板高度
int: w; set of int: W = 1..w; % 板宽度
array[H,W] of -1..5: b;      % 板
int: E = -1;                  % 空白格
set of int: N = 0..4;         % 填充并含有数字的网格
int: F = 5;                   % 填充但不含有数字的网格

% 位置 (i1,j1) 对位置 (i2,j2) 可见
test visible(int: i1, int: j1, int: i2, int: j2) =
  ((i1 == i2) /\ forall(j in min(j1,j2)..max(j1,j2))(b[i1,j] == E))
  \/ ((j1 == j2) /\ forall(i in min(i1,i2)..max(i1,i2))(b[i,j1] == E));

array[H,W] of var bool: l; % is there a light

% 填充的网格没有灯泡
constraint forall(i in H, j in W, where b[i,j] != E)(l[i,j] == false);
% lights next to filled numbered square agree
include "boolsum.mzn";
constraint forall(i in H, j in W where b[i,j] in N)(
  bool_sum_eq([ l[i1,j1] | i1 in i-1..i+1, j1 in j-1..j+1 where
                abs(i1 - i) + abs(j1 - j) == 1 /\ 
                i1 in H /\ j1 in W ], b[i,j]));
% 每个空白网格是被照亮的
constraint forall(i in H, j in W where b[i,j] == E)(
  exists(j1 in W where visible(i,j,i,j1))(l[i,j1]) \/
  exists(i1 in H where visible(i,j,i1,j))(l[i1,j]));
);

% 任何两个灯泡看不到彼此
constraint forall(i1,i2 in H, j1,j2 in W where
  (i1 != i2 \/ j1 != j2) /\ b[i1,j1] == E
  /\ b[i2,j2] == E \/ visible(i1,j1,i2,j2))(not l[i1,j1] \/ not l[i2,j2]);
);

solve satisfy;
output [ if b[i,j] != E then show(b[i,j]) 
```

```

else if fix(l[i,j]) then "L" else "." endif
endif ++ if j == w then "\n" else " " endif |
i in H, j in W];

```

图 Listing 2.7.3 中给出了这个问题的一个模型。图 Fig. 2.7.1 中给出的问题的数据文件在图 Listing 2.7.4 中给出。

Listing 2.7.4: 点灯游戏的 Fig. 2.7.1 中实例的数据文件

```

h = 7;
w = 7;
b = [| -1,-1,-1,-1, 0,-1,-1
      | -1,-1,-1,-1,-1,-1,-1
      | 0,-1,-1, 3,-1,-1,-1
      | -1,-1, 2,-1, 4,-1,-1
      | -1,-1,-1, 5,-1,-1, 1
      | -1,-1,-1,-1,-1,-1,-1
      | 1,-1, 2,-1,-1,-1,-1 |];

```

模型利用了一个布尔型求和谓词

```
predicate bool_sum_eq(array[int] of var bool:x, int:s);
```

使得一个布尔型数组的和等于一个固定的整数。多种方式都能使用布尔型变量给 *cardinality* 约束建模。

- 加法器网络：我们可以使用包含加法器的一个网络给布尔型总和建立一个二元布尔型表达式
- 排序网络：我们可以通过使用一个排序网络去分类布尔型数组来创建一个布尔型总和的一元表达式
- 二元决策图：我们可以创建一个二维决策图（BDD）来编码势约束。

Listing 2.7.5: 使用二元加法器网络表示势约束 (bboolsum.mzn).

```

% 布尔型变量 x 的总和 = s
predicate bool_sum_eq(array[int] of var bool:x, int:s) =
    let { int: c = length(x) } in
        if s < 0 then false
        elseif s == 0 then

```

```

    forall(i in 1..c)(x[i] == false)
elseif s < c then
    let { % cp = number of bits required for representing 0..c
        int: cp = floor(log2(int2float(c))),
        % z is sum of x in binary
        array[0..cp] of var bool:z } in
    binary_sum(x, z) /\
    % z == s
    forall(i in 0..cp)(z[i] == ((s div pow(2,i)) mod 2 == 1))
elseif s == c then
    forall(i in 1..c)(x[i] == true)
else false endif;

include "binarysum.mzn";

```

Listing 2.7.6: 创建二元求和网络的代码 (binarysum.mzn).

```

% 位 x 的总和 = 二元表示的 s。
%           s[0], s[1], ..., s[k] 其中 2^k >= length(x) > 2^(k-1)
predicate binary_sum(array[int] of var bool:x,
                     array[int] of var bool:s)=
    let { int: l = length(x) } in
    if l == 1 then s[0] = x[1]
    elseif l == 2 then
        s[0] = (x[1] xor x[2]) /\ s[1] = (x[1] /\ x[2])
    else let { int: ll = (l div 2),
               array[1..ll] of var bool: f = [ x[i] | i in 1..ll ],
               array[1..ll] of var bool: t = [x[i]| i in ll+1..2*ll],
               var bool: b = if ll*2 == l then false else x[1] endif,
               int: cp = floor(log2(int2float(ll))),
               array[0..cp] of var bool: fs,
               array[0..cp] of var bool: ts } in
        binary_sum(f, fs) /\ binary_sum(t, ts) /\
        binary_add(fs, ts, b, s)
    endif;

% 把两个二元数值 x 和 y 加起来，位 ci 用来表示进位，来得到二元 s
predicate binary_add(array[int] of var bool: x,
                     array[int] of var bool: y,

```

```

        var bool: ci,
        array[int] of var bool: s) =
let { int:l = length(x),
      int:n = length(s), } in
assert(l == length(y),
      "length of binary_add input args must be same",
assert(n == l \& n == l+1, "length of binary_add output " ++
      "must be equal or one more than inputs",
let { array[0..l] of var bool: c } in
full_adder(x[0], y[0], ci, s[0], c[0]) /\ 
forall(i in 1..l)(full_adder(x[i], y[i], c[i-1], s[i], c[i])) /\ 
if n > l then s[n] = c[l] else c[l] == false endif );

```

**predicate full\_adder(var bool: x, var bool: y, var bool: ci,**

**var bool: s, var bool: co) =**

**let { var bool: xy = x xor y } in**

**s = (xy xor ci) /\ co = ((x /\ y) \& (ci /\ xy));**

我们可以使用图 Listing 2.7.5 给出的二元加法器网络代码实现 `bool_sum_eq`。图 Listing 2.7.6 中定义的谓词 `binary_sum` 创建了一个 `x` 总和的二维表示法。它把列表分为两部分，把每一部分分别加起来得到它们的一个二元表示，然后用 `binary_add` 把这两个二元数值加起来。如果 `x` 列大小是奇数，则最后一位被保存起来作为二元加法时的进位来使用。

Listing 2.7.7: 使用二元加法器网络表示势约束 (`uboolsum.mzn`).

```

% 布尔型变量 x 的总和 = s
predicate bool_sum_eq(array[int] of var bool:x, int:s) =
let { int: c = length(x) } in
if s < 0 then false
elseif s == 0 then forall(i in 1..c)(x[i] == false)
elseif s < c then
let { % cp = nearest power of 2 >= c
      int: cp = pow(2,ceil(log2(int2float(c)))),
      array[1..cp] of var bool:y, % y is padded version of x
      array[1..cp] of var bool:z } in
forall(i in 1..c)(y[i] == x[i]) /\ 
forall(i in c+1..cp)(y[i] == false) /\ 
oesort(y, z) /\ z[s] == true /\ z[s+1] == false
elseif s == c then forall(i in 1..c)(x[i] == true)

```

```

else false endif;

include "oesort.mzn";

```

Listing 2.7.8: 奇偶归并排序网络 (oesort.mzn).

```

%% 奇偶排序
%% y 是 x 的有序版本，所有的真都在假之前
predicate oesort(array[int] of var bool:x, array[int] of var bool:y)=
let { int: c = card(index_set(x)) } in
if c == 1 then x[1] == y[1]
elseif c == 2 then comparator(x[1],x[2],y[1],y[2])
else
let {
array[1..c div 2] of var bool:xf = [x[i] | i in 1..c div 2],
array[1..c div 2] of var bool:xl = [x[i] | i in c div 2 +1..c],
array[1..c div 2] of var bool:tf,
array[1..c div 2] of var bool:tl } in
oesort(xf,tf) /\ oesort(xl,tl) /\ oemerge(tf ++ tl, y)
endif;

%% 奇偶合并
%% y 是 x 的有序版本，所有的真都在假之前
%% 假设 x 的前一半是有序的，后一半也是
predicate oemerge(array[int] of var bool:x, array[int] of var bool:y)=
let { int: c = card(index_set(x)) } in
if c == 1 then x[1] == y[1]
elseif c == 2 then comparator(x[1],x[2],y[1],y[2])
else
let { array[1..c div 2] of var bool:xo =
[x[i] | i in 1..c where i mod 2 == 1],
array[1..c div 2] of var bool:xe =
[x[i] | i in 1..c where i mod 2 == 0],
array[1..c div 2] of var bool:to,
array[1..c div 2] of var bool:te } in
oemerge(xo,to) /\ oemerge(xe,te) /\
y[1] = to[1] /\
forall(i in 1..c div 2 -1)(
comparator(te[i],to[i+1],y[2*i],y[2*i+1])) /\

```

```

y[c] = te[c div 2]
endif);

% 比较器 o1 = max(i1,i2), o2 = min(i1,i2)
predicate comparator(var bool:i1,var bool:i2,var bool:o1,var bool:o2)=
(o1 = (i1 \vee i2)) /\ (o2 = (i1 /\ i2));

```

我们可以使用图 Listing 2.7.7 中给出的一元排序网络代码来实现 `bool_sum_eq`。势约束通过扩展输入 `x` 长度为 2 的次幂，然后使用奇偶归并排序网络给得到的位排序来实现。奇偶归并排序工作方式在图 Listing 2.7.8 中给出，它递归地把输入列表拆为两部分，给每一部分排序，然后再把有序的两部分归并起来。

Listing 2.7.9: 使用二元加法器网络表示势约束 (`bddsum.mzn`).

```

% the sum of booleans x = s
predicate bool_sum_eq(array[int] of var bool:x, int:s) =
let { int: c = length(x),
      array[1..c] of var bool: y = [x[i] | i in index_set(x)]
} in
rec_bool_sum_eq(y, 1, s);

predicate rec_bool_sum_eq(array[int] of var bool:x, int: f, int:s) =
let { int: c = length(x) } in
if s < 0 then false
elseif s == 0 then
  forall(i in f..c)(x[i] == false)
elseif s < c - f + 1 then
  (x[f] == true /\ rec_bool_sum_eq(x,f+1,s-1)) \/
  (x[f] == false /\ rec_bool_sum_eq(x,f+1,s))
elseif s == c - f + 1 then
  forall(i in f..c)(x[i] == true)
else false endif;

```

我们可以使用图 Listing 2.7.9 中给出的二元决策图代码来实现 `bool_sum_eq`。势约束被分为两种情况：或者第一个元素 `x[1]` 为 `true` 并且剩下位的总和是 `s-1`，或者 `x[1]` 为 `false` 并且剩下位的总和是 `s`。它的效率的提高依赖于去除共同子表达式来避免产生太多的相同的约束。

# CHAPTER 2.8

---

## FlatZinc 和展平

---

约束求解器不会直接支持 MiniZinc 模型. 为了运行一个 MiniZinc 模型, 它被翻译成一个 MiniZinc 的简单子集叫 FlatZinc. FlatZinc 反映了大多数约束求解器只会求解具有  $\exists c_1 \wedge \dots \wedge c_m$  的满足问题或者有  $\text{minimize } z \text{ subject to } c_1 \wedge \dots \wedge c_m$  的优化问题, 其中  $c_i$  是基本的约束而  $z$  是一个具有某些限定形式的整型或者浮点表达式.

`minizinc` 工具包含了 MiniZinc 编译器, 它可以用一个 MiniZinc 模型和数据文件来创建一个展平后的 FlatZinc 模型, 它等价于给定数据的 MiniZinc 模型表达为之前提到的受限制的形式. 通常来说构建一个给求解器的 FlatZinc 模型是对用户隐藏的, 不过你也可以通过以下命令查看结合数据 `data.dzn` 来展平一个模型 `model.mzn` 的结果:

```
minizinc -c model.mzn data.dzn
```

这会创建一个 FlatZinc 模型叫 `model.fzn`.

在这一章中我们探索把 MiniZinc 翻译成 FlatZinc 的过程.

### 2.8.1 展平表达式

底层求解器的限制意味着复杂的 MiniZinc 表达式需要被 展平 为内部不具有更复杂结构项的基本约束的合取式.

思考以下保证两个在长方形箱子的两个圆不会重叠的模型:

Listing 2.8.1: 两个不会重叠的圆的模型 (cnonoverlap.mzn).

```

float: width;           % 包含圆的长方形的宽
float: height;          % 包含圆的长方形的高
float: r1;
var r1..width-r1: x1;  % (x1,y1) 是第一个圆的中心
var r1..height-r1: y1;
float: r2;
var r2..width-r2: x2;  % (x2,y2) 是第二个圆的中心
var r2..height-r2: y2;
                           % 中心之间至少有 r1 + r2 的距离
constraint (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) >= (r1+r2)*(r1+r2);
solve satisfy;
```

### 2.8.1.1 简化和求值

给定数据文件

```

width = 10.0;
height = 8.0;
r1 = 2.0;
r2 = 3.0;
```

转换到 FlatZinc 首先通过替换所有参数为它们的值来简化模型, 然后求出所有取值已经固定的表达式的值. 在这步简化后参数的值将不再需要. 一个例外是大型数组的参数值. 如果它们被适用多于一次, 那么为了避免重复大的表达式这个参数会被保留.

在简化后, Listing 2.8.1 的模型的变量和参数声明部分变为

```

var 2.0 .. 8.0: x1;
var 2.0 .. 6.0: y1;
var 3.0 .. 7.0: x2;
var 3.0 .. 5.0: y2;
```

### 2.8.1.2 定义子表达式

现在没有约束求解器可以直接处理像在 Listing 2.8.1 里复杂的约束表达式。作为替代，我们可以命名表达式中的每一个子表达式，我们创建约束来构建及代表子表达式的数值。让我们来看看约束表达式的子表达式。 $(x_1 - x_2)$  是一个子表达式，如果我们将它命名为 FLOAT01 我们可以定义它为 `constraint FLOAT01 = x1 - x2;`。注意到这个表达式只在模型中出现两次。我们只需要构建这个值一次，然后我们可以重复使用它。这就是 共同子表达式消除。子表达式  $(x_1 - x_2) * (x_1 - x_2)$  可以命名为 FLOAT02，而我们可以定义它为 `constraint FLOAT02 = FLOAT01 * FLOAT01;`。我们可以命名 `constraint FLOAT03 = y1 - y2;` 和 `constraint FLOAT04 = FLOAT03 * FLOAT03;` 最后 `constraint FLOAT05 = FLOAT02 * FLOAT04;`。不等约束本身则变成 `constraint FLOAT05 >= 25.0;` 因为  $(r_1 + r_2) * (r_1 + r_2)$  计算出结果为 `25.0`。于是这个约束被展平为

```
constraint FLOAT01 = x1 - x2;
constraint FLOAT02 = FLOAT01 * FLOAT01;
constraint FLOAT03 = y1 - y2;
constraint FLOAT04 = FLOAT03 * FLOAT03;
constraint FLOAT05 = FLOAT02 * FLOAT04;
constraint FLOAT05 >= 25.0
```

### 2.8.1.3 FlatZinc 约束形式

展平的最后步骤是把约束的形式转换成标准的 FlatZinc 形式，它总是以  $p(a_1, \dots, a_n)$  的形式出现。其中的  $p$  是某个基础约束， $a_1, \dots, a_n$  是参数。FlatZinc 尝试使用最少的不同约束形式。所以  $FLOAT01 = x_1 - x_2$  首先尝试重写为  $FLOAT01 + x_2 = x_1$  然后使用 `float_plus` 输出基本的约束。得出的约束形式如下：

```
constraint float_plus(FLOAT01, x2, x1);
constraint float_plus(FLOAT03, y2, y1);
constraint float_plus(FLOAT02, FLOAT04, FLOAT05);
constraint float_times(FLOAT01, FLOAT01, FLOAT02);
constraint float_times(FLOAT03, FLOAT03, FLOAT04);
```

### 2.8.1.4 边界分析

我们仍然缺少一项, 声明引入的变量 `FLOAT01`,  $\dots$ , `FLOAT05`. 这些可以被声明为 `var float`. 不过为了令求解器的任务更简单, MiniZinc 尝试通过简单的分析确定新引入变量的上界和下界. 比如因为 `FLOAT01 = x1 - x2` 和  $2.0 \leq x1 \leq 8.0$  还有  $3.0 \leq x2 \leq 7.0$ , 所以可以得出  $-5.0 \leq FLOAT01 \leq 5.0$  然后我们可以看到  $-25.0 \leq FLOAT02 \leq 25.0$  (尽管注意到如果我们发现相乘实际上是平方, 我们可以给出更精确的边界  $0.0 \leq FLOAT02 \leq 25.0$  ).

细心的读者会发现在定义子表达式 (page 129) 和 FlatZinc 约束形式 (page 129) 里约束的展平形式的一点不同. 在后面没有非等约束. 因为一元的不等式可以完全被一个变量的边界表示出来, 不等关系可以令 `FLOAT05` 的下界变为 `25.0`, 然后这会变得冗余. 最后 Listing 2.8.1 的展平后形式是:

```
% 变量
var 2.0 .. 8.0: x1;
var 2.0 .. 6.0: y1;
var 3.0 .. 7.0: x2;
var 3.0 .. 5.0: y2;
%
var -5.0..5.0: FLOAT01;
var -25.0..25.0: FLOAT02;
var -3.0..3.0: FLOAT03;
var -9.0..9.0: FLOAT04;
var 25.0..34.0: FLOAT05;
%
% 约束
constraint float_plus(FLOAT01, x2, x1);
constraint float_plus(FLOAT03, y2, y1);
constraint float_plus(FLOAT02, FLOAT04, FLOAT05);
constraint float_times(FLOAT01, FLOAT01, FLOAT02);
constraint float_times(FLOAT03, FLOAT03, FLOAT04);
%
solve satisfy;
```

### 2.8.1.5 目标函数

MiniZinc 就像展平约束一样, 展平最小化和最大化目标函数. 跟其他表达式一样, 目标表达式被展平时创建一个变量. 在 FlatZinc 输出求解项永远是单一变量. 看一下例子 `Let` 表达式 (page 141).

## 2.8.2 线性表达式

约束的一个最重要的而广泛用于建模的形式是线性约束

$$\begin{aligned} &= \\ a_1x_1 + \dots + a_nx_n &\leq a_0 \\ &< \end{aligned}$$

其中  $a_i$  是整数或者浮点数约束, 而  $x_i$  是整数或者浮点数变量. 它们非常有表达能力, 同时也是(整数)线性规划约束求解器唯一支持的形式. 从 MiniZinc 到 FlatZinc 的翻译器尝试创建线性约束, 而不是把线性约束变成许多子表达式.

Listing 2.8.2: 说明线性约束展平的 MiniZinc 模型 (linear.mzn).

```
int:      d = -1;
var 0..10: x;
var -3..6: y;
var 3..8: z;
constraint 3*x - y + x * z <= 19 + d * (x + y + z) - 4*d;
solve satisfy;
```

考虑在 Listing 2.8.2 中的模型. 这里并没有为所有子表达式  $3*x$ ,  $3*x - y$ ,  $x*z$ ,  $3*x - y + x*z$ ,  $x + y + z$ ,  $d*(x + y + z)$ ,  $19 + d*(x + y + z)$ , 和  $19 + d*(x + y + z) - 4*d$  创建一个变量. 这里的翻译在创建一个 FlatZinc 约束时, 会尝试创建一个尽可能记录大部分原约束内容的线性约束.

展平会创建线性表达式并将其视为一个单位, 而不视为每个字表达式构建中间变量. 这也使创建的表达式简单化. 从约束中抽取出线性表达式为

```
var 0..80: INT01;
constraint 4*x + z + INT01 <= 23;
constraint INT01 = x * z;
```

注意到 非线性表达式  $x * z$  是如何被抽取出来作为一个新的子表达式并赋予名字的, 与此同时剩下的项会被收集起来从而使每个变量只出现一次 (的确变量  $y$  的项被移除了)

最后每个约束被写到 FlatZinc 形式, 从而得到:

```
var 0..80: INT01;
constraint int_lin_le([1,4,1],[INT01,x,z],23);
constraint int_times(x,z,INT01);
```

## 2.8.3 展开表达式

大多数的模型需要创建一些基于输入数据的约束。 MiniZinc 通过数组类型，列表和列生成解析还有聚合函数来支持这些模型。

考虑以下从生产调度例子 Listing 2.2.2 中出现的聚合函数表达式

```
int: mproducts = max (p in Products)
    (min (r in Resources where consumption[p,r] > 0)
        (capacity[r] div consumption[p,r]));
```

由于这用到生成器语法，我们可以把它重写成可以被编译器处理的相等的形式：

```
int: mproducts = max([ min [ capacity[r] div consumption[p,r]
    | r in Resources where consumption[p,r] > 0])
    | p in Products]);
```

给定数据

```
nproducts = 2;
nresources = 5;
capacity = [4000, 6, 2000, 500, 500];
consumption= [| 250, 2, 75, 100, 0,
    | 200, 0, 150, 150, 75 |];
```

这首先创建  $p = 1$  的数组

```
[ capacity[r] div consumption[p,r]
    | r in 1..5 where consumption[p,r] > 0]
```

也就是  $[16, 3, 26, 5]$  然后计算最小值为 3。它之后为  $p = 2$  建立相同的数组  $[20, 13, 3, 6]$ ，并计算最小的数值为 3。然后创建数组  $[3, 3]$  并计算最大值为 3。在 FlatZinc 里面没有 `mproducts` 的表示，这种通过计算数值 3 的方法会被用来代替 `mproducts`。

在约束模型中最常见的聚合表达式是 `forall`。Forall 表达式会被展开成为多个约束。

考虑以下在 SEND-MORE-MONEY 例子 Listing 2.2.4 中使用预设的分解 `alldifferent` 出现的 MiniZinc 片段。

```
array[1..8] of var 0..9: v = [S,E,N,D,M,O,R,Y];
constraint forall(i,j in 1..8 where i < j)(v[i] != v[j])
```

`forall` 表达式为每一对需要满足  $i < j$  的  $i, j$  创建一个约束, 所以创建

```
constraint v[1] != v[2]; % S != E
constraint v[1] != v[3]; % S != N
...
constraint v[1] != v[8]; % S != Y
constraint v[2] != v[3]; % E != N
...
constraint v[7] != v[8]; % R != Y
```

在 FlatZinc 中形成

```
constraint int_neq(S,E);
constraint int_neq(S,N);
...
constraint int_neq(S,Y);
constraint int_neq(E,N);
...
constraint int_neq(R,Y);
```

注意到临时的数组变量 `v[i]` 是如何在 FlatZinc 的输出中被原来的变量替换的.

## 2.8.4 数组

一维变量在 MiniZinc 中可以有任意的下标, 只要它们是相邻的整数. 在 FlatZinc 所有数组都被 `1..1` 标注, 其中 1 是数组的长度. 这意味着数组查询时需要被转换成 FlatZinc 下标的形式.

考虑以下 MiniZinc 模型来使用 `m` 个 1kg 磕码来平衡一个长为 `2 * 12` 的跷跷板, 其中上面有一个重为 `cw` kg 小孩.

```
int: cw;                                % 小孩重量
int: 12;                                 % 一半跷跷板长度
int: m;                                  % 1kg 磕码的数量
array[-12..12] of var 0..max(m,cw): w; % 在每个点的重量
```

```

var -12..12: p;                                % 孩子的位置
constraint sum(i in -12..12)(i * w[i]) = 0; % 平衡
constraint sum(i in -12..12)(w[i]) = m + cw; % 所有使用的砝码
constraint w[p] = cw;                          % 孩子在位置 p
solve satisfy;

```

给定  $cw = 2$ ,  $12 = 2$ , 和  $m = 3$ , 展开可以产生约束

```

array[-2..2] of var 0..3: w;
var -2..2: p
constraint -2*w[-2] + -1*w[-1] + 0*w[0] + 1*w[1] + 2*w[2] = 0;
constraint w[-2] + w[-1] + w[0] + w[1] + w[2] = 5;
constraint w[p] = 2;

```

不过 FlatZinc 坚持  $w$  数组从下标 1 开始. 这意味着我们需要重写所有数组获取来使用新的下标数值. 对于固定值数组查找这很简单, 对于变量值数组查找我们可能需要创建一个新的变量. 以上公式的结果为

```

array[1..5] of var 0..3: w;
var -2..2: p
var 1..5: INT01;
constraint -2*w[1] + -1*w[2] + 0*w[3] + 1*w[4] + 2*w[5] = 0;
constraint w[1] + w[2] + w[3] + w[4] + w[5] = 5;
constraint w[INT01] = 2;
constraint INT01 = p + 3;

```

最后我们重写约束成 FlatZinc 的形式. 注意到变量数组下标查找的形式是如何映射到 `array_var_int_element` 上的.

```

array[1..5] of var 0..3: w;
var -2..2: p
var 1..5: INT01;
constraint int_lin_eq([-2, 1, -1, -2], [w[1], w[2], w[4], w[5]], 0);
constraint int_lin_eq([1, 1, 1, 1, 1], [w[1], w[2], w[3], w[4], w[5]], 5);
constraint array_var_int_element(INT01, w, 2);
constraint int_lin_eq([-1, 1], [INT01, p], -3);

```

MiniZinc 支持多维数组, 但是 (目前来说)FlatZinc 只支持单维度数组. 这意味着多维度数组必须

映射到单维度数组上，而且多维度数组访问必须映射到单维度数组访问。

考虑在有限元平面模型 Listing 2.2.1: 的 Laplace 等式约束：

```

set of int: HEIGHT = 0..h;
set of int: CHEIGHT = 1..h-1;
set of int: WIDTH = 0..w;
set of int: CWIDTH = 1..w-1;
array[HEIGHT,WIDTH] of var float: t; % 在点 (i,j) 处的温度
var float: left;    % 左侧温度
var float: right;   % 右侧温度
var float: top;     % 顶部温度
var float: bottom;  % 底部温度

% 拉普拉斯方程：每一个内部点温度是它相邻点的平均值
constraint forall(i in CHEIGHT, j in CWIDTH)(
    4.0*t[i,j] = t[i-1,j] + t[i,j-1] + t[i+1,j] + t[i,j+1]);

```

假设  $w = 4$  和  $h = 4$ ，这会创建约束

```

array[0..4,0..4] of var float: t; % temperature at point (i,j)
constraint 4.0*t[1,1] = t[0,1] + t[1,0] + t[2,1] + t[1,2];
constraint 4.0*t[1,2] = t[0,2] + t[1,1] + t[2,2] + t[1,3];
constraint 4.0*t[1,3] = t[0,3] + t[1,2] + t[2,3] + t[1,4];
constraint 4.0*t[2,1] = t[1,1] + t[2,0] + t[3,1] + t[2,2];
constraint 4.0*t[2,2] = t[1,2] + t[2,1] + t[3,2] + t[2,3];
constraint 4.0*t[2,3] = t[1,3] + t[2,2] + t[3,3] + t[2,4];
constraint 4.0*t[3,1] = t[2,1] + t[3,0] + t[4,1] + t[3,2];
constraint 4.0*t[3,2] = t[2,2] + t[3,1] + t[4,2] + t[3,3];
constraint 4.0*t[3,3] = t[2,3] + t[3,2] + t[4,3] + t[3,4];

```

25 个元素的 2 维数组被转换成一维数组，而且下标也会相应改变：所以  $[i,j]$  下标会变成  $[i * 5 + j + 1]$ 。

```

array [1..25] of var float: t;
constraint 4.0*t[7] = t[2] + t[6] + t[12] + t[8];
constraint 4.0*t[8] = t[3] + t[7] + t[13] + t[9];
constraint 4.0*t[9] = t[4] + t[8] + t[14] + t[10];
constraint 4.0*t[12] = t[7] + t[11] + t[17] + t[13];

```

```

constraint 4.0*t[13] = t[8] + t[12] + t[18] + t[14];
constraint 4.0*t[14] = t[9] + t[13] + t[19] + t[15];
constraint 4.0*t[17] = t[12] + t[16] + t[22] + t[18];
constraint 4.0*t[18] = t[13] + t[17] + t[23] + t[19];
constraint 4.0*t[19] = t[14] + t[18] + t[24] + t[20];

```

## 2.8.5 具体化

FlatZinc 模型包含了只有变量和参数声明, 和一系列原始的约束. 所以当我们在 MiniZinc 用布尔连接符而不是析取式来建模时, 需要进行一些处理. 处理使用连接符不只是析取式来构建的复杂公式, 其核心的方法是具体化. 具体化一个约束  $c$  创建新的约束等价于  $b \leftrightarrow c$ , 即如果约束满足则布尔变量  $b$  的值是 `true`, 否则为 `false`.

当我们有能力 具体化 约束, 对待复杂公式的方式跟数学表达式并无不同. 我们为子表达式创建了一个名称和一个展平的约束来约束子表达式的数值.

考虑以下任务调度例子 Listing 2.2.8 中出现在约束表达式:

```

constraint % 保证任务之间没有重叠
forall(j in 1..tasks) (
    forall(i,k in 1..jobs where i < k) (
        s[i,j] + d[i,j] <= s[k,j] \/
        s[k,j] + d[k,j] <= s[i,j]
    ) );

```

对于数据文件

```

jobs = 2;
tasks = 3;
d = [| 5, 3, 4 | 2, 6, 3 |]

```

然后展开过程生成

```

constraint s[1,1] + 5 <= s[2,1] \/ s[2,1] + 2 <= s[1,1];
constraint s[1,2] + 3 <= s[2,2] \/ s[2,2] + 6 <= s[1,2];
constraint s[1,3] + 4 <= s[2,3] \/ s[2,3] + 3 <= s[1,3];

```

具体化在析取式中出现的约束创建新的布尔变量来定义每个表达式的数值.

```

array[1..2,1..3] of var 0..23: s;
constraint BOOL01 <-> s[1,1] + 5 <= s[2,1];
constraint BOOL02 <-> s[2,1] + 2 <= s[1,1];
constraint BOOL03 <-> s[1,2] + 3 <= s[2,2];
constraint BOOL04 <-> s[2,2] + 6 <= s[1,2];
constraint BOOL05 <-> s[1,3] + 4 <= s[2,3];
constraint BOOL06 <-> s[2,3] + 3 <= s[1,3];
constraint BOOL01 /\ BOOL02;
constraint BOOL03 /\ BOOL04;
constraint BOOL05 /\ BOOL06;

```

每个基础的约束现在会映射到 FlatZinc 形式下。注意到二维数组 `s` 是如何映射到一维数组里面的。

```

array[1..6] of var 0..23: s;
constraint int_lin_le_reif([1, -1], [s[1], s[4]], -5, BOOL01);
constraint int_lin_le_reif([-1, 1], [s[1], s[4]], -2, BOOL02);
constraint int_lin_le_reif([1, -1], [s[2], s[5]], -3, BOOL03);
constraint int_lin_le_reif([-1, 1], [s[2], s[5]], -6, BOOL04);
constraint int_lin_le_reif([1, -1], [s[3], s[6]], -4, BOOL05);
constraint int_lin_le_reif([-1, 1], [s[3], s[6]], -3, BOOL06);
constraint array_bool_or([BOOL01, BOOL02], true);
constraint array_bool_or([BOOL03, BOOL04], true);
constraint array_bool_or([BOOL05, BOOL06], true);

```

`int_lin_le_reif` 是线性约束 `int_lin_le` 的具体化形式。

大多数 FlatZinc 基本约束  $p(\bar{x})$  有一个具体化形式  $p\_reif(\bar{x}, b)$ ，它利用最后额外的参数  $b$  来定义一个约束  $b \leftrightarrow p(\bar{x})$ 。定义像 `int_plus` 和 `int_minus` 的函数式关系的 FlatZinc 基本约束不需要支持具体化。反而，有结果的函数的等式被具体化了。

具体化的另外一个重要作用出现在当我们使用强制转换函数 `bool2int` (可能是显式地或者隐式地把布尔表达式使用成整数表达式使用)。平整过程将创建一个布尔变量来保存一个布尔表达式参数，以及一个整型变量 (限制到 `0..1`) 来保存这个数值。

考虑 Listing 2.2.12 中的魔术序列问题。

```

int: n;
array[0..n-1] of var 0..n: s;

```

```
constraint forall(i in 0..n-1) (
    s[i] = (sum(j in 0..n-1)(bool2int(s[j]=i))));
```

给定  $n = 2$ ，展开创造了

```
constraint s[0] = bool2int(s[0] = 0) + bool2int(s[1] = 0);
constraint s[1] = bool2int(s[0] = 1) + bool2int(s[1] = 1);
```

和展平创造了

```
constraint BOOL01 <-> s[0] = 0;
constraint BOOL03 <-> s[1] = 0;
constraint BOOL05 <-> s[0] = 1;
constraint BOOL07 <-> s[1] = 1;
constraint INT02 = bool2int(BOOL01);
constraint INT04 = bool2int(BOOL03);
constraint INT06 = bool2int(BOOL05);
constraint INT08 = bool2int(BOOL07);
constraint s[0] = INT02 + INT04;
constraint s[1] = INT06 + INT08;
```

最后的 FlatZinc 形式是

```
var bool: BOOL01;
var bool: BOOL03;
var bool: BOOL05;
var bool: BOOL07;
var 0..1: INT02;
var 0..1: INT04;
var 0..1: INT06;
var 0..1: INT08;
array [1..2] of var 0..2: s;
constraint int_eq_reif(s[1], 0, BOOL01);
constraint int_eq_reif(s[2], 0, BOOL03);
constraint int_eq_reif(s[1], 1, BOOL05);
constraint int_eq_reif(s[2], 1, BOOL07);
constraint bool2int(BOOL01, INT02);
```

```

constraint bool2int(BOOL03, INT04);
constraint bool2int(BOOL05, INT06);
constraint bool2int(BOOL07, INT08);
constraint int_lin_eq([-1, -1, 1], [INT02, INT04, s[1]], 0);
constraint int_lin_eq([-1, -1, 1], [INT06, INT08, s[2]], 0);
solve satisfy;

```

## 2.8.6 谓词

MiniZinc 支持许多不同求解器的一个重要的因素是全局约束 (还有真正的 FlatZinc 约束) 可以根据不同的求解器专业化.

每一个求解器生命一个谓词有时会, 但有时并不会提供具体的定义. 举个例子一个求解器有一个内建的全局 `alldifferent` 谓词, 会包含定义

```
predicate alldifferent(array[int] of var int:x);
```

在全局约束库中, 同时一个求解器预设的分解会有定义

```

predicate alldifferent(array[int] of var int:x) =
  forall(i,j in index_set(x) where i < j)(x[i] != x[j]);

```

谓词调用  $p(\bar{t})$  在平整时, 首先为每个参数项  $t_i$  创建对应变量  $v_i$ . 如果谓词没有定义我们只需要使用创建的参数  $p(\bar{v})$  来调用谓词. 如果一个谓词有一个定义  $p(\bar{x}) = \phi(\bar{x})$  然后我们将用谓词的定义来替换这个谓词调用  $p(\bar{t})$  当中形式参数被替换为对应的参数变量, 即  $\phi(\bar{v})$ . 注意到如果一个谓词调用  $p(\bar{t})$  出现在具体化位置而且它没有定义, 我们则检查我们适用这个谓词的具体化版本  $p\_reif(\bar{x}, b)$ .

考虑在 SEND-MORE-MONEY 例子 Listing 2.2.4 中 `alldifferent` 约束的:

```
constraint alldifferent([S,E,N,D,M,O,R,Y]);
```

如果这个求解器有一个内建的 `alldifferent` 我们只需要为这个参数创建一个新的变量, 然后在调用时替换它.

```

array[1..8] of var 0..9: v = [S,E,N,D,M,O,R,Y];
constraint alldifferent(v);

```

注意到边界分析尝试在新的数组变量上找到一个紧的边界。建造这个数组参数的理由就是，如果我们中使用相同的数组两次，FlatZinc 求解器不会创建它两次。在这种情况下因为它不是使用两次，后面的转换会把 `v` 替换成它的定义。

如果求解器使用预设的定义 `alldifferent` 呢？然后变量 `v` 会正常地定义，谓词调用会被替换为一个当中变量被重命名的版本，其中 `v` 替换了形式参数 `x`。结果的程序是

```
array[1..8] of var 0..9: v = [S,E,N,D,M,O,R,Y];
constraint forall(i,j in 1..8 where i < j)(v[i] != v[j])
```

我们可以在展开表达式 (page 132) 中看到。

考虑到以下约束，其中 `alldifferent` 在一个具体化位置出现。

```
constraint alldifferent([A,B,C]) \vee alldifferent([B,C,D]);
```

如果求解器有 `alldifferent` 的具体化形式，这将会被展平为

```
constraint alldifferent_reif([A,B,C],BOOL01);
constraint alldifferent_reif([B,C,D],BOOL02);
constraint array_bool_or([BOOL01,BOOL02],true);
```

适用这个预设的分解，谓词替换会首先创建

```
array[1..3] of var int: v1 = [A,B,C];
array[1..3] of var int: v2 = [B,C,D];
constraint forall(i,j in 1..3 where i<j)(v1[i] != v1[j]) \/
    forall(i,j in 1..3 where i<j)(v2[i] != v2[j]);
```

它最终会展平成 FlatZinc 形式

```
constraint int_neq_reif(A,B,BOOL01);
constraint int_neq_reif(A,C,BOOL02);
constraint int_neq_reif(B,C,BOOL03);
constraint array_bool_and([BOOL01,BOOL02,BOOL03],BOOL04);
constraint int_neq_reif(B,D,BOOL05);
constraint int_neq_reif(C,D,BOOL06);
constraint array_bool_and([BOOL03,BOOL05,BOOL06],BOOL07);
constraint array_bool_or([BOOL04,BOOL07],true);
```

注意到共同子表达式消除是如何利用具体化不等式  $B \neq C$  的. (虽然有一个更好的转换把共同约束提升到最顶层的合取式中)

## 2.8.7 Let 表达式

Let 表达式是 MiniZinc 中可用于引入新的变量的非常强大的工具. 在展平时, let 表达式被转换成变量和约束声明. 这个 MiniZinc 的关系语义意味着这些约束必须像在第一个包含的布尔表达式中出现.

let 表达式的一个重要特征是每一次它们被使用时它们都创建新的变量.

考虑一下展平的代码

```
constraint even(u) \vee even(v);
predicate even(var int: x) =
    let { var int: y } in x = 2 * y;
```

首先谓词调用被他们的定义取代.

```
constraint (let { var int: y} in u = 2 * y) \vee
    (let { var int: y} in v = 2 * y);
```

然后 let 变量会另外被重命名

```
constraint (let { var int: y1} in u = 2 * y1) \vee
    (let { var int: y2} in v = 2 * y2);
```

最后变量声明会被抽取到第一层

```
var int: y1;
var int: y2;
constraint u = 2 * y1 \vee v = 2 * y2;
```

一旦 let 表达式被清除我们可以像之前那样展平.

记住 let 表达式可以定义新引入的变量 (对某些参数的确需要这样做). 这些隐式地定义了必须满足的约束.

考虑婚礼座位问题 Listing 2.3.10 的复杂的目标函数.

```

solve maximize sum(h in Hatreds)(
    let { var Seats: p1 = pos[h1[h]],
          var Seats: p2 = pos[h2[h]],
          var 0..1: same = bool2int(p1 <= 6 <-> p2 <= 6) } in
    same * abs(p1 - p2) + (1-same) * (abs(13 - p1 - p2) + 1));

```

为了简介我们假设只使用前两个相互敌视的人，所以

```

set of int: Hatreds = 1..2;
array[Hatreds] of Guests: h1 = [groom, carol];
array[Hatreds] of Guests: h2 = [clara, bestman];

```

展平的第一步是展开 `sum` 表达式, 给定 (为了简洁我们保留客人名字和参数 `Seats` , 在实际中他们会被他们的定义取代):

```

solve maximize
  (let { var Seats: p1 = pos[groom],
         var Seats: p2 = pos[clara],
         var 0..1: same = bool2int(p1 <= 6 <-> p2 <= 6) } in
    same * abs(p1 - p2) + (1-same) * (abs(13 - p1 - p2) + 1))
  +
  (let { var Seats: p1 = pos[carol],
         var Seats: p2 = pos[bestman],
         var 0..1: same = bool2int(p1 <= 6 <-> p2 <= 6) } in
    same * abs(p1 - p2) + (1-same) * (abs(13 - p1 - p2) + 1));

```

然后每一个在 `let` 表达式的新变量会被分别命名为

```

solve maximize
  (let { var Seats: p11 = pos[groom],
         var Seats: p21 = pos[clara],
         var 0..1: same1 = bool2int(p11 <= 6 <-> p21 <= 6) } in
    same1 * abs(p11 - p21) + (1-same1) * (abs(13 - p11 - p21) + 1))
  +
  (let { var Seats: p12 = pos[carol],
         var Seats: p22 = pos[bestman],
         var 0..1: same2 = bool2int(p12 <= 6 <-> p22 <= 6) } in
    same2 * abs(p12 - p22) + (1-same2) * (abs(13 - p12 - p22) + 1));

```

在 let 表达式的变量会被抽取到第一层，并且定义约束会被抽取到正确的层 (在这里是最顶层).

```

var Seats: p11;
var Seats: p21;
var 0..1: same1;
constraint p12 = pos[clara];
constraint p11 = pos[groom];
constraint same1 = bool2int(p11 <= 6 <-> p21 <= 6);
var Seats p12;
var Seats p22;
var 0..1: same2;
constraint p12 = pos[carol];
constraint p22 = pos[bestman];
constraint same2 = bool2int(p12 <= 6 <-> p22 <= 6) } in
solve maximize
    same1 * abs(p11 - p21) + (1-same1) * (abs(13 - p11 - p21) + 1)
+
    same2 * abs(p12 - p22) + (1-same2) * (abs(13 - p12 - p22) + 1));

```

现在我们已经构成不需要使用 let 表达式的等价的 MiniZinc 代码, 和展平可以正常进行.

为了说明没有出现在最顶层的 let 表达式的情况, 看看以下模型

```

var 0..9: x;
constraint x >= 1 -> let { var 2..9: y = x - 1 } in
    y + (let { var int: z = x * y } in z * z) < 14;

```

我们抽取变量定义到最顶层, 约束到第一个围住的布尔语境, 这里是蕴含的右手边.

```

var 0..9: x;
var 2..9: y;
var int: z;
constraint x >= 1 -> (y = x - 1 /\ z = x * y /\ y + z * z < 14);

```

注意到如果我们知道定义一个变量的等式的真值不会为假, 我们可以抽取它到最顶层. 这通常可以是求解大幅加快.

对于上面的例子, 因为  $y$  的值域对于  $x - 1$  并不够大, 所以约束  $y = x - 1$  可能失败. 不过约束  $z = x * y$  不可以 (实际上边界分析会给予  $z$  足够大的边界来包含  $x * y$  所有的可能值). 一个更好的展平可以给出

```

var 0..9: x;
var 2..9: y;
var int: z;
constraint z = x * y;
constraint x >= 1 -> (y = x - 1 /\ y + z * z < 14);

```

现在 MiniZinc 编译器通过总是使引入变量声明的边界足够大, 它应该可以包含所有它定义的表达式的值. 然后在正确的语境中为 let 表达式加入边界约束. 在上面的例子中这个结果是

```

var 0..9: x;
var -1..8: y;
var -9..72: z;
constraint y = x - 1;
constraint z = x * y;
constraint x >= 1 -> (y >= 2 /\ y + z * z < 14);

```

这个转换可以使求解更加高效, 因为 let 变量的所有可能的复杂计算并没有被具体化.

这种方法的另外一个原因是在引入变量出现在取反语境的时候它也可以被使用 (只要它们有一个定义). 考虑一下与之前相似的这个例子:

```

var 0..9: x;
constraint (let { var 2..9: y = x - 1 } in
            y + (let { var int: z = x * y } in z * z) > 14) -> x >= 5;

```

这个 let 表达式出现在否定语境中, 不过每个引入变量都被定义了. 展平后的代码是

```

var 0..9: x;
var -1..8: y;
var -9..72: z;
constraint y = x - 1;
constraint z = x * y;
constraint (y >= 2 /\ y + z * z > 14) -> x >= 5;

```

注意到作为对比的 let 消除方法不能给出一个正确的转换:

```

var 0..9: x;
var 2..9: y;

```

```
var int: z;
constraint (y = x - 1 /\ z = x * y /\ y + z * z > 14) -> x >= 5;
```

以上转换对于所有  $x$  的可能值给出结果, 而原来的约束除掉了  $x = 4$  的可能性.

对于在 let 表达式中的处理跟对定义的变量的处理是相似的. 你可以认为一个约束等价于定义一个新的布尔变量. 新的布尔变量定义可以从最顶层中抽取出来, 而布尔保存在正确的语境下.

```
constraint z > 1 -> let { var int: y,
                           constraint (x >= 0) -> y = x,
                           constraint (x < 0) -> y = -x
                         } in y * (y - 2) >= z;
```

可以处理成

```
constraint z > 1 -> let { var int: y,
                           var bool: b1 = ((x >= 0) -> y = x),
                           var bool: b2 = ((x < 0) -> y = -x),
                           constraint b1 /\ b2
                         } in y * (y - 2) >= z;
```

然后展平成

```
constraint b1 = ((x >= 0) -> y = x);
constraint b2 = ((x < 0) -> y = -x);
constraint z > 1 -> (b1 /\ b2 /\ y * (y - 2) >= z);
```



## **Part 3**

# **User Manual**



# CHAPTER 3.1

---

## The MiniZinc Command Line Tool

---

The core of the MiniZinc constraint modelling system is the `minizinc` tool. You can use it directly from the command line, through the MiniZinc IDE, or through a programmatic interface (API). This chapter summarises the options accepted by the tool, and explains how it interacts with target solvers.

### 3.1.1 Basic Usage

The `minizinc` tool performs three basic functions: it *compiles* a MiniZinc model (plus instance data), it *runs* an external solver, and it *translates solver output* into the form specified in the model. Most users would use all three functions at the same time. For example, let us assume that we want to solve the following simple problem, given as two files (`model.mzn` and `data.dzn`):

```
int: n;
array[1..n] of var 1..2*n: x;
include "alldifferent.mzn";
constraint alldifferent(x);
solve maximize sum(x);
output ["The resulting values are \$(x).\n"];
```

```
n = 5;
```

To run the model file `model.mzn` with data file `data.dzn` using the Gecode solver, you can use the following command line:

```
$ minizinc --solver Gecode model.mzn data.dzn
```

This would result in the output

```
The resulting values are [10, 9, 8, 7, 6].
```

```
-----
```

```
=====
```

However, each of the three functions can also be accessed individually. For example, to compile the same model and data, use the `-c` option:

```
$ minizinc -c --solver Gecode model.mzn data.dzn
```

This will result in two new files, `model.fzn` and `model.ozn`, being output in the same directory as `model.mzn`. You could then run a target solver directly on the `model.fzn` file, or use `minizinc`:

```
$ minizinc --solver Gecode model.fzn
```

You will see that the solver produces output in a standardised form, but not the output prescribed by the `output` item in the model:

```
x = array1d(1..5 ,[10, 9, 8, 7, 6]);
```

```
-----
```

```
=====
```

The translation from this output to the form specified in the model is encoded in the `model.ozn` file. You can use `minizinc` to execute the `.ozn` file. In this mode, it reads a stream of solutions from standard input, so we need to pipe the solver output into `minizinc`:

```
$ minizinc --solver Gecode model.fzn | minizinc --ozn-file model.ozn
```

These are the most basic command line options that you need in order to compile and run models and translate their output. The next section lists all available command line options in detail. Section 4.3.5 explains how new solvers can be added to the system.

### 3.1.2 Adding Solvers

Solvers that support MiniZinc typically consist of two parts: a solver *executable*, which can be run on the FlatZinc output of the MiniZinc compiler, and a *solver library*, which consists of a set of MiniZinc files that define the constraints that the solver supports natively. This section deals with making existing solvers available to the MiniZinc tool chain. For information on how to add FlatZinc support to a solver, refer to [Section 4.3](#).

#### 3.1.2.1 Configuration files

In order for MiniZinc to be able to find both the solver library and the executable, the solver needs to be described in a *solver configuration file* (see [Section 4.3.5](#) for details). If the solver you want to install comes with a configuration file (which has the file extension `.msc` for MiniZinc Solver Configuration), it has to be in one of the following locations:

- In the `minizinc/solvers/` directory of the MiniZinc installation. If you install MiniZinc from the binary distribution, this directory can be found at `/usr/share/minizinc/solvers` on Linux systems, inside the MiniZincIDE application on macOS system, and in the `Program Files\\MiniZinc IDE` (bundled) folder on Windows.
- In the directory `$HOME/.minizinc/solvers` on Linux and macOS systems, and the Application Data directory on Windows systems.
- In any directory listed on the `MZN_SOLVER_PATH` environment variable (directories are separated by `:` on Linux and macOS, and by `;` on Windows systems).
- In any directory listed in the `mzn_solver_path` option of the global or user-specific configuration file (see [Section 3.1.4](#))
- Alternatively, you can use the MiniZinc IDE to create solver configuration files, see [Section 3.2.5.2](#) for details.

After adding a solver, it will be listed in the output of the `minizinc --solvers` command.

#### 3.1.2.2 Configuration for MIP solvers

Some solvers require additional configuration flags before they can be used. For example, the binary bundle of MiniZinc comes with interfaces to the CPLEX and Gurobi Mixed Integer Programming solvers. However, due to licensing restrictions, the solvers themselves are not part of the bundled release. Depending on where CPLEX or Gurobi is installed on your system, MiniZinc

may be able to find the solvers automatically, or it may require an additional option to point it to the shared library.

In case the libraries cannot be found automatically, you can use one of the following:

- CPLEX: Specify the location of the shared library using the `--cplex-dll` command line option. On Windows, the library is called `cplexXXXX.dll` and typically found in same directory as the `cplex` executable. On Linux it is `libcplexXXX.so`, and on macOS `libcplexXXXX.jnilib`, where `XXX` and `XXXX` stand for the version number.
- Gurobi: The command line option for Gurobi is `--gurobi-dll`. On Windows, the library is called `gurobiXX.dll` (in the same directory as the `gurobi` executable), and on Linux and macOS is it `libgurobiXX.so` (in the `lib` directory of your Gurobi installation).
- You can define these paths as defaults in your user configuration file, see [Section 3.1.4](#).

### 3.1.3 Options

You can get a list of all the options supported by the `minizinc` tool using the `--help` flag.

#### 3.1.3.1 General options

These options control the general behaviour of the `minizinc` tool.

`--help, -h`

Print a help message.

`--version`

Print version information.

`--solvers`

Print list of available solvers.

`--solver <id>, --solver <solver configuration file>.msc`

Select solver to use. The first form of the command selects one of the solvers known to MiniZinc (that appear in the list of the `--solvers` command). You can select a solver by name, id, or tag, and add a specific version. For example, to select a mixed-integer programming solver, identified by the `mip` tag, you can use `--solver mip`. To select a specific version of Gurobi (in case you have two versions installed), use `--solver Gurobi@7.5.2`. Instead of the name you can also use the solver's identifier, e.g. `--solver org.gecode.gecode`.

The second form of the command selects the solver from the given configuration file (see [Section 4.3.5](#)).

--help <id>

Print help for a particular solver. The scheme for selecting a solver is the same as for the --solver option.

-v, -l, --verbose

Print progress/log statements (for both compilation and solver). Note that some solvers may log to stdout.

--verbose-compilation

Print progress/log statements for compilation only.

-s, --statistics

Print statistics (for both compilation and solving).

--compiler-statistics

Print statistics for compilation.

-c, --compile

Compile only (do not run solver).

--config-dirs

Output configuration directories.

--solvers-json

Print configurations of available solvers as a JSON array.

### 3.1.3.2 Solving options

Each solver may support specific command line options for controlling its behaviour. These can be queried using the --help <id> flag, where <id> is the name or identifier of a particular solver. Most solvers will support some or all of the following options.

-a, --all-solutions

Report *all* solutions in the case of satisfaction problems, or print *intermediate* solutions of increasing quality in the case of optimisation problems.

-n <i>, --num-solutions <i>

Stop after reporting i solutions (only used with satisfaction problems).

-f, --free-search

Instructs the solver to conduct a “free search”, i.e., ignore any search annotations. The solver is not *required* to ignore the annotations, but it is *allowed* to do so.

```
--solver-statistics
    Print statistics during and/or after the search for solutions.

--verbose-solving
    Print log messages (verbose solving) to the standard error stream.

-p <i>, --parallel <i>
    Run with i parallel threads (for multi-threaded solvers).

-r <i>, --random-seed <i>
    Use i as the random seed (for any random number generators the solver may be using).
```

### 3.1.3.3 Flattener input options

These options control aspects of the MiniZinc compiler.

```
--ignore-stdlib
    Ignore the standard libraries stdlib.mzn and builtins.mzn

--instance-check-only
    Check the model instance (including data) for errors, but do not convert to FlatZinc.

-e, --model-check-only
    Check the model (without requiring data) for errors, but do not convert to FlatZinc.

--model-interface-only
    Only extract parameters and output variables.

--model-types-only
    Only output variable (enum) type information.

--no-optimize
    Do not optimize the FlatZinc

-d <file>, --data <file>
    File named <file> contains data used by the model.

-D <data>, --cmdline-data <data>
    Include the given data assignment in the model.

--stdlib-dir <dir>
    Path to MiniZinc standard library directory

-G --globals-dir --mzn-globals-dir <dir>
    Search for included globals in <stdlib>/<dir>.
```

---

```

- --input-from-stdin
  Read problem from standard input

-I --search-dir
  Additionally search for included files in <dir>.

-D "fMIPdomains=false"
  No domain unification for MIP

--MIPDMaxIntvEE <n>
  Max integer domain subinterval length to enforce equality encoding, default 0

--MIPDMaxDensEE <n>
  Max domain cardinality to N subintervals ratio to enforce equality encoding, default 0,
  either condition triggers

--only-range-domains
  When no MIPdomains: all domains contiguous, holes replaced by inequalities

--allow-multiple-assignments
  Allow multiple assignments to the same variable (e.g. in dzn)

--compile-solution-checker <file>.mzc.mzn
  Compile solution checker model.

```

### Flattener two-pass options

Two-pass compilation means that the MiniZinc compiler will first compile the model in order to collect some global information about it, which it can then use in a second pass to improve the resulting FlatZinc. For some combinations of model and target solver, this can lead to substantial improvements in solving time. However, the additional time spent on the first compilation pass does not always pay off.

```

--two-pass
  Flatten twice to make better flattening decisions for the target

--use-gecode
  Perform root-node-propagation with Gecode (adds --two-pass)

--shave
  Probe bounds of all variables at the root node (adds --use-gecode)

--sac
  Probe values of all variables at the root node (adds --use-gecode)

```

--pre-passes <n>  
Number of times to apply shave/sac pass (0 = fixed-point, 1 = default)

-0<n>  
Two-pass optimisation levels:

-O0: Disable optimize (-no-optimize) -O1: Single pass (default) -O2: Same as: -two-pass  
-O3: Same as: -use-gecode -O4: Same as: -shave -O5: Same as: -sac

## Flattener output options

These options control how the MiniZinc compiler produces the resulting FlatZinc output. If you run the solver directly through the `minizinc` command or the MiniZinc IDE, you do not need to use any of these options.

--no-output-ozn, -0-  
Do not output ozn file

--output-base <name>  
Base name for output files

--fzn <file>, --output-fzn-to-file <file>  
Filename for generated FlatZinc output

-0, --ozn, --output-ozn-to-file <file>  
Filename for model output specification (-O- for none)

--keep-paths  
Don't remove path annotations from FlatZinc

--output-paths  
Output a symbol table (.paths file)

--output-paths-to-file <file>  
Output a symbol table (.paths file) to <file>

--output-to-stdout, --output-fzn-to-stdout  
Print generated FlatZinc to standard output

--output-ozn-to-stdout  
Print model output specification to standard output

--output-paths-to-stdout  
Output symbol table to standard output

```
--output-mode <item|dzn|json>
  Create output according to output item (default), or output compatible with dzn or json
  format

--output-objective
  Print value of objective function in dzn or json output

-Werror
  Turn warnings into errors
```

### 3.1.3.4 Solution output options

These options control how solutions are output. Some of these options only apply if `minizinc` is used to translate a stream of solutions coming from a solver into readable output (using a `.oзн` file generated by the compiler).

```
--ozn-file <file>
  Read output specification from ozn file.

-o <file>, --output-to-file <file>
  Filename for generated output.

-i <n>, --ignore-lines <n>, --ignore-leading-lines <n>
  Ignore the first <n> lines in the FlatZinc solution stream.

--soln-sep <s>, --soln-separator <s>, --solution-separator <s>
  Specify the string printed after each solution (as a separate line). The default is to use the
  same as FlatZinc, “-----”.

--soln-comma <s>, --solution-comma <s>
  Specify the string used to separate solutions. The default is the empty string.

--unsat-msg (--unsatisfiable-msg)
  Specify status message for unsatisfiable problems (default: "=====UNSATISFIABLE=====")
```

```
--unbounded-msg
  Specify status message for unbounded problems (default: "=====UNBOUNDED=====")
```

```
--unsatOrunbnd-msg
  Specify status message for unsatisfiable or unbounded problems (default:
  "=====UNSATOrUNBOUNDED=====")
```

```
--unknown-msg
  Specify status message if search finished before determining status (default:
  "=====UNKNOWN=====")
```

```
--error-msg
    Specify status message if search resulted in an error (default: "=====ERROR=====")  
  
--search-complete-msg <msg>
    Specify status message if when search exhausted the entire search space (default:
    "=====")  
  
--non-unique
    Allow duplicate solutions.  
  
-c, --canonicalize
    Canonicalize the output solution stream (i.e., buffer and sort).  
  
--output-non-canonical <file>
    Non-buffered solution output file in case of canonicalization.  
  
--output-raw <file>
    File to dump the solver's raw output (not for hard-linked solvers)  
  
--no-output-comments
    Do not print comments in the FlatZinc solution stream.  
  
--output-time
    Print timing information in the FlatZinc solution stream.  
  
--no-flush-output
    Don't flush output stream after every line.
```

### 3.1.4 User Configuration Files

The `minizinc` tool reads a system-wide and a user-specific configuration file to determine default paths, solvers and solver options. The files are called `Preferences.json`, and you can find out the locations for your platform using the option `--config-dirs`:

```
$ minizinc --config-dirs
{
    "globalConfigFile" : "/Applications/MiniZincIDE.app/Contents/Resources/
    ↪share/minizinc/Preferences.json",
    "userConfigFile" : "/Users/Joe/.minizinc/Preferences.json",
    "userSolverConfigDir" : "/Users/Joe/.minizinc/solvers",
    "mznStdlibDir" : "/Applications/MiniZincIDE.app/Contents/Resources/share/
    ↪minizinc"
```

```
}
```

The configuration files are simple JSON files that can contain the following configuration options:

- `mzn_solver_path` (list of strings): Additional directories to scan for solver configuration files.
- `mzn_lib_dir` (string): Location of the MiniZinc standard library.
- `tagDefaults` (list of lists of strings): Each entry maps a tag to the default solver for that tag. For example, `[["cp", "org.chuffed.chuffed"], ["mip", "org.minizinc.gurobi"]]` would declare that whenever a solver with tag "cp" is requested, Chuffed should be used, and for the "mip" tag, Gurobi is the default. The empty tag ("") can

be used to define the system-wide default solver (i.e., the solver that is chosen when running `minizinc` without the `--solver` argument). - `solverDefaults` (list of lists of strings): Each entry consists of a list of three strings: a solver identifier, a command line option, and a value for that command line option. For example, `[["org.minizinc.gurobi", "--gurobi-dll", "/Library/gurobi752/mac64/lib/libgurobi75.so"]]` would specify the Gurobi shared library to use (on a macOS system with Gurobi 7.5.2). For command line options that don't take a value, you have to specify an empty string, e.g. `[["org.minizinc.gurobi", "--uniform-search", ""]]`.

Here is a sample configuration file:

```
{
  "mzn_solver_path": ["/usr/share/choco"],
  "tagDefaults": [[{"cp": "org.choco-solver.choco"}, {"mip": "org.minizinc.cplex"}, {"": "org.gecode.gecode"}]],
  "solverDefaults": [{"cplex": ["--cplex-dll", "/opt/CPLEX_Studio128/cplex/bin/x86-64_sles10_4.1/libcplex128.so"]}]
}
```

Configuration values in the user-specific configuration file override the global values, except for solver default arguments, which are only overridden if the name of the option is the same, and otherwise get added to the command line.

Note: Due to current limitations in MiniZinc's JSON parser, we use lists of strings rather than objects for the default mappings. This may change in a future release, but the current syntax will remain valid. The location of the global configuration is currently the `share/minizinc` directory of the MiniZinc installation. This may change in future versions to be more consistent with file system standards (e.g., to use `/etc` on Linux and `/Library/Preferences` on macOS).



# CHAPTER 3.2

---

## The MiniZinc IDE

---

The MiniZinc IDE lets you edit and run MiniZinc models. It requires a working installation of the MiniZinc tool chain (which is included when you download the *bundled version* of the IDE). For installation instructions, see [Section 1.2](#). This document assumes that you have installed the IDE from the bundled binary distribution, which means that it will already be configured correctly when you start it for the first time. The configuration options are described in [Section 3.2.5](#).

Section 1.3 contains an introduction to the absolute basics of working with the IDE. This document goes into a bit more detail.

### 3.2.1 Editing files

The basic editor provides the usual functionality of a simple text editor. You can edit MiniZinc models (file extension `.mzn`) and data files (`.dzn`). When you first open the MiniZinc IDE, you are presented with an empty *Playground* editor, which lets you quickly try out simple MiniZinc models (it does not have to be saved to a file before running a solver on it).

Each file will be opened in a separate tab. To switch between files, click on the tab, select the file from the *Window* menu, or use the *Previous tab/Next tab* options from the *View* menu.

When saving a file, make sure that you select the correct file type (represented by the file extension). Models should be saved as `.mzn` files, while data files should have a `.dzn` extension. This is important because the IDE will only let you invoke the solver on model files.

### 3.2.1.1 Editing functions

The *Edit* menu contains the usual functions for editing text files, such as undo/redo, copy/cut/paste, and find/replace. It also allows you to jump to a particular line number (*Go to line*), and to shift the currently selected text (or the current line if nothing is selected) right or left by two spaces. The (*Un*)comment option will turn the current selection (or current line) into comments, or remove the comment symbols if it is already commented.

### 3.2.1.2 Fonts and dark mode

You can select the font and font size in the *View* menu. We recommend to use a fixed-width font (the IDE should pick such a font by default).

The *View* menu also lets you activate “dark mode”, which switches the colour scheme to a dark background.

## 3.2.2 Configuring and Running a Solver

The MiniZinc IDE automatically detects which solvers are available to MiniZinc. You can select the solver to use from the solver selection drop-down menu next to the *Run* icon in the tool bar:



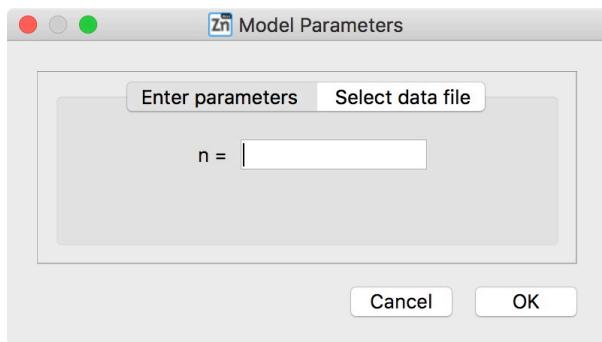
### 3.2.2.1 Running a model

MiniZinc models can be compiled and run by clicking the *Run* icon, selecting *Run* from the *MiniZinc* menu, or using the keyboard shortcut **Ctrl+R** (**Cmd+R** on macOS). The IDE will use the currently selected solver for compiling and running the model.

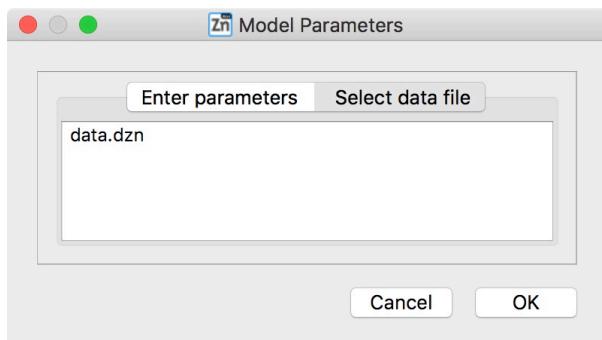
Running a model will open the *Output* window, usually located at the bottom of the IDE’s main window. MiniZinc displays progress messages as well as any output received from the solver there. If compilation resulted in an error message, clicking on the error will jump to the corresponding location in the model or data file.

The current run can be aborted by clicking the *Stop* icon, selecting *Stop* from the *MiniZinc* menu, or using the keyboard shortcut **Ctrl+E** (**Cmd+E** on macOS).

If the selected model requires some parameters to be set before it can be run, the MiniZinc IDE will open a parameter dialog. It has two tabs. The left tab lets you enter the parameters manually:



The second tab lets you select one or several of the data files that are currently open:



### 3.2.2.2 Solver configurations

Selecting one of the built-in solvers from the drop-down menu activates its default configuration. In order to change the solver’s behaviour, open the solver configuration editor by clicking on the icon in the tool bar, selecting *Show configuration editor* from the *MiniZinc/Solver configurations* menu, or using the keyboard shortcut `Ctrl+Shift+C` (`Cmd+Shift+C` on macOS).

Fig. 3.2.1 shows the configuration window. The first section (marked with a 1 in a red circle) contains a drop-down menu to select the *solver configuration*. In this case, a built-in configuration for the OSI-CBC solver was selected. You can make this configuration the default (the MiniZinc IDE will remember this setting), you can reset all values to the defaults, and you can make a clone of the configuration. Cloning a configuration is useful if you want to be able to quickly switch between different sets of options.

Note that any changes to the built-in configurations will be lost when you close the IDE. Any changes to a cloned configuration are saved as part of the *project* (see Section 3.2.3).

The *Solving* section below contains a number of general options. First of all, it shows the concrete solver used in this configuration. Below that, you can set a time limit, after which the execution will be stopped. The built-in configurations all use the “default behaviour” (marked with a 2), which is to print all intermediate solutions for optimisation problems, and stop after the first found solution for satisfaction problems. To change this, you can select *User-defined behavior* instead (marked with a 3).

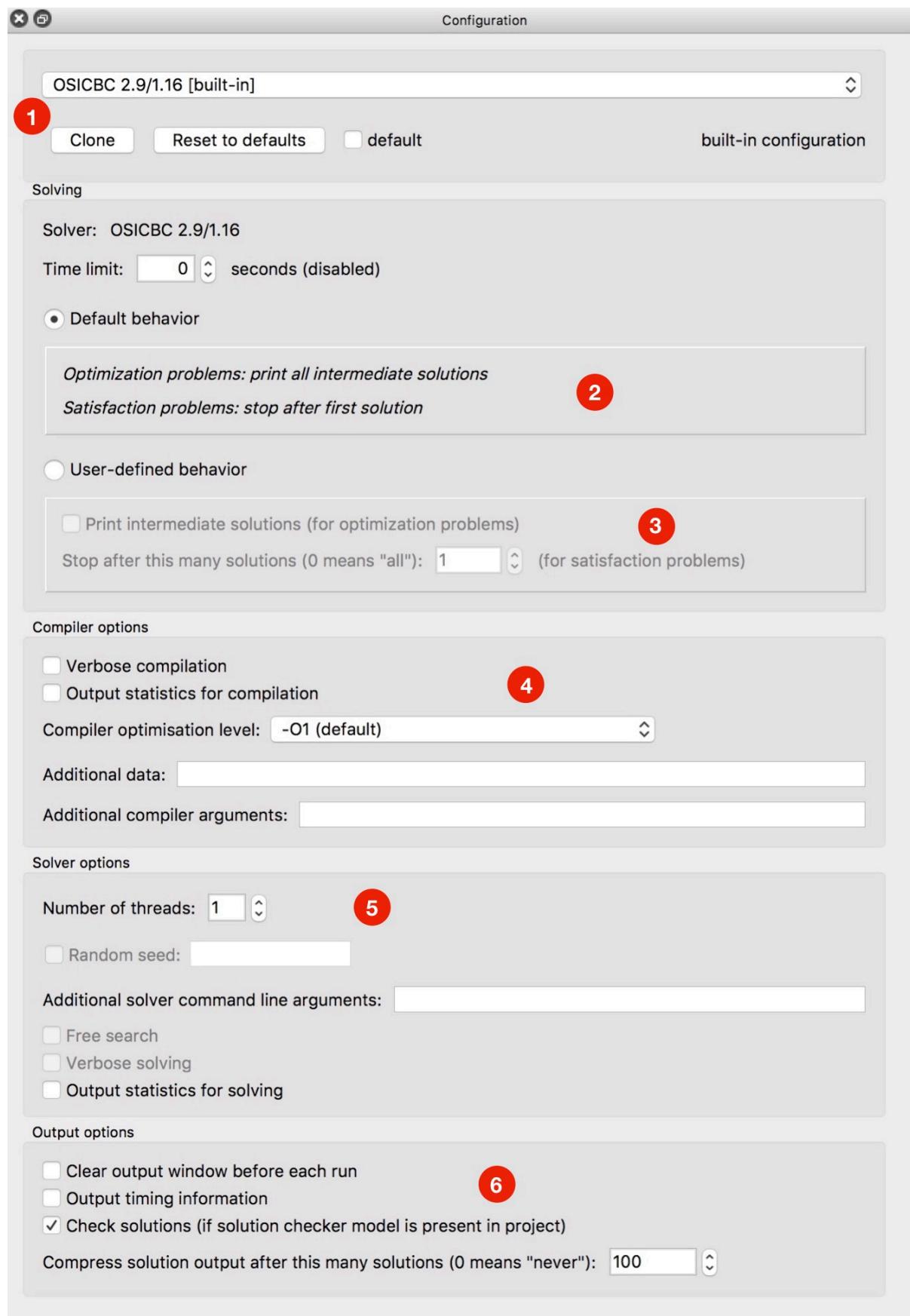


Fig. 3.2.1: The solver configuration window

The next section, *Compiler options* (marked with a 4), controls different aspects of the compilation from MiniZinc to FlatZinc for the selected solver. The first two checkboxes control verbosity and statistics output of the compiler. The drop-down below controls the optimisation level of the compiler, i.e., how much effort it should spend on creating the best possible FlatZinc representation for the given model. The two input fields below allow you to specify additional data (passed into the compilation as if it was part of a .dzn file) and additional command line options for the compiler.

The *Solver options* section (marked with a 5) contains configuration options for the selected solver. Only options that are supported by the solver will be available, others will be grayed out (e.g., the selected solver in Fig. 3.2.1 does not support setting a random seed, or free search).

Finally, the *Output options* section gives you control over the output behaviour. The first tick box enables you to clear the *Output* window automatically every time you run a model. The second option inserts timing information into the stream of solutions. The third check box (*Check solutions*) is described in Section 3.2.2.3 below. The *Compress solution output* option is useful for problems that produce a lot (read: thousands) of solutions, which can slow down and clutter the output window. The compression works by printing just the number of solutions rather than the solutions themselves. For example, the following model would produce 1000 solutions when run with *User-defined behavior* and solution limit set to 0:

```
var 1..1000: x;
solve satisfy;
```

When running with compression set to 100, MiniZinc will output the first 100 solutions, and then a sequence of output like this:

```
[ 100 more solutions ]
[ 200 more solutions ]
[ 400 more solutions ]
[ 199 more solutions ]
x = 1000;
-----
=====
```

The number of solutions captured by one of the ... more solutions lines is doubled each time, in order to keep the overall output low. The last solution produced by the solver will always be printed (since, in the case of optimisation problems, the last solution is the best one found).

### 3.2.2.3 Automatic Solution Checking

MiniZinc can automatically run the output of a model through a *solution checker*, another MiniZinc model that verifies that the solution satisfies a given set of rules. This can be useful for teaching constraint modelling, if the solution checker is given to students. Another use case is to use a simple checker while working on a more complex model, to ensure that the complex model still meets the specification.

The default behaviour of the MiniZinc IDE is to run a solution checker if one is present. For a model `abc.mzn`, a solution checker must be called `abc.mzc` or `abc.mzc.mzn`. If a checker is present, the *Run* icon will turn into a *Run + check* icon instead. The output of the solution checker is displayed together with the normal solution output in the *Output* window.

You can disable solution checkers by deselecting the *Check solutions* option in the solver configuration window.

### 3.2.2.4 Compiling a model

It can sometimes be useful to look at the FlatZinc code generated by the MiniZinc compiler for a particular model. You can use the *Compile* option from the *MiniZinc* menu to compile the model without solving. The generated FlatZinc will be opened in a new tab. You can edit and save the FlatZinc to a file, and run it directly (without recompiling).

## 3.2.3 Working With Projects

Each main window of the MiniZinc IDE corresponds to a *project*, a collection of files and settings that belong together. A project can be saved to and loaded from a file.

You can open a new project by selecting the *New project* option from the *File* menu, or using the `Ctrl+Shift+N` keyboard shortcut (`Cmd+Shift+N` on macOS).

All the files that belong to the current project are shown in the *Project explorer* (see Fig. 3.2.2), which can be opened using the tool bar icon, or using the *Show project explorer* option in the *View* menu. The project explorer lets you run the model in the currently active tab with any of the data files by right-clicking on a `.dzn` file and selecting *Run model with this data*. Right-clicking any file presents a number of options: opening it, removing it from the project, renaming it, running it, and adding new files to the project.

A saved project contains the following pieces of information:

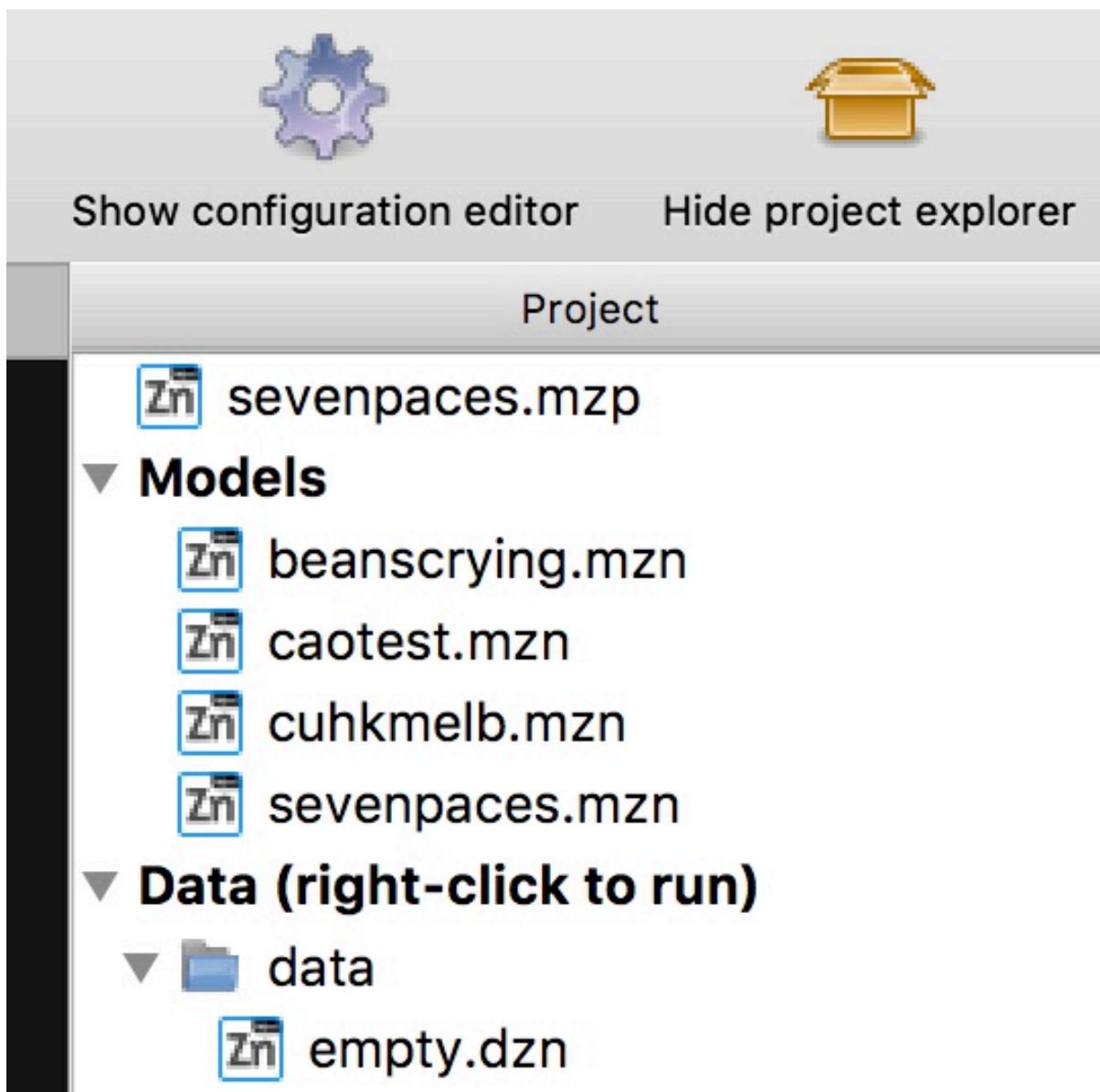


Fig. 3.2.2: Project explorer

- The names of all files in the project. These are stored as relative paths (relative to the project file).
- Which files were open in tabs (and in which order) at the time the project was saved.
- The active solver configuration.
- The state of any cloned solver configuration.

The following will *not* be saved as part of the project:

- The contents of the *Output* window.
- The state of the built-in solver configurations.

### 3.2.4 Submitting Solutions to Online Courses

The MiniZinc IDE has built-in support for submitting solutions and models to online courses, including the Coursera courses that introduce modelling in MiniZinc:

- Basic Modeling for Discrete Optimization<sup>7</sup>
- Advanced Modeling for Discrete Optimization<sup>8</sup>

The submission system is controlled by a file called `_mooc`, which is typically part of the projects that you can download for workshops and assignments. When a project contains this file, a new submission icon will appear in the tool bar, together with an option in the *MiniZinc* menu.

Clicking the icon (or selecting the menu option) opens the submission dialog (see Fig. 3.2.3). It lets you select the problems that you would like to run on your machine, after which the solutions will be sent to the online course auto-grading system. Some projects may also contain model submissions, which are not run on your machine, but are evaluated by the online auto-grader on new data that was not available to you for testing.

You will have to enter the assignment-specific login details. By clicking the *Run and submit* button, you start the solving process. When it finishes, the MiniZinc IDE will upload the solutions to the auto-grading platform.

---

<sup>7</sup> <https://www.coursera.org/learn/basic-modeling>

<sup>8</sup> <https://www.coursera.org/learn/advanced-modeling>

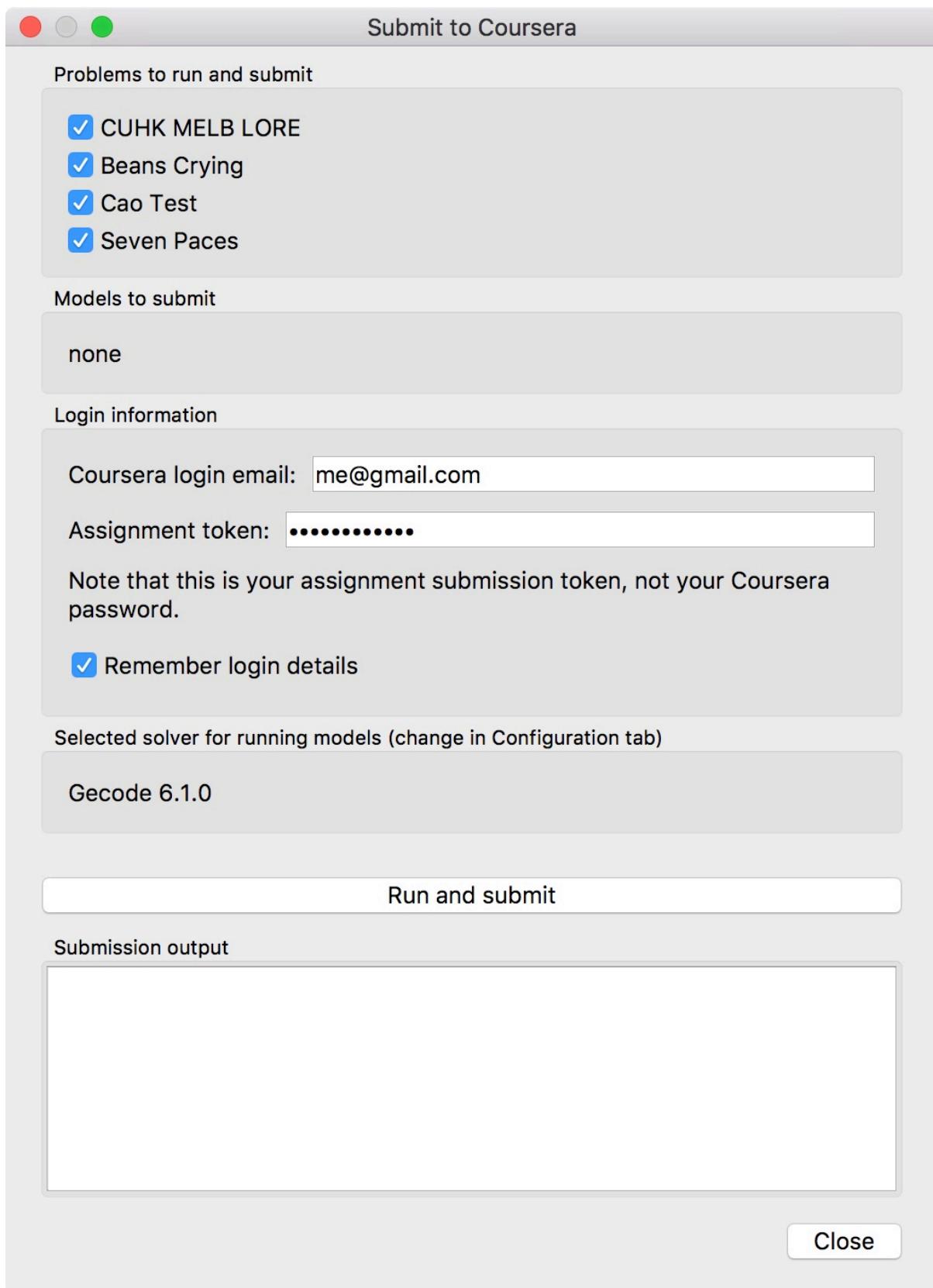


Fig. 3.2.3: Submitting to Coursera

## 3.2.5 Configuration Options

The MiniZinc IDE can be configured through the *Preferences* dialog in the *MiniZinc* menu (on Windows and Linux) or the *MiniZincIDE* menu (on macOS), as shown in Fig. ??.

### 3.2.5.1 Locating the MiniZinc installation

The most important configuration option is the path to the `minizinc` executable. In the example in Fig. ??, this field has been left empty, in which case `minizinc` is assumed to be on the standard search path (usually the PATH environment variable). Typically, in a bundled binary installation of MiniZinc, this field can therefore be left empty.

If you installed MiniZinc from sources, or want to switch between different versions of the compiler, you can add the path to the directory that contains the `minizinc` executable here. You can select a directory from a file dialog using the *Select* button, or enter it manually. Clicking the *Check* button will check that `minizinc` can in fact be run, and has the right version. The version of `minizinc` that was found is displayed below the path input field. Fig. ?? below shows an example where MiniZinc is located at `/home/me/minizinc-2.2.0/bin`.

You can have the MiniZinc IDE check once a day whether a new version of MiniZinc is available.

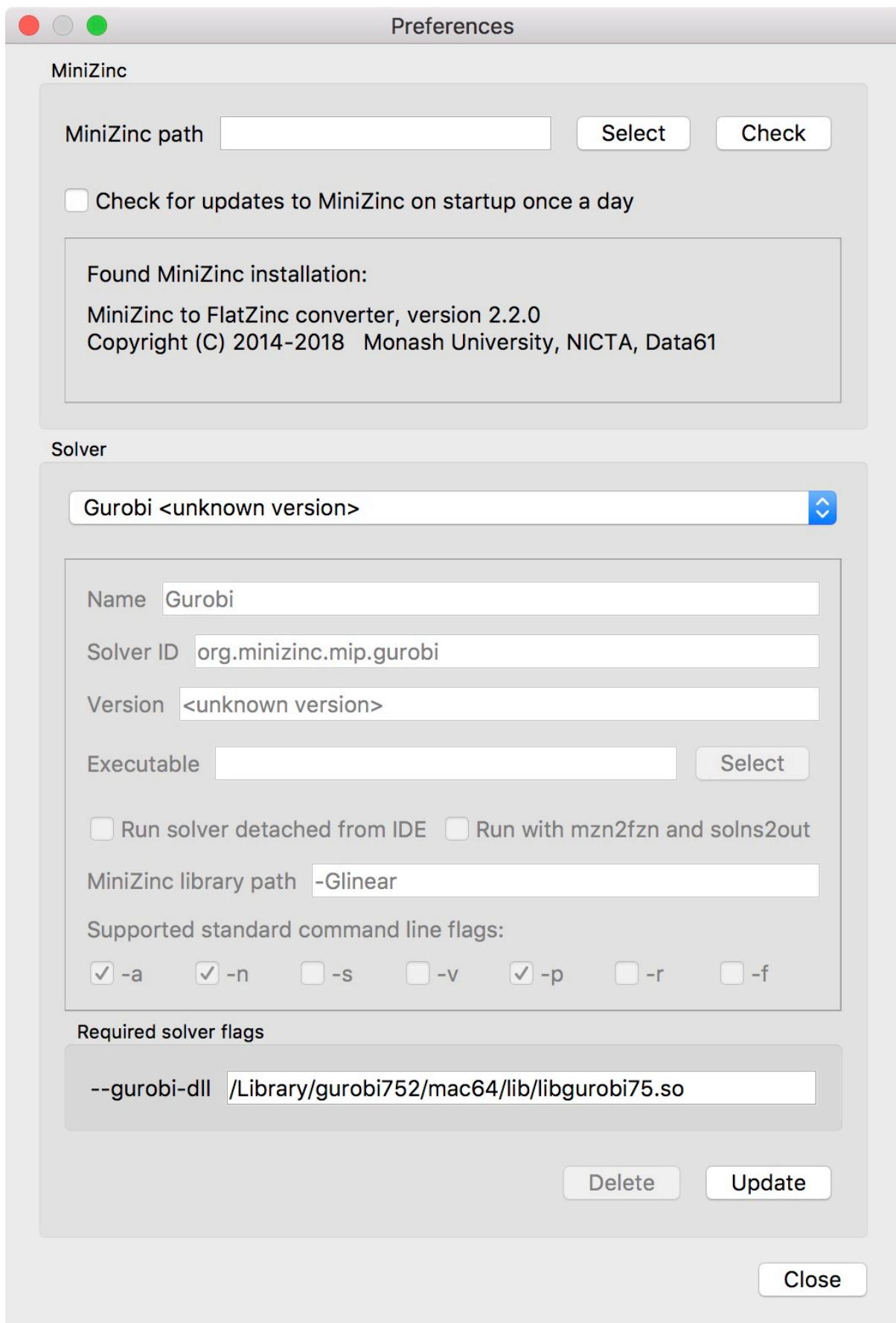
### 3.2.5.2 Adding Third-Party Solvers

The *Solver* section of the configuration dialog can be used to inspect the solvers that are currently available to MiniZinc, and to add new solvers to the system.

#### Configuring existing solvers

You can use the configuration dialog to set up defaults for the installed solvers. In the current version of the MiniZinc IDE, this is limited to configuring the CPLEX and Gurobi backends. The bundled binary version of MiniZinc comes with support for loading CPLEX and Gurobi as *plugins*, i.e., MiniZinc does not ship with the code for these solvers but can load them dynamically if they are installed.

For example, Fig. ?? shows a potential configuration for Gurobi. On Windows, the library is called `gurobiXX.dll` (in the same directory as the `gurobi` executable), and on Linux and macOS is it `libgurobiXX.so` (in the `lib` directory of your Gurobi installation), where `XX` stands for the version number of Gurobi.



If you select the CPLEX solver, a similar option appears (`--cplex-dll`). On Windows, the CPLEX library is called `cplexXXXX.dll` and typically found in same directory as the `cplex` executable. On Linux it is `libcplexXXX.so`, and on macOS `libcplexXXXX.jnilib`, where `XXX` and `XXXX` stand for the version number of CPLEX.

## Adding new solvers

The example in Fig. ?? shows a potential configuration for Gecode, which was installed in `/home/me/gecode`.

Each solver needs to be given

- a name;
- a unique identifier (usually in reverse domain name notation);
- a version string; and
- the executable that can run FlatZinc.

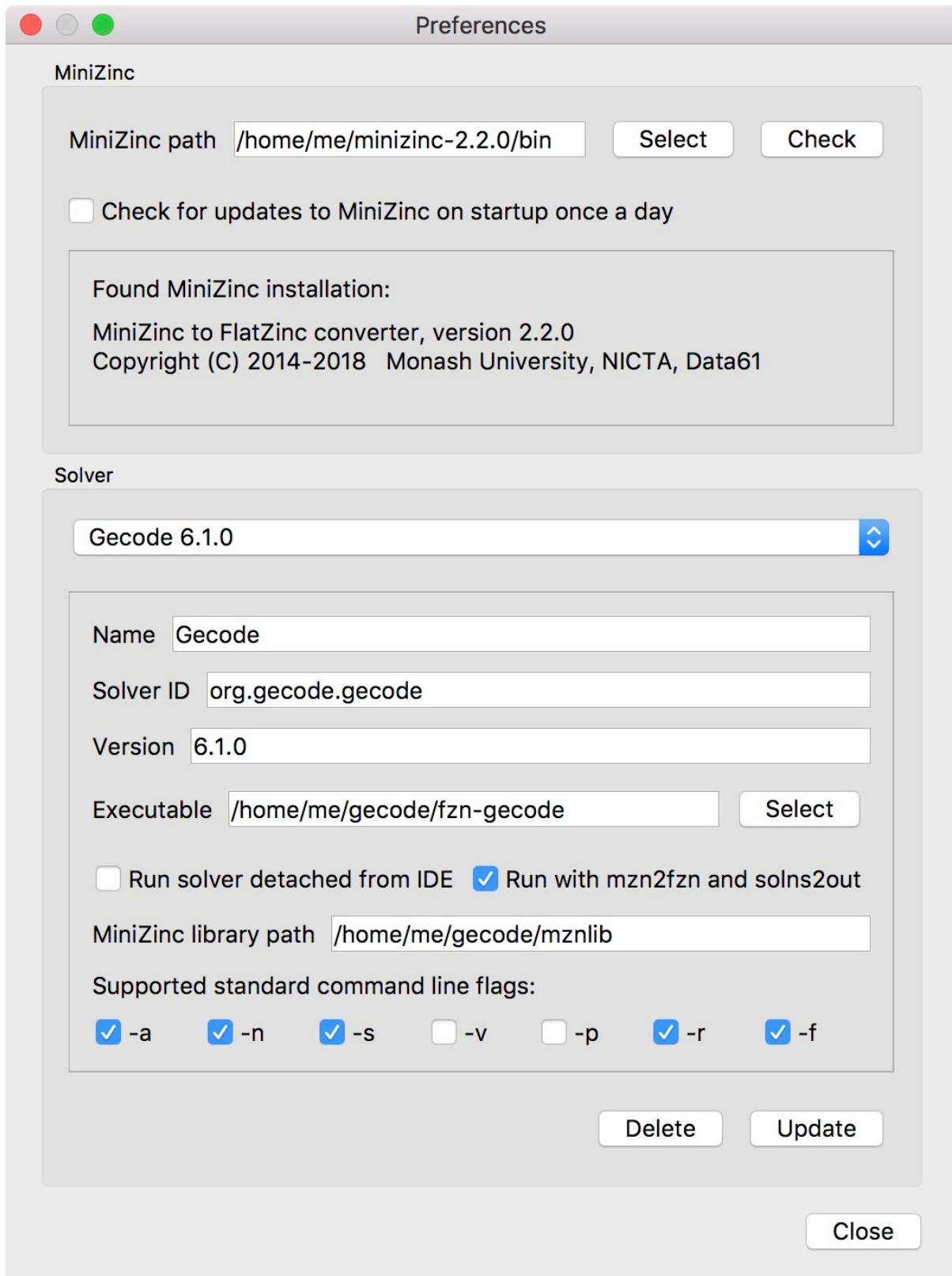
In addition, you can specify the location of a solver-specific MiniZinc library (see Section 4.3.3 for details). If you leave this field empty, the MiniZinc standard library will be used. The path entered into this field should be an absolute path in the file system, without extra quoting, and without any command line arguments (such as `-I`).

Most solvers will require compilation and output processing, since they only deal with FlatZinc files. For these solvers, the *Run with mzn2fzn and solns2out* option must be selected. For solvers that can deal with MiniZinc models natively, this option can be deselected.

Some solvers open an independent application with its own graphical user interface. One such example is the *Gecode (Gist)* solver that comes with the bundled version of the IDE. For these solvers, select the *Run solver detached from IDE* option, so that the IDE does not wait for solver output.

Finally, you can select which command line flags are supported by the solver. This controls which options will be available in the solver configuration window.

Solver configurations that are edited or created through the IDE are saved in a configuration file in a standard location. These solvers are therefore available the next time the IDE is started, as well as through the `minizinc` command line tool.





# CHAPTER 3.3

---

## Globalizer

---

Globalizer<sup>1</sup> analyses a model and set of data to provide suggestions of global constraints that can replace or enhance a user’s model.

### 3.3.1 Basic Usage

To use Globalizer simply execute it on a model and set of data files:

```
minizinc --solver org.minizinc.globalizer model.mzn data-1.dzn data-2.dzn
```

Note: Globalizer may also be executed on a non-parameterised model with no data files.

The following demonstrates the basic usage of Globalizer. Below, we see a simple model for the car sequencing problem<sup>2</sup>, cars.mzn.

Listing 3.3.1: Model for the car sequencing problem (cars.mzn).

```
% cars.mzn
include "globals.mzn";

int: n_cars; int: n_options; int: n_classes;
```

---

<sup>1</sup> Leo, K. et al., “Globalizing Constraint Models”, 2013.

<sup>2</sup> <http://www.csplib.org/Problems/prob001/>

```

set of int: steps = 1..n_cars;
set of int: options = 1..n_options;
set of int: classes = 1..n_classes;

array [options] of int: max_per_block;
array [options] of int: block_size;
array [classes] of int: cars_in_class;
array [classes, options] of 0..1: need;

% The class of car being started at each step.
array [steps] of var classes: class;

% Which options are required by the car started at each step.
array [steps, options] of var 0..1: used;

% Block p must be used at step s if the class of the car to be
% produced at step s needs it.
constraint forall (s in steps, p in options) (used[s, p]=need[class[s], p]);

% For each option p with block size b and maximum limit m, no consecutive
% sequence of b cars contains more than m that require option p.
constraint
  forall (p in options, i in 1..(n_cars - (block_size[p] - 1))) (
    sum (j in 0..(block_size[p] - 1)) (used[i + j, p])
    <= max_per_block[p]);
  
% Require that the right number of cars in each class are produced.
constraint forall (c in classes) (count(class, c, cars_in_class[c]));

solve satisfy; % Find any solution.

```

And here we have a data file `cars_data.dzn`.

Listing 3.3.2: Data for the car sequencing problem (`cars_data.dzn`).

```
% cars_data.dzn
n_cars = 10;
n_options = 5;
n_classes = 6;
```

```

max_per_block = [1, 2, 1, 2, 1];
block_size = [2, 3, 3, 5, 5];
cars_in_class = [1, 1, 2, 2, 2, 2];

need = array2d(1..6, 1..5, [
    1, 0, 1, 1, 0,
    0, 0, 0, 1, 0,
    0, 1, 0, 0, 1,
    0, 1, 0, 1, 0,
    1, 0, 1, 0, 0,
    1, 1, 0, 0, 0
]);

```

Executing Globalizer on this model and data file we get the following output:

```

% minizinc --solver globalizer cars.mzn cars_data.dzn
cars.mzn|33|12|33|69 [ ] gcc(class,cars_in_class)
cars.mzn|33|35|33|68 [ ] count(class,c,cars_in_class[c])
cars.mzn|28|27|28|65;cars.mzn|29|9|30|32 [ ] sliding_sum(0,max_per_block[p],
→block_size[p],used[_, p])

```

Each line of output is comprised of three elements.

1. Expression locations (separated by semicolon ‘;’ ) that the constraint might be able to replace.
2. (Between square brackets ‘[ ‘ ‘ ]’ ) the context under which the constraint might replace the expressions of 1.
3. The replacement constraint.

From the example above we see that a `gcc` constraint and a `sliding_sum` constraint can replace some constraints in the model. Taking the `sliding_sum` case, we see that it replaces the expression `cars.mzn|28|27|28|65` which corresponds to `i` in `1..(n_cars - (block_size[p] - 1))` and `cars.mzn|29|27|28|65` which corresponds to the `<=` expression including the `sum`. The original constraint can be replaced with:

```

constraint forall (p in options) (
    sliding_sum(0, max_per_block[p], block_size[p], used[..,p]));

```

### 3.3.2 Caveats

**MiniZinc syntax support.** Globalizer was implemented with support for a subset of an early version of the MiniZinc 2.0 language. As a result there are some limitations. First, Globalizer does not support set variables or enum types. The array slicing syntax supported in MiniZinc was not decided upon when Globalizer was implemented so it uses an incompatible syntax. Globalizer uses `_` instead of `...`. This can be seen above in the translation of `used[_,p]` to `used[...,p]`.

**New constraint.** A special two argument version of the `global_cardinality` constraint called `gcc` has been added which is not in the standard library. It is defined as follows:

```
predicate gcc(array[int] of var int: x, array[int] of var int: counts) =  
    global_cardinality(x,  
        [ i | i in index_set(counts) ],  
        array1d(counts));
```

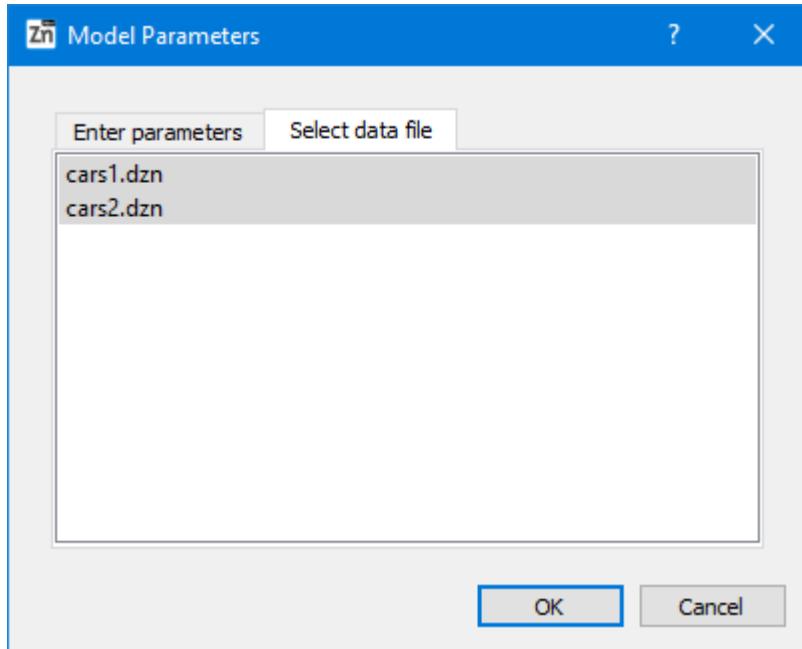
### 3.3.3 Supported Constraints

Globalizer currently supports detection of the following constraints:

alldifferent	alldifferent_except_0	all_equal_int
bin_packing	bin_packing_capa	bin_packing_load
binaries_represent_int →3B	binaries_represent_int_3A	binaries_represent_int_
binaries_represent_int_3C	channel	channelACB
count_geq	cumulative_assert	decreasing
gcc	global_cardinality	inverse
lex_lesseq_int_checking	lex2_checking	maximum_int_checking
minimum_int_checking	member	nvalue
strict_lex2_checking	subcircuit_checking	true
atleast	atmost	bin_packing_load_ub
circuit_checking	count	diffn
distribute	element	increasing
lex_less_int_checking	sliding_sum	sort_checking
unary	value_precede_checking	

### 3.3.4 Using Globalizer in the MiniZinc IDE

To use the Globalizer in the MiniZinc IDE, open a model, and several data files. Select Globalizer from the solver configuration dropdown menu. Click the solve button (play symbol). If you did not select any data file, a dialog should pop up that allows you to select data files. To select multiple data files hold the **Ctrl** or **Cmd** key while clicking on each data file.



Click run. While processing your model and data files the MiniZinc IDE will display a progress bar on the toolbar.

The screenshot shows the MiniZinc IDE interface. The main window displays a MiniZinc model named 'cars.mzn'. The code is as follows:

```

15 % The class of car being started at each step.
16 array [steps] of var classes: class;
17
18 % Which options are required by the car started at each step.
19 array [steps, options] of var 0..1: used;
20
21 % Block p must be used at step s if the class of the car to be
22 % produced at step s needs it.
23 constraint forall (s in steps, p in options) (used[s, p]=need[class[s], p]);
24
25 % For each option p with block size b and maximum limit m, no consecutive
26 % sequence of b cars contains more than m that require option p.
27 constraint
28   forall (p in options, i in 1..(n_cars - (block_size[p] - 1))) (
29     sum (j in 0..(block_size[p] - 1)) (used[i + j, p])
30       <= max_per_block[p]);
31
32 % Require that the right number of cars in each class are produced.
33 constraint forall (c in classes) (count(class, c, cars_in_class[c]));
34
35 solve satisfy; % Find any solution.

```

The output pane below the code lists discovered global constraints:

**Found Globals:**

- `gcc(class,[1, 1, 2, 2, 2, 2])`
- `gcc(class,cars_in_class)`
- `count(class,c,cars_in_class[c])`
- `sliding_sum(0,max_per_block[p],block_size[p],used[ , p])`  
% NUMCALLS: 610

At the bottom of the output pane, it says "Ready." and shows a progress bar indicating "1m 25s" and "100%".

Any discovered global constraints will be listed in the output pane of the IDE. Clicking on one of these constraints will highlight in the model the expressions that might be replaceable or strengthened by the constraint.

### 3.3.5 How it works

A summary of the algorithm is presented here. For a more detailed exploration of the approach see the Globalizing Constraint Models paper<sup>1</sup>.

- **Normalize.** In this step the model is transformed to make analysis easier.
  - Conjunctions of constraints are broken into individual constraints. For example:  $C1 \wedge C2$ ; becomes  $C1;$   $C2;$
  - Conjunctions under `forall` constraints are broken into individual `forall` constraints.

For example: `forall(...)(C1 /\ C2);` becomes `forall(...)(C1); forall(...)(C2)`

- **Generate submodels.** In this step all subsets containing 1 or 2 constraints (this can be configured using `--numConstraints` argument) are enumerated.
- **Instantiate and unroll into groups.** Each submodel is instantiated with the provided data files. These instantiated submodels are further used to produce more submodels by unrolling loops. For example, a constraint such as `forall(i in 1..n)(c(i));` will be used to produce the constraints: `c(1)` and `c(n)` for each instantiation of `n` in the data files. All instantiated submodels are then grouped together.
- **Process groups.** For each submodel in a group a set of 30 random solutions are found. (configurable using the `--randomSolutions` argument). A template model with all of these random solutions is created.

The different variables (including arrays and array accesses) and parameters used by a submodel are collected into a set of potential arguments along with the constant 0 and a special blank symbol representing an argument that can be inferred based on others used.

The list of constraints above is filtered based on the arguments available. For example, the `alldifferent` constraint will be removed from the list of candidates if the submodel does not reference any arrays of variables.

Finally the set of constraints are filtered by adding them to the template and solving it. If the template is satisfiable, the constraint accepts all of the random solutions.

30 sample solutions for each candidate constraint are generated. (configurable using the `--sampleSolutions` argument). The candidate is then ranked based on how many of these solutions are also solutions to the original submodel. If its score is less than some threshold the candidate is removed from the set of candidates for this group and will not be tested on later submodels.

- **Report.** The remaining candidates are presented to the user.

### 3.3.6 Performance tips

Globalizing constraint models can be a time consuming task. If

- **Use small or relatively easy instances.** Globalizer solves many subproblems while processing your model and data files. Using easier instances can speed up the process considerably.

- **Disable the initial pass.** As discussed above, Globalizer performs two passes. The first pass tries to detect alternate viewpoints that can be added to your model. If you are confident that this will not be useful we recommend disabling this first pass using the `--no-initial-pass` argument.
- **Narrow search using filters.** Globalizer attempts to match a large selection of global constraints to subproblems in your model. If you wish to check if only a few specific constraints are present you can focus Globalizer using the `--constraintFilter` or `-f` arguments followed by a comma separated list of strings. Only global constraints where one of the strings is a substring will be included in the search.
- **Disable implies check.** The implication check can also be disabled to improve performance. This results in less accurate results but may still help a user to understand the structure of their model.
- **Free-search.** To improve accuracy and to avoid false positives, subproblems solved by Globalizer are solved using a random heuristic and restarts. If the subproblems are particularly difficult to solve a random heuristic may be prohibitive and the solver may not be able to produce enough samples within the solving timelimit. In these circumstances a user can either increase the solver timeout using the `-t` argument followed by the number of milliseconds a solver should be given to find samples. Alternatively the `--free-search` argument can be used to force Globalizer to use the solver's free search to find samples. This has the downside of reducing the diversity of samples but allows enough samples to be found to allow suggested globals to be found.

### 3.3.7 Limitations / Future work

- Globalizer supports only a subset of the MiniZinc language and as such cannot be executed on all MiniZinc models.
- There are some relatively cheap syntactic approaches that should be performed before Globalization that currently is not implemented. For example, there are several common formulations of an `alldifferent` constraint that can be detected syntactically. This would be much cheaper than using Globalizer.

# CHAPTER 3.4

---

## FindMUS

---

FindMUS<sup>1</sup> lists unsatisfiable subsets of constraints in your MiniZinc model. These subsets, called Minimal Unsatisfiable Subsets can help you to find faults in your unsatisfiable constraint model. FindMUS uses the hierarchical structure provided by the model to guide its search.

### 3.4.1 Basic Usage

To use FindMUS on the command line simply execute it on a model and set of data files by typing:

```
minizinc --solver findMUS model.mzn data-1.dzn
```

Note: FindMUS requires a fully instantiated constraint model.

#### 3.4.1.1 Commandline arguments

The FindMUS tool supports the following arguments:

##### Driver options

The driver creates the map solver and sub-solver and requests MUSes from FindMUS' s enumeration algorithm HierMUS.

---

<sup>1</sup> Leo, K. et al., “Debugging Unsatisfiable Constraint Models” , 2017.

`-n <count>`; Stop after the  $n$  th MUS is found (Default: 1)

`-a` Find all MUSes

`--timeout <s>` Stop search after  $s$  seconds (Default: 1800)

### Enumeration options

The enumeration algorithm (HierMUS) explores the constraint hierarchy provided in the user's model, proposes potential MUSes to the sub-solver. The default algorithm is internally referred to as `stackMUS`. This algorithm can be replaced with either `MARCO` or `ReMUS` which have their own strengths. Our implementation of these algorithms in turn utilizes a simple linear deletion based 'shrink' method. This can be replaced with the binary-splitting `QuickXplain` which can be much quicker.

`--marco` Use the `MARCO`<sup>3</sup> algorithm as sub-enumerator

`--remus` Use the `ReMUS`<sup>4</sup> algorithm as sub-enumerator (not compatible with Hierarchical Search)

`--qx` Use `QuickXplain`<sup>5</sup> for shrink step of MARCO or ReMUS

`--depth mzn,fzn,<n>` How deep in the tree should search explore. (Default: 1)

`mzn` expands the search as far as the point when the compiler leaves the MiniZinc model.

`fzn` expands search as far as the FlatZinc constraints.

`<n>` expand search to the  $n$  level of the hierarchy.

### Subsolver options

FindMUS can be used in conjunction with any FlatZinc solver. These options mimic the `minizinc` arguments `--solver` and `--fzn-flags`. The behavior of these arguments is likely to change in later versions of the tool.

`--solver <s>` Use solver  $s$  for SAT checking. (Default: "gecode")

`--solver-flags <f>` Pass flags  $f$  to sub-solver. (Default: empty)

`--solver-timelimit <ms>` Set hard time limit for solver in milliseconds. (Default: 1100)

`--soft-defines` Consider functional constraints as part of MUSes

### Filtering options

---

<sup>3</sup> Liffiton, M. H. et al., "Fast, Flexible MUS Enumeration", 2016.

<sup>4</sup> Bendík, J. et al., "Recursive Online Enumeration of All Minimal Unsatisfiable Subsets", 2018.

<sup>5</sup> Junker, U. et al., "QUICKXPLAIN: preferred explanations and relaxations for over-constrained problems", 2004.

FindMUS can include or exclude constraints from its search based on the expression and constraint name annotations as well as properties of their paths (for example, line numbers). These filters are currently based on substrings but in future may support text spans and regular expressions.

```
--named-only Only consider constraints annotated with string annotations  
--filter-named <names>; --filter-named-exclude <names>; Include/exclude  
constraints with names that match sep separated names  
--filter-path <paths>; --filter-path-exclude <paths>; Include/exclude based  
on paths  
--filter-sep <sep>; Separator used for named and path filters
```

### Structure options

The structure coming from a user's model can significantly impact the performance of a MUS enumeration algorithm. Here we allow the structure to be generalized in various ways and extra structure can be injected in the form of binarization of the tree.

```
--structure flat,gen,normal,mix  
  
Alters initial structure: (Default: normal)  
  
flat Remove all structure  
  
gen Remove instance specific structure  
  
normal No change  
  
mix Apply gen before normal  
  
--binarize normal,leaves,all  
  
Add additional structure: (Default: normal)  
  
normal No change  
  
leaves Introduce structure at the leaves  
  
all Introduce structure throughout tree
```

### Verbosity options

```
--verbose-{map,enum,subsolve} <n> Set verbosity level for different components  
--verbose Set verbosity level of all components to 1
```

### Misc options

```
--dump-dot <dot> Write tree in GraphViz format to file <dot>
```

### 3.4.1.2 Example

The following demonstrates the basic usage of FindMUS on a simple example. Below, we see a model for the latin squares puzzle<sup>2</sup> with some incompatible symmetry breaking constraints added.

Listing 3.4.1: Faulty model for Latin Squares (latin\_squares.mzn).

```
% latin_squares.mzn
include "globals.mzn";

int: n = 3;
set of int: N = 1..n;
array[N, N] of var N: X;

constraint :: "ADRows"
  forall (i in N)
    (alldifferent(row(X, i)) :: "AD(row \((i))");
constraint :: "ADCols"
  forall (j in N)
    (alldifferent(col(X, j)) :: "AD(col \((j))");

constraint :: "LLRows"
  forall (i in 1..n-1)
    (lex_less(row(X, i), row(X, i+1)) :: "LL(rows \((i)) \((i+1))");
constraint :: "LGcols"
  forall (j in 1..n-1)
    (lex_greater(col(X, j), col(X, j+1)) :: "LG(cols \((j)) \((j+1))");

solve satisfy;

output [ show2d(X) ];
```

Here we have used the new constraint and expression annotations added in MiniZinc 2.2.0. Note that these annotations are not necessary for FindMUS to work but may help with interpreting the output. The first two constraints: ADRows and ADCols define the alldifferent constraints

---

<sup>2</sup> [https://en.wikipedia.org/wiki/Latin\\_square](https://en.wikipedia.org/wiki/Latin_square)

on the respective rows and columns of the latin square. The next two constraints LLRows and LGcols post lex constraints that order the rows to be increasing and the columns to be increasing. Certain combinations of these constraints are going to be in conflict.

Executing the command `minizinc --solver findMUS -a latin_squares.mzn` returns the following output. Note that the `-a` argument requests all MUSes that can be found with the default settings (more detail below).

```
FznSubProblem: hard cons: 36 soft cons: 26 leaves: 26 branches: 21
↳ Built tree in 0.03100 seconds.

SubsetMap: nleaves: 4 nbranches: 1

MUS: 0 1 2 21 22 3 32 33 4 43 44 5 54 55 6 7 8

Brief: exists;@{LG(cols 1 2@LGcols)}:() exists;@{LG(cols 2 3@LGcols)}:()
↳ exists;@{LL(rows 1 2)@LLRows}:() exists;@{LL(rows 2 3)@LLRows}:() int_lin_
↳ le;@{LG(cols 1 2@LGcols)}:() int_lin_le;@{LG(cols 2 3@LGcols)}:() int_lin_le;
↳ @{LL(rows 1 2)@LLRows}:() int_lin_le;@{LL(rows 2 3)@LLRows}:() int_lin_ne;@
↳ {AD(row 1)@ADRows}:() int_lin_ne;@{AD(row 1)@ADRows}:() int_lin_ne;@
↳ {AD(row 1)@ADRows}:() int_lin_ne;@{AD(row 2)@ADRows}:() int_lin_ne;@
↳ {AD(row 2)@ADRows}:() int_lin_ne;@{AD(row 2)@ADRows}:() int_lin_ne;@
↳ {AD(row 3)@ADRows}:() int_lin_ne;@{AD(row 3)@ADRows}:() int_lin_ne;@
↳ {AD(row 3)@ADRows}:()

Traces:
latin_squares.mzn|9|5|10|51|ca|forall
latin_squares.mzn|16|5|17|68|ca|forall
latin_squares.mzn|19|5|20|70|ca|forall
=====

MUS: 10 11 12 13 14 15 16 17 21 22 32 33 43 44 54 55 9

Brief: exists;@{LG(cols 1 2@LGcols)}:() exists;@{LG(cols 2 3@LGcols)}:()
↳ exists;@{LL(rows 1 2)@LLRows}:() exists;@{LL(rows 2 3)@LLRows}:() int_lin_
↳ le;@{LG(cols 1 2@LGcols)}:() int_lin_le;@{LG(cols 2 3@LGcols)}:() int_lin_le;
↳ @{LL(rows 1 2)@LLRows}:() int_lin_le;@{LL(rows 2 3)@LLRows}:() int_lin_ne;@
↳ {AD(col 1)@ADCols}:() int_lin_ne;@{AD(col 1)@ADCols}:() int_lin_ne;@
↳ {AD(col 1)@ADCols}:() int_lin_ne;@{AD(col 2)@ADCols}:() int_lin_ne;@
↳ {AD(col 2)@ADCols}:() int_lin_ne;@{AD(col 2)@ADCols}:() int_lin_ne;@
↳ {AD(col 3)@ADCols}:() int_lin_ne;@{AD(col 3)@ADCols}:() int_lin_ne;@
↳ {AD(col 3)@ADCols}:()
```

```
Traces:  
latin_squares.mzn|16|5|17|68|ca|forall  
latin_squares.mzn|12|5|13|51|ca|forall  
latin_squares.mzn|19|5|20|70|ca|forall  
=====  
Total Time: 0.24700      nmuses: 2      map: 10 sat: 6  total: 16  
=====UNKNOWN=====
```

The first two lines, starting with `FznSubProblem:` and `SubsetMap` provide some useful information for debugging the `findMUS` tool. Next we have the list of MUSes separated by a series of equals = signs. Each MUS is described with three sections:

1. `MUS`: lists the indices of FlatZinc constraints involved in this MUS.
2. `Brief`: lists the FlatZinc constraint name, the expression name, and the constraint name for each involved FlatZinc constraint.
3. `Traces`: lists the MiniZinc paths corresponding to the constraints of the MUS. Each path typically contains a list of path elements separated by semi-colons ;. Each element includes a file path, a start line, start column, end line and end column denoting a span of text from the listed file. And finally, a description of the element. In the examples above all paths point to calls to a forall on different lines of the model. (`ca|forall`)

The final line of output lists the `Total Time`, the number of MUSes found, and some statistics about the number of times the internal map solver `map` was called, and the number of times the subproblem solver was called `sat`.

Interpreting the two MUSes listed here we see that the `lex` constraints from lines 16 and 19 were included in both and only one of the `alldifferent` constraints from line 9 and 12 are required for the model to be unsatisfiable. The `lex` constraints being involved in every MUS make them a strong candidate for being the source of unsatisfiability in the user's model.

### 3.4.2 Using FindMUS in the MiniZinc IDE

To use FindMUS in the MiniZinc IDE, upon discovering that a model is unsatisfiable. Select `FindMUS` from the solver configuration dropdown menu and click the solve button (play symbol). By default FindMUS is configured to return a single MUS at a depth of '1'. This should be relatively fast and help locate the relevant constraint items. The following shows the result of

running FindMUS with the default options.

```

% latin_squares.mzn
include "globals.mzn";

int: n = 3;
set of int: N = 1..n;
array[N, N] of var N: X;

constraint :: "ADRows"
  forall (i in N)
    (alldifferent(row(X, i)) :: "AD(row \((i))");

constraint :: "ADCols"
  forall (j in N)
    (alldifferent(col(X, j)) :: "AD(col \((j))");

constraint :: "LLRows"
  forall (i in 1..n-1)
    (lex_less(row(X, i), row(X, i+1)) :: "LL(rows \((i) \((i+1))");

constraint :: "LGCols"
  forall (j in 1..n-1)
    (lex_greater(col(X, j), col(X, j+1)) :: "LG(cols \((j) \((j+1))");

solve satisfy;

output [ show2d(X) ];

```

**Output**

```

int lin_ne;@{AD(row 2)@ADRows}:( )
int lin_ne;@{AD(row 2)@ADRows}:( )
int lin_ne;@{AD(row 2)@ADRows}:( )
int lin_ne;@{AD(row 3)@ADRows}:( )
int lin_ne;@{AD(row 3)@ADRows}:( )
int lin_ne;@{AD(row 3)@ADRows}:( )

Total Time: 0.19000          nmuses: 1          map: 6   sat: 5   total: 11
Finished in 475msec

```

Ready. 475msec

Selecting the returned MUS highlights three top level constraints as shown: ADRows, LLRows and LGCols. To get a more specific MUS we can instruct FindMUS to go deeper than the top level constraints by clicking the “Show configuration editor” button in the top right hand corner

of the MiniZinc IDE window, and adding `--depth mzn` to the “Additional solver command line arguments” textbox in the “Solver options” section. The following shows a more specific MUS in this model.

The screenshot shows the MiniZinc IDE interface with the following details:

- Title Bar:** Shows the file name "latin\_squares.mzn — Untitled Project".
- Toolbar:** Includes standard file operations (New model, Open, Save), text editing (Copy, Cut, Paste, Undo, Redo), navigation (Shift left, Shift right), and execution (Run, Stop) buttons, along with a "Show configuration editor" icon.
- Code Editor:** Displays the MiniZinc code for a Latin Squares problem. The code defines variables, constraints (ADRows, ADCols, LLRows, LGCols), and a solve clause. Lines 16 and 20 are highlighted in pink.

```

1 % latin_squares.mzn
2 include "globals.mzn";
3
4 int: n = 3;
5 set of int: N = 1..n;
6 array[N, N] of var N: X;
7
8 constraint :: "ADRows"
9     forall (i in N)
10        (alldifferent(row(X, i)) :: "AD(row \((i))");
11 constraint :: "ADCols"
12     forall (j in N)
13        (alldifferent(col(X, j)) :: "AD(col \((j))");
14
15 constraint :: "LLRows"
16     forall [i in 1..n-1]
17        (lex_less(row(X, i), row(X, i+1)) :: "LL(rows \((i) \((i+1))");
18 constraint :: "LGcols"
19     forall (j in 1..n-1)
20        (lex_greater(col(X, j), col(X, j+1)) :: "LG(cols \((j) \((j+1))");
21
22 solve satisfy;
23

```

- Output Pane:** Shows the MUS output. It lists all constraints and their corresponding loop variables. The output is as follows:

```

10 11 12 13 14 21 22 32 33 43 44 9 Brief: exists;@{LG(cols 1 2@LGCols):(j=1)
exists;@{LL(rows 1 2)@LLRows}:(i=1)
exists;@{LL(rows 2 3)@LLRows}:(i=2)
int lin le;@{LG(cols 1 2@LGCols):(j=1)
int lin le;@{LL(rows 1 2)@LLRows}:(i=1)
int lin le;@{LL(rows 2 3)@LLRows}:(i=2)
int lin ne;@{AD(col 1)@ADCols}:(j=1)
int lin ne;@{AD(col 1)@ADCols}:(j=1)
int lin ne;@{AD(col 1)@ADCols}:(j=1)
int lin ne;@{AD(col 2)@ADCols}:(j=2)
int lin ne;@{AD(col 2)@ADCols}:(j=2)
int lin ne;@{AD(col 2)@ADCols}:(j=2)

```

In this case we can see that the output pane lists more specific information about the constraints involved in the MUS. After each listed constraint name we see what any loop variables were assigned to when the constraint was added to the FlatZinc. For example `(j=2)`.

### 3.4.3 How it works

A simple explanation of the algorithm is presented here. For a more detailed exploration of an earlier version of the approach see the Debugging Unsatisfiable Constraint Models paper<sup>1</sup>.

The approach takes the full FlatZinc program and partitions the constraints into groups based on the hierarchy provided in the user's model. To begin with (at depth '1') we search for MUSes in the set of top level constraint items. If we are not at the target depth we recursively select a found MUS, split its constituent constraints into lower level constraints based on the hierarchy and begin another search for MUSes underneath this high-level MUS. If any MUSes are found we know that the high-level MUS is not minimal and so it should not be reported. This process is repeated on any found MUSes until we reach the required depth at which point we will start to report MUSes. If in the recursive search we return to a high-level MUS without finding any sub-MUSes we can report this MUS as a real MUS. This recursive process is referred to as HierMUS. At each stage when we request the enumeration of a set of MUSes underneath a high-level MUS we can use one of several MUS enumeration algorithms. By default we use the internally developed StackMUS. We can also utilize MARCO and ReMUS for this role.

### 3.4.4 Performance tips

If you are trying to find MUSes in a very large instance it is advised to make use of the filtering tools available. Use the default settings to find a very high-level MUS and then use the --depth option to find lower-level, more specific MUSes in conjunction with the --filter-name and --filter-path options to focus on finding specific sub-MUSes of a found high-level MUS.

### 3.4.5 Limitations / Future work

There are several features that we aim to include quite soon:

**Regular expression based filtering** This will allow more complex filtering to be used.

**Text span based filtering** This will allow a user to simply click-and-drag a selection around the parts of a constraint model they wish to analyse.

**Single MUS focus mode** This mode would perform the process outlined in the 'Performance tips' section automatically making it easier for users to find detailed MUSes.



# CHAPTER 3.5

---

## Using MiniZinc in Jupyter Notebooks

---

You can use MiniZinc inside a Jupyter / IPython notebook using the `iminizinc` Python module. The module provides a “cell magic” extension that lets you solve MiniZinc models.

The module requires an existing installation of MiniZinc.

### 3.5.1 Installation

You can install or upgrade this module via pip:

```
pip install -U iminizinc
```

Consult your Python documentation to find out if you need any extra options (e.g. you may want to use the `--user` flag to install only for the current user, or you may want to use virtual environments).

Make sure that the `minizinc` binary are on the PATH environment variable when you start the notebook server. The easiest way to do that is to get the “bundled installation” that includes the MiniZinc IDE and a few solvers, available from GitHub here: <https://github.com/MiniZinc/MiniZincIDE/releases/latest> You then need to change your PATH environment variable to include the MiniZinc installation.

### 3.5.2 Basic usage

After installing the module, you have to load the extension using `%load_ext iminizinc`. This will enable the cell magic `%%minizinc`, which lets you solve MiniZinc models. Here is a simple example:

```
In[1]: %load_ext iminizinc

In[2]: n=8

In[3]: %%minizinc

    include "globals.mzn";
    int: n;
    array[1..n] of var 1..n: queens;
    constraint all_different(queens);
    constraint all_different([queens[i]+i | i in 1..n]);
    constraint all_different([queens[i]-i | i in 1..n]);
    solve satisfy;

Out[3]: {u'queens': [4, 2, 7, 3, 6, 8, 5, 1]}
```

As you can see, the model binds variables in the environment (in this case, `n`) to MiniZinc parameters, and returns an object with fields for all declared decision variables.

Alternatively, you can bind the decision variables to Python variables:

```
In[1]: %load_ext iminizinc

In[2]: n=8

In[3]: %%minizinc -m bind

    include "globals.mzn";
    int: n;
    array[1..n] of var 1..n: queens;
    constraint all_different(queens);
    constraint all_different([queens[i]+i | i in 1..n]);
    constraint all_different([queens[i]-i | i in 1..n]);
    solve satisfy;
```

```
In[4]: queens
```

```
Out[4]: [4, 2, 7, 3, 6, 8, 5, 1]
```

If you want to find all solutions of a satisfaction problem, or all intermediate solutions of an optimisation problem, you can use the `-a` flag:

```
In[1]: %load_ext iminizinc
```

```
In[2]: n=6
```

```
In[3]: %%minizinc -a
```

```
include "globals.mzn";
int: n;
array[1..n] of var 1..n: queens;
constraint all_different(queens);
constraint all_different([queens[i]+i | i in 1..n]);
constraint all_different([queens[i]-i | i in 1..n]);
solve satisfy;
```

```
Out[3]: [{u'queens': [5, 3, 1, 6, 4, 2]}, {u'queens': [4, 1, 5, 2, 6, 3]}, {u'queens': [3, 6, 2, 5, 1, 4]}, {u'queens': [2, 4, 6, 1, 3, 5]}]
```

The magic supports a number of additional options, in particular loading MiniZinc models and data from files. Some of these may only work with the development version of MiniZinc (i.e., not the one that comes with the bundled binary releases). You can take a look at the help using

```
In[1]: %%minizinc?
```



## **Part 4**

# **Reference Manual**



# CHAPTER 4.1

---

## Specification of MiniZinc

---

### 4.1.1 Introduction

This document defines MiniZinc, a language for modelling constraint satisfaction and optimisation problems.

MiniZinc is a high-level, typed, mostly first-order, functional, modelling language. It provides:

- mathematical notation-like syntax (automatic coercions, overloading, iteration, sets, arrays);
- expressive constraints (finite domain, set, linear arithmetic, integer);
- support for different kinds of problems (satisfaction, explicit optimisation);
- separation of data from model;
- high-level data structures and data encapsulation (sets, arrays, enumerated types, constrained type- insts);
- extensibility (user-defined functions and predicates);
- reliability (type checking, instantiation checking, assertions);
- solver-independent modelling;
- simple, declarative semantics.

MiniZinc is similar to OPL and moves closer to CLP languages such as ECLiPSe.

This document has the following structure. *Notation* (page 200) introduces the syntax notation used throughout the specification. *Overview of a Model* (page 201) provides a high-level overview of MiniZinc models. *Syntax Overview* (page 204) covers syntax basics. *High-level Model Structure* (page 205) covers high-level structure: items, multi-file models, namespaces, and scopes. *Types and Type-insts* (page 207) introduces types and type-insts. *Expressions* (page 218) covers expressions. *Items* (page 235) describes the top-level items in detail. *Annotations* (page 244) describes annotations. *Partiality* (page 245) describes how partiality is handled in various cases. *Built-in Operations* (page 248) describes the language built-ins. *Full grammar* (page 260) gives the MiniZinc grammar. *Content-types* (page 258) defines content-types used in this specification.

This document also provides some explanation of why certain design decisions were made. Such explanations are marked by the word *Rationale* and written in italics, and do not constitute part of the specification as such. *Rationale: These explanations are present because they are useful to both the designers and the users of MiniZinc.*

#### 4.1.1.1 Original authors.

The original version of this document was prepared by Nicholas Nethercote, Kim Marriott, Reza Rafeh, Mark Wallace and Maria Garcia de la Banda. MiniZinc is evolving, however, and so is this document.

For a formally published paper on the MiniZinc language and the superset Zinc language, please see:

- N. Nethercote, P.J. Stuckey, R. Becket, S. Brand, G.J. Duck, and G. Tack. Minizinc: Towards a standard CP modelling language. In C. Bessiere, editor, *Proceedings of the 13th International Conference on Principles and Practice of Constraint Programming*, volume 4741 of *LNCS*, pages 529–543. Springer-Verlag, 2007.
- K. Marriott, N. Nethercote, R. Rafeh, P.J. Stuckey, M. Garcia de la Banda, and M. Wallace. The Design of the Zinc Modelling Language. *Constraints*, 13(3):229-267, 2008.

#### 4.1.2 Notation

The basics of the EBNF used in this specification are as follows.

- Non-terminals are written between angle brackets, <item>.

- Terminals are written in double quotes, e.g. `"constraint"`. A double quote terminal is written as a sequence of three double quotes: `"""`.
- Optional items are written in square brackets, e.g. `[ "var" ]`.
- Sequences of zero or more items are written with parentheses and a star, e.g. `( "," <ident> )*`.
- Sequences of one or more items are written with parentheses and a plus, e.g. `( <msg> )+`.
- Non-empty lists are written with an item, a separator/terminator terminal, and three dots. For example, this:

```
<expr> ", " ...
```

is short for this:

```
<expr> ( "," <expr> )* [ "," ]
```

The final terminal is always optional in non-empty lists.

- Regular expressions are used in some productions, e.g. `[+-]?[0-9]+`.

MiniZinc's grammar is presented piece-by-piece throughout this document. It is also available as a whole in [Full grammar](#) (page 260). The output grammar also includes some details of the use of whitespace. The following conventions are used:

- A newline character or CRLF sequence is written `\n`.

### 4.1.3 Overview of a Model

Conceptually, a MiniZinc problem specification has two parts.

1. The *model*: the main part of the problem specification, which describes the structure of a particular class of problems.
2. The *data*: the input data for the model, which specifies one particular problem within this class of problems.

The pairing of a model with a particular data set is a *model instance* (sometimes abbreviated to *instance*).

The model and data may be separated, or the data may be “hard-wired” into the model. [Model Instance Files](#) (page 206) specifies how the model and data can be structured within files in a

model instance.

There are two broad classes of problems: satisfaction and optimisation. In satisfaction problems all solutions are considered equally good, whereas in optimisation problems the solutions are ordered according to an objective and the aim is to find a solution whose objective is optimal. *Solve Items* (page 238) specifies how the class of problem is chosen.

#### 4.1.3.1 Evaluation Phases

A MiniZinc model instance is evaluated in two distinct phases.

1. Instance-time: static checking of the model instance.
2. Run-time: evaluation of the instance (i.e., constraint solving).

The model instance may not compile due to a problem with the model and/or data, detected at instance-time. This could be caused by a syntax error, a type-inst error, the use of an unsupported feature or operation, etc. In this case the outcome of evaluation is a static error; this must be reported prior to run-time. The form of output for static errors is implementation-dependent, although such output should be easily recognisable as a static error.

An implementation may produce warnings during all evaluation phases. For example, an implementation may be able to determine that unsatisfiable constraints exist prior to run-time, and the resulting warning given to the user may be more helpful than if the unsatisfiability is detected at run-time.

An implementation must produce a warning if the objective for an optimisation problem is unbounded.

#### 4.1.3.2 Run-time Outcomes

Assuming there are no static errors, the output from the run-time phase has the following abstract form:

```
<output> ::= <no-solutions> [ <warnings> ] <free-text>
          | ( <solution> )* [ <complete> ] [ <warnings> ] <free-text>
```

If a solution occurs in the output then it must be feasible. For optimisation problems, each solution must be strictly better than any preceding solution.

If there are no solutions in the output, the outcome must indicate that there are no solutions.

If the search is complete the output may state this after the solutions. The absence of the completeness message indicates that the search is incomplete.

Any warnings produced during run-time must be summarised after the statement of completeness. In particular, if there were any warnings at all during run-time then the summary must indicate this fact.

The implementation may produce text in any format after the warnings. For example, it may print a summary of benchmarking statistics or resources used.

#### 4.1.3.3 Output

Implementations must be capable of producing output of content type application/x-zinc-output, which is described below and also in [Content-types](#) (page 258). Implementations may also produce output in alternative formats. Any output should conform to the abstract format from the previous section and must have the semantics described there.

Content type application/x-zinc-output extends the syntax from the previous section as follows:

```
<solution> ::= <solution-text> [ \n ] "-----" \n
```

The solution text for each solution must be as described in [Output Items](#) (page 239). A newline must be appended if the solution text does not end with a newline. *Rationale: This allows solutions to be extracted from output without necessarily knowing how the solutions are formatted.* Solutions end with a sequence of ten dashes followed by a newline.

```
<no-solutions> ::= "=====UNSATISFIABLE=====" \n
```

The completeness result is printed on a separate line. *Rationale: The strings are designed to clearly indicate the end of the solutions.*

```
<complete> ::= "===== " \n
```

If the search is complete, a statement corresponding to the outcome is printed. For an outcome of no solutions the statement is that the model instance is unsatisfiable, for an outcome of no more solutions the statement is that the solution set is complete, and for an outcome of no better solutions the statement is that the last solution is optimal. *Rationale: These are the logical implications of a search being complete.*

```
<warnings> ::= ( <message> )+  
  
<message> ::= ( <line> )+
```

If the search is incomplete, one or more messages describing reasons for incompleteness may be printed. Likewise, if any warnings occurred during search they are repeated after the completeness message. Both kinds of message should have lines that start with % so they are recognized as comments by post-processing. *Rationale:* *This allows individual messages to be easily recognised.*

For example, the following may be output for an optimisation problem:

```
=====UNSATISFIABLE=====  
% trentin.fzn:4: warning: model inconsistency detected before search.
```

Note that, as in this case, an unbounded objective is not regarded as a source of incompleteness.

## 4.1.4 Syntax Overview

### 4.1.4.1 Character Set

The input files to MiniZinc must be encoded as UTF-8.

MiniZinc is case sensitive. There are no places where upper-case or lower-case letters must be used.

MiniZinc has no layout restrictions, i.e., any single piece of whitespace (containing spaces, tabs and newlines) is equivalent to any other.

### 4.1.4.2 Comments

A % indicates that the rest of the line is a comment. MiniZinc also has block comments, using symbols /\* and \*/ to mark the beginning and end of a comment.

### 4.1.4.3 Identifiers

Identifiers have the following syntax:

```
<ident> ::= [A-Za-z][A-Za-z0-9_]*           % excluding keywords
          | """ [^\xa\xd\x0]* """

```

```
my_name_2
MyName2
'An arbitrary identifier'
```

A number of keywords are reserved and cannot be used as identifiers. The keywords are: `ann`, `annotation`, `any`, `array`, `bool`, `case`, `constraint`, `diff`, `div`, `else`, `elseif`, `endif`, `enum`, `false`, `float`, `function`, `if`, `in`, `include`, `int`, `intersect`, `let`, `list`, `maximize`, `minimize`, `mod`, `not`, `of`, `op`, `opt`, `output`, `par`, `predicate`, `record`, `satisfy`, `set`, `solve`, `string`, `subset`, `superset`, `symdiff`, `test`, `then`, `true`, `tuple`, `type`, `union`, `var`, `where`, `xor`.

A number of identifiers are used for built-ins; see *Built-in Operations* (page 248) for details.

#### 4.1.5 High-level Model Structure

A MiniZinc model consists of multiple *items*:

```
<model> ::= [ <item> ";" ... ]
```

Items can occur in any order; identifiers need not be declared before they are used. Items have the following top-level syntax:

```
<item> ::= <include-item>
          | <var-decl-item>
          | <enum-item>
          | <assign-item>
          | <constraint-item>
          | <solve-item>
          | <output-item>
          | <predicate-item>
          | <test-item>
          | <function-item>
          | <annotation-item>

<ti-expr-and-id> ::= <ti-expr> ":" <ident>
```

Include items provide a way of combining multiple files into a single instance. This allows a model to be split into multiple files ([Include Items](#) (page 235)).

Variable declaration items introduce new global variables and possibly bind them to a value ([Variable Declaration Items](#) (page 235)).

Assignment items bind values to global variables ([Assignment Items](#) (page 238)).

Constraint items describe model constraints ([Constraint Items](#) (page 238)).

Solve items are the “starting point” of a model, and specify exactly what kind of solution is being looked for: plain satisfaction, or the minimization/maximization of an expression. Each model must have exactly one solve item ([Solve Items](#) (page 238)).

Output items are used for nicely presenting the result of a model execution ([Output Items](#) (page 239)).

Predicate items, test items (which are just a special type of predicate) and function items introduce new user-defined predicates and functions which can be called in expressions ([User-defined Operations](#) (page 240)). Predicates, functions, and built-in operators are described collectively as *operations*.

Annotation items augment the `ann` type, values of which can specify non-declarative and/or solver-specific information in a model.

#### 4.1.5.1 Model Instance Files

MiniZinc models can be constructed from multiple files using include items (see [Include Items](#) (page 235)). MiniZinc has no module system as such; all the included files are simply concatenated and processed as a whole, exactly as if they had all been part of a single file. *Rationale:* *We have not found much need for one so far. If bigger models become common and the single global namespace becomes a problem, this should be reconsidered.*

Each model may be paired with one or more data files. Data files are more restricted than model files. They may only contain variable assignments (see [Assignment Items](#) (page 238)).

Data files may not include calls to user-defined operations.

Models do not contain the names of data files; doing so would fix the data file used by the model and defeat the purpose of allowing separate data files. Instead, an implementation must allow one or more data files to be combined with a model for evaluation via a mechanism such as the command-line.

When checking a model with data, all global variables with fixed type-insts must be assigned, unless they are not used (in which case they can be removed from the model without effect).

A data file can only be checked for static errors in conjunction with a model, since the model contains the declarations that include the types of the variables assigned in the data file.

A single data file may be shared between multiple models, so long as the definitions are compatible with all the models.

#### 4.1.5.2 Namespaces

All names declared at the top-level belong to a single namespace. It includes the following names.

1. All global variable names.
2. All function and predicate names, both built-in and user-defined.
3. All enumerated type names and enum case names.
4. All annotation names.

Because multi-file MiniZinc models are composed via concatenation (*Model Instance Files* (page 206)), all files share this top-level namespace. Therefore a variable `x` declared in one model file could not be declared with a different type in a different file, for example.

MiniZinc supports overloading of built-in and user-defined operations.

#### 4.1.5.3 Scopes

Within the top-level namespace, there are several kinds of local scope that introduce local names:

- Comprehension expressions (*Set Comprehensions* (page 226)).
- Let expressions (*Let Expressions* (page 232)).
- Function and predicate argument lists and bodies (*User-defined Operations* (page 240)).

The listed sections specify these scopes in more detail. In each case, any names declared in the local scope overshadow identical global names.

### 4.1.6 Types and Type-insts

MiniZinc provides four scalar built-in types: Booleans, integers, floats, and strings; enumerated types; two compound built-in types: sets and multi-dimensional arrays; and the user extensible annotation type `ann`.

Each type has one or more possible *instantiations*. The instantiation of a variable or value indicates if it is fixed to a known value or not. A pairing of a type and instantiation is called a *type-inst*.

We begin by discussing some properties that apply to every type. We then introduce instantiations in more detail. We then cover each type individually, giving: an overview of the type and its possible instantiations, the syntax for its type-insts, whether it is a finite type (and if so, its domain), whether it is varifiable, the ordering and equality operations, whether its variables must be initialised at instance-time, and whether it can be involved in automatic coercions.

#### 4.1.6.1 Properties of Types

The following list introduces some general properties of MiniZinc types.

- Currently all types are monotypes. In the future we may allow types which are polymorphic in other types and also the associated constraints.
- We distinguish types which are *finite types*. In MiniZinc, finite types include Booleans, enums, types defined via set expression type-insts such as range types (see *Set Expression Type-insts* (page 217)), as well as sets and arrays, composed of finite types. Types that are not finite types are unconstrained integers, unconstrained floats, unconstrained strings, and `ann`. Finite types are relevant to sets (`spec-Sets`) and array indices (`spec-Arrays`). Every finite type has a *domain*, which is a set value that holds all the possible values represented by the type.
- Every first-order type (this excludes `ann`) has a built-in total order and a built-in equality; `>`, `<`, `==/=`, `!=`, `<=` and `>=` comparison operators can be applied to any pair of values of the same type. *Rationale:* This facilitates the specification of symmetry breaking and of polymorphic predicates and functions. Note that, as in most languages, using equality on floats or types that contain floats is generally not reliable due to their inexact representation. An implementation may choose to warn about the use of equality with floats or types that contain floats.

#### 4.1.6.2 Instantiations

When a MiniZinc model is evaluated, the value of each variable may initially be unknown. As it runs, each variable's *domain* (the set of values it may take) may be reduced, possibly to a single value.

An *instantiation* (sometimes abbreviated to *inst*) describes how fixed or unfixed a variable is at instance-time. At the most basic level, the instantiation system distinguishes between two kinds

of variables:

1. *Parameters*, whose values are fixed at instance-time (usually written just as “fixed” ).
2. *Decision variables* (often abbreviated to *variables*), whose values may be completely unfixed at instance-time, but may become fixed at run-time (indeed, the fixing of decision variables is the whole aim of constraint solving).

In MiniZinc decision variables can have the following types: Booleans, integers, floats, and sets of integers, and enums. Arrays and `ann` can contain decision variables.

#### 4.1.6.3 Type-insts

Because each variable has both a type and an inst, they are often combined into a single *type-inst*. Type-insts are primarily what we deal with when writing models, rather than types.

A variable’s type-inst *never changes*. This means a decision variable whose value becomes fixed during model evaluation still has its original type-inst (e.g. `var int`), because that was its type-inst at instance-time.

Some type-insts can be automatically coerced to another type-inst. For example, if a `par int` value is used in a context where a `var int` is expected, it is automatically coerced to a `var int`. We write this  $\text{par int} \xrightarrow{c} \text{var int}$ . Also, any type-inst can be considered coercible to itself. MiniZinc allows coercions between some types as well.

Some type-insts can be *varified*, i.e., made unfixed at the top-level. For example, `par int` is varified to `var int`. We write this  $\text{par int} \xrightarrow{v} \text{var int}$ .

Type-insts that are varifiable include the type-insts of the types that can be decision variables (Booleans, integers, floats, sets, enumerated types). Varification is relevant to type-inst synonyms and array accesses.

#### 4.1.6.4 Type-inst expression overview

This section partly describes how to write type-insts in MiniZinc models. Further details are given for each type as they are described in the following sections.

A type-inst expression specifies a type-inst. Type-inst expressions may include type-inst constraints. Type-inst expressions appear in variable declarations (*Variable Declaration Items* (page 235)) and user-defined operation items (*User-defined Operations* (page 240)).

Type-inst expressions have this syntax:

```
<ti-expr> ::= <base-ti-expr>

<base-ti-expr> ::= <var-par> <base-ti-expr-tail>

<var-par> ::= "var" | "par" | ε

<base-type> ::= "bool"
               | "int"
               | "float"
               | "string"

<base-ti-expr-tail> ::= <ident>
                         | <base-type>
                         | <set-ti-expr-tail>
                         | <ti-variable-expr-tail>
                         | <array-ti-expr-tail>
                         | "ann"
                         | "opt" <base-ti-expr-tail>
                         | { <expr> ", " ... }
                         | <num-expr> ".." <num-expr>
```

(The final alternative, for range types, uses the numeric-specific `<num-expr>` non-terminal, defined in [Expressions Overview](#) (page 218), rather than the `<expr>` non-terminal. If this were not the case, the rule would never match because the `..` operator would always be matched by the first `<expr>`.)

This fully covers the type-inst expressions for scalar types. The compound type-inst expression syntax is covered in more detail in [Built-in Compound Types and Type-insts](#) (page 213).

The `par` and `var` keywords (or lack of them) determine the instantiation. The `par` annotation can be omitted; the following two type-inst expressions are equivalent:

```
par int
int
```

*Rationale:* The use of the explicit `var` keyword allows an implementation to check that all parameters are initialised in the model or the instance. It also clearly documents which variables are parameters, and allows more precise type-inst checking.

A type-inst is fixed if it does not contain `var` or `any`, with the exception of `ann`.

Note that several type-inst expressions that are syntactically expressible represent illegal type-insts. For example, although the grammar allows `var` in front of all these base type-inst expression tails, it is a type-inst error to have `var` in the front of a string or array expression.

#### 4.1.6.5 Built-in Scalar Types and Type-insts

##### Booleans

*Overview.* Booleans represent truthhood or falsity. *Rationale:* Boolean values are not represented by integers. Booleans can be explicit converted to integers with the `bool2int` function, which makes the user's intent clear.

*Allowed Insts.* Booleans can be fixed or unfixed.

*Syntax.* Fixed Booleans are written `bool` or `par bool`. Unfixed Booleans are written as `var bool`.

*Finite?* Yes. The domain of a Boolean is `false`, `true`.

*Varifiable?*  $\text{par bool} \xrightarrow{v} \text{var bool}$ ,  $\text{var bool} \xrightarrow{v} \text{var bool}$ .

*Ordering.* The value `false` is considered smaller than `true`.

*Initialisation.* A fixed Boolean variable must be initialised at instance-time; an unfixed Boolean variable need not be.

*Coercions.*  $\text{par bool} \xrightarrow{c} \text{var bool}$ .

Also Booleans can be automatically coerced to integers; see [Integers](#) (page 211).

##### Integers

*Overview.* Integers represent integral numbers. Integer representations are implementation-defined. This means that the representable range of integers is implementation-defined. However, an implementation should abort at run-time if an integer operation overflows.

*Allowed Insts.* Integers can be fixed or unfixed.

*Syntax.* Fixed integers are written `int` or `par int`. Unfixed integers are written as `var int`.

*Finite?* Not unless constrained by a set expression (see [Set Expression Type-insts](#) (page 217)).

*Varifiable?*  $\text{par int} \xrightarrow{v} \text{var int}$ ,  $\text{var int} \xrightarrow{v} \text{var int}$ .

*Ordering.* The ordering on integers is the standard one.

*Initialisation.* A fixed integer variable must be initialised at instance-time; an unfixed integer variable need not be.

*Coercions.*  $\text{par int} \xrightarrow{c} \text{var int}$ ,  $\text{par bool} \xrightarrow{c} \text{par int}$ ,  $\text{par bool} \xrightarrow{c} \text{var int}$ ,  $\text{var bool} \xrightarrow{c} \text{var int}$ .

Also, integers can be automatically coerced to floats; see [Floats](#) (page 212).

## FLOATS

*Overview.* Floats represent real numbers. Float representations are implementation-defined. This means that the representable range and precision of floats is implementation-defined. However, an implementation should abort at run-time on exceptional float operations (e.g., those that produce NaN, if using IEEE754 floats).

*Allowed Insts.* Floats can be fixed or unfixed.

*Syntax.* Fixed floats are written `float` or `par float`. Unfixed floats are written as `var float`.

*Finite?* Not unless constrained by a set expression (see [Set Expression Type-insts](#) (page 217)).

*Varifiable?*  $\text{par float} \xrightarrow{v} \text{var float}$ ,  $\text{var float} \xrightarrow{v} \text{var float}$ .

*Ordering.* The ordering on floats is the standard one.

*Initialisation.* A fixed float variable must be initialised at instance-time; an unfixed float variable need not be.

*Coercions.*  $\text{par int} \xrightarrow{c} \text{par float}$ ,  $\text{par int} \xrightarrow{c} \text{var float}$ ,  $\text{var int} \xrightarrow{c} \text{var float}$ ,  $\text{par float} \xrightarrow{c} \text{var float}$ .

## ENUMERATED TYPES

*Overview.* Enumerated types (or *enums* for short) provide a set of named alternatives. Each alternative is identified by its *case name*. Enumerated types, like in many other languages, can be used in the place of integer types to achieve stricter type checking.

*Allowed Insts.* Enums can be fixed or unfixed.

*Syntax.* Variables of an enumerated type named `X` are represented by the term `X` or `par X` if fixed, and `var X` if unfixed.

*Finite?* Yes.

The domain of an enum is the set containing all of its case names.

*Varifiable?*  $\text{par X} \xrightarrow{v} \text{var X}$ ,  $\text{var X} \xrightarrow{v} \text{var X}$ .

*Ordering.* When two enum values with different case names are compared, the value with the case name that is declared first is considered smaller than the value with the case name that is declared second.

*Initialisation.* A fixed enum variable must be initialised at instance-time; an unfixed enum variable need not be.

*Coercions.* `par X`  $\xrightarrow{c}$  `par int`, `var X`  $\xrightarrow{c}$  `var int`.

## Strings

*Overview.* Strings are primitive, i.e., they are not lists of characters.

String expressions are used in assertions, output items and annotations, and string literals are used in include items.

*Allowed Insts.* Strings must be fixed.

*Syntax.* Fixed strings are written `string` or `par string`.

*Finite?* Not unless constrained by a set expression (see *Set Expression Type-insts* (page 217)).

*Varifiable?* No.

*Ordering.* Strings are ordered lexicographically using the underlying character codes.

*Initialisation.* A string variable (which can only be fixed) must be initialised at instance-time.

*Coercions.* None automatic. However, any non-string value can be manually converted to a string using the built-in `show` function or using string interpolation (see *String Literals and String Interpolation* (page 224)).

### 4.1.6.6 Built-in Compound Types and Type-insts

#### Sets

*Overview.* A set is a collection with no duplicates.

*Allowed Insts.* The type-inst of a set's elements must be fixed. *Rationale:* This is because current solvers are not powerful enough to handle sets containing decision variables. Sets may contain any type, and may be fixed or unfixed. If a set is unfixed, its elements must be finite, unless it occurs in one of the following contexts:

- the argument of a predicate, function or annotation.

- the declaration of a variable or let local variable with an assigned value.

*Syntax.* A set base type-inst expression tail has this syntax:

```
<set-ti-expr-tail> ::= "set" "of" <base-type>
```

Some example set type-inst expressions:

```
set of int  
var set of bool
```

*Finite?* Yes, if the set elements are finite types. Otherwise, no.

The domain of a set type that is a finite type is the powerset of the domain of its element type. For example, the domain of `set of 1..2` is `powerset(1..2)`, which is `{}, {1}, {1,2}, {2}`.

*Varifiable?* `par set of TI`  $\xrightarrow{v}$  `var set of TI`, `var set of TI`  $\xrightarrow{v}$  `var set of TI`.

*Ordering.* The pre-defined ordering on sets is a lexicographic ordering of the *sorted set form*, where `{1,2}` is in sorted set form, for example, but `{2,1}` is not. This means, for instance, `{} < {1,3} < {2}`.

*Initialisation.* A fixed set variable must be initialised at instance-time; an unfixed set variable need not be.

*Coercions.* `par set of TI`  $\xrightarrow{c}$  `par set of UI` and `par set of TI`  $\xrightarrow{c}$  `var set of UI` and `var set of TI`  $\xrightarrow{c}$  `var set of UI`, if `TI`  $\xrightarrow{c}$  `UI`.

## Arrays

*Overview.* MiniZinc arrays are maps from fixed integers to values. Values can be of any type. The values can only have base type-insts. Arrays-of-arrays are not allowed. Arrays can be multi-dimensional.

MiniZinc arrays can be declared in two different ways.

- *Explicitly-indexed* arrays have index types in the declaration that are finite types. For example:

```
array[0..3] of int: a1;  
array[1..5, 1..10] of var float: a5;
```

For such arrays, the index type specifies exactly the indices that will be in the array - the array's index set is the *domain* of the index type - and if the indices of the value assigned do not match then it is a run-time error.

For example, the following assignments cause run-time errors:

```
a1 = [4,6,4,3,2];    % too many elements
a5 = [];
```

- *Implicitly-indexed* arrays have index types in the declaration that are not finite types. For example:

```
array[int,int] of int: a6;
```

No checking of indices occurs when these variables are assigned.

In MiniZinc all index sets of an array must be contiguous ranges of integers, or enumerated types. The expression used for initialisation of an array must have matching index sets. An array expression with an enum index set can be assigned to an array declared with an integer index set, but not the other way around. The exception are array literals, which can be assigned to arrays declared with enum index sets.

For example:

```
enum X = {A,B,C};
enum Y = {D,E,F};
array[X] of int: x = array1d(X, [5,6,7]); % correct
array[Y] of int: y = x;                      % index set mismatch: Y != X
array[int] of int: z = x;                      % correct: assign X index set to_
→int
array[X] of int: x2 = [10,11,12];            % correct: automatic coercion for_
→array literals
```

The initialisation of an array can be done in a separate assignment statement, which may be present in the model or a separate data file.

Arrays can be accessed. See *Array Access Expressions* (page 229) for details.

*Allowed Insts.* An array's size must be fixed. Its indices must also have fixed type-insts. Its elements may be fixed or unfixed.

*Syntax.* An array base type-inst expression tail has this syntax:

```
<array-ti-expr-tail> ::= "array" [ <ti-expr> "," ... ] "of" <ti-expr>
| "list" "of" <ti-expr>
```

Some example array type-inst expressions:

```
array[1..10] of int
list of var int
```

Note that `list of <T>` is just syntactic sugar for `array[int] of <T>`. *Rationale: Integer-indexed arrays of this form are very common, and so worthy of special support to make things easier for modellers. Implementing it using syntactic sugar avoids adding an extra type to the language, which keeps things simple for implementers.*

Because arrays must be fixed-size it is a type-inst error to precede an array type-inst expression with `var`.

*Finite?* Yes, if the index types and element type are all finite types. Otherwise, no.

The domain of an array type that is a finite array is the set of all distinct arrays whose index set equals the domain of the index type and whose elements are of the array element type.

*Varifiable?* No.

*Ordering.* Arrays are ordered lexicographically, taking absence of a value for a given key to be before any value for that key. For example, `[1, 1]` is less than `[1, 2]`, which is less than `[1, 2, 3]` and `array1d(2..4,[0, 0, 0])` is less than `[1, 2, 3]`.

*Initialisation.* An explicitly-indexed array variable must be initialised at instance-time only if its elements must be initialised at instance time. An implicitly-indexed array variable must be initialised at instance-time so that its length and index set is known.

*Coercions.* `array[TI0] of TI`  $\xrightarrow{c}$  `array[UI0] of UI` if `TI0`  $\xrightarrow{c}$  `UI0` and `TI`  $\xrightarrow{c}$  `UI`.

## Option Types

*Overview.* Option types defined using the `opt` type constructor, define types that may or may not be there. They are similar to Maybe types of Haskell implicitly adding a new value `<>` to the type.

*Allowed Insts.* The argument of an option type must be one of the base types `bool`, `int` or `float`.

*Syntax.* The option type is written `opt <T>` where `<T>` is one of the three base types, or one of their constrained instances.

*Finite?* Yes if the underlying type is finite, otherwise no.

*Varifiable?* Yes.

*Ordering.*  $\text{<}$  is always less than any other value in the type. But beware that overloading of operators like  $<$  is different for option types.

*Initialisation.* An `opt` type variable does not need to be initialised at instance-time. An uninitialised `opt` type variable is automatically initialised to  $\text{<}$ .

*Coercions.*  $\text{TI} \xrightarrow{c} \text{opt UI}$  if  $\text{TI} \xrightarrow{c} \text{UI..}$

## The Annotation Type

*Overview.* The annotation type, `ann`, can be used to represent arbitrary term structures. It is augmented by annotation items (*Annotation Items* (page 239)).

*Allowed Insts.* `ann` is always considered unfixed, because it may contain unfixed elements. It cannot be preceded by `var`.

*Syntax.* The annotation type is written `ann`.

*Finite?* No.

*Varifiable?* No.

*Ordering.* N/A. Annotation types do not have an ordering defined on them.

*Initialisation.* An `ann` variable must be initialised at instance-time.

*Coercions.* None.

### 4.1.6.7 Constrained Type-insts

One powerful feature of MiniZinc is *constrained type-insts*. A constrained type-inst is a restricted version of a *base* type-inst, i.e., a type-inst with fewer values in its domain.

#### Set Expression Type-insts

Three kinds of expressions can be used in type-insts.

1. Integer ranges: e.g. `1..3`.
2. Set literals: e.g. `var {1,3,5}`.

3. Identifiers: the name of a set parameter (which can be global, let-local, the argument of a predicate or function, or a generator value) can serve as a type-inst.

In each case the base type is that of the set's elements, and the values within the set serve as the domain. For example, whereas a variable with type-inst `var int` can take any integer value, a variable with type-inst `var 1..3` can only take the value 1, 2 or 3.

All set expression type-insts are finite types. Their domain is equal to the set itself.

## Float Range Type-insts

Float ranges can be used as type-insts, e.g. `1.0 .. 3.0`. These are treated similarly to integer range type-insts, although `1.0 .. 3.0` is not a valid expression whereas `1 .. 3` is.

Float ranges are not finite types.

## 4.1.7 Expressions

### 4.1.7.1 Expressions Overview

Expressions represent values. They occur in various kinds of items. They have the following syntax:

```
<expr> ::= <expr-atom> <expr-binop-tail>

<expr-atom> ::= <expr-atom-head> <expr-atom-tail> <annotations>

<expr-binop-tail> ::= "[" <bin-op> <expr> "]"

<expr-atom-head> ::= <builtin-un-op> <expr-atom>
                     | "(" <expr> ")"
                     | <ident-or-quoted-op>
                     | "_"
                     | <bool-literal>
                     | <int-literal>
                     | <float-literal>
                     | <string-literal>
                     | <set-literal>
                     | <set-comp>
```

```

| <array-literal>
| <array-literal-2d>
| <array-comp>
| <ann-literal>
| <if-then-else-expr>
| <let-expr>
| <call-expr>
| <gen-call-expr>

<expr-atom-tail> ::= ε
| <array-access-tail> <expr-atom-tail>

```

Expressions can be composed from sub-expressions combined with operators. All operators (binary and unary) are described in *Operators* (page 220), including the precedences of the binary operators. All unary operators bind more tightly than all binary operators.

Expressions can have one or more annotations. Annotations bind more tightly than unary and binary operator applications, but less tightly than access operations and non-operator applications. In some cases this binding is non-intuitive. For example, in the first three of the following lines, the annotation `a` binds to the identifier expression `x` rather than the operator application. However, the fourth line features a non-operator application (due to the single quotes around the `not`) and so the annotation binds to the whole application.

```

not x:::a;
not (x):::a;
not(x):::a;
'not'(x):::a;

```

*Annotations* (page 244) has more on annotations.

Expressions can be contained within parentheses.

The array access operations all bind more tightly than unary and binary operators and annotations. They are described in more detail in *Array Access Expressions* (page 229).

The remaining kinds of expression atoms (from `<ident>` to `<gen-call-expr>`) are described in *Identifier Expressions and Quoted Operator Expressions* (page 223) to *Generator Call Expressions* (page 234).

We also distinguish syntactically valid numeric expressions. This allows range types to be parsed correctly.

```

<num-expr> ::= <num-expr-atom> <num-expr-binop-tail>

<num-expr-atom> ::= <num-expr-atom-head> <expr-atom-tail> <annotations>

<num-expr-binop-tail> ::= "[" <num-bin-op> <num-expr> "]"

<num-expr-atom-head> ::= <builtin-num-un-op> <num-expr-atom>
| "(" <num-expr> ")"
| <ident-or-quoted-op>
| <int-literal>
| <float-literal>
| <if-then-else-expr>
| <let-expr>
| <call-expr>
| <gen-call-expr>

```

#### 4.1.7.2 Operators

Operators are functions that are distinguished by their syntax in one or two ways. First, some of them contain non-alphanumeric characters that normal functions do not (e.g. `+`). Second, their application is written in a manner different to normal functions.

We distinguish between binary operators, which can be applied in an infix manner (e.g. `3 + 4`), and unary operators, which can be applied in a prefix manner without parentheses (e.g. `not x`). We also distinguish between built-in operators and user-defined operators. The syntax is the following:

```

<builtin-op> ::= <builtin-bin-op> | <builtin-un-op>

<bin-op> ::= <builtin-bin-op> | '<ident>' ' 

<builtin-bin-op> ::= "<->" | ">->" | "<-<" | "\/" | "xor" | "/\" | "<<" | ">>" |
→ "<=>" | ">=>" | "==" | "=" | "!="
| "in" | "subset" | "superset" | "union" | "diff" |
→ "symdiff"
| ".." | "intersect" | "++" | <builtin-num-bin-op>

<builtin-un-op> ::= "not" | <builtin-num-un-op>

```

Again, we syntactically distinguish numeric operators.

```
<num-bin-op> ::= <builtin-num-bin-op> | '<ident>' ···  

<builtin-num-bin-op> ::= "+" | "-" | "*" | "/" | "div" | "mod"  

<builtin-num-un-op> ::= "+" | "-"
```

Some operators can be written using their unicode symbols, which are listed in Table 4.1.7.2 (recall that MiniZinc input is UTF-8).

Table 1.1: Unicode equivalents of binary operators

Operator	Unicode symbol	UTF-8 code
<code>&lt;-&gt;</code>	$\leftrightarrow$	E2 86 94
<code>-&gt;</code>	$\rightarrow$	E2 86 92
<code>&lt;-</code>	$\leftarrow$	E2 86 90
<code>not</code>	$\neg$	C2 AC
<code>\vee</code>	$\vee$	E2 88 A8
<code>\wedge</code>	$\wedge$	E2 88 A7
<code>!=</code>	$\neq$	E2 89 A0
<code>&lt;=</code>	$\leq$	E2 89 A4
<code>&gt;=</code>	$\geq$	E2 89 A5
<code>in</code>	$\in$	E2 88 88
<code>subset</code>	$\subseteq$	E2 8A 86
<code>superset</code>	$\supseteq$	E2 8A 87
<code>union</code>	$\cup$	E2 88 AA
<code>intersect</code>	$\cap$	E2 88 A9

The binary operators are listed in Table 4.1.7.2. A lower precedence number means tighter binding; for example, `1+2*3` is parsed as `1+(2*3)` because `*` binds tighter than `+`. Associativity indicates how chains of operators with equal precedences are handled; for example, `1+2+3` is parsed as `(1+2)+3` because `+` is left-associative, `a++b++c` is parsed as `a++(b++c)` because `++` is right-associative, and `1<x<2` is a syntax error because `<` is non-associative.

Table 1.2: Binary infix operators

Symbol(s)	Assoc.	Prec.
<code>&lt;-&gt;</code>	left	1200
<code>-&gt;</code>	left	1100
<code>&lt;-</code>	left	1100
<code>\/</code>	left	1000
<code>xor</code>	left	1000
<code>/\</code>	left	900
<code>&lt;</code>	none	800
<code>&gt;</code>	none	800
<code>&lt;=</code>	none	800
<code>&gt;=</code>	none	800
<code>==,</code>		
<code>=</code>	none	800
<code>!=</code>	none	800
<code>in</code>	none	700
<code>subset</code>	none	700
<code>superset</code>	none	700
<code>union</code>	left	600
<code>diff</code>	left	600
<code>symdiff</code>	left	600
<code>..</code>	none	500
<code>+</code>	left	400
<code>-</code>	left	400
<code>*</code>	left	300
<code>div</code>	left	300
<code>mod</code>	left	300
<code>/</code>	left	300
<code>intersect</code>	left	300
<code>++</code>	right	200
<code>` &lt;ident&gt; `</code>	left	100

A user-defined binary operator is created by backquoting a normal identifier, for example:

```
A `min2` B
```

This is a static error if the identifier is not the name of a binary function or predicate.

The unary operators are: `+`, `-` and `not`. User-defined unary operators are not possible.

As [Identifiers](#) (page 204) explains, any built-in operator can be used as a normal function identifier by quoting it, e.g. `'+'`(`3`, `4`) is equivalent to `3 + 4`.

The meaning of each operator is given in [Built-in Operations](#) (page 248).

### 4.1.7.3 Expression Atoms

#### Identifier Expressions and Quoted Operator Expressions

Identifier expressions and quoted operator expressions have the following syntax:

```
<ident-or-quoted-op> ::= <ident>
| ' <builtin-op> '
```

Examples of identifiers were given in [Identifiers](#) (page 204). The following are examples of quoted operators:

```
'+'
'union'
```

In quoted operators, whitespace is not permitted between either quote and the operator. [Operators](#) (page 220) lists MiniZinc's built-in operators.

Syntactically, any identifier or quoted operator can serve as an expression. However, in a valid model any identifier or quoted operator serving as an expression must be the name of a variable.

#### Anonymous Decision Variables

There is a special identifier, `_`, that represents an unfixed, anonymous decision variable. It can take on any type that can be a decision variable. It is particularly useful for initialising decision variables within compound types. For example, in the following array the first and third elements are fixed to 1 and 3 respectively and the second and fourth elements are unfixed:

```
array[1..4] of var int: xs = [1, _, 3, _];
```

Any expression that does not contain `_` and does not involve decision variables is fixed.

## Boolean Literals

Boolean literals have this syntax:

```
<bool-literal> ::= "false" | "true"
```

## Integer and Float Literals

There are three forms of integer literals - decimal, hexadecimal, and octal - with these respective forms:

```
<int-literal> ::= [0-9]+  
| 0x[0-9A-Fa-f]+  
| 0o[0-7]+
```

For example: 0, 005, 123, 0x1b7, 0o777; but not -1.

Float literals have the following form:

```
<float-literal> ::= [0-9]+.[0-9]+  
| [0-9]+.[0-9]+[Ee][-+]?[0-9]+  
| [0-9]+[Ee][-+]?[0-9]+
```

For example: 1.05, 1.3e-5, 1.3+e5; but not 1., .5, 1.e5, .1e5, -1.0, -1E05. A - symbol preceding an integer or float literal is parsed as a unary minus (regardless of intervening whitespace), not as part of the literal. This is because it is not possible in general to distinguish a - for a negative integer or float literal from a binary minus when lexing.

## String Literals and String Interpolation

String literals are written as in C:

```
<string-contents> ::= ([^"\n\"] | \\[^\\n\\])*  
  
<string-literal> ::= ""<string-contents> ""  
| ""<string-contents> "\\("<string-interpolate-tail>  
  
<string-interpolate-tail> ::= <expr> ")"<string-contents>""
```

```
| <expr> ")"<string-contents>"\("_
↪<string-interpolate-tail>
```

This includes C-style escape sequences, such as `\"` for double quotes, `\\"` for backslash, and `\n` for newline.

For example: `"Hello, world!\n"`.

String literals must fit on a single line.

Long string literals can be split across multiple lines using string concatenation. For example:

```
string: s = "This is a string literal "
      ++ "split across two lines.;"
```

A string expression can contain an arbitrary MiniZinc expression, which will be converted to a string similar to the builtin `show` function and inserted into the string.

For example:

```
var set of 1..10: q;
solve satisfy;
output [show("The value of q is \$(q), and it has \$(card(q)) elements.")];
```

## Set Literals

Set literals have this syntax:

```
<set-literal> ::= "{" [ <expr> "," ... ] "}"
```

For example:

```
{ 1, 3, 5 }
{ }
{ 1, 2.0 }
```

The type-insts of all elements in a literal set must be the same, or coercible to the same type-inst (as in the last example above, where the integer `1` will be coerced to a `float`).

## Set Comprehensions

Set comprehensions have this syntax:

```
<set-comp> ::= "{" <expr> " | " <comp-tail> "}"  
  
<comp-tail> ::= <generator> [ "where" <expr> ] "," ...  
  
<generator> ::= <ident> "," ... "in" <expr>
```

For example (with the literal equivalent on the right):

```
{ 2*i | i in 1..5 }      % { 2, 4, 6, 8, 10 }  
{ 1  | i in 1..5 }      % { 1 }    (no duplicates in sets)
```

The expression before the `|` is the *head expression*. The expression after the `in` is a *generator expression*. Generators can be restricted by a *where-expression*. For example:

```
{ i | i in 1..10 where (i mod 2 = 0) }      % { 2, 4, 6, 8, 10 }
```

When multiple generators are present, the right-most generator acts as the inner-most one. For example:

```
{ 3*i+j | i in 0..2, j in {0, 1} }      % { 0, 1, 3, 4, 6, 7 }
```

The scope of local generator variables is given by the following rules:

- They are visible within the head expression (before the `|`).
- They are visible within the where-expression of their own generator.
- They are visible within generator expressions and where-expressions in any subsequent generators.

The last of these rules means that the following set comprehension is allowed:

```
{ i+j | i in 1..3, j in 1..i }  % { 1+1, 2+1, 2+2, 3+1, 3+2, 3+3 }
```

Multiple where-expressions are allowed, as in the following example:

```
[f(i, j) | i in A1 where p(i), j in A2 where q(i,j)]
```

A generator expression must be an array or a fixed set.

*Rationale:* For set comprehensions, set generators would suffice, but for array comprehensions, array generators are required for full expressivity (e.g., to provide control over the order of the elements in the resulting array). Set comprehensions have array generators for consistency with array comprehensions, which makes implementations simpler.

The where-expression (if present) must be Boolean. It can be var, in which case the type of the comprehension is lifted to an optional type.

## Array Literals

Array literals have this syntax:

```
<array-literal> ::= "[" [ <expr> "," ... ] "]"
```

For example:

```
[1, 2, 3, 4]
[]
[1, _]
```

In a array literal all elements must have the same type-inst, or be coercible to the same type-inst (as in the last example above, where the fixed integer 1 will be coerced to a `var int`).

The indices of a array literal are implicitly 1..n, where n is the length of the literal.

## 2d Array Literals

Simple 2d array literals have this syntax:

```
<array-literal-2d> ::= "[" [ (<expr> "," ... ) "|" ... ] "]"
```

For example:

```
[| 1, 2, 3
 | 4, 5, 6
 | 7, 8, 9 |]      % array[1..3, 1..3]
 [| x, y, z |]      % array[1..1, 1..3]
 [| 1 | _ | _ |]    % array[1..3, 1..1]
```

In a 2d array literal, every sub-array must have the same length.

In a 2d array literal all elements must have the same type-inst, or be coercible to the same type-inst (as in the last example above, where the fixed integer 1 will be coerced to a `var int`).

The indices of a 2d array literal are implicitly  $(1, 1) \dots (m, n)$ , where  $m$  and  $n$  are determined by the shape of the literal.

## Array Comprehensions

Array comprehensions have this syntax:

```
<array-comp> ::= "[" <expr> "|" <comp-tail> "]"
```

For example (with the literal equivalents on the right):

```
[2*i | i in 1..5]      % [2, 4, 6, 8, 10]
```

Array comprehensions have more flexible type and inst requirements than set comprehensions (see *Set Comprehensions* (page 226)).

Array comprehensions are allowed over a variable set with finite type, the result is an array of optional type, with length equal to the cardinality of the upper bound of the variable set. For example:

```
var set of 1..5: x;
array[int] of var opt int: y = [ i * i | i in x];
```

The length of array will be 5.

Array comprehensions are allowed where the where-expression is a `var bool`. Again the resulting array is of optional type, and of length equal to that given by the generator expressions. For example:

```
var int x;
array[int] of var opt int: y = [ i | i in 1..10 where i != x ];
```

The length of the array will be 10.

The indices of an evaluated simple array comprehension are implicitly  $1..n$ , where  $n$  is the length of the evaluated comprehension.

## Array Access Expressions

Array elements are accessed using square brackets after an expression:

```
<array-access-tail> ::= "[" <expr> "," ... "]"
```

For example:

```
int: x = a1[1];
```

If all the indices used in an array access are fixed, the type-inst of the result is the same as the element type-inst. However, if any indices are not fixed, the type-inst of the result is the varified element type-inst. For example, if we have:

```
array[1..2] of int: a2 = [1, 2];
var int: i;
```

then the type-inst of  $a2[i]$  is `var int`. If the element type-inst is not varifiable, such an access causes a static error.

Multi dimensional arrays are accessed using comma separated indices.

```
array[1..3,1..3] of int: a3;
int: y = a3[1, 2];
```

Indices must match the index set type of the array. For example, an array declared with an enum index set can only be accessed using indices from that enum.

```
enum X = {A,B,C};
array[X] of int: a4 = [1,2,3];
```

```
int: y = a4[1];          % wrong index type
int: z = a4[B];          % correct
```

## Array Slice Expressions

Arrays can be *sliced* in order to extract individual rows, columns or blocks. The syntax is that of an array access expression (see above), but where one or more of the expressions inside the square brackets are set-valued.

For example, the following extracts row 2 from a two-dimensional array:

```
array[1..n,4..8] of int: x;
array[int] of int: row_2_of_x = x[2,4..8];
```

Note that the resulting array `row_2_of_x` will have index set `4..8`.

A short-hand for all indices of a particular dimension is to use just dots:

```
array[1..n,4..8] of int: x;
array[int] of int: row_2_of_x = x[2,...];
```

You can also restrict the index set by giving a sub-set of the original index set as the slice:

```
array[1..n,4..8] of int: x;
array[int] of int: row_2_of_x = x[2,5..6];
```

The resulting array `row_2_of_x` will now have length 2 and index set `5..6`.

The dots notation also allows for partial bounds, for example:

```
array[1..n,4..8] of int: x;
array[int] of int: row_2_of_x = x[2,...6];
```

The resulting array will have length 3 and index set `4..6`. Of course `6..` would also be allowed and result in an array with index set `6..8`.

## Annotation Literals

Literals of the `ann` type have this syntax:

```
<ann-literal> ::= <ident> [ "(" <expr> "," ... ")" ]
```

For example:

```
foo
cons(1, cons(2, cons(3, nil)))
```

There is no way to inspect or deconstruct annotation literals in a MiniZinc model; they are intended to be inspected only by an implementation, e.g., to direct compilation.

## If-then-else Expressions

MiniZinc provides if-then-else expressions, which provide selection from two alternatives based on a condition. They have this syntax:

```
<if-then-else-expr> ::= "if" <expr> "then" <expr> [ "elseif" <expr> "then"_
↳<expr> ]* "else" <expr> "endif"
```

For example:

```
if x <= y then x else y endif
if x < 0 then -1 elseif x > 0 then 1 else 0 endif
```

The presence of the `endif` avoids possible ambiguity when an if-then-else expression is part of a larger expression.

The type-inst of the `if` expression must be `par bool` or `var bool`. The `then` and `else` expressions must have the same type-inst, or be coercible to the same type-inst, which is also the type-inst of the whole expression.

If the `if` expression is `var bool` then the type-inst of the `then` and `else` expressions must be varifiable.

If the `if` expression is `par bool` then evaluation of if-then-else expressions is lazy: the condition is evaluated, and then only one of the `then` and `else` branches are evaluated, depending on whether the condition succeeded or failed. This is not the case if it is `var bool`.

## Let Expressions

Let expressions provide a way of introducing local names for one or more expressions and local constraints that can be used within another expression. They are particularly useful in user-defined operations.

Let expressions have this syntax:

```
<let-expr> ::= "let" "{" <let-item> ";" ... "}" "in" <expr>  
  
<let-item> ::= <var-decl-item>  
           | <constraint-item>
```

For example:

```
let { int: x = 3; int: y = 4; } in x + y;  
let { var int: x;  
      constraint x >= y /\ x >= -y /\ (x = y \/\ x = -y); }  
in x
```

The scope of a let local variable covers:

- The type-inst and initialisation expressions of any subsequent variables within the let expression (but not the variable's own initialisation expression).
- The expression after the `in`, which is parsed as greedily as possible.

A variable can only be declared once in a let expression.

Thus in the following examples the first is acceptable but the rest are not:

```
let { int: x = 3; int: y = x; } in x + y; % ok  
let { int: y = x; int: x = 3; } in x + y; % x not visible in y's defn.  
let { int: x = x; } in x; % x not visible in x's defn.  
let { int: x = 3; int: x = 4; } in x; % x declared twice
```

The initialiser for a let local variable can be omitted only if the variable is a decision variable. For example:

```
let { var int: x; } in ...; % ok  
let {     int: x; } in ...; % illegal
```

The type-inst of the entire let expression is the type-inst of the expression after the `in` keyword.

There is a complication involving let expressions in negative contexts. A let expression occurs in a negative context if it occurs in an expression of the form `not X, X <-> Y` or in the sub-expression `X` in `X -> Y` or `Y <- X`, or in a subexpression `bool2int(X)`.

If a let expression is used in a negative context, then any let-local decision variables must be defined only in terms of non-local variables and parameters. This is because local variables are implicitly existentially quantified, and if the let expression occurred in a negative context then the local variables would be effectively universally quantified which is not supported by MiniZinc.

Constraints in let expressions float to the nearest enclosing Boolean context. For example

```
constraint b -> x + let { var 0..2: y; constraint y != -1;} in y >= 4;
```

is equivalent to

```
var 0..2: y;
constraint b -> (x + y >= 4 /\ y != 1);
```

For backwards compatibility with older versions of MiniZinc, items inside the let can also be separated by commas instead of semicolons.

## Call Expressions

Call expressions are used to call predicates and functions.

Call expressions have this syntax:

```
<call-expr> ::= <ident-or-quoted-op> [ "(" <expr> "," ... ")" ]
```

For example:

```
x = min(3, 5);
```

The type-insts of the expressions passed as arguments must match the argument types of the called predicate/function. The return type of the predicate/function must also be appropriate for the calling context.

Note that a call to a function or predicate with no arguments is syntactically indistinguishable from the use of a variable, and so must be determined during type-inst checking.

Evaluation of the arguments in call expressions is strict: all arguments are evaluated before the call itself is evaluated. Note that this includes Boolean operations such as `/\`, `\/`, `->` and `<-` which could be lazy in one argument. The one exception is `assert`, which is lazy in its third argument (*Other Operations* (page 257)).

*Rationale:* Boolean operations are strict because: (a) this minimises exceptional cases; (b) in an expression like `A -> B` where `A` is not fixed and `B` causes an abort, the appropriate behaviour is unclear if laziness is present; and (c) if a user needs laziness, an if-then-else can be used.

The order of argument evaluation is not specified. *Rationale:* Because MiniZinc is declarative, there is no obvious need to specify an evaluation order, and leaving it unspecified gives implementors some freedom.

## Generator Call Expressions

MiniZinc has special syntax for certain kinds of call expressions which makes models much more readable.

Generator call expressions have this syntax:

```
<gen-call-expr> ::= <ident-or-quoted-op> "(" <comp-tail> ")" "(" <expr> ")"
```

A generator call expression `P(Gs)(E)` is equivalent to the call expression `P([E | Gs])`. For example, the expression:

```
forall(i,j in Domain where i<j)
  (noattack(i, j, queens[i], queens[j]));
```

(in a model specifying the N-queens problem) is equivalent to:

```
forall( [ noattack(i, j, queens[i], queens[j])
  | i,j in Domain where i<j ] );
```

The parentheses around the latter expression are mandatory; this avoids possible confusion when the generator call expression is part of a larger expression.

The identifier must be the name of a unary predicate or function that takes an array argument.

The generators and where-expression (if present) have the same requirements as those in array comprehensions ([Array Comprehensions](#) (page 228)).

## 4.1.8 Items

This section describes the top-level program items.

### 4.1.8.1 Include Items

Include items allow a model to be split across multiple files. They have this syntax:

```
<include-item> ::= "include" <string-literal>
```

For example:

```
include "foo.mzn";
```

includes the file `foo.mzn`.

Include items are particularly useful for accessing libraries or breaking up large models into small pieces. They are not, as [Model Instance Files](#) (page 206) explains, used for specifying data files.

If the given name is not a complete path then the file is searched for in an implementation-defined set of directories. The search directories must be able to be altered with a command line option.

### 4.1.8.2 Variable Declaration Items

Variable declarations have this syntax:

```
<var-decl-item> ::= <ti-expr-and-id> <annotations> [ "=" <expr> ]
```

For example:

```
int: A = 10;
```

It is a type-inst error if a variable is declared and/or defined more than once in a model.

A variable whose declaration does not include an assignment can be initialised by a separate assignment item ([Assignment Items](#) (page 238)). For example, the above item can be separated into the following two items:

```
int: A;  
...  
A = 10;
```

All variables that contain a parameter component must be defined at instance-time.

Variables can have one or more annotations. [Annotations](#) (page 244) has more on annotations.

#### 4.1.8.3 Enum Items

Enumerated type items have this syntax:

```
<enum-item> ::= "enum" <ident> <annotations> [ "=" <enum-cases> ]  
  
<enum-cases> ::= "{" <ident> "," ... "}"
```

An example of an enum:

```
enum country = {Australia, Canada, China, England, USA};
```

Each alternative is called an *enum case*. The identifier used to name each case (e.g. `Australia`) is called the *enum case name*.

Because enum case names all reside in the top-level namespace ([Namespaces](#) (page 207)), case names in different enums must be distinct.

An enum can be declared but not defined, in which case it must be defined elsewhere within the model, or in a data file. For example, a model file could contain this:

```
enum Workers;  
enum Shifts;
```

and the data file could contain this:

```
Workers = { welder, driller, stamper };
Shifts  = { idle, day, night };
```

Sometimes it is useful to be able to refer to one of the enum case names within the model. This can be achieved by using a variable. The model would read:

```
enum Shifts;
Shifts: idle;           % Variable representing the idle constant.
```

and the data file:

```
enum Shifts = { idle_const, day, night };
idle = idle_const;      % Assignment to the variable.
```

Although the constant `idle_const` cannot be mentioned in the model, the variable `idle` can be.

All enums must be defined at instance-time.

Enum items can be annotated. [Annotations](#) (page 244) has more details on annotations.

Each case name can be coerced automatically to the integer corresponding to its index in the type.

```
int: oz = Australia;  % oz = 1
```

For each enumerated type `T`, the following functions exist:

```
% Return next greater enum value of x in enum type X
function T: enum_next(set of T: X, T: x);
function var T: enum_next(set of T: X, var T: x);

% Return next smaller enum value of x in enum type X
function T: enum_prev(set of T: X, T: x);
function var T: enum_prev(set of T: X, var T: x);

% Convert x to enum type X
function T: to_enum(set of T: X, int: x);
function var T: to_enum(set of T: X, var int: x);
```

#### 4.1.8.4 Assignment Items

Assignments have this syntax:

```
<assign-item> ::= <ident> "=" <expr>
```

For example:

```
A = 10;
```

#### 4.1.8.5 Constraint Items

Constraint items form the heart of a model. Any solutions found for a model will satisfy all of its constraints.

Constraint items have this syntax:

```
<constraint-item> ::= "constraint" <string-annotation> <expr>
```

For example:

```
constraint a*x < b;
```

The expression in a constraint item must have type-inst `par bool` or `var bool`; note however that constraints with fixed expressions are not very useful.

#### 4.1.8.6 Solve Items

Every model must have exactly one or no solve item. Solve items have the following syntax:

```
<solve-item> ::= "solve" <annotations> "satisfy"  
           | "solve" <annotations> "minimize" <expr>  
           | "solve" <annotations> "maximize" <expr>
```

Example solve items:

```
solve satisfy;
solve maximize a*x + y - 3*z;
```

The solve item determines whether the model represents a constraint satisfaction problem or an optimisation problem. If there is no solve item, the model is assumed to be a satisfaction problem. For optimisation problems, the given expression is the one to be minimized/maximized.

The expression in a minimize/maximize solve item can have integer or float type.

*Rationale:* This is possible because all type-insts have a defined order. Note that having an expression with a fixed type-inst in a solve item is not very useful as it means that the model requires no optimisation.

Solve items can be annotated. [Annotations](#) (page 244) has more details on annotations.

#### 4.1.8.7 Output Items

Output items are used to present the solutions of a model instance. They have the following syntax:

```
<output-item> ::= "output" <expr>
```

For example:

```
output ["The value of x is ", show(x), "!\\n"];
```

The expression must have type-inst `array[int] of par string`. It can be composed using the built-in operator `++` and the built-in functions `show`, `show_int`, and `show_float` ([Built-in Operations](#) (page 248)), as well as string interpolations ([String Literals and String Interpolation](#) (page 224)). The output is the concatenation of the elements of the array. If multiple output items exist, the output is the concatenation of all of their outputs, in the order in which they appear in the model.

If no output item is present, the implementation should print all the global variables and their values in a readable format.

#### 4.1.8.8 Annotation Items

Annotation items are used to augment the `ann` type. They have the following syntax:

```
<annotation-item> ::= "annotation" <ident> <params>
```

For example:

```
annotation solver(int: kind);
```

It is a type-inst error if an annotation is declared and/or defined more than once in a model.

The use of annotations is described in *Annotations* (page 244).

#### 4.1.8.9 User-defined Operations

MiniZinc models can contain user-defined operations. They have this syntax:

```
<predicate-item> ::= "predicate" <operation-item-tail>

<test-item> ::= "test" <operation-item-tail>

<function-item> ::= "function" <ti-expr> ":" <operation-item-tail>

<operation-item-tail> ::= <ident> <params> <annotations> [ "=" <expr> ]

<params> ::= [ ( <ti-expr-and-id> ", " ... ) ]
```

The type-inst expressions can include type-inst variables in the function and predicate declaration.

For example, predicate even checks that its argument is an even number.

```
predicate even(var int: x) =
    x mod 2 = 0;
```

A predicate supported natively by the target solver can be declared as follows:

```
predicate alldifferent(array [int] of var int: xs);
```

Predicate declarations that are natively supported in MiniZinc are restricted to using FlatZinc types (for instance, multi-dimensional and non-1-based arrays are forbidden). .. % pjs{need to

```
fix this if we allow2d arrays in FlatZinc!}
```

Declarations for user-defined operations can be annotated. [Annotations](#) (page 244) has more details on annotations.

## Basic Properties

The term “predicate” is generally used to refer to both test items and predicate items. When the two kinds must be distinguished, the terms “test item” and “predicate item” can be used.

The return type-inst of a test item is implicitly `par bool`. The return type-inst of a predicate item is implicitly `var bool`.

Predicates and functions are allowed to be recursive. Termination of a recursive function call depends solely on its fixed arguments, i.e., recursive functions and predicates cannot be used to define recursively constrained variables. ... % Rationale{This ensures that the satisfiability of models is decidable.}

Predicates and functions introduce their own local names, being those of the formal arguments. The scope of these names covers the predicate/function body. Argument names cannot be repeated within a predicate/function declaration.

## Ad-hoc polymorphism

MiniZinc supports ad-hoc polymorphism via overloading. Functions and predicates (both built-in and user-defined) can be overloaded. A name can be overloaded as both a function and a predicate.

It is a type-inst error if a single version of an overloaded operation with a particular type-inst signature is defined more than once in a model. For example:

```
predicate p(1..5: x);
predicate p(1..5: x) = false;      % ok:    first definition
predicate p(1..5: x) = true;       % error: repeated definition
```

The combination of overloading and coercions can cause problems. Two overloadings of an operation are said to *overlap* if they could match the same arguments. For example, the following overloading of `p` overlap, as they both match the call `p(3)`.

```
predicate p(par int: x);
predicate p(var int: x);
```

However, the following two predicates do not overlap because they cannot match the same argument:

```
predicate q(int: x);
predicate q(set of int: x);
```

We avoid two potential overloading problems by placing some restrictions on overlapping over-loadings of operations.

1. The first problem is ambiguity. Different placement of coercions in operation arguments may allow different choices for the overloaded function. For instance, if a MiniZinc function `f` is overloaded like this:

```
function int: f(int: x, float: y) = 0;
function int: f(float: x, int: y) = 1;
```

then `f(3,3)` could be either 0 or 1 depending on coercion/overloading choices.

To avoid this problem, any overlapping over-loadings of an operation must be semantically equivalent with respect to coercion. For example, the two over-loadings of the predicate `p` above must have bodies that are semantically equivalent with respect to overloading.

Currently, this requirement is not checked and the modeller must satisfy it manually. In the future, we may require the sharing of bodies among different versions of overloaded operations, which would provide automatic satisfaction of this requirement.

2. The second problem is that certain combinations of over-loadings could require a MiniZinc implementation to perform combinatorial search in order to explore different choices of coercions and overloading. For example, if function `g` is overloaded like this:

```
function float: g(int: t1, float: t2) = t2;
function int : g(float: t1, int: t2) = t1;
```

then how the overloading of `g(3,4)` is resolved depends upon its context:

```
float: s = g(3,4);
int: t = g(3,4);
```

In the definition of `s` the first overloaded definition must be used while in the definition of `t` the second must be used.

To avoid this problem, all overlapping overloading of an operation must be closed under intersection of their input type-insts. That is, if overloaded versions have input type-inst  $(S_1, \dots, S_n)$  and  $(T_1, \dots, T_n)$  then there must be another overloaded version with input type-inst  $(R_1, \dots, R_n)$  where each  $R_i$  is the greatest lower bound (*glb*) of  $S_i$  and  $T_i$ .

Also, all overlapping overloading of an operation must be monotonic. That is, if there are overloaded versions with input type-insts  $(S_1, \dots, S_n)$  and  $(T_1, \dots, T_n)$  and output type-inst  $S$  and  $T$ , respectively, then  $S_i \preceq T_i$  for all  $i$ , implies  $S \preceq T$ . At call sites, the matching overloading that is lowest on the type-inst lattice is always used.

For `g` above, the type-inst intersection (or *glb*) of `(int, float)` and `(float, int)` is `(int, int)`. Thus, the overloaded versions are not closed under intersection and the user needs to provide another overloading for `g` with input type-inst `(int, int)`. The natural definition is:

```
function int: g(int: t1, int: t2) = t1;
```

Once `g` has been augmented with the third overloading, it satisfies the monotonicity requirement because the output type-inst of the third overloading is `int` which is less than the output type-inst of the original overloading.

Monotonicity and closure under type-inst conjunction ensure that whenever an overloaded function or predicate is reached during type-inst checking, there is always a unique and safe “minimal” version to choose, and so the complexity of type-inst checking remains linear. Thus in our example `g(3,4)` is always resolved by choosing the new overloaded definition.

## Local Variables

Local variables in operation bodies are introduced using let expressions. For example, the predicate `have_common_divisor` takes two integer values and checks whether they have a common divisor greater than one:

```
predicate have_common_divisor(int: A, int: B) =
  let {
    var 2..min(A,B): C;
  } in
  A mod C = 0 /\ B mod C = 0;
```

However, as [Let Expressions](#) (page 232) explained, because `C` is not defined, this predicate cannot be called in a negative context. The following is a version that could be called in a negative context:

```
predicate have_common_divisor(int: A, int: B) =  
    exists(C in 2..min(A,B))  
        (A mod C = 0 /\ B mod C = 0);
```

## 4.1.9 Annotations

Annotations allow a modeller to specify non-declarative and solver-specific information that is beyond the core language. Annotations do not change the meaning of a model, however, only how it is solved.

Annotations can be attached to variables (on their declarations), constraints, expressions, type-inst synonyms, enum items, solve items and on user defined operations. They have the following syntax:

```
<annotations> ::= [ ":" <annotation> ]*  
  
<annotation> ::= <expr-atom-head> <expr-atom-tail>  
  
<string-annotation> ::= ":" <string-literal>
```

For example:

```
int: x::foo;  
x = (3 + 4)::bar("a", 9)::baz("b");  
solve :: blah(4)  
minimize x;
```

The types of the argument expressions must match the argument types of the declared annotation. Like user-defined predicates and functions, annotations can be overloaded.

Annotation signatures can contain type-inst variables.

The order and nesting of annotations do not matter. For the expression case it can be helpful to view the annotation connector `::` as an overloaded operator:

```
ann: '::'(any $T: e, ann: a);      % associative
ann: '::'(ann:     a, ann: b);      % associative + commutative
```

Both operators are associative, the second is commutative. This means that the following expressions are all equivalent:

```
e :: a :: b
e :: b :: a
(e :: a) :: b
(e :: b) :: a
e :: (a :: b)
e :: (b :: a)
```

This property also applies to annotations on solve items and variable declaration items. *Rationale*: *This property make things simple, as it allows all nested combinations of annotations to be treated as if they are flat, thus avoiding the need to determine what is the meaning of an annotated annotation. It also makes the MiniZinc abstract syntax tree simpler by avoiding the need to represent nesting.*

Annotations have to be values of the `ann` type or string literals. The latter are used for *naming* constraints and expressions, for example

```
constraint ::"first constraint" alldifferent(x);
constraint ::"second constraint" alldifferent(y);
constraint forall (i in 1..n) (my_constraint(x[i],y[i])::"constraint \$(i)");
```

Note that constraint items can *only* be annotated with string literals.

*Rationale*: *Allowing arbitrary annotations on constraint items makes the grammar ambiguous, and seems unnecessary since we can just as well annotate the constraint expression.*

## 4.1.10 Partiality

The presence of constrained type-insts in MiniZinc means that various operations are potentially *partial*, i.e., not clearly defined for all possible inputs. For example, what happens if a function expecting a positive argument is passed a negative argument? What happens if a variable is assigned a value that does not satisfy its type-inst constraints? What happens if an array index is out of bounds? This section describes what happens in all these cases.

In general, cases involving fixed values that do not satisfy constraints lead to run-time aborts.  
*Rationale:* Our experience shows that if a fixed value fails a constraint, it is almost certainly due to a programming error. Furthermore, these cases are easy for an implementation to check.

But cases involving unfixed values vary, as we will see. *Rationale:* The best thing to do for unfixed values varies from case to case. Also, it is difficult to check constraints on unfixed values, particularly because during search a decision variable might become fixed and then backtracking will cause this value to be reverted, in which case aborting is a bad idea.

#### 4.1.10.1 Partial Assignments

The first operation involving partiality is assignment. There are four distinct cases for assignments.

- A value assigned to a fixed, constrained global variable is checked at run-time; if the assigned value does not satisfy its constraints, it is a run-time error. In other words, this:

```
1..5: x = 3;
```

is equivalent to this:

```
int: x = 3;
constraint assert(x in 1..5,
    "assignment to global parameter 'x' failed")
```

- A value assigned to an unfixed, constrained global variable makes the assignment act like a constraint; if the assigned value does not satisfy the variable's constraints, it causes a run-time model failure. In other words, this:

```
var 1..5: x = 3;
```

is equivalent to this:

```
var int: x = 3;
constraint x in 1..5;
```

*Rationale:* This behaviour is easy to understand and easy to implement.

- A value assigned to a fixed, constrained let-local variable is checked at run-time; if the assigned value does not satisfy its constraints, it is a run-time error. In other words, this:

```
let { 1..5: x = 3; } in x+1
```

is equivalent to this:

```
let { int: x = 3; } in
  assert(x in 1..5,
    "assignment to let parameter 'x' failed", x+1)
```

- A value assigned to an unfixed, constrained let-local variable makes the assignment act like a constraint; if the constraint fails at run-time, the failure “bubbles up” to the nearest enclosing Boolean scope, where it is interpreted as false.

*Rationale: This behaviour is consistent with assignments to global variables.*

Note that in cases where a value is partly fixed and partly unfixed, e.g., some arrays, the different elements are checked according to the different cases, and fixed elements are checked before unfixed elements. For example:

```
u = [ let { var 1..5: x = 6} in x, let { par 1..5: y = 6; } in y ];
```

This causes a run-time abort, because the second, fixed element is checked before the first, unfixed element. This ordering is true for the cases in the following sections as well. *Rationale: This ensures that failures cannot mask aborts, which seems desirable.*

#### 4.1.10.2 Partial Predicate/Function and Annotation Arguments

The second kind of operation involving partiality is calls and annotations.

The semantics is similar to assignments: fixed arguments that fail their constraints will cause aborts, and unfixed arguments that fail their constraints will cause failure, which bubbles up to the nearest enclosing Boolean scope.

#### 4.1.10.3 Partial Array Accesses

The third kind of operation involving partiality is array access. There are two distinct cases.

- A fixed value used as an array index is checked at run-time; if the index value is not in the index set of the array, it is a run-time error.

- An unfixed value used as an array index makes the access act like a constraint; if the access fails at run-time, the failure “bubbles up” to the nearest enclosing Boolean scope, where it is interpreted as false. For example:

```
array[1..3] of int: a = [1,2,3];
var int: i;
constraint (a[i] + 3) > 10 \vee i = 99;
```

Here the array access fails, so the failure bubbles up to the disjunction, and `i` is constrained to be 99. *Rationale: Unlike predicate/function calls, modellers in practice sometimes do use array accesses that can fail. In such cases, the “bubbling up” behaviour is a reasonable one.*

## 4.1.11 Built-in Operations

This appendix lists built-in operators, functions and predicates. They may be implemented as true built-ins, or in libraries that are automatically imported for all models. Many of them are overloaded.

Operator names are written within single quotes when used in type signatures, e.g.  
`bool: '\>'(bool, bool).`

We use the syntax `TI: f(TI1, ..., TIn)` to represent an operation named `f` that takes arguments with type-args `TI, ..., TIn` and returns a value with type-inst `TI`. This is slightly more compact than the usual MiniZinc syntax, in that it omits argument names.

### 4.1.11.1 Comparison Operations

Less than. Other comparisons are similar: greater than (`>`), less than or equal (`<=`), greater than or equal (`>=`), equality (`==`, `=`), and disequality (`!=`).

```
bool: '<'(    $T,    $T)
var bool: '<'(var $T, var $T)
```

### 4.1.11.2 Arithmetic Operations

Addition. Other numeric operations are similar: subtraction (`-`), and multiplication (`*`).

```

int: '+'( int,      int)
var int: '+'(var int,  var int)
float: '+'( float,    float)
var float: '+'(var float, var float)

```

Unary minus. Unary plus (+) is similar.

```

int: '-'( int)
var int: '-'(var int)
float: '-'( float)
var float: '-'(var float)

```

Integer and floating-point division and modulo.

```

int: 'div'( int,      int)
var int: 'div'(var int,  var int)
int: 'mod'( int,      int)
var int: 'mod'(var int,  var int)
float: '/' ( float,    float)
var float: '/' (var float, var float)

```

The result of the modulo operation, if non-zero, always has the same sign as its first operand. The integer division and modulo operations are connected by the following identity:

$$x = (x \text{ div } y) * y + (x \text{ mod } y)$$

Some illustrative examples:

$7 \text{ div } 4 = 1$	$7 \text{ mod } 4 = 3$
$-7 \text{ div } 4 = -1$	$-7 \text{ mod } 4 = -3$
$7 \text{ div } -4 = -1$	$7 \text{ mod } -4 = 3$
$-7 \text{ div } -4 = 1$	$-7 \text{ mod } -4 = -3$

Sum multiple numbers. Product ([product](#)) is similar. Note that the sum of an empty array is 0, and the product of an empty array is 1.

```
int: sum(array[$T] of int)
var int: sum(array[$T] of var int)
float: sum(array[$T] of float)
var float: sum(array[$T] of var float)
```

Minimum of two values; maximum (`max`) is similar.

```
any $T: min(any $T, any $T)
```

Minimum of an array of values; maximum (`max`) is similar. Aborts if the array is empty.

```
any $U: min(array[$T] of any $U)
```

Minimum of a fixed set; maximum (`max`) is similar. Aborts if the set is empty.

```
$T: min(set of $T)
```

Absolute value of a number.

```
int: abs(int)
var int: abs(var int)
float: abs(float)
var float: abs(var float)
```

Square root of a float. Aborts if argument is negative.

```
float: sqrt(float)
var float: sqrt(var float)
```

Power operator. E.g. `pow(2, 5)` gives `32`.

```
int: pow(int, int)
float: pow(float, float)
```

Natural exponent.

```
float: exp(float)
var float: exp(var float)
```

Natural logarithm. Logarithm to base 10 (`log10`) and logarithm to base 2 (`log2`) are similar.

```
float: ln(float)
var float: ln(var float)
```

General logarithm; the first argument is the base.

```
float: log(float, float)
```

Sine. Cosine (`cos`), tangent (`tan`), inverse sine (`asin`), inverse cosine (`acos`), inverse tangent (`atan`), hyperbolic sine (`sinh`), hyperbolic cosine (`cosh`), hyperbolic tangent (`tanh`), inverse hyperbolic sine (`asinh`), inverse hyperbolic cosine (`acosh`) and inverse hyperbolic tangent (`atanh`) are similar.

```
float: sin(float)
var float: sin(var float)
```

### 4.1.11.3 Logical Operations

Conjunction. Other logical operations are similar: disjunction (`\vee`) reverse implication (`<-`), forward implication (`->`), bi-implication (`<->`), exclusive disjunction (`xor`), logical negation (`not`).

Note that the implication operators are not written using `=>`, `<=` and `<=>` as is the case in some languages. This allows `<=` to instead represent “less than or equal” .

```
bool: '/\\'(    bool,      bool)
var bool: '/\\'(var bool, var bool)
```

Universal quantification. Existential quantification (`exists`) is similar. Note that, when applied to an empty list, `forall` returns true, and `exists` returns false.

```
bool: forall(array[$T]  of      bool)
var bool: forall(array[$T]  of var bool)
```

N-ary exclusive disjunction. N-ary bi-implication (`iffall`) is similar, with `true` instead of `false`.

```
bool: xorall(array[$T] of bool: bs) = foldl('xor', false, bs)
var bool: xorall(array[$T] of var bool: bs) = foldl('xor', false, bs)
```

#### 4.1.11.4 Set Operations

Set membership.

```
bool: 'in'( $T, set of $T )
var bool: 'in'(var int, var set of int)
```

Non-strict subset. Non-strict superset (superset) is similar.

```
bool: 'subset'( set of $T , set of $T )
var bool: 'subset'(var set of int, var set of int)
```

Set union. Other set operations are similar: intersection (`intersect`), difference (`diff`), symmetric difference (`symdiff`).

```
set of $T: 'union'( set of $T, set of $T )
var set of int: 'union'(var set of int, var set of int)
```

Set range. If the first argument is larger than the second (e.g. `1..0`), it returns the empty set.

```
set of int: '..'(int, int)
```

Cardinality of a set.

```
int: card( set of $T )
var int: card(var set of int)
```

Union of an array of sets. Intersection of multiple sets (`array_intersect`) is similar.

```
set of $U: array_union(array[$T] of set of $U)
var set of int: array_union(array[$T] of var set of int)
```

#### 4.1.11.5 Array Operations

Length of an array.

```
int: length(array[$T] of any $U)
```

List concatenation. Returns the list (integer-indexed array) containing all elements of the first argument followed by all elements of the second argument, with elements occurring in the same order as in the arguments. The resulting indices are in the range  $1..n$ , where  $n$  is the sum of the lengths of the arguments. *Rationale:* This allows list-like arrays to be concatenated naturally and avoids problems with overlapping indices. The resulting indices are consistent with those of implicitly indexed array literals. Note that '`++`' also performs string concatenation.

```
array[int] of any $T: '++'(array[int] of any $T, array[int] of any $T)
```

Index sets of arrays. If the argument is a literal, returns  $1..n$  where  $n$  is the (sub-)array length. Otherwise, returns the declared or inferred index set. This list is only partial, it extends in the obvious way, for arrays of higher dimensions.

```
set of $T: index_set      (array[$T]      of any $V)
set of $T: index_set_1of2(array[$T, $U]  of any $V)
set of $U: index_set_2of2(array[$T, $U]  of any $V)
...
...
```

Replace the indices of the array given by the last argument with the Cartesian product of the sets given by the previous arguments. Similar versions exist for arrays up to 6 dimensions.

```
array[$T1] of any $V: array1d(set of $T1, array[$U] of any $V)
array[$T1,$T2] of any $V:
    array2d(set of $T1, set of $T2, array[$U] of any $V)
array[$T1,$T2,$T3] of any $V:
    array3d(set of $T1, set of $T2, set of $T3, array[$U] of any $V)
```

#### 4.1.11.6 Coercion Operations

Round a float towards  $+\infty$ ,  $-\infty$ , and the nearest integer, respectively.

```
int: ceil (float)
int: floor(float)
int: round(float)
```

Explicit casts from one type-inst to another.

```
int:          bool2int(  bool)
var int:      bool2int(var bool)
float:        int2float(   int)
var float:    int2float(var int)
array[int] of $T: set2array(set of $T)
```

#### 4.1.11.7 String Operations

To-string conversion. Converts any value to a string for output purposes. The exact form of the resulting string is implementation-dependent.

```
string: show(any $T)
```

Formatted to-string conversion for integers. Converts the integer given by the second argument into a string right justified by the number of characters given by the first argument, or left justified if that argument is negative. If the second argument is not fixed, the form of the string is implementation-dependent.

```
string: show_int(int, var int);
```

Formatted to-string conversion for floats. Converts the float given by the third argument into a string right justified by the number of characters given by the first argument, or left justified if that argument is negative. The number of digits to appear after the decimal point is given by the second argument. It is a run-time error for the second argument to be negative. If the third argument is not fixed, the form of the string is implementation-dependent.

```
string: show_float(int, int, var float)
```

String concatenation. Note that `'++'` also performs array concatenation.

```
string: '+'(string, string)
```

Concatenate an array of strings. Equivalent to folding `'+'` over the array, but may be implemented more efficiently.

```
string: concat(array[$T] of string)
```

Concatenate an array of strings, putting a separator between adjacent strings. Returns the empty string if the array is empty.

```
string: join(string, array[$T] of string)
```

#### 4.1.11.8 Bound and Domain Operations

The bound operations `lb` and `ub` return fixed, correct lower/upper bounds to the expression. For numeric types, they return a lower/upper bound value, e.g. the lowest/highest value the expression can take. For set types, they return a subset/superset, e.g. the intersection/union of all possible values of the set expression.

The bound operations abort on expressions that have no corresponding finite bound. For example, this would be the case for a variable declared without bounds in an implementation that does not assign default bounds. (Set expressions always have a finite lower bound of course, namely `{}`, the empty set.)

Numeric lower/upper bound:

```
int: lb(var int)
float: lb(var float)
int: ub(var int)
float: ub(var float)
```

Set lower/upper bound:

```
set of int: lb(var set of int)
set of int: ub(var set of int)
```

Versions of the bound operations that operate on arrays are also available, they return a safe lower bound or upper bound for all members of the array - they abort if the array is empty:

```
int:           lb_array(array[$T] of var int)
float:         lb_array(array[$T] of var float)
set of int:   lb_array(array[$T] of var set of int)
int:           ub_array(array[$T] of var int)
float:         ub_array(array[$T] of var float)
set of int:   ub_array(array[$T] of var set of int)
```

Integer domain:

```
set of int: dom(var int)
```

The domain operation `dom` returns a fixed superset of the possible values of the expression.

Integer array domain, returns a superset of all possible values that may appear in the array - this aborts if the array is empty:

```
set of int: dom_array(array[$T] of var int)
```

Domain size for integers:

```
int: dom_size(var int)
```

The domain size operation `dom_size` is equivalent to `card(dom(x))`.

Note that these operations can produce different results depending on when they are evaluated and what form the argument takes. For example, consider the numeric lower bound operation.

- If the argument is a fixed expression, the result is the argument's value.
- If the argument is a decision variable, then the result depends on the context.
  - If the implementation can determine a lower bound for the variable, the result is that lower bound. The lower bound may be from the variable's declaration, or higher than that due to preprocessing, or lower than that if an implementation-defined lower bound is applied (e.g. if the variable was declared with no lower bound, but the implementation imposes a lowest possible bound).
  - If the implementation cannot determine a lower bound for the variable, the operation aborts.
- If the argument is any other kind of unfixed expression, the lower bound depends on the bounds of unfixed subexpressions and the connecting operators.

### 4.1.11.9 Option Type Operations

The option type value ( $\top$ ) is written

```
opt $T: '<>';
```

One can determine if an option type variable actually occurs or not using `occurs` and `absent`

```
par bool: occurs(par opt $T);
var bool: occurs(var opt $T);
par bool: absent(par opt $T);
var bool: absent(var opt $T);
```

One can return the non-optional value of an option type variable using the function `deopt`

```
par $T: deopt{par opt $T};
var $T: deopt(var opt $T);
```

### 4.1.11.10 Other Operations

Check a Boolean expression is true, and abort if not, printing the second argument as the error message. The first one returns the third argument, and is particularly useful for sanity-checking arguments to predicates and functions; importantly, its third argument is lazy, i.e. it is only evaluated if the condition succeeds. The second one returns `true` and is useful for global sanity-checks (e.g. of instance data) in constraint items.

```
any $T: assert(bool, string, any $T)
par bool: assert(bool, string)
```

Abort evaluation, printing the given string.

```
any $T: abort(string)
```

Return `true`. As a side-effect, an implementation may print the first argument.

```
bool: trace(string)
```

Return the second argument. As a side-effect, an implementation may print the first argument.

```
any $T: trace(string, any $T)
```

Check if the argument's value is fixed at this point in evaluation. If not, abort; if so, return its value. This is most useful in output items when decision variables should be fixed: it allows them to be used in places where a fixed value is needed, such as if-then-else conditions.

```
$T: fix(any $T)
```

As above, but return `false` if the argument's value is not fixed.

```
par bool: is_fixed(any $T)
```

## 4.1.12 Content-types

The content-type application/x-zinc-output defines a text output format for Zinc. The format extends the abstract syntax and semantics given in *Run-time Outcomes* (page 202), and is discussed in detail in *Output* (page 203).

The full syntax is as follows:

```
% Output
<output> ::= <no-solutions> [ <warnings> ] <free-text>
           | ( <solution> )* [ <complete> ] [ <warnings> ] <free-text>

% Solutions
<solution> ::= <solution-text> [ \n ] "-----" \n

% Unsatisfiable
<no-solutions> ::= "=====UNSATISFIABLE=====" \n

% Complete
<complete> ::= "======" \n

% Messages
<warnings> ::= ( <message> )+
```

```
<message> ::= ( <line> )+
<line>    ::= "%" [^\n]* \n
```

The solution text for each solution must be as described in [Output Items](#) (page 239). A newline must be appended if the solution text does not end with a newline.

### 4.1.13 JSON support

MiniZinc can support reading input parameters and providing output formatted as JSON objects. A JSON input file needs to have the following structure:

- Consist of a single top-level object
- The members of the object (the key-value pairs) represent model parameters
- Each member key must be a valid MiniZinc identifier (and it supplies the value for the corresponding parameter of the model)
- Each member value can be one of the following:
  - A string (assigned to a MiniZinc string parameter)
  - A number (assigned to a MiniZinc int or float parameter)
  - The values true or false (assigned to a MiniZinc bool parameter)
  - An array of values. Arrays of arrays are supported only if all inner arrays are of the same length, so that they can be mapped to multi-dimensional MiniZinc arrays.
  - A set of values encoded as an object with a single member with key "set" and a list of values (the elements of the set).

This is an example of a JSON parameter file using all of the above features:

```
{
  "n" : 3,
  "distances" : [ [1,2,3],
                  [4,5,6]],
  "patterns" : [ {"set" : [1,3,5]}, {"set" : [2,4,6]} ]
}
```

The first parameter declares a simple integer `n`. The `distances` parameter is a two-dimensional array; note that all inner arrays must be of the same size in order to map to a (rectangular) MiniZinc two-dimensional array. The third parameter is an array of sets of integers.

**Note:** The JSON input and output currently does not support enumerated types. This will be added in a future release.

## 4.1.14 Full grammar

### 4.1.14.1 Items

```
% A MiniZinc model
<model> ::= [ <item> ";" ... ]

% Items
<item> ::= <include-item>
          | <var-decl-item>
          | <enum-item>
          | <assign-item>
          | <constraint-item>
          | <solve-item>
          | <output-item>
          | <predicate-item>
          | <test-item>
          | <function-item>
          | <annotation-item>

<ti-expr-and-id> ::= <ti-expr> ":" <ident>

% Include items
<include-item> ::= "include" <string-literal>

% Variable declaration items
<var-decl-item> ::= <ti-expr-and-id> <annotations> [ "=" <expr> ]

% Enum items
<enum-item> ::= "enum" <ident> <annotations> [ "=" <enum-cases> ]
```

```

<enum-cases> ::= "{" <ident> "," ... "}"

% Assign items
<assign-item> ::= <ident> "=" <expr>

% Constraint items
<constraint-item> ::= "constraint" <string-annotation> <expr>

% Solve item
<solve-item> ::= "solve" <annotations> "satisfy"
               | "solve" <annotations> "minimize" <expr>
               | "solve" <annotations> "maximize" <expr>

% Output items
<output-item> ::= "output" <expr>

% Annotation items
<annotation-item> ::= "annotation" <ident> <params>

% Predicate, test and function items
<predicate-item> ::= "predicate" <operation-item-tail>

<test-item> ::= "test" <operation-item-tail>

<function-item> ::= "function" <ti-expr> ":" <operation-item-tail>

<operation-item-tail> ::= <ident> <params> <annotations> [ "=" <expr> ]

<params> ::= [ ( <ti-expr-and-id> "," ... ) ]

```

#### 4.1.14.2 Type-Inst Expressions

```

<ti-expr> ::= <base-ti-expr>

<base-ti-expr> ::= <var-par> <base-ti-expr-tail>

<var-par> ::= "var" | "par" | ε

```

```

<base-type> ::= "bool"
  | "int"
  | "float"
  | "string"

<base-ti-expr-tail> ::= <ident>
  | <base-type>
  | <set-ti-expr-tail>
  | <ti-variable-expr-tail>
  | <array-ti-expr-tail>
  | "ann"
  | "opt" <base-ti-expr-tail>
  | { <expr> "," ... }
  | <num-expr> ".." <num-expr>

% Type-inst variables

<ti-variable-expr-tail> ::= ${A-Za-z}[A-Za-z0-9_]*

% Set type-inst expressions

<set-ti-expr-tail> ::= "set" "of" <base-type>

% Array type-inst expressions

<array-ti-expr-tail> ::= "array" [ <ti-expr> "," ... ] "of" <ti-expr>
  | "list" "of" <ti-expr>

```

#### 4.1.14.3 Expressions

```

<expr> ::= <expr-atom> <expr-binop-tail>

<expr-atom> ::= <expr-atom-head> <expr-atom-tail> <annotations>

<expr-binop-tail> ::= "[" <bin-op> <expr> "]"

<expr-atom-head> ::= <builtin-un-op> <expr-atom>
  | "(" <expr> ")"
  | <ident-or-quoted-op>

```

```

    | " "
    | <bool-literal>
    | <int-literal>
    | <float-literal>
    | <string-literal>
    | <set-literal>
    | <set-comp>
    | <array-literal>
    | <array-literal-2d>
    | <array-comp>
    | <ann-literal>
    | <if-then-else-expr>
    | <let-expr>
    | <call-expr>
    | <gen-call-expr>

<expr-atom-tail> ::= ε
    | <array-access-tail> <expr-atom-tail>

% Numeric expressions

<num-expr> ::= <num-expr-atom> <num-expr-binop-tail>

<num-expr-atom> ::= <num-expr-atom-head> <expr-atom-tail> <annotations>

<num-expr-binop-tail> ::= "[" <num-bin-op> <num-expr> "]"

<num-expr-atom-head> ::= <builtin-num-un-op> <num-expr-atom>
    | "(" <num-expr> ")"
    | <ident-or-quoted-op>
    | <int-literal>
    | <float-literal>
    | <if-then-else-expr>
    | <let-expr>
    | <call-expr>
    | <gen-call-expr>

% Built-in operators

<builtin-op> ::= <builtin-bin-op> | <builtin-un-op>

```

```

<bin-op> ::= <builtin-bin-op> | '<ident>' '


<builtin-bin-op> ::= "<->" | "->" | "<->" | "\/" | "xor" | "/\\" | "<" | ">" |
→ "<=" | ">=" | "==" | "=" | "!="
          | "in" | "subset" | "superset" | "union" | "diff" |
→ "symdiff"
          | ".." | "intersect" | "++" | <builtin-num-bin-op>

<builtin-un-op> ::= "not" | <builtin-num-un-op>

% Built-in numeric operators

<num-bin-op> ::= <builtin-num-bin-op> | '<ident>' '


<builtin-num-bin-op> ::= "+" | "-" | "*" | "/" | "div" | "mod"

<builtin-num-un-op> ::= "+" | "-"

% Boolean literals

<bool-literal> ::= "false" | "true"

% Integer literals

<int-literal> ::= [0-9]+
                  | 0x[0-9A-Fa-f]+
                  | 0o[0-7]+

% Float literals

<float-literal> ::= [0-9]+.[0-9]+
                  | [0-9]+.[0-9]+[Ee][-+]?[0-9]+
                  | [0-9]+[Ee][-+]?[0-9]+

% String literals

<string-contents> ::= ([^\n] | \[^n])*
                      | " " <string-contents> "
                      | " " <string-contents> "\(" <string-interpolate-tail>

<string-interpolate-tail> ::= <expr> ")" <string-contents> "

```

```

| <expr> ")"<string-contents>"\("_
↳<string-interpolate-tail>

% Set literals
<set-literal> ::= "{" [ <expr> "," ... ] "}"

% Set comprehensions
<set-comp> ::= "{" <expr> "|" <comp-tail> "}"

<comp-tail> ::= <generator> [ "where" <expr> ] "," ...
<generator> ::= <ident> "," ... "in" <expr>

% Array literals
<array-literal> ::= "[" [ <expr> "," ... ] "]"

% 2D Array literals
<array-literal-2d> ::= "[" [ (<expr> "," ...) "|" ... ] "]"

% Array comprehensions
<array-comp> ::= "[" <expr> "|" <comp-tail> "]"

% Array access
<array-access-tail> ::= "[" <expr> "," ... "]"

% Annotation literals
<ann-literal> ::= <ident> [ "(" <expr> "," ... ")" ]"

% If-then-else expressions
<if-then-else-expr> ::= "if" <expr> "then" <expr> [ "elseif" <expr> "then"_
↳<expr> ]* "else" <expr> "endif"

% Call expressions
<call-expr> ::= <ident-or-quoted-op> [ "(" <expr> "," ... ")" ]"

% Let expressions
<let-expr> ::= "let" "{" <let-item> ";" ... "}" "in" <expr>

```

```
<let-item> ::= <var-decl-item>
             | <constraint-item>

% Generator call expressions
<gen-call-expr> ::= <ident-or-quoted-op> "(" <comp-tail> ")" "(" <expr> ")"
```

#### 4.1.14.4 Miscellaneous Elements

```
% Identifiers
<ident> ::= [A-Za-z][A-Za-z0-9_]* | '^' \xa\xd\x0]*'

% Identifiers and quoted operators
<ident-or-quoted-op> ::= <ident>
                         | '<builtin-op>'

% Annotations
<annotations> ::= [ ":" <annotation> ]*

<annotation> ::= <expr-atom-head> <expr-atom-tail>

<string-annotation> ::= ":" <string-literal>
```

# CHAPTER 4.2

---

## The MiniZinc library

---

### 4.2.1 Global constraints

These constraints represent high-level modelling abstractions, for which many solvers implement special, efficient inference algorithms.

#### 4.2.1.1 All-Different and related constraints

```
predicate all_different(array [$X] of var int: x)
```

Constrain the array of integers  $x$  to be all different.

```
predicate all_different(array [$X] of var set of int: x)
```

Constrain the array of sets of integers  $x$  to be all different.

```
predicate all_disjoint(array [int] of var set of int: S)
```

Constrain the array of sets of integers  $S$  to be pairwise disjoint.

```
predicate all_equal(array [$X] of var int: x)
```

Constrain the array of integers  $x$  to be all equal

```
predicate all_equal(array [$X] of var set of int: x)
```

Constrain the array of sets of integers  $x$  to be all equal

```
predicate alldifferent_except_0(array [int] of var int: vs)
```

Constrain the array of integers  $vs$  to be all different except those elements that are assigned the value 0.

```
predicate nvalue(var int: n, array [int] of var int: x)
```

Requires that the number of distinct values in  $x$  is  $n$ .

```
function var int: nvalue(array [int] of var int: x)
```

Returns the number of distinct values in  $x$ .

```
predicate symmetric_all_different(array [int] of var int: x)
```

Requires the array of integers  $x$  to be all different, and for all  $i$ ,  $x[i]=j \rightarrow x[j]=i$ .

#### 4.2.1.2 Lexicographic constraints

```
predicate lex2(array [int,int] of var int: x)
```

Require adjacent rows and adjacent columns in the array  $x$  to be lexicographically ordered. Adjacent rows and adjacent columns may be equal.

```
predicate lex_greater(array [int] of var bool: x,
                     array [int] of var bool: y)
```

Requires that the array  $x$  is strictly lexicographically greater than array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_greater(array [int] of var int: x,
                     array [int] of var int: y)
```

Requires that the array  $x$  is strictly lexicographically greater than array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_greater(array [int] of var float: x,
                     array [int] of var float: y)
```

Requires that the array  $x$  is strictly lexicographically greater than array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_greater(array [int] of var set of int: x,
                     array [int] of var set of int: y)
```

Requires that the array  $x$  is strictly lexicographically greater than array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_greatereq(array [int] of var bool: x,
                       array [int] of var bool: y)
```

Requires that the array  $x$  is lexicographically greater than or equal to array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_greatereq(array [int] of var int: x,
                       array [int] of var int: y)
```

Requires that the array  $x$  is lexicographically greater than or equal to array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_greatereq(array [int] of var float: x,
                       array [int] of var float: y)
```

Requires that the array  $x$  is lexicographically greater than or equal to array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_greatereq(array [int] of var set of int: x,
                        array [int] of var set of int: y)
```

Requires that the array  $x$  is lexicographically greater than or equal to array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_less(array [int] of var bool: x,
                  array [int] of var bool: y)
```

Requires that the array  $x$  is strictly lexicographically less than array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_less(array [int] of var int: x,
                  array [int] of var int: y)
```

Requires that the array  $x$  is strictly lexicographically less than array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_less(array [int] of var float: x,
                  array [int] of var float: y)
```

Requires that the array  $x$  is strictly lexicographically less than array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_less(array [int] of var set of int: x,
                  array [int] of var set of int: y)
```

Requires that the array  $x$  is strictly lexicographically less than array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_lesseq(array [int] of var bool: x,
                     array [int] of var bool: y)
```

Requires that the array  $x$  is lexicographically less than or equal to array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_lesseq(array [int] of var float: x,
                     array [int] of var float: y)
```

Requires that the array  $x$  is lexicographically less than or equal to array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_lesseq(array [int] of var int: x,
                     array [int] of var int: y)
```

Requires that the array  $x$  is lexicographically less than or equal to array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate lex_lesseq(array [int] of var set of int: x,
                     array [int] of var set of int: y)
```

Requires that the array  $x$  is lexicographically less than or equal to array  $y$ . Compares them from first to last element, regardless of indices.

```
predicate strict_lex2(array [int,int] of var int: x)
```

Require adjacent rows and adjacent columns in the array  $x$  to be lexicographically ordered. Adjacent rows and adjacent columns cannot be equal.

```
predicate value_precede(int: s, int: t, array [int] of var int: x)
```

Requires that  $s$  precede  $t$  in the array  $x$ .

Precedence means that if any element of  $x$  is equal to  $t$ , then another element of  $x$  with a lower index is equal to  $s$ .

```
predicate value_precede(int: s,
                       int: t,
                       array [int] of var set of int: x)
```

Requires that  $s$  precede  $t$  in the array  $x$ .

Precedence means that if an element of  $x$  contains  $t$  but not  $s$ , then another element of  $x$  with lower index contains  $s$  but not  $t$ .

```
predicate value_precede_chain(array [int] of int: c,
                               array [int] of var int: x)
```

Requires that  $c[i]$  precedes  $c[i+1]$  in the array  $x$ .

Precedence means that if any element of  $x$  is equal to  $c[i+1]$ , then another element of  $x$  with a lower index is equal to  $c[i]$ .

```
predicate value_precede_chain(array [int] of int: c,
                               array [int] of var set of int: x)
```

Requires that  $c[i]$  precedes  $c[i+1]$  in the array  $x$ .

Precedence means that if an element of  $x$  contains  $c[i+1]$  but not  $c[i]$ , then another element of  $x$  with lower index contains  $c[i]$  but not  $c[i+1]$ .

#### 4.2.1.3 Sorting constraints

```
function array [int] of var int: arg_sort(array [int] of var int: x)
```

Returns the permutation  $p$  which causes  $x$  to be in sorted order hence  $x[p[i]] \leq x[p[i+1]]$ .

The permutation is the stable sort hence  $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$ .

```
function array [int] of var int: arg_sort(array [int] of var float: x)
```

Returns the permutation  $p$  which causes  $x$  to be in sorted order hence  $x[p[i]] \leq x[p[i+1]]$ .

The permutation is the stable sort hence  $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$ .

```
predicate arg_sort(array [int] of var int: x,
                  array [int] of var int: p)
```

Constrains  $p$  to be the permutation which causes  $x$  to be in sorted order hence  $x[p[i]] \leq x[p[i+1]]$ .

The permutation is the stable sort hence  $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$ .

```
predicate arg_sort(array [int] of var float: x,
                  array [int] of var int: p)
```

Constrains  $p$  to be the permutation which causes  $x$  to be in sorted order hence  $x[p[i]] \leq x[p[i+1]]$ .

The permutation is the stable sort hence  $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$ .

```
predicate decreasing(array [int] of var bool: x)
```

Requires that the array  $x$  is in decreasing order (duplicates are allowed).

```
predicate decreasing(array [int] of var float: x)
```

Requires that the array  $x$  is in decreasing order (duplicates are allowed).

```
predicate decreasing(array [int] of var int: x)
```

Requires that the array  $x$  is in decreasing order (duplicates are allowed).

```
predicate decreasing(array [int] of var set of int: x)
```

Requires that the array  $x$  is in decreasing order (duplicates are allowed).

```
predicate increasing(array [int] of var bool: x)
```

Requires that the array  $x$  is in increasing order (duplicates are allowed).

```
predicate increasing(array [int] of var float: x)
```

Requires that the array  $x$  is in increasing order (duplicates are allowed).

```
predicate increasing(array [int] of var int: x)
```

Requires that the array  $x$  is in increasing order (duplicates are allowed).

```
predicate increasing(array [int] of var set of int: x)
```

Requires that the array  $x$  is in increasing order (duplicates are allowed).

```
predicate sort(array [int] of var int: x, array [int] of var int: y)
```

Requires that the multiset of values in  $x$  are the same as the multiset of values in  $y$  but  $y$  is in sorted order.

```
function array [int] of var int: sort(array [int] of var int: x)
```

Return a multiset of values that is the same as the multiset of values in  $x$  but in sorted order.

#### 4.2.1.4 Channeling constraints

```
predicate int_set_channel(array [int] of var int: x,
                           array [int] of var set of int: y)
```

Requires that array of int variables  $x$  and array of set variables  $y$  are related such that ( $x[i] = j$ ) if and only if ( $i$  in  $y[j]$ ).

```
predicate inverse(array [int] of var int: f,
                  array [int] of var int: invf)
```

Constrains two arrays of int variables,  $f$  and  $invf$ , to represent inverse functions. All the values in each array must be within the index set of the other array.

```
function array [int] of var int: inverse(array [int] of var int: f)
```

Given a function  $f$  represented as an array, return the inverse function.

```
predicate inverse_set(array [int] of var set of int: f,
                     array [int] of var set of int: invf)
```

Constrains two arrays of set of int variables,  $f$  and  $invf$ , so that  $a[j] \in f[i]$  iff  $i \in invf[j]$ . All the values in each array's sets must be within the index set of the other array.

```
predicate link_set_to_booleans(var set of int: s,
                                array [int] of var bool: b)
```

Constrain the array of Booleans b to be a representation of the set  $s : i \in s$  if and only if  $b[i]$ .

The index set of b must be a superset of the possible values of s.

#### 4.2.1.5 Counting constraints

```
predicate among(var int: n, array [int] of var int: x, set of int: v)
```

Requires exactly n variables in x to take one of the values in v.

```
function var int: among(array [int] of var int: x, set of int: v)
```

Returns the number of variables in x that take one of the values in v.

```
predicate at_least(int: n, array [int] of var int: x, int: v)
```

Requires at least n variables in x to take the value v.

```
predicate at_least(int: n,
                  array [int] of var set of int: x,
                  set of int: v)
```

Requires at least n variables in x to take the value v.

```
predicate at_most(int: n, array [int] of var int: x, int: v)
```

Requires at most n variables in x to take the value v.

```
predicate at_most(int: n,
                  array [int] of var set of int: x,
                  set of int: v)
```

Requires at most n variables in x to take the value v.

```
predicate at_most1(array [int] of var set of int: s)
```

Requires that each pair of sets in  $s$  overlap in at most one element.

```
predicate count(array [int] of var int: x, var int: y, var int: c)
```

Constrains  $c$  to be the number of occurrences of  $y$  in  $x$ .

```
function var int: count(array [int] of var int: x, var int: y)
```

Returns the number of occurrences of  $y$  in  $x$ .

```
predicate count_eq(array [int] of var int: x, var int: y, var int: c)
```

Constrains  $c$  to be the number of occurrences of  $y$  in  $x$ .

```
predicate count_geq(array [int] of var int: x, var int: y, var int: c)
```

Constrains  $c$  to be greater than or equal to the number of occurrences of  $y$  in  $x$ .

```
predicate count_gt(array [int] of var int: x, var int: y, var int: c)
```

Constrains  $c$  to be strictly greater than the number of occurrences of  $y$  in  $x$ .

```
predicate count_leq(array [int] of var int: x, var int: y, var int: c)
```

Constrains  $c$  to be less than or equal to the number of occurrences of  $y$  in  $x$ .

```
predicate count_lt(array [int] of var int: x, var int: y, var int: c)
```

Constrains  $c$  to be strictly less than the number of occurrences of  $y$  in  $x$ .

```
predicate count_neq(array [int] of var int: x, var int: y, var int: c)
```

Constrains  $c$  to be not equal to the number of occurrences of  $y$  in  $x$ .

```
predicate distribute(array [int] of var int: card,
                     array [int] of var int: value,
                     array [int] of var int: base)
```

Requires that `card [ i ]` is the number of occurrences of `value [ i ]` in `base`. The values in `value` need not be distinct.

```
function array [int] of var int: distribute(array [int] of var int: value,
                                             array [int] of var int: base)
```

Returns an array of the number of occurrences of `value [ i ]` in `base`. The values in `value` need not be distinct.

```
predicate exactly(int: n, array [int] of var int: x, int: v)
```

Requires exactly `n` variables in `x` to take the value `v`.

```
predicate exactly(int: n,
                 array [int] of var set of int: x,
                 set of int: v)
```

Requires exactly `n` variables in `x` to take the value `v`.

```
predicate global_cardinality(array [int] of var int: x,
                            array [int] of int: cover,
                            array [int] of var int: counts)
```

Requires that the number of occurrences of `cover [ i ]` in `x` is `counts [ i ]`.

```
function array [int] of var int: global_cardinality(array [int] of var int:_  
↳ x,  
                                              array [int] of int:_  
↳ cover)
```

Returns the number of occurrences of `cover [ i ]` in `x`.

```
predicate global_cardinality_closed(array [int] of var int: x,
                                    array [int] of int: cover,
                                    array [int] of var int: counts)
```

Requires that the number of occurrences of  $i$  in  $x$  is  $\text{counts}[i]$ .

The elements of  $x$  must take their values from  $\text{cover}$ .

```
function array [int] of var int: global_cardinality_closed(array [int] of_
    ↪var int: x,
                           array [int] of_
    ↪int: cover)
```

Returns an array with number of occurrences of  $i$  in  $x$ .

The elements of  $x$  must take their values from  $\text{cover}$ .

```
predicate global_cardinality_low_up(array [int] of var int: x,
                                    array [int] of int: cover,
                                    array [int] of int: lbound,
                                    array [int] of int: ubound)
```

Requires that for all  $i$ , the value  $\text{cover}[i]$  appears at least  $\text{lbound}[i]$  and at most  $\text{ubound}[i]$  times in the array  $x$ .

```
predicate global_cardinality_low_up_closed(array [int] of var int: x,
                                            array [int] of int: cover,
                                            array [int] of int: lbound,
                                            array [int] of int: ubound)
```

Requires that for all  $i$ , the value  $\text{cover}[i]$  appears at least  $\text{lbound}[i]$  and at most  $\text{ubound}[i]$  times in the array  $x$ .

The elements of  $x$  must take their values from  $\text{cover}$ .

#### 4.2.1.6 Packing constraints

```

predicate bin_packing(int: c,
                      array [int] of var int: bin,
                      array [int] of int: w)

```

Requires that each item  $i$  with weight  $w[i]$ , be put into bin  $[i]$  such that the sum of the weights of the items in each bin does not exceed the capacity  $c$ .

Assumptions:

- $\forall i, w[i] \geq 0$
- $c \geq 0$

```

predicate bin_packing_capa(array [int] of int: c,
                           array [int] of var int: bin,
                           array [int] of int: w)

```

Requires that each item  $i$  with weight  $w[i]$ , be put into bin  $[i]$  such that the sum of the weights of the items in each bin  $b$  does not exceed the capacity  $c[b]$ .

Assumptions:

- $\forall i, w[i] \geq 0$
- $\forall b, c[b] \geq 0$

```

predicate bin_packing_load(array [int] of var int: load,
                           array [int] of var int: bin,
                           array [int] of int: w)

```

Requires that each item  $i$  with weight  $w[i]$ , be put into bin  $[i]$  such that the sum of the weights of the items in each bin  $b$  is equal to  $load[b]$ .

Assumptions:

- $\forall i, w[i] \geq 0$

```

function array [int] of var int: bin_packing_load(array [int] of var int:_
→bin,
                                                 array [int] of int: w)

```

Returns the load of each bin resulting from packing each item  $i$  with weight  $w[i]$  into bin  $[i]$ , where the load is defined as the sum of the weights of the items in each bin.

Assumptions:

- `forall i, w [ i ] >=0`

```
predicate diffn(array [int] of var int: x,
               array [int] of var int: y,
               array [int] of var int: dx,
               array [int] of var int: dy)
```

Constrains rectangles  $i$ , given by their origins ( $x[i], y[i]$ ) and sizes ( $dx[i], dy[i]$ ), to be non-overlapping. Zero-width rectangles can still not overlap with any other rectangle.

```
predicate diffn_k(array [int,int] of var int: box_posn,
                  array [int,int] of var int: box_size)
```

Constrains  $k$ -dimensional boxes to be non-overlapping. For each box  $i$  and dimension  $j$ ,  $box\_posn[i, j]$  is the base position of the box in dimension  $j$ , and  $box\_size[i, j]$  is the size in that dimension. Boxes whose size is 0 in any dimension still cannot overlap with any other box.

```
predicate diffn_nonstrict(array [int] of var int: x,
                           array [int] of var int: y,
                           array [int] of var int: dx,
                           array [int] of var int: dy)
```

Constrains rectangles  $i$ , given by their origins ( $x[i], y[i]$ ) and sizes ( $dx[i], dy[i]$ ), to be non-overlapping. Zero-width rectangles can be packed anywhere.

```
predicate diffn_nonstrict_k(array [int,int] of var int: box_posn,
                            array [int,int] of var int: box_size)
```

Constrains  $k$ -dimensional boxes to be non-overlapping. For each box  $i$  and dimension  $j$ ,  $box\_posn[i, j]$  is the base position of the box in dimension  $j$ , and  $box\_size[i, j]$  is the size in that dimension. Boxes whose size is 0 in at least one dimension can be packed anywhere.

```
predicate geost(int: k,
                array [int,int] of int: rect_size,
                array [int,int] of int: rect_offset,
                array [int] of set of int: shape,
                array [int,int] of var int: x,
```

```
array [int] of var int: kind)
```

A global non-overlap constraint for k dimensional objects. It enforces that no two objects overlap.

Parameters:

- k: the number of dimensions
- rect\_size: the size of each box in k dimensions
- rect\_offset: the offset of each box from the base position in k dimensions
- shape: the set of rectangles defining the i -th shape. Assumption: Each pair of boxes in a shape must not overlap.
- x: the base position of each object.  $x [ i , j ]$  is the position of object i in dimension j .
- kind: the shape used by each object.

```
predicate geost_bb(int: k,
                    array [int,int] of int: rect_size,
                    array [int,int] of int: rect_offset,
                    array [int] of set of int: shape,
                    array [int,int] of var int: x,
                    array [int] of var int: kind,
                    array [int] of var int: l,
                    array [int] of var int: u)
```

A global non-overlap constraint for k dimensional objects. It enforces that no two objects overlap, and that all objects fit within a global k dimensional bounding box.

Parameters:

- k: the number of dimensions
- rect\_size: the size of each box in k dimensions
- rect\_offset: the offset of each box from the base position in k dimensions
- shape: the set of rectangles defining the i -th shape. Assumption: Each pair of boxes in a shape must not overlap.
- x: the base position of each object.  $x [ i , j ]$  is the position of object i in dimension j .
- kind: the shape used by each object.

- $l$ : is an array of lower bounds,  $l[i]$  is the minimum bounding box for all objects in dimension  $i$ .
- $u$ : is an array of upper bounds,  $u[i]$  is the maximum bounding box for all objects in dimension  $i$ .

```
predicate geost_smallest_bb(int: k,
                           array [int,int] of int: rect_size,
                           array [int,int] of int: rect_offset,
                           array [int] of set of int: shape,
                           array [int,int] of var int: x,
                           array [int] of var int: kind,
                           array [int] of var int: l,
                           array [int] of var int: u)
```

A global non-overlap constraint for  $k$  dimensional objects. It enforces that no two objects overlap, and that all objects fit within a global  $k$  dimensional bounding box. In addition, it enforces that the bounding box is the smallest one containing all objects, i.e., each of the  $2k$  boundaries is touched by at least by one object.

Parameters:

- $k$ : the number of dimensions
- $\text{rect\_size}$ : the size of each box in  $k$  dimensions
- $\text{rect\_offset}$ : the offset of each box from the base position in  $k$  dimensions
- $\text{shape}$ : the set of rectangles defining the  $i$ -th shape. Assumption: Each pair of boxes in a shape must not overlap.
- $x$ : the base position of each object.  $x[i, j]$  is the position of object  $i$  in dimension  $j$ .
- $\text{kind}$ : the shape used by each object.
- $l$ : is an array of lower bounds,  $l[i]$  is the minimum bounding box for all objects in dimension  $i$ .
- $u$ : is an array of upper bounds,  $u[i]$  is the maximum bounding box for all objects in dimension  $i$ .

```
predicate knapsack(array [int] of int: w,
                   array [int] of int: p,
                   array [int] of var int: x,
```

```
var int: W,
var int: P)
```

Requires that items are packed in a knapsack with certain weight and profit restrictions.

Assumptions:

- Weights w and profits p must be non-negative
- w , p and x must have the same index sets

Parameters:

- w: weight of each type of item
- p: profit of each type of item
- x: number of items of each type that are packed
- W: sum of sizes of all items in the knapsack
- P: sum of profits of all items in the knapsack

#### 4.2.1.7 Scheduling constraints

```
predicate alternative(var opt int: s0,
                     var int: d0,
                     array [int] of var opt int: s,
                     array [int] of var int: d)
```

Alternative constraint for optional tasks. Task ( s<sub>0</sub> , d<sub>0</sub> ) spans the optional tasks ( s [ i ], d [ i ] ) in the array arguments and at most one can occur

```
predicate cumulative(array [int] of var int: s,
                     array [int] of var int: d,
                     array [int] of var int: r,
                     var int: b)
```

Requires that a set of tasks given by start times s , durations d , and resource requirements r , never require more than a global resource bound b at any one time.

Assumptions:

- $\text{forall } i, d[i] \geq 0 \text{ and } r[i] \geq 0$

```
predicate cumulative(array [int] of var opt int: s,
                     array [int] of var int: d,
                     array [int] of var int: r,
                     var int: b)
```

Requires that a set of tasks given by start times  $s$ , durations  $d$ , and resource requirements  $r$ , never require more than a global resource bound  $b$  at any one time. Start times are optional variables, so that absent tasks do not need to be scheduled.

Assumptions: -  $\text{forall } i, d[i] \geq 0 \text{ and } r[i] \geq 0$

```
predicate disjunctive(array [int] of var int: s,
                      array [int] of var int: d)
```

Requires that a set of tasks given by start times  $s$  and durations  $d$  do not overlap in time. Tasks with duration 0 can be scheduled at any time, even in the middle of other tasks.

Assumptions:

- $\text{forall } i, d[i] \geq 0$

```
predicate disjunctive(array [int] of var opt int: s,
                      array [int] of var int: d)
```

Requires that a set of tasks given by start times  $s$  and durations  $d$  do not overlap in time. Tasks with duration 0 can be scheduled at any time, even in the middle of other tasks. Start times are optional variables, so that absent tasks do not need to be scheduled.

Assumptions:

- $\text{forall } i, d[i] \geq 0$

```
predicate disjunctive_strict(array [int] of var int: s,
                             array [int] of var int: d)
```

Requires that a set of tasks given by start times  $s$  and durations  $d$  do not overlap in time. Tasks with duration 0 CANNOT be scheduled at any time, but only when no other task is running.

Assumptions:

- `forall i, d [ i ] >= 0`

```
predicate disjunctive_strict(array [int] of var opt int: s,
                             array [int] of var int: d)
```

Requires that a set of tasks given by start times `s` and durations `d` do not overlap in time. Tasks with duration 0 CANNOT be scheduled at any time, but only when no other task is running. Start times are optional variables, so that absent tasks do not need to be scheduled.

Assumptions:

- `forall i, d [ i ] >= 0`

```
predicate span(var opt int: s0,
              var int: d0,
              array [int] of var opt int: s,
              array [int] of var int: d)
```

Span constraint for optional tasks. Task (`s0, d0`) spans the optional tasks (`s [ i ], d [ i ]`) in the array arguments.

#### 4.2.1.8 Extensional constraints (table, regular etc.)

```
predicate regular(array [int] of var int: x,
                  int: Q,
                  int: S,
                  array [int,int] of int: d,
                  int: q0,
                  set of int: F)
```

The sequence of values in array `x` (which must all be in the range `1.. S`) is accepted by the DFA of `Q` states with input `1.. S` and transition function `d` (which maps `(1.. Q, 1.. S) -> 0.. Q`) and initial state `q0` (which must be in `1.. Q`) and accepting states `F` (which all must be in `1.. Q`). We reserve state 0 to be an always failing state.

```
predicate regular(array [int] of var int: x, string: r)
```

The sequence of values in array  $x$  is accepted by the regular expression  $r$ . This constraint generates its DFA equivalent.

Regular expressions can use the following syntax:

- Selection:
  - Concatenation: “12 34”, 12 followed by 34. (Characters are assumed to be the part of the same number unless split by syntax or whitespace.)
  - Union: “7|11”, a 7 or 11.
  - Groups: “7(6|8)”, a 7 followed by a 6 or an 8.
  - Wildcard: “.”, any value within the domain.
  - Classes: “[3-6 7]”, a 3,4,5,6, or 7.
  - Negated classes: “[^3 5]”, any value within the domain except for a 3 or a 5.
- Quantifiers:
  - Asterisk: “12\*”, 0 or more times a 12.
  - Question mark: “5?”, 0 or 1 times a 5. (optional)
  - Plus sign: “42+”, 1 or more time a 42.
  - Exact: “1{3}”, exactly 3 times a 1.
  - At least: “9{5,}”, 5 or more times a 9.
  - Between: “7{3,5}”, at least 3 times, but at most 5 times a 7.

Members of enumerated types can be used in place of any integer (e.g., “A B”, A followed by B). Enumerated identifiers still use whitespace for concatenation.

```
predicate regular_nfa(array [int] of var int: x,
                      int: Q,
                      int: S,
                      array [int,int] of set of int: d,
                      int: q0,
                      set of int: F)
```

The sequence of values in array  $x$  (which must all be in the range  $1..S$ ) is accepted by the NFA of  $Q$  states with input  $1..S$  and transition function  $d$  (which maps  $(1..Q, 1..S) \rightarrow \text{set of } 1..Q$ )

)) and initial state  $q_0$  (which must be in  $1.. Q$ ) and accepting states  $F$  (which all must be in  $1.. Q$ ).

```
predicate table(array [int] of var bool: x, array [int,int] of bool: t)
```

Represents the constraint  $x$  in  $t$  where we consider each row in  $t$  to be a tuple and  $t$  as a set of tuples.

```
predicate table(array [int] of var int: x, array [int,int] of int: t)
```

Represents the constraint  $x$  in  $t$  where we consider each row in  $t$  to be a tuple and  $t$  as a set of tuples.

#### 4.2.1.9 Other declarations

```
function var int: arg_max(array [int] of var int: x)
```

Returns the index of the maximum value in the array  $x$ . When breaking ties the least index is returned.

```
function var int: arg_max(array [int] of var float: x)
```

Returns the index of the maximum value in the array  $x$ . When breaking ties the least index is returned.

```
function var int: arg_min(array [int] of var int: x)
```

Returns the index of the minimum value in the array  $x$ . When breaking ties the least index is returned.

```
function var int: arg_min(array [int] of var float: x)
```

Returns the index of the minimum value in the array  $x$ . When breaking ties the least index is returned.

```
predicate circuit(array [int] of var int: x)
```

Constrains the elements of  $x$  to define a circuit where  $x[i] = j$  means that  $j$  is the successor of  $i$ .

```
predicate disjoint(var set of int: s1, var set of int: s2)
```

Requires that sets  $s1$  and  $s2$  do not intersect.

```
predicate maximum(var int: m, array [int] of var int: x)
```

Constrains  $m$  to be the maximum of the values in  $x$ .

Assumptions:  $|x| > 0$ .

```
predicate maximum(var float: m, array [int] of var float: x)
```

Constrains  $m$  to be the maximum of the values in  $x$ .

Assumptions:  $|x| > 0$ .

```
predicate maximum_arg(array [int] of var int: x, var int: i)
```

Constrain  $i$  to be the index of the maximum value in the array  $x$ . When breaking ties the least index is returned.

Assumption:  $|x| > 0$

```
predicate maximum_arg(array [int] of var float: x, var int: i)
```

Constrain  $i$  to be the index of the maximum value in the array  $x$ . When breaking ties the least index is returned.

Assumption:  $|x| > 0$

```
predicate member(array [int] of var bool: x, var bool: y)
```

Requires that  $y$  occurs in the array  $x$ .

```
predicate member(array [int] of var float: x, var float: y)
```

Requires that y occurs in the array x .

```
predicate member(array [int] of var int: x, var int: y)
```

Requires that y occurs in the array x .

```
predicate member(array [int] of var set of int: x, var set of int: y)
```

Requires that y occurs in the array x .

```
predicate member(var set of int: x, var int: y)
```

Requires that y occurs in the set x .

```
predicate minimum(var float: m, array [int] of var float: x)
```

Constrains m to be the minimum of the values in x .

Assumptions:  $|x| > 0$ .

```
predicate minimum(var int: m, array [int] of var int: x)
```

Constrains m to be the minimum of the values in x .

Assumptions:  $|x| > 0$ .

```
predicate minimum_arg(array [int] of var int: x, var int: i)
```

Constrain i to be the index of the minimum value in the array x . When breaking ties the least index is returned.

Assumption:  $|x| > 0$

```
predicate minimum_arg(array [int] of var float: x, var int: i)
```

Constrain  $i$  to be the index of the minimum value in the array  $x$ . When breaking ties the least index is returned.

Assumption:  $|x| > 0$

```
predicate network_flow(array [int,1..2] of int: arc,
                      array [int] of int: balance,
                      array [int] of var int: flow)
```

Defines a network flow constraint.

Parameters:

- $arc$ : a directed arc of the flow network. Arc  $i$  connects node  $arc[i,1]$  to node  $arc[i,2]$ .
- $balance$ : the difference between input and output flow for each node.
- $flow$ : the flow going through each arc.

```
predicate network_flow_cost(array [int,1..2] of int: arc,
                           array [int] of int: balance,
                           array [int] of int: weight,
                           array [int] of var int: flow,
                           var int: cost)
```

Defines a network flow constraint with cost.

Parameters:

- $arc$ : a directed arc of the flow network. Arc  $i$  connects node  $arc[i,1]$  to node  $arc[i,2]$ .
- $balance$ : the difference between input and output flow for each node.
- $weight$ : the unit cost of the flow through the arc.
- $flow$ : the flow going through each arc.
- $cost$ : the overall cost of the flow.

```
predicate partition_set(array [int] of var set of int: S,
                       set of int: universe)
```

Constrains the sets in array S to partition the universe .

```
predicate range(array [int] of var int: x,
               var set of int: s,
               var set of int: t)
```

Requires that the image of function x (represented as an array) on set of values s is t . ub( s ) must be a subset of index\_set( x ) otherwise an assertion failure will occur.

```
function var set of int: range(array [int] of var int: x,
                               var set of int: s)
```

Returns the image of function x (represented as an array) on set of values s . ub( s ) must be a subset of index\_set( x ) otherwise an assertion failure will occur.

```
predicate roots(array [int] of var int: x,
               var set of int: s,
               var set of int: t)
```

Requires that x [ i ] in t for all i in s

```
function var set of int: roots(array [int] of var int: x,
                               var set of int: t)
```

Returns s such that x [ i ] in t for all i in s

```
predicate sliding_sum(int: low,
                     int: up,
                     int: seq,
                     array [int] of var int: vs)
```

Requires that in each subsequence vs [ i ], ⋯, vs [ i + seq - 1] the sum of the values belongs to the interval [ low , up ].

```
predicate subcircuit(array [int] of var int: x)
```

Constrains the elements of x to define a subcircuit where x [ i ] = j means that j is the successor of i and x [ i ] = i means that i is not in the circuit.

```
predicate sum_pred(var int: i,
                    array [int] of set of int: sets,
                    array [int] of int: cs,
                    var int: s)
```

Requires that the sum of  $cs[i_1]..cs[i_N]$  equals  $s$ , where  $i_1..i_N$  are the elements of the  $i$ th set in  $sets$ .

Nb: not called ‘sum’ as in the constraints catalog because ‘sum’ is a MiniZinc built-in function.

## 4.2.2 Annotations

These annotations control evaluation and solving behaviour.

### 4.2.2.1 General annotations

#### Parameters

```
annotation add_to_output
```

Declare that the annotated variable should be added to the output of the model. This annotation only has an effect when the model does not have an output item.

```
annotation is_defined_var
```

Declare the annotated variable as being functionally defined. This annotation is introduced into FlatZinc code by the compiler.

```
annotation is_reverse_map
```

Declare that the annotated expression is used to map an expression back from FlatZinc to MiniZinc.

```
annotation maybe_partial
```

Declare that expression may have undefined result (to avoid warnings)

```
annotation mzn_break_here
```

With debug build of mzn2fzn, call MiniZinc::mzn\_break\_here when flattening this expression to make debugging easier. This annotation is ignored by the release build.

```
annotation mzn_check_var
```

Declare that the annotated variable is required for checking solutions.

```
annotation mzn_rhs_from_assignment
```

Used internally by the compiler

```
annotation output_only
```

Declare that the annotated variable should be only used for output. This annotation can be used to define variables that are required for solution checkers, or that are necessary for the output item. The annotated variable must be par.

```
annotation output_var
```

Declare that the annotated variable should be printed by the solver. This annotation is introduced into FlatZinc code by the compiler.

```
annotation promise_total
```

Declare function as total, i.e. it does not put any constraints on its arguments.

```
annotation var_is_introduced
```

Declare a variable as being introduced by the compiler.

## Functions and Predicates

```
annotation constraint_name(string: s)
```

Used to attach a name s to a constraint and its decomposition. String annotations on constraint keywords are re-written as constraint\_name annotations

```
annotation defines_var(var $t: c)
```

Declare variable: c as being functionally defined by the annotated constraint. This annotation is introduced into FlatZinc code by the compiler.

```
annotation doc_comment(string: s)
```

Document the function or variable declaration item with the string s .

```
annotation expression_name(string: s)
```

Used to attach a name s to an expression, this should also propagate to any sub-expressions or decomposition of the annotated expression. String annotations on expressions are re-written as expression\_name annotations

```
annotation mzn_check_enum_var(set of int)
```

Declare that the annotated variable is required for checking solutions and has an enum type.

```
annotation mzn_constraint_name(string)
```

Declare a name for the annotated constraint.

```
annotation mzn_expression_name(string)
```

Declare a name for the annotated expression.

```
annotation mzn_path(string: s)
```

Representation of the call-stack when the annotated item was introduced, as a string s . Can be used to uniquely identify variables and constraints across different compilations of a model that may have different names. This annotation is introduced into FlatZinc code by the compiler and is retained if –keep-paths argument is used.

```
annotation output_array(array [$u] of set of int: a)
```

Declare that the annotated array should be printed by the solver with the given index sets `a`. This annotation is introduced into FlatZinc code by the compiler.

#### 4.2.2.2 Propagation strength annotations

```
annotation bounds
```

Annotate a constraint to use bounds propagation

```
annotation domain
```

Annotate a constraint to use domain propagation

#### 4.2.2.3 Search annotations

##### Variable selection annotations

```
annotation anti_first_fail
```

Choose the variable with the largest domain

```
annotation dom_w_deg
```

Choose the variable with largest domain, divided by the number of attached constraints weighted by how often they have caused failure

```
annotation first_fail
```

Choose the variable with the smallest domain

```
annotation impact
```

Choose the variable with the highest impact so far during the search

```
annotation input_order
```

Search variables in the given order

```
annotation largest
```

Choose the variable with the largest value in its domain

```
annotation max_regret
```

Choose the variable with largest difference between the two smallest values in its domain

```
annotation most_constrained
```

Choose the variable with the smallest domain, breaking ties using the number of attached constraints

```
annotation occurrence
```

Choose the variable with the largest number of attached constraints

```
annotation smallest
```

Choose the variable with the smallest value in its domain

### Value choice annotations

```
annotation indomain
```

Assign values in ascending order

```
annotation indomain_interval
```

If the domain consists of several contiguous intervals, reduce the domain to the first interval. Otherwise bisect the domain.

```
annotation indomain_max
```

Assign the largest value in the domain

```
annotation indomain_median
```

Assign the middle value in the domain

```
annotation indomain_middle
```

Assign the value in the domain closest to the mean of its current bounds

```
annotation indomain_min
```

Assign the smallest value in the domain

```
annotation indomain_random
```

Assign a random value from the domain

```
annotation indomain_reverse_split
```

Bisect the domain, excluding the lower half first

```
annotation indomain_split
```

Bisect the domain, excluding the upper half first

```
annotation indomain_split_random
```

Bisect the domain, randomly selecting which half to exclude first

```
annotation outdomain_max
```

Exclude the largest value from the domain

```
annotation outdomain_median
```

Exclude the middle value from the domain

```
annotation outdomain_min
```

Exclude the smallest value from the domain

```
annotation outdomain_random
```

Exclude a random value from the domain

### Exploration strategy annotations

```
annotation complete
```

Perform a complete search

### Restart annotations

### Parameters

```
annotation restart_none
```

Do not restart

### Functions and Predicates

```
annotation restart_constant(int: scale)
```

MINIMIZER Handbook, Release 2.2.2

Restart after constant number of nodes scale

```
annotation restart_geometric(float: base, int: scale)
```

Restart with geometric sequence with parameters base and scale

```
annotation restart_linear(int: scale)
```

Restart with linear sequence scaled by scale

```
annotation restart_luby(int: scale)
```

Restart with Luby sequence scaled by scale

## Other declarations

```
annotation bool_search(array [$X] of var bool: x,  
                      ann: select,  
                      ann: choice,  
                      ann: explore)
```

Specify search on variables `x` , with variable selection strategy `select` , value choice strategy `choice` , and exploration strategy `explore` . If `x` is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the `array1d` function).

```
annotation bool_search(array [$X] of var bool: x,  
                      ann: select,  
                      ann: choice)
```

Specify search on variables `x` , with variable selection strategy `select` , and value choice strategy `choice` . If `x` is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the `array1d` function).

```
annotation float_search(array [$X] of var float: x,  
                       float: prec,  
                       ann: select,  
                       ann: choice,
```

```
    ann: explore)
```

Specify search on variables `x` , with precision `prec` , variable selection strategy `select` , value choice strategy `choice` , and exploration strategy `explore` . If `x` is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the `array1d` function).

```
annotation float_search(array [$X] of var float: x,
                        float: prec,
                        ann: select,
                        ann: choice)
```

Specify search on variables `x` , with precision `prec` , variable selection strategy `select` , and value choice strategy `choice` . If `x` is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the `array1d` function).

```
annotation int_search(array [$X] of var int: x,
                      ann: select,
                      ann: choice,
                      ann: explore)
```

Specify search on variables `x` , with variable selection strategy `select` , value choice strategy `choice` , and exploration strategy `explore` . If `x` is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the `array1d` function).

```
annotation int_search(array [$X] of var int: x,
                      ann: select,
                      ann: choice)
```

Specify search on variables `x` , with variable selection strategy `select` , and value choice strategy `choice` . If `x` is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the `array1d` function).

```
annotation seq_search(array [int] of ann: s)
```

Sequentially perform the searches specified in array `s`

```
annotation set_search(array [$X] of var set of int: x,
                     ann: select,
                     ann: choice,
                     ann: explore)
```

Specify search on variables  $x$  , with variable selection strategy `select` , value choice strategy `choice` , and exploration strategy `explore` . If  $x$  is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the `array1d` function).

```
annotation set_search(array [$X] of var set of int: x,
                     ann: select,
                     ann: choice)
```

Specify search on variables  $x$  , with variable selection strategy `select` , and value choice strategy `choice` . If  $x$  is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the `array1d` function).

#### 4.2.2.4 Warm start annotations

To be put on the solve item, similar to search annotations. A variable can be mentioned several times and in different annotations but only one of the values is taken

##### **Warm start annotations with optional values**

The value arrays can contain `<>` elements (absent values). The following decompositions eliminate those elements because FlatZinc 1.6 does not support optionals.

```
annotation warm_start(array [int] of var bool: x,
                      array [int] of opt bool: v)
```

Specify warm start values  $v$  for an array of booleans  $x$

```
annotation warm_start(array [int] of var int: x,
                      array [int] of opt int: v)
```

Specify warm start values  $v$  for an array of integers  $x$

```
annotation warm_start(array [int] of var float: x,
                      array [int] of opt float: v)
```

Specify warm start values v for an array of floats x

```
annotation warm_start(array [int] of var set of int: x,
                      array [int] of opt set of int: v)
```

Specify warm start values v for an array of sets x

## Other declarations

```
annotation warm_start(array [int] of var bool: x,
                      array [int] of bool: v)
```

Specify warm start values v for an array of booleans x

```
annotation warm_start(array [int] of var int: x, array [int] of int: v)
```

Specify warm start values v for an array of integers x

```
annotation warm_start(array [int] of var float: x,
                      array [int] of float: v)
```

Specify warm start values v for an array of floats x

```
annotation warm_start(array [int] of var set of int: x,
                      array [int] of set of int: v)
```

Specify warm start values v for an array of sets x

```
annotation warm_start_array(array [int] of ann: w)
```

Specify an array w of warm\_start annotations or other warm\_start\_array annotations. Can be useful to keep the annotation order in FlatZinc for manual updating.

Note: if you have search annotations as well, put `warm_starts` into `seq_search` in order to have precedence between both, which may matter.

### 4.2.3 Option type support

These functions and predicates implement the standard library for working with option types.  
Note that option type support is still incomplete.

#### 4.2.3.1 Option type support for Booleans

```
predicate 'not'(var opt bool: x)
```

Usage: `not x`

True iff `x` is absent or false

```
predicate absent(var opt bool: x)
```

True iff `x` is absent

```
predicate absent(var opt int: x)
```

True iff `x` is absent

```
predicate bool_eq(var opt bool: b0, var opt bool: b1)
```

True iff both `b0` and `b1` are absent or both are present and have the same value.

```
predicate bool_eq(var opt bool: b0, var bool: b1)
```

True iff `b0` occurs and is equal to `b1`

```
predicate bool_eq(var bool: b0, var opt bool: b1)
```

True iff `b1` occurs and is equal to `b0`

```
predicate deopt(var opt bool: x)
```

Return value of x (assumes that x is not absent)

```
function var int: deopt(var opt int: x)
```

Return value of x (assumes that x is not absent)

```
function var opt bool: element(var opt int: idx,
                                array [int] of var bool: x)
```

Return absent if idx is absent, otherwise return x [ idx ]

```
function var opt bool: element(var opt int: idx1,
                               var opt int: idx2,
                               array [int,int] of var bool: x)
```

Return absent if idx1 or idx2 is absent, otherwise return x [ idx1 , idx2 ]

```
function var opt bool: element(var int: idx,
                               array [int] of var opt bool: x)
```

Return x [ idx ]

```
function var opt bool: element(var int: idx1,
                               var int: idx2,
                               array [int,int] of var opt bool: x)
```

Return x [ idx1 , idx2 ]

```
function var opt bool: element(var opt int: idx,
                               array [int] of var opt bool: x)
```

Return absent if idx is absent, otherwise return x [ idx ]

```
function var opt bool: element(var opt int: idx1,
                               var opt int: idx2,
                               array [int,int] of var opt bool: x)
```

Return absent if idx1 or idx2 is absent, otherwise return x [ idx1 , idx2 ]

```
predicate exists(array [int] of var opt bool: x)
```

True iff for at least one i , x [i] occurs and is true

```
predicate forall(array [int] of var opt bool: x)
```

True iff for any i , x [i] is absent or true

```
predicate occurs(var opt bool: x)
```

True iff x is not absent

#### 4.2.3.2 Option type support for integers

```
predicate '<'(var opt int: x, var opt int: y)
```

Usage: x < y

Weak comparison: true iff either x or y is absent, or both occur and the value of x is less than the value of y .

```
test '<'(opt int: x, opt int: y)
```

Usage: x < y

Weak comparison: true iff either x or y is absent, or both occur and the value of x is less than the value of y .

```
predicate '<='(var opt int: x, var opt int: y)
```

Usage: `x <= y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is less than or equal to the value of `y`.

```
test '<='(opt int: x, opt int: y)
```

Usage: `x <= y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is less than or equal to the value of `y`.

```
predicate '>'(var opt int: x, var opt int: y)
```

Usage: `x > y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is greater than the value of `y`.

```
test '>'(opt int: x, opt int: y)
```

Usage: `x > y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is greater than the value of `y`.

```
predicate '>='(var opt int: x, var opt int: y)
```

Usage: `x >= y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is greater than or equal to the value of `y`.

```
test '>='(opt int: x, opt int: y)
```

Usage: `x >= y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is greater than or equal to the value of `y`.

```
function var opt int: bool2int(var opt bool: x)
```

Return optional 0/1 integer that is absent iff x is absent, and 1 iff x occurs and is true.

```
function var opt int: element(var opt int: idx,
                           array [int] of var int: x)
```

Return absent if idx is absent, otherwise return x [ idx ]

```
function var opt int: element(var opt int: idx1,
                           var opt int: idx2,
                           array [int,int] of var int: x)
```

Return absent if idx1 or idx2 is absent, otherwise return x [ idx1 , idx2 ]

```
function var opt int: element(var int: idx,
                           array [int] of var opt int: x)
```

Return x [ idx ]

```
function var opt int: element(var int: idx1,
                           var int: idx2,
                           array [int,int] of var opt int: x)
```

Return x [ idx1 , idx2 ]

```
function var opt int: element(var opt int: idx,
                           array [int] of var opt int: x)
```

Return absent if idx is absent, otherwise return x [ idx ]

```
function var opt int: element(var opt int: idx1,
                           var opt int: idx2,
                           array [int,int] of var opt int: x)
```

Return absent if idx1 or idx2 is absent, otherwise return x [ idx1 , idx2 ]

```
predicate int_eq(var opt int: x, var opt int: y)
```

True iff both  $x$  and  $y$  are absent or both are present and have the same value.

```
predicate int_ne(var opt int: x, var opt int: y)
```

True iff only one of  $x$  and  $y$  is absent or both are present and have different values.

```
function var opt int: max(array [int] of var opt int: x)
```

Return maximum of elements in  $x$  that are not absent, or absent if all elements in  $x$  are absent.

```
function var opt int: min(array [int] of var opt int: x)
```

Return minimum of elements in  $x$  that are not absent, or absent if all elements in  $x$  are absent.

```
predicate occurs(var opt int: x)
```

True iff  $x$  is not absent

```
function var int: product(array [int] of var opt int: x)
```

Return product of non-absent elements of  $x$ .

```
function var int: sum(array [int] of var opt int: x)
```

Return sum of non-absent elements of  $x$ .

```
function var opt int: ~*(var opt int: x, var opt int: y)
```

Weak multiplication. Return product of  $x$  and  $y$  if both are present, otherwise return absent.

```
function var opt int: ~+(var opt int: x, var opt int: y)
```

Weak addition. Return sum of  $x$  and  $y$  if both are present, otherwise return absent.

```
function var opt int: ~-(var opt int: x, var opt int: y)
```

Weak subtraction. Return difference of  $x$  and  $y$  if both are present, otherwise return absent.

```
predicate ~=(var opt int: x, var opt int: y)
```

Weak equality. True if either  $x$  or  $y$  are absent, or present and equal.

#### 4.2.3.3 Other declarations

```
test absent(var $T: x)
```

Test if  $x$  is absent (always returns false)

```
test absent(opt $T: x)
```

Test if  $x$  is absent

```
function $T: deopt(opt $T: x)
```

Return value of  $x$  if  $x$  is not absent. Aborts when evaluated on absent value.

```
function var $T: deopt(var $T: x)
```

Return value  $x$  unchanged (since  $x$  is guaranteed to be non-optional).

```
test occurs(var $T: x)
```

Test if  $x$  is not absent (always returns true)

```
test occurs(opt $T: x)
```

Test if  $x$  is not absent

## 4.2.4 Compiler options

### 4.2.4.1 Parameters

```
opt bool: mzn_opt_only_range_domains
```

Whether to only generate domains that are contiguous ranges

```
opt int: mzn_min_version_required
```

If defined, this can be used to check that the MiniZinc compiler supports all the features used in the model.

### 4.2.4.2 Functions and Predicates

```
test mzn_check_only_range_domains()
```

Check whether to only generate domains that are contiguous ranges

## 4.2.5 Builtins

These functions and predicates define built-in operations of the MiniZinc language.

### 4.2.5.1 Comparison Builtins

These builtins implement comparison operations.

```
test '!='($T: x, $T: y)
```

Usage: `x != y`

Return if `x` is not equal to `y`

```
predicate '!='(var $T: x, var $T: y)
```

Usage: `x != y`

Return if `x` is not equal to `y`

```
test '!='(array [$U] of $T: x, array [$U] of $T: y)
```

Usage: `x != y`

Return if array `x` is not equal to array `y`

```
predicate '!='(array [$U] of var $T: x, array [$U] of var $T: y)
```

Usage: `x != y`

Return if array `x` is not equal to array `y`

```
test '<'($T: x, $T: y)
```

Usage: `x < y`

Return if `x` is less than `y`

```
predicate '<'(var $T: x, var $T: y)
```

Usage: `x < y`

Return if `x` is less than `y`

```
test '<'(array [$U] of $T: x, array [$U] of $T: y)
```

Usage: `x < y`

Return if array `x` is lexicographically smaller than array `y`

```
predicate '<'(array [$U] of var $T: x, array [$U] of var $T: y)
```

Usage: `x < y`

Return if array  $x$  is lexicographically smaller than array  $y$

```
test '<='($T: x, $T: y)
```

Usage:  $x \leq y$

Return if  $x$  is less than or equal to  $y$

```
predicate '<='(var $T: x, var $T: y)
```

Usage:  $x \leq y$

Return if  $x$  is less than or equal to  $y$

```
test '<='(array [$U] of $T: x, array [$U] of $T: y)
```

Usage:  $x \leq y$

Return if array  $x$  is lexicographically smaller than or equal to array  $y$

```
predicate '<='(array [$U] of var $T: x, array [$U] of var $T: y)
```

Usage:  $x \leq y$

Return if array  $x$  is lexicographically smaller than or equal to array  $y$

```
test '='($T: x, $T: y)
```

Usage:  $x = y$

Return if  $x$  is equal to  $y$

```
test '='(opt $T: x, opt $T: y)
```

Usage:  $x = y$

Return if  $x$  is equal to  $y$

```
predicate '='(var $T: x, var $T: y)
```

Usage: `x = y`

Return if `x` is equal to `y`

```
predicate '='(var opt $T: x, var opt $T: y)
```

Usage: `x = y`

Return if `x` is equal to `y`

```
test '='(array [$T] of int: x, array [$T] of int: y)
```

Usage: `x = y`

Return if array `x` is equal to array `y`

```
predicate '='(array [$T] of var int: x, array [$T] of var int: y)
```

Usage: `x = y`

Return if array `x` is equal to array `y`

```
test '='(array [$T] of bool: x, array [$T] of bool: y)
```

Usage: `x = y`

Return if array `x` is equal to array `y`

```
predicate '='(array [$T] of var bool: x, array [$T] of var bool: y)
```

Usage: `x = y`

Return if array `x` is equal to array `y`

```
test '='(array [$T] of set of int: x, array [$T] of set of int: y)
```

Usage: `x = y`

Return if array `x` is equal to array `y`

```
predicate '='(array [\$T] of var set of int: x,
              array [\$T] of var set of int: y)
```

Usage: `x = y`

Return if array `x` is equal to array `y`

```
test '='(array [\$T] of float: x, array [\$T] of float: y)
```

Usage: `x = y`

Return if array `x` is equal to array `y`

```
predicate '='(array [\$T] of var float: x, array [\$T] of var float: y)
```

Usage: `x = y`

Return if array `x` is equal to array `y`

```
test '>'(\$T: x, \$T: y)
```

Usage: `x > y`

Return if `x` is greater than `y`

```
predicate '>'(var \$T: x, var \$T: y)
```

Usage: `x > y`

Return if `x` is greater than `y`

```
test '>'(array [\$U] of \$T: x, array [\$U] of \$T: y)
```

Usage: `x > y`

Return if array `x` is lexicographically greater than array `y`

```
predicate '>'(array [\$U] of var \$T: x, array [\$U] of var \$T: y)
```

Usage: `x > y`

Return if array `x` is lexicographically greater than array `y`

```
test '>='($T: x, $T: y)
```

Usage: `x >= y`

Return if `x` is greater than or equal to `y`

```
predicate '>='(var $T: x, var $T: y)
```

Usage: `x >= y`

Return if `x` is greater than or equal to `y`

```
test '>='(array [$U] of $T: x, array [$U] of $T: y)
```

Usage: `x >= y`

Return if array `x` is lexicographically greater than or equal to array `y`

#### 4.2.5.2 Arithmetic Builtins

These builtins implement arithmetic operations.

```
function int: '*'(int: x, int: y)
```

Usage: `x * y`

Return `x * y`

```
function var int: '*'(var int: x, var int: y)
```

Usage: `x * y`

Return `x * y`

```
function float: '*'(float: x, float: y)
```

Usage:  $x * y$

Return  $x * y$

```
function var float: '*'(var float: x, var float: y)
```

Usage:  $x * y$

Return  $x * y$

```
function int: '+'(int: x, int: y)
```

Usage:  $x + y$

Return  $x + y$

```
function var int: '+'(var int: x, var int: y)
```

Usage:  $x + y$

Return  $x + y$

```
function float: '+'(float: x, float: y)
```

Usage:  $x + y$

Return  $x + y$

```
function var float: '+'(var float: x, var float: y)
```

Usage:  $x + y$

Return  $x + y$

```
function int: '-'(int: x, int: y)
```

Usage:  $x - y$

Return  $x - y$

```
function var int: '-'(var int: x, var int: y)
```

Usage:  $x - y$

Return  $x - y$

```
function float: '-'(float: x, float: y)
```

Usage:  $x - y$

Return  $x - y$

```
function var float: '-'(var float: x, var float: y)
```

Usage:  $x - y$

Return  $x - y$

```
function int: '-'(int: x)
```

Usage:  $- x$

Return negative  $x$

```
function var int: '-'(var int: x)
```

Usage:  $- x$

Return negative  $x$

```
function float: '-'(float: x)
```

Usage:  $- x$

Return negative  $x$

```
function var float: '-'(var float: x)
```

Usage:  $- x$

Return negative x

```
function float: '/'(float: x, float: y)
```

Usage: x / y

Return result of floating point division x / y

```
function var float: '/'(var float: x, var float: y)
```

Usage: x / y

Return result of floating point division x / y

```
function int: '^'(int: x, int: y)
```

Usage: x ^ y

Return x ^ y

```
function var int: '^'(var int: x, var int: y)
```

Usage: x ^ y

Return x ^ y

```
function float: '^'(float: x, float: y)
```

Usage: x ^ y

Return x ^ y

```
function var float: '^'(var float: x, var float: y)
```

Usage: x ^ y

Return x ^ y

```
function int: 'div'(int: x, int: y)
```

Usage: `x div y`

Return result of integer division  $x / y$

```
function var int: 'div'(var int: x, var int: y)
```

Usage: `x div y`

Return result of integer division  $x / y$

```
function int: 'mod'(int: x, int: y)
```

Usage: `x mod y`

Return remainder of integer division  $x \% y$

```
function var int: 'mod'(var int: x, var int: y)
```

Usage: `x mod y`

Return remainder of integer division  $x \% y$

```
function int: abs(int: x)
```

Return absolute value of  $x$

```
function var int: abs(var int: x)
```

Return absolute value of  $x$

```
function float: abs(float: x)
```

Return absolute value of  $x$

```
function var float: abs(var float: x)
```

Return absolute value of  $x$

```
function $$E: arg_max(array [$$E] of int: x)
```

Return index of maximum of elements in array x

```
function $$E: arg_max(array [$$E] of float: x)
```

Return index of maximum of elements in array x

```
function $$E: arg_min(array [$$E] of int: x)
```

Return index of minimum of elements in array x

```
function $$E: arg_min(array [$$E] of float: x)
```

Return index of minimum of elements in array x

```
function $T: max($T: x, $T: y)
```

Return maximum of x and y

```
function $T: max(array [$U] of $T: x)
```

Return maximum of elements in array x

```
function $$E: max(set of $$E: x)
```

Return maximum of elements in set x

```
function var int: max(var int: x, var int: y)
```

Return maximum of x and y

```
function var int: max(array [$U] of var int: x)
```

Return maximum of elements in array x

```
function var float: max(var float: x, var float: y)
```

Return maximum of x and y

```
function var float: max(array [$U] of var float: x)
```

Return maximum of elements in array x

```
function $T: min($T: x, $T: y)
```

Return minimum of x and y

```
function $T: min(array [$U] of $T: x)
```

Return minimum of elements in array x

```
function $$E: min(set of $$E: x)
```

Return minimum of elements in set x

```
function var int: min(var int: x, var int: y)
```

Return minimum of x and y

```
function var int: min(array [$U] of var int: x)
```

Return minimum of elements in array x

```
function var float: min(var float: x, var float: y)
```

Return minimum of x and y

```
function var float: min(array [$U] of var float: x)
```

Return minimum of elements in array x

```
function int: pow(int: x, int: y)
```

Return  $x^y$

```
function var int: pow(var int: x, var int: y)
```

Return  $x^y$

```
function float: pow(float: x, float: y)
```

Return  $x^y$

```
function var float: pow(var float: x, var float: y)
```

Return  $x^y$

```
function int: product(array [$T] of int: x)
```

Return product of elements in array x

```
function var int: product(array [$T] of var int: x)
```

Return product of elements in array x

```
function float: product(array [$T] of float: x)
```

Return product of elements in array x

```
function var float: product(array [$T] of var float: x)
```

Return product of elements in array x

```
function float: sqrt(float: x)
```

Return  $\sqrt{x}$

```
function var float: sqrt(var float: x)
```

Return  $\sqrt{x}$

```
function int: sum(array [\$T] of int: x)
```

Return sum of elements in array x

```
function var int: sum(array [\$T] of var int: x)
```

Return sum of elements in array x

```
function float: sum(array [\$T] of float: x)
```

Return sum of elements in array x

```
function var float: sum(array [\$T] of var float: x)
```

Return sum of elements in array x

#### 4.2.5.3 Exponential and logarithmic builtins

These builtins implement exponential and logarithmic functions.

```
function float: exp(float: x)
```

Return  $e^x$

```
function var float: exp(var float: x)
```

Return  $e^x$

```
function float: ln(float: x)
```

Return  $\ln x$

```
function var float: ln(var float: x)
```

Return  $\ln x$

```
function float: log(float: x, float: y)
```

Return  $\log_x y$

```
function float: log10(float: x)
```

Return  $\log_{10} x$

```
function var float: log10(var float: x)
```

Return  $\log_{10} x$

```
function float: log2(float: x)
```

Return  $\log_2 x$

```
function var float: log2(var float: x)
```

Return  $\log_2 x$

#### 4.2.5.4 Trigonometric functions

These builtins implement the standard trigonometric functions.

```
function float: acos(float: x)
```

Return  $\arccos x$

```
function var float: acos(var float: x)
```

Return  $\arccos x$

```
function float: acosh(float: x)
```

Return  $\text{acosh } x$

```
function var float: acosh(var float: x)
```

Return  $\text{acosh } x$

```
function float: asin(float: x)
```

Return  $\text{asin } x$

```
function var float: asin(var float: x)
```

Return  $\text{asin } x$

```
function float: asinh(float: x)
```

Return  $\text{asinh } x$

```
function var float: asinh(var float: x)
```

Return  $\text{asinh } x$

```
function float: atan(float: x)
```

Return  $\text{atan } x$

```
function var float: atan(var float: x)
```

Return  $\text{atan } x$

```
function float: atanh(float: x)
```

Return  $\text{atanh } x$

```
function var float: atanh(var float: x)
```

Return  $\text{atanh } x$

```
function float: cos(float: x)
```

Return  $\cos x$

```
function var float: cos(var float: x)
```

Return  $\cos x$

```
function float: cosh(float: x)
```

Return  $\cosh x$

```
function var float: cosh(var float: x)
```

Return  $\cosh x$

```
function float: sin(float: x)
```

Return  $\sin x$

```
function var float: sin(var float: x)
```

Return  $\sin x$

```
function float: sinh(float: x)
```

Return  $\sinh x$

```
function var float: sinh(var float: x)
```

Return  $\sinh x$

```
function float: tan(float: x)
```

Return  $\tan x$

```
function var float: tan(var float: x)
```

Return  $\tan x$

```
function float: tanh(float: x)
```

Return  $\tanh x$

```
function var float: tanh(var float: x)
```

Return  $\tanh x$

#### 4.2.5.5 Logical operations

Logical operations are the standard operators of Boolean logic.

```
test ' $\rightarrow$ '(bool: x, bool: y)
```

Usage:  $x \rightarrow y$

Return truth value of  $x$  implies  $y$

```
predicate ' $\rightarrow$ '(var bool: x, var bool: y)
```

Usage:  $x \rightarrow y$

Return truth value of  $x$  implies  $y$

```
test ' $/\wedge$ '(bool: x, bool: y)
```

Usage:  $x /\wedge y$

Return truth value of  $x \& y$

```
predicate '/\\'(var bool: x, var bool: y)
```

Usage:  $x \wedge y$

Return truth value of  $x \wedge y$

```
test '<-'(bool: x, bool: y)
```

Usage:  $x <- y$

Return truth value of  $y \rightarrow x$

```
predicate '<-'(var bool: x, var bool: y)
```

Usage:  $x <- y$

Return truth value of  $y \rightarrow x$

```
test '<->'(bool: x, bool: y)
```

Usage:  $x \leftrightarrow y$

Return truth value of  $x \leftrightarrow y$

```
predicate '<->'(var bool: x, var bool: y)
```

Usage:  $x \leftrightarrow y$

Return truth value of  $x \leftrightarrow y$

```
test '\/\''(bool: x, bool: y)
```

Usage:  $x \vee y$

Return truth value of  $x \vee y$

```
predicate '\/\''(var bool: x, var bool: y)
```

Usage:  $x \vee y$

Return truth value of  $x \&or; y$

```
test 'not'(bool: x)
```

Usage: `not x`

Return truth value of the negation of  $x$

```
predicate 'not'(var bool: x)
```

Usage: `not x`

Return truth value of the negation of  $x$

```
test 'xor'(bool: x, bool: y)
```

Usage: `x xor y`

Return truth value of  $x \text{ xor } y$

```
predicate 'xor'(var bool: x, var bool: y)
```

Usage: `x xor y`

Return truth value of  $x \text{ xor } y$

```
predicate clause(array [$T] of var bool: x, array [$T] of var bool: y)
```

Return truth value of  $(\bigvee_i x[i]) \vee (\bigvee_j \neg y[j])$

```
predicate clause(array [$T] of bool: x, array [$T] of bool: y)
```

Return truth value of  $(\bigvee_i x[i]) \vee (\bigvee_j \neg y[j])$

```
test exists(array [$T] of bool: x)
```

Return truth value of  $\bigvee_i x[i]$

```
predicate exists(array [\$T] of var bool: x)
```

Return truth value of  $\bigvee_i \mathbf{x}[i]$

```
test forall(array [\$T] of bool: x)
```

Return truth value of  $\bigwedge_i \mathbf{x}[i]$

```
predicate forall(array [\$T] of var bool: x)
```

Return truth value of  $\bigwedge_i \mathbf{x}[i]$

```
test iffall(array [\$T] of bool: x)
```

Return truth value of true  $\oplus (\bigoplus_i \mathbf{x}[i])$

```
predicate iffall(array [\$T] of var bool: x)
```

Return truth value of true  $\oplus (\bigoplus_i \mathbf{x}[i])$

```
test xorall(array [\$T] of bool: x)
```

Return truth value of  $\bigoplus_i \mathbf{x}[i]$

```
predicate xorall(array [\$T] of var bool: x)
```

Return truth value of  $\bigoplus_i \mathbf{x}[i]$

#### 4.2.5.6 Set operations

These functions implement the basic operations on sets.

```
function set of $$E: '..'($$E: a, $$E: b)
```

Usage: a .. b

Return the set  $\{a, \dots, b\}$

```
function set of float: '..'(float: a, float: b)
```

Usage:  $a \dots b$

Return the set  $\{a, \dots, b\}$

```
function set of $T: 'diff'(set of $T: x, set of $T: y)
```

Usage:  $x \text{ diff } y$

Return the set difference of sets  $x \setminus y$

```
function var set of $$T: 'diff'(var set of $$T: x, var set of $$T: y)
```

Usage:  $x \text{ diff } y$

Return the set difference of sets  $x \setminus y$

```
test 'in'(int: x, set of int: y)
```

Usage:  $x \text{ in } y$

Test if  $x$  is an element of the set  $y$

```
predicate 'in'(var int: x, var set of int: y)
```

Usage:  $x \text{ in } y$

$x$  is an element of the set  $y$

```
test 'in'(float: x, set of float: y)
```

Usage:  $x \text{ in } y$

Test if  $x$  is an element of the set  $y$

```
predicate 'in'(var float: x, set of float: y)
```

Usage: `x in y`

Test if `x` is an element of the set `y`

```
function set of $T: 'intersect'(set of $T: x, set of $T: y)
```

Usage: `x intersect y`

Return the intersection of sets `x` and `y`

```
function var set of $$T: 'intersect'(var set of $$T: x,
                                         var set of $$T: y)
```

Usage: `x intersect y`

Return the intersection of sets `x` and `y`

```
test 'subset'(set of $T: x, set of $T: y)
```

Usage: `x subset y`

Test if `x` is a subset of `y`

```
predicate 'subset'(var set of int: x, var set of int: y)
```

Usage: `x subset y`

`x` is a subset of `y`

```
test 'superset'(set of $T: x, set of $T: y)
```

Usage: `x superset y`

Test if `x` is a superset of `y`

```
predicate 'superset'(var set of int: x, var set of int: y)
```

Usage: `x superset y`

`x` is a superset of `y`

```
function set of $T: 'symdiff'(set of $T: x, set of $T: y)
```

Usage: `x symdiff y`

Return the symmetric set difference of sets `x` and `y`

```
function var set of $$T: 'symdiff'(var set of $$T: x,
                                    var set of $$T: y)
```

Usage: `x symdiff y`

Return the symmetric set difference of sets `x` and `y`

```
function set of $T: 'union'(set of $T: x, set of $T: y)
```

Usage: `x union y`

Return the union of sets `x` and `y`

```
function var set of $$T: 'union'(var set of $$T: x, var set of $$T: y)
```

Usage: `x union y`

Return the union of sets `x` and `y`

```
function set of $U: array_intersect(array [$T] of set of $U: x)
```

Return the intersection of the sets in array `x`

```
function var set of int: array_intersect(array [int] of var set of int: x)
```

Return the intersection of the sets in array `x`

```
function set of $U: array_union(array [$T] of set of $U: x)
```

Return the union of the sets in array `x`

```
function var set of int: array_union(array [int] of var set of int: x)
```

Return the union of the sets in array x

```
function int: card(set of $T: x)
```

Return the cardinality of the set x

```
function var int: card(var set of int: x)
```

Return the cardinality of the set x

```
function var $$E: max(var set of $$E: s)
```

Return the maximum of the set s

```
function var $$E: min(var set of $$E: s)
```

Return the minimum of the set s

#### 4.2.5.7 Array operations

These functions implement the basic operations on arrays.

```
function array [int] of $T: '+'(array [int] of $T: x,
                                array [int] of $T: y)
```

Usage: x ++ y

Return the concatenation of arrays x and y

```
function array [int] of opt $T: '+'(array [int] of opt $T: x,
                                         array [int] of opt $T: y)
```

Usage: x ++ y

Return the concatenation of arrays x and y

```
function array [int] of var $T: '+'(array [int] of var $T: x,
                                         array [int] of var $T: y)
```

Usage: `x ++ y`

Return the concatenation of arrays `x` and `y`

```
function array [int] of var opt $T: '+'(array [int] of var opt $T: x,
                                         array [int] of var opt $T: y)
```

Usage: `x ++ y`

Return the concatenation of arrays `x` and `y`

```
function array [int] of $V: array1d(array [$U] of $V: x)
```

Return array `x` coerced to index set `1..length(x)`. Coercions are performed by considering the array `x` in row-major order.

```
function array [int] of opt $V: array1d(array [$U] of opt $V: x)
```

Return array `x` coerced to index set `1..length(x)`. Coercions are performed by considering the array `x` in row-major order.

```
function array [int] of var $V: array1d(array [$U] of var $V: x)
```

Return array `x` coerced to index set `1..length(x)`. Coercions are performed by considering the array `x` in row-major order.

```
function array [int] of var opt $V: array1d(array [$U] of var opt $V: x)
```

Return array `x` coerced to index set `1..length(x)`. Coercions are performed by considering the array `x` in row-major order.

```
function array [$$E] of $V: array1d(set of $$E: S, array [$U] of $V: x)
```

Return array `x` coerced to one-dimensional array with index set `S`. Coercions are performed by considering the array `x` in row-major order.

```
function array [$$E] of opt $V: array1d(set of $$E: S,
                                         array [$U] of opt $V: x)
```

Return array  $x$  coerced to one-dimensional array with index set  $S$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [$$E] of var $V: array1d(set of $$E: S,
                                         array [$U] of var $V: x)
```

Return array  $x$  coerced to one-dimensional array with index set  $S$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [$$E] of var opt $V: array1d(set of $$E: S,
                                              array [$U] of var opt $V: x)
```

Return array  $x$  coerced to one-dimensional array with index set  $S$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [$$E,$$F] of $V: array2d(set of $$E: S1,
                                         set of $$F: S2,
                                         array [$U] of $V: x)
```

Return array  $x$  coerced to two-dimensional array with index sets  $S1$  and  $S2$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [$$E,$$F] of opt $V: array2d(set of $$E: S1,
                                              set of $$F: S2,
                                              array [$U] of opt $V: x)
```

Return array  $x$  coerced to two-dimensional array with index sets  $S1$  and  $S2$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [$$E,$$F] of var $V: array2d(set of $$E: S1,
                                              set of $$F: S2,
                                              array [$U] of var $V: x)
```

Return array  $x$  coerced to two-dimensional array with index sets  $S1$  and  $S2$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [ $$E, $$F] of var opt $V: array2d(set of $$E: S1,
                                                 set of $$F: S2,
                                                 array [$U] of var opt $V: x)
```

Return array  $x$  coerced to two-dimensional array with index sets  $S1$  and  $S2$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [ $$E, $$F, $$G] of $V: array3d(set of $$E: S1,
                                                set of $$F: S2,
                                                set of $$G: S3,
                                                array [$U] of $V: x)
```

Return array  $x$  coerced to three-dimensional array with index sets  $S1$ ,  $S2$  and  $S3$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [ $$E, $$F, $$G] of opt $V: array3d(set of $$E: S1,
                                                 set of $$F: S2,
                                                 set of $$G: S3,
                                                 array [$U] of opt $V: x)
```

Return array  $x$  coerced to three-dimensional array with index sets  $S1$ ,  $S2$  and  $S3$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [ $$E, $$F, $$G] of var $V: array3d(set of $$E: S1,
                                                 set of $$F: S2,
                                                 set of $$G: S3,
                                                 array [$U] of var $V: x)
```

Return array  $x$  coerced to three-dimensional array with index sets  $S1$ ,  $S2$  and  $S3$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [ $$E, $$F, $$G] of var opt $V: array3d(set of $$E: S1,
                                                       set of $$F: S2,
                                                       set of $$G: S3,
```

```
array [$$U] of var opt  
↪$V: x)
```

Return array  $x$  coerced to three-dimensional array with index sets  $S_1$ ,  $S_2$  and  $S_3$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [ $$E, $$F, $$G, $$H] of $V: array4d(set of $$E: S1,  
set of $$F: S2,  
set of $$G: S3,  
set of $$H: S4,  
array [$$U] of $V: x)
```

Return array  $x$  coerced to 4-dimensional array with index sets  $S_1$ ,  $S_2$ ,  $S_3$  and  $S_4$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [ $$E, $$F, $$G, $$H] of opt $V: array4d(set of $$E: S1,  
set of $$F: S2,  
set of $$G: S3,  
set of $$H: S4,  
array [$$U] of opt $V: x)
```

Return array  $x$  coerced to 4-dimensional array with index sets  $S_1$ ,  $S_2$ ,  $S_3$  and  $S_4$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [ $$E, $$F, $$G, $$H] of var $V: array4d(set of $$E: S1,  
set of $$F: S2,  
set of $$G: S3,  
set of $$H: S4,  
array [$$U] of var $V: x)
```

Return array  $x$  coerced to 4-dimensional array with index sets  $S_1$ ,  $S_2$ ,  $S_3$  and  $S_4$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [ $$E, $$F, $$G, $$H] of var opt $V: array4d(set of $$E: S1,  
set of $$F: S2,  
set of $$G: S3,  
set of $$H: S4,
```

```

array [$$U] of var_
→opt $$V: x)

```

Return array  $x$  coerced to 4-dimensional array with index sets  $S_1, S_2, S_3$  and  $S_4$ . Coercions are performed by considering the array  $x$  in row-major order.

```

function array [$$E,$$F,$$G,$$H,$$I] of $$V: array5d(set of $$E: S1,
                                                 set of $$F: S2,
                                                 set of $$G: S3,
                                                 set of $$H: S4,
                                                 set of $$I: S5,
                                                 array [$$U] of $$V: x)

```

Return array  $x$  coerced to 5-dimensional array with index sets  $S_1, S_2, S_3, S_4$  and  $S_5$ . Coercions are performed by considering the array  $x$  in row-major order.

```

function array [$$E,$$F,$$G,$$H,$$I] of opt $$V: array5d(set of $$E: S1,
                                                       set of $$F: S2,
                                                       set of $$G: S3,
                                                       set of $$H: S4,
                                                       set of $$I: S5,
                                                       array [$$U] of opt
→$$V: x)

```

Return array  $x$  coerced to 5-dimensional array with index sets  $S_1, S_2, S_3, S_4$  and  $S_5$ . Coercions are performed by considering the array  $x$  in row-major order.

```

function array [$$E,$$F,$$G,$$H,$$I] of var $$V: array5d(set of $$E: S1,
                                                       set of $$F: S2,
                                                       set of $$G: S3,
                                                       set of $$H: S4,
                                                       set of $$I: S5,
                                                       array [$$U] of var
→$$V: x)

```

Return array  $x$  coerced to 5-dimensional array with index sets  $S_1, S_2, S_3, S_4$  and  $S_5$ . Coercions are performed by considering the array  $x$  in row-major order.

```
function array [$$E,$$F,$$G,$$H,$$I] of var opt $V: array5d(set of $$E: S1,  
                                                 set of $$F: S2,  
                                                 set of $$G: S3,  
                                                 set of $$H: S4,  
                                                 set of $$I: S5,  
                                                 array [$U] of _  
→var opt $V: x)
```

Return array `x` coerced to 5-dimensional array with index sets `S1` , `S2` , `S3` , `S4` and `S5` . Coercions are performed by considering the array `x` in row-major order.

```
function array [$$E,$$F,$$G,$$H,$$I,$$J] of $V: array6d(set of $$E: S1,  
                                                 set of $$F: S2,  
                                                 set of $$G: S3,  
                                                 set of $$H: S4,  
                                                 set of $$I: S5,  
                                                 set of $$J: S6,  
                                                 array [$U] of $V: x)
```

Return array `x` coerced to 6-dimensional array with index sets `S1` , `S2` , `S3` , `S4` , `S5` and `S6` . Coercions are performed by considering the array `x` in row-major order.

```
function array [$$E,$$F,$$G,$$H,$$I,$$J] of opt $V: array6d(set of $$E: S1,  
                                                 set of $$F: S2,  
                                                 set of $$G: S3,  
                                                 set of $$H: S4,  
                                                 set of $$I: S5,  
                                                 set of $$J: S6,  
                                                 array [$U] of _  
→opt $V: x)
```

Return array `x` coerced to 6-dimensional array with index sets `S1` , `S2` , `S3` , `S4` , `S5` and `S6` . Coercions are performed by considering the array `x` in row-major order.

```
function array [$$E,$$F,$$G,$$H,$$I,$$J] of var $V: array6d(set of $$E: S1,  
                                                 set of $$F: S2,  
                                                 set of $$G: S3,
```

```

set of $$H: S4,
set of $$I: S5,
set of $$J: S6,
array [\$U] of_
↪var \$V: x)

```

Return array `x` coerced to 6-dimensional array with index sets `S1` , `S2` , `S3` , `S4` , `S5` and `S6` . Coercions are perfomed by considering the array `x` in row-major order.

```

function array [$$E,$$F,$$G,$$H,$$I,$$J] of var opt $V: array6d(set of $$E:_  

↪S1,  

                           set of $$F:_  

↪S2,  

                           set of $$G:_  

↪S3,  

                           set of $$H:_  

↪S4,  

                           set of $$I:_  

↪S5,  

                           set of $$J:_  

↪S6,  

                           array [\$U]_  

↪of var opt $V: x)

```

Return array `x` coerced to 6-dimensional array with index sets `S1` , `S2` , `S3` , `S4` , `S5` and `S6` . Coercions are perfomed by considering the array `x` in row-major order.

```

function array [$T] of $V: arrayXd(array [$T] of var opt $X: x,
                                         array [\$U] of $V: y)

```

Return array `y` coerced to array with same number of dimensions and same index sets as array `x` . Coercions are perfomed by considering the array `y` in row-major order.

```

function array [$T] of opt $V: arrayXd(array [$T] of var opt $X: x,
                                         array [\$U] of opt $V: y)

```

Return array `y` coerced to array with same number of dimensions and same index sets as array `x` . Coercions are perfomed by considering the array `y` in row-major order.

```
function array [\$T] of var \$V: arrayXd(array [\$T] of var opt \$X: x,
                                         array [\$U] of var \$V: y)
```

Return array y coerced to array with same number of dimensions and same index sets as array x . Coercions are performed by considering the array y in row-major order.

```
function array [\$T] of var opt \$V: arrayXd(array [\$T] of var opt \$X: x,
                                             array [\$U] of var opt \$V: y)
```

Return array y coerced to array with same number of dimensions and same index sets as array x . Coercions are performed by considering the array y in row-major order.

```
function array [ $$E] of $T: col(array [ $$E,int] of $T: x, int: c)
```

Return column c of array x

```
function array [ $$E] of opt $T: col(array [ $$E,int] of opt $T: x,
                                         int: c)
```

Return column c of array x

```
function array [ $$E] of var $T: col(array [ $$E,int] of var $T: x,
                                         int: c)
```

Return column c of array x

```
function array [ $$E] of var opt $T: col(array [ $$E,int] of var opt $T: x,
                                         int: c)
```

Return column c of array x

```
test has_element($T: e, array [int] of $T: x)
```

Test if e is an element of array x

```
test has_element($T: e, array [int] of opt $T: x)
```

Test if e is an element of array x

```
predicate has_element($T: e, array [$$E] of var opt $T: x)
```

Test if e is an element of array x

```
test has_index(int: i, array [int] of var opt $T: x)
```

Test if i is in the index set of x

```
function set of $$E: index_set(array [$$E] of var opt $U: x)
```

Return index set of one-dimensional array x

```
function set of $$E: index_set_1of2(array [$$E,int] of var opt $U: x)
```

Return index set of first dimension of two-dimensional array x

```
function set of $$E: index_set_1of3(array [$$E,int,int] of var opt $U: x)
```

Return index set of first dimension of 3-dimensional array x

```
function set of $$E: index_set_1of4(array [$$E,int,int,int] of var opt $U: x)
```

Return index set of first dimension of 4-dimensional array x

```
function set of $$E: index_set_1of5(array [$$E,int,int,int,int] of var opt  
→$U: x)
```

Return index set of first dimension of 5-dimensional array x

```
function set of $$E: index_set_1of6(array [$$E,int,int,int,int,int] of var  
→opt $U: x)
```

Return index set of first dimension of 6-dimensional array  $x$

```
function set of $$E: index_set_2of2(array [int,$$E] of var opt $U: x)
```

Return index set of second dimension of two-dimensional array  $x$

```
function set of $$E: index_set_2of3(array [int,$$E,int] of var opt $U: x)
```

Return index set of second dimension of 3-dimensional array  $x$

```
function set of $$E: index_set_2of4(array [int,$$E,int,int] of var opt $U: x)
```

Return index set of second dimension of 4-dimensional array  $x$

```
function set of $$E: index_set_2of5(array [int,$$E,int,int,int] of var opt  
→ $U: x)
```

Return index set of second dimension of 5-dimensional array  $x$

```
function set of $$E: index_set_2of6(array [int,$$E,int,int,int,int] of var  
→ opt $U: x)
```

Return index set of second dimension of 6-dimensional array  $x$

```
function set of $$E: index_set_3of3(array [int,int,$$E] of var opt $U: x)
```

Return index set of third dimension of 3-dimensional array  $x$

```
function set of $$E: index_set_3of4(array [int,int,$$E,int] of var opt $U: x)
```

Return index set of third dimension of 4-dimensional array  $x$

```
function set of $$E: index_set_3of5(array [int,int,$$E,int,int] of var opt  
→ $U: x)
```

Return index set of third dimension of 5-dimensional array  $x$

```
function set of $$E: index_set_3of6(array [int,int,$$E,int,int,int] of var_
↪opt $U: x)
```

Return index set of third dimension of 6-dimensional array x

```
function set of $$E: index_set_4of4(array [int,int,int,$$E] of var opt $U: x)
```

Return index set of fourth dimension of 4-dimensional array x

```
function set of $$E: index_set_4of5(array [int,int,int,$$E,int] of var opt
↪$U: x)
```

Return index set of fourth dimension of 5-dimensional array x

```
function set of $$E: index_set_4of6(array [int,int,int,$$E,int,int] of var_
↪opt $U: x)
```

Return index set of fourth dimension of 6-dimensional array x

```
function set of $$E: index_set_5of5(array [int,int,int,int,$$E] of var opt
↪$U: x)
```

Return index set of fifth dimension of 5-dimensional array x

```
function set of $$E: index_set_5of6(array [int,int,int,int,$$E,int] of var_
↪opt $U: x)
```

Return index set of fifth dimension of 6-dimensional array x

```
function set of $$E: index_set_6of6(array [int,int,int,int,int,$$E] of var_
↪opt $U: x)
```

Return index set of sixth dimension of 6-dimensional array x

```
test index_sets_agree(array [$T] of var opt $U: x,
                      array [$T] of var opt $W: y)
```

Test if  $x$  and  $y$  have the same index sets

```
function int: length(array [T] of var opt U: x)
```

Return the length of array  $x$

Note that the length is defined as the number of elements in the array, regardless of its dimensionality.

```
function array [E] of T: reverse(array [E] of T: x)
```

Return the array  $x$  in reverse order

The resulting array has the same index set as  $x$ .

```
function array [E] of opt T: reverse(array [E] of opt T: x)
```

Return the array  $x$  in reverse order

The resulting array has the same index set as  $x$ .

```
function array [E] of var T: reverse(array [E] of var T: x)
```

Return the array  $x$  in reverse order

The resulting array has the same index set as  $x$ .

```
function array [E] of var opt T: reverse(array [E] of var opt T: x)
```

Return the array  $x$  in reverse order

The resulting array has the same index set as  $x$ .

```
function array [E] of T: row(array [int,E] of T: x, int: r)
```

Return row  $r$  of array  $x$

```
function array [E] of opt T: row(array [int,E] of opt T: x,
                                int: r)
```

Return row  $r$  of array  $x$

```
function array [$$E] of var $T: row(array [int,$$E] of var $T: x,
                                         int: r)
```

Return row  $r$  of array  $x$

```
function array [$$E] of var opt $T: row(array [int,$$E] of var opt $T: x,
                                         int: r)
```

Return row  $r$  of array  $x$

```
function array [int] of $T: slice_1d(array [$E] of $T: x,
                                         array [int] of set of int: s,
                                         set of int: dims1)
```

Return slice of array  $x$  specified by sets  $s$  , coerced to new 1d array with index set  $\text{dims1}$

```
function array [int] of opt $T: slice_1d(array [$E] of opt $T: x,
                                         array [int] of set of int: s,
                                         set of int: dims1)
```

Return slice of array  $x$  specified by sets  $s$  , coerced to new 1d array with index set  $\text{dims1}$

```
function array [int] of var $T: slice_1d(array [$E] of var $T: x,
                                         array [int] of set of int: s,
                                         set of int: dims1)
```

Return slice of array  $x$  specified by sets  $s$  , coerced to new 1d array with index set  $\text{dims1}$

```
function array [int] of var opt $T: slice_1d(array [$E] of var opt $T: x,
                                         array [int] of set of int: s,
                                         set of int: dims1)
```

Return slice of array  $x$  specified by sets  $s$  , coerced to new 1d array with index set  $\text{dims1}$

```
function array [int,int] of $T: slice_2d(array [$E] of $T: x,
                                         array [int] of set of int: s,
                                         set of int: dims1,
                                         set of int: dims2)
```

Return slice of array x specified by sets s , coerced to new 2d array with index sets dims1 and dims2

```
function array [int,int] of opt $T: slice_2d(array [$E] of opt $T: x,
                                             array [int] of set of int: s,
                                             set of int: dims1,
                                             set of int: dims2)
```

Return slice of array x specified by sets s , coerced to new 2d array with index sets dims1 and dims2

```
function array [int,int] of var $T: slice_2d(array [$E] of var $T: x,
                                              array [int] of set of int: s,
                                              set of int: dims1,
                                              set of int: dims2)
```

Return slice of array x specified by sets s , coerced to new 2d array with index sets dims1 and dims2

```
function array [int,int] of var opt $T: slice_2d(array [$E] of var opt $T: x,
                                                 array [int] of set of int:_
                                                 ↪s,
                                                 set of int: dims1,
                                                 set of int: dims2)
```

Return slice of array x specified by sets s , coerced to new 2d array with index sets dims1 and dims2

```
function array [int,int,int] of $T: slice_3d(array [$E] of $T: x,
                                              array [int] of set of int: s,
                                              set of int: dims1,
                                              set of int: dims2,
                                              set of int: dims3)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims1}$ ,  $\text{dims2}$  and  $\text{dims3}$

```
function array [int,int,int] of opt $T: slice_3d(array [$E] of opt $T: x,
                                                array [int] of set of int:_  

                                                ↵s,  

                                                set of int: dims1,  

                                                set of int: dims2,  

                                                set of int: dims3)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims1}$ ,  $\text{dims2}$  and  $\text{dims3}$

```
function array [int,int,int] of var $T: slice_3d(array [$E] of var $T: x,
                                                array [int] of set of int:_  

                                                ↵s,  

                                                set of int: dims1,  

                                                set of int: dims2,  

                                                set of int: dims3)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims1}$ ,  $\text{dims2}$  and  $\text{dims3}$

```
function array [int,int,int] of var opt $T: slice_3d(array [$E] of var opt  

                                                ↵$T: x,  

                                                array [int] of set of _  

                                                ↵int: s,  

                                                set of int: dims1,  

                                                set of int: dims2,  

                                                set of int: dims3)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims1}$ ,  $\text{dims2}$  and  $\text{dims3}$

```
function array [int,int,int] of $T: slice_4d(array [$E] of $T: x,  

                                                array [int] of set of int: s,  

                                                set of int: dims1,  

                                                set of int: dims2,
```

```
    set of int: dims3,  
    set of int: dims4)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims1}$ ,  $\text{dims2}$ ,  $\text{dims3}$ ,  $\text{dims4}$

```
function array [int,int,int] of opt $T: slice_4d(array [$E] of opt $T: x,  
                                              array [int] of set of int:  
                                              ↳s,  
                                              set of int: dims1,  
                                              set of int: dims2,  
                                              set of int: dims3,  
                                              set of int: dims4)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims1}$ ,  $\text{dims2}$ ,  $\text{dims3}$ ,  $\text{dims4}$

```
function array [int,int,int] of var $T: slice_4d(array [$E] of var $T: x,  
                                              array [int] of set of int:  
                                              ↳s,  
                                              set of int: dims1,  
                                              set of int: dims2,  
                                              set of int: dims3,  
                                              set of int: dims4)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims1}$ ,  $\text{dims2}$ ,  $\text{dims3}$ ,  $\text{dims4}$

```
function array [int,int,int] of var opt $T: slice_4d(array [$E] of var opt  
                                              ↳$T: x,  
                                              array [int] of set of  
                                              ↳int: s,  
                                              set of int: dims1,  
                                              set of int: dims2,  
                                              set of int: dims3,  
                                              set of int: dims4)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims1}$ ,  $\text{dims2}$

, dims3 , dims4

```
function array [int,int,int] of $T: slice_5d(array [$E] of $T: x,
                                              array [int] of set of int: s,
                                              set of int: dims1,
                                              set of int: dims2,
                                              set of int: dims3,
                                              set of int: dims4,
                                              set of int: dims5)
```

Return slice of array x specified by sets s , coerced to new 3d array with index sets dims1 , dims2 , dims3 , dims4 , dims5

```
function array [int,int,int] of opt $T: slice_5d(array [$E] of opt $T: x,
                                                array [int] of set of int:_
                                                ↵s,
                                                set of int: dims1,
                                                set of int: dims2,
                                                set of int: dims3,
                                                set of int: dims4,
                                                set of int: dims5)
```

Return slice of array x specified by sets s , coerced to new 3d array with index sets dims1 , dims2 , dims3 , dims4 , dims5

```
function array [int,int,int] of var $T: slice_5d(array [$E] of var $T: x,
                                                array [int] of set of int:_
                                                ↵s,
                                                set of int: dims1,
                                                set of int: dims2,
                                                set of int: dims3,
                                                set of int: dims4,
                                                set of int: dims5)
```

Return slice of array x specified by sets s , coerced to new 3d array with index sets dims1 , dims2 , dims3 , dims4 , dims5

```
function array [int,int,int] of var opt $T: slice_5d(array [$E] of var opt
    ↵$T: x,
                                array [int] of set of_
    ↵int: s,
                                set of int: dims1,
                                set of int: dims2,
                                set of int: dims3,
                                set of int: dims4,
                                set of int: dims5)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims}_1, \text{dims}_2, \text{dims}_3, \text{dims}_4, \text{dims}_5$

```
function array [int,int,int] of $T: slice_6d(array [$E] of $T: x,
                                array [int] of set of int: s,
                                set of int: dims1,
                                set of int: dims2,
                                set of int: dims3,
                                set of int: dims4,
                                set of int: dims5,
                                set of int: dims6)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims}_1, \text{dims}_2, \text{dims}_3, \text{dims}_4, \text{dims}_5, \text{dims}_6$

```
function array [int,int,int] of opt $T: slice_6d(array [$E] of opt $T: x,
                                array [int] of set of int:_
    ↵s,
                                set of int: dims1,
                                set of int: dims2,
                                set of int: dims3,
                                set of int: dims4,
                                set of int: dims5,
                                set of int: dims6)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims}_1, \text{dims}_2, \text{dims}_3, \text{dims}_4, \text{dims}_5, \text{dims}_6$

```
function array [int,int,int] of var $T: slice_6d(array [$E] of var $T: x,
                                                array [int] of set of int:_  

                                                ↵s,  

                                                set of int: dims1,  

                                                set of int: dims2,  

                                                set of int: dims3,  

                                                set of int: dims4,  

                                                set of int: dims5,  

                                                set of int: dims6)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims}_1, \text{dims}_2, \text{dims}_3, \text{dims}_4, \text{dims}_5, \text{dims}_6$

```
function array [int,int,int] of var opt $T: slice_6d(array [$E] of var opt  

                                                 ↵$T: x,  

                                                 array [int] of set of _  

                                                 ↵int: s,  

                                                 set of int: dims1,  

                                                 set of int: dims2,  

                                                 set of int: dims3,  

                                                 set of int: dims4,  

                                                 set of int: dims5,  

                                                 set of int: dims6)
```

Return slice of array  $x$  specified by sets  $s$ , coerced to new 3d array with index sets  $\text{dims}_1, \text{dims}_2, \text{dims}_3, \text{dims}_4, \text{dims}_5, \text{dims}_6$

#### 4.2.5.8 Array sorting operations

```
function array [int] of $$E: arg_sort(array [$$E] of int: x)
```

Returns the permutation  $p$  which causes  $x$  to be in sorted order hence  $x[p[i]] \leq x[p[i+1]]$ .

The permutation is the stable sort hence  $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$ .

```
function array [int] of $$E: arg_sort(array [$$E] of float: x)
```

Returns the permutation  $p$  which causes  $x$  to be in sorted order hence  $x[p[i]] \leq x[p[i+1]]$ .

The permutation is the stable sort hence  $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$ .

```
function array [$$E] of int: sort(array [$$E] of int: x)
```

Return values from array  $x$  sorted in non-decreasing order

```
function array [$$E] of float: sort(array [$$E] of float: x)
```

Return values from array  $x$  sorted in non-decreasing order

```
function array [$$E] of bool: sort(array [$$E] of bool: x)
```

Return values from array  $x$  sorted in non-decreasing order

```
function array [$$E] of var opt $T: sort_by(array [$$E] of var opt $T: x,
                                              array [$$E] of int: y)
```

Return array  $x$  sorted by the values in  $y$  in non-decreasing order

The sort is stable, i.e. if  $y[i] = y[j]$  with  $i < j$ , then  $x[i]$  will appear in the output before  $x[j]$ .

```
function array [$$E] of var $T: sort_by(array [$$E] of var $T: x,
                                         array [$$E] of int: y)
```

Return array  $x$  sorted by the values in  $y$  in non-decreasing order

The sort is stable, i.e. if  $y[i] = y[j]$  with  $i < j$ , then  $x[i]$  will appear in the output before  $x[j]$ .

```
function array [$$E] of $T: sort_by(array [$$E] of $T: x,
                                       array [$$E] of int: y)
```

Return array  $x$  sorted by the values in  $y$  in non-decreasing order

The sort is stable, i.e. if  $y[i] = y[j]$  with  $i < j$ , then  $x[i]$  will appear in the output before  $x[j]$ .

```
function array [ $$E ] of var opt $T: sort_by(array [ $$E ] of var opt $T: x,
                                              array [ $$E ] of float: y)
```

Return array  $x$  sorted by the values in  $y$  in non-decreasing order

The sort is stable, i.e. if  $y[i] = y[j]$  with  $i < j$ , then  $x[i]$  will appear in the output before  $x[j]$ .

```
function array [ $$E ] of var $T: sort_by(array [ $$E ] of var $T: x,
                                           array [ $$E ] of float: y)
```

Return array  $x$  sorted by the values in  $y$  in non-decreasing order

The sort is stable, i.e. if  $y[i] = y[j]$  with  $i < j$ , then  $x[i]$  will appear in the output before  $x[j]$ .

```
function array [ $$E ] of $T: sort_by(array [ $$E ] of $T: x,
                                       array [ $$E ] of float: y)
```

Return array  $x$  sorted by the values in  $y$  in non-decreasing order

The sort is stable, i.e. if  $y[i] = y[j]$  with  $i < j$ , then  $x[i]$  will appear in the output before  $x[j]$ .

#### 4.2.5.9 Coercions

These functions implement coercions, or channeling, between different types.

```
function float: bool2float(bool: b)
```

Return Boolean  $b$  coerced to a float

```
function array [ $T ] of float: bool2float(array [ $T ] of bool: x)
```

Return array of Booleans x coerced to an array of floats

```
function array [\$T] of var float: bool2float(array [\$T] of var bool: x)
```

Return array of Booleans x coerced to an array of floats

```
function var float: bool2float(var bool: b)
```

Return Boolean b coerced to a float

```
function int: bool2int(bool: b)
```

Return Boolean b coerced to an integer

```
function var int: bool2int(var bool: b)
```

Return Boolean b coerced to an integer

```
function array [ $$E ] of var int: bool2int(array [ $$E ] of var bool: b)
```

Return array of Booleans b coerced to an array of integers

```
function array [ \$T ] of int: bool2int(array [ \$T ] of bool: x)
```

Return array of Booleans x coerced to an array of integers

```
function array [ \$T ] of set of int: bool2int(array [ \$T ] of set of bool: x)
```

Return array of sets of Booleans x coerced to an array of sets of integers

```
function array [ \$T ] of var int: bool2int(array [ \$T ] of var bool: x)
```

Return array of Booleans x coerced to an array of integers

```
function array [ \$T ] of var opt int: bool2int(array [ \$T ] of var opt bool: x)
```

Return array of Booleans  $x$  coerced to an array of integers

```
function int: ceil(float: x)
```

Return  $\lceil x \rceil$

```
function int: floor(float: x)
```

Return  $\lfloor x \rfloor$

```
function float: int2float(int: x)
```

Return integer  $x$  coerced to a float

```
function var float: int2float(var int: x)
```

Return integer  $x$  coerced to a float

```
function array [T] of float: int2float(array [T] of int: x)
```

Return array of integers  $x$  coerced to an array of floats

```
function array [T] of var float: int2float(array [T] of var int: x)
```

Return array of integers  $x$  coerced to an array of floats

```
function int: round(float: x)
```

Return  $x$  rounded to nearest integer

```
function array [int] of set of E: set2array(set of E: x)
```

Return a set of integers  $x$  coerced to an array of integers

#### 4.2.5.10 String operations

These functions implement operations on strings.

```
function string: '++'(string: s1, string: s2)
```

Usage: `s1 ++ s2`

Return concatenation of `s1` and `s2`

```
function string: concat(array [$T] of string: s)
```

Return concatenation of strings in array `s`

```
function string: file_path()
```

Return path of file where this function is called

```
function string: format(var opt $T: x)
```

Convert `x` into a string

```
function string: format(var opt set of $T: x)
```

Convert `x` into a string

```
function string: format(array [$U] of var opt $T: x)
```

Convert `x` into a string

```
function string: format(int: w, int: p, var opt $T: x)
```

Formatted to-string conversion

Converts the value `x` into a string right justified by the number of characters given by `w`, or left justified if `w` is negative.

The maximum length of the string representation of `x` is given by `p`, or the maximum number of digits after the decimal point for floating point numbers. It is a run-time error for `p` to be negative.

```
function string: format(int: w, int: p, var opt set of $T: x)
```

Formatted to-string conversion

Converts the value  $x$  into a string right justified by the number of characters given by  $w$ , or left justified if  $w$  is negative.

The maximum length of the string representation of  $x$  is given by  $p$ . It is a run-time error for  $p$  to be negative.

```
function string: format(int: w, int: p, array [$U] of var opt $T: x)
```

Formatted to-string conversion

Converts the value  $x$  into a string right justified by the number of characters given by  $w$ , or left justified if  $w$  is negative.

The maximum length of the string representation of  $x$  is given by  $p$ . It is a run-time error for  $p$  to be negative.

```
function string: format(int: w, var opt $T: x)
```

Formatted to-string conversion

Converts the value  $x$  into a string right justified by the number of characters given by  $w$ , or left justified if  $w$  is negative.

```
function string: format(int: w, var opt set of $T: x)
```

Formatted to-string conversion

Converts the value  $x$  into a string right justified by the number of characters given by  $w$ , or left justified if  $w$  is negative.

```
function string: format(int: w, array [$U] of var opt $T: x)
```

Formatted to-string conversion

Converts the value  $x$  into a string right justified by the number of characters given by  $w$ , or left justified if  $w$  is negative.

```
function string: format_justify_string(int: w, string: x)
```

String justification

Returns the string *x* right justified by the number of characters given by *w*, or left justified if *w* is negative.

```
function string: join(string: d, array [$T] of string: s)
```

Join string in array *s* using delimiter *d*

```
function array [int] of string: outputJSON()
```

Return array for output of all variables in JSON format

```
function array [int] of string: outputJSONParameters()
```

Return array for output of all parameters in JSON format

```
function string: show(var opt set of $T: x)
```

Convert *x* into a string

```
function string: show(var opt $T: x)
```

Convert *x* into a string

```
function string: show(array [$U] of var opt $T: x)
```

Convert *x* into a string

```
function string: show2d(array [int,int] of var opt $T: x)
```

Convert two-dimensional array *x* into a string

```
function string: show3d(array [int,int,int] of var opt $T: x)
```

Convert three-dimensional array x into a string

```
function string: showJSON(var opt $T: x)
```

Convert x into JSON string

```
function string: showJSON(array [$U] of var opt $T: x)
```

Convert x into JSON string

```
function string: show_float(int: w, int: p, var float: x)
```

Formatted to-string conversion for floats.

Converts the float x into a string right justified by the number of characters given by w , or left justified if w is negative. The number of digits to appear after the decimal point is given by p . It is a run-time error for p to be negative.

```
function string: show_int(int: w, var int: x)
```

Formatted to-string conversion for integers

Converts the integer x into a string right justified by the number of characters given by w , or left justified if w is negative.

```
function int: string_length(string: s)
```

Return length of s

#### 4.2.5.11 Reflection operations

These functions return information about declared or inferred variable bounds and domains.

```
function set of int: dom(var int: x)
```

Return domain of  $x$

```
function set of int: dom_array(array [$T] of var int: x)
```

Return union of all domains of the elements in array  $x$

```
function set of int: dom_bounds_array(array [$T] of var int: x)
```

Return approximation of union of all domains of the elements in array  $x$

```
function int: dom_size(var int: x)
```

Return cardinality of the domain of  $x$

```
function $T: fix(var opt $T: x)
```

Check if the value of  $x$  is fixed at this point in evaluation. If it is fixed, return its value, otherwise abort.

```
function array [$U] of $T: fix(array [$U] of var opt $T: x)
```

Check if the value of every element of the array  $x$  is fixed at this point in evaluation. If all are fixed, return an array of their values, otherwise abort.

```
test has_bounds(var int: x)
```

Test if variable  $x$  has declared, finite bounds

```
test has_bounds(var float: x)
```

Test if variable  $x$  has declared, finite bounds

```
test has_ub_set(var set of int: x)
```

Test if variable  $x$  has a declared, finite upper bound

```
test is_fixed(var opt $T: x)
```

Test if  $x$  is fixed

```
test is_fixed(array [$U] of var opt $T: x)
```

Test if every element of array  $x$  is fixed

```
function int: lb(var int: x)
```

Return lower bound of  $x$

```
function int: lb(var opt int: x)
```

Return lower bound of  $x$

```
function float: lb(var float: x)
```

Return lower bound of  $x$

```
function set of int: lb(var set of int: x)
```

Return lower bound of  $x$

```
function array [$U] of int: lb(array [$U] of var int: x)
```

Return array of lower bounds of the elements in array  $x$

```
function array [$U] of float: lb(array [$U] of var float: x)
```

Return array of lower bounds of the elements in array  $x$

```
function array [$U] of set of int: lb(array [$U] of var set of int: x)
```

Return array of lower bounds of the elements in array  $x$

```
function int: lb_array(array [$U] of var opt int: x)
```

Return minimum of all lower bounds of the elements in array x

```
function float: lb_array(array [$U] of var float: x)
```

Return minimum of all lower bounds of the elements in array x

```
function set of int: lb_array(array [$U] of var set of int: x)
```

Return intersection of all lower bounds of the elements in array x

```
function int: ub(var int: x)
```

Return upper bound of x

```
function int: ub(var opt int: x)
```

Return upper bound of x

```
function float: ub(var float: x)
```

Return upper bound of x

```
function set of int: ub(var set of int: x)
```

Return upper bound of x

```
function array [$U] of int: ub(array [$U] of var int: x)
```

Return array of upper bounds of the elements in array x

```
function array [$U] of float: ub(array [$U] of var float: x)
```

Return array of upper bounds of the elements in array x

```
function array [\$U] of set of int: ub(array [\$U] of var set of int: x)
```

Return array of upper bounds of the elements in array x

```
function int: ub_array(array [\$U] of var opt int: x)
```

Return maximum of all upper bounds of the elements in array x

```
function float: ub_array(array [\$U] of var float: x)
```

Return maximum of all upper bounds of the elements in array x

```
function set of int: ub_array(array [\$U] of var set of int: x)
```

Return union of all upper bounds of the elements in array x

#### 4.2.5.12 Assertions and debugging functions

These functions help debug models and check that input data conforms to the expectations.

```
test abort(string: msg)
```

Abort evaluation and print message msg .

```
function \$T: assert(bool: b, string: msg, \$T: x)
```

If b is true, return x , otherwise abort with message msg .

```
function var \$T: assert(bool: b, string: msg, var \$T: x)
```

If b is true, return x , otherwise abort with message msg .

```
function var opt \$T: assert(bool: b, string: msg, var opt \$T: x)
```

If b is true, return x , otherwise abort with message msg .

```
function array [\$U] of \$T: assert(bool: b,
                                    string: msg,
                                    array [\$U] of \$T: x)
```

If `b` is true, return `x` , otherwise abort with message `msg` .

```
function array [\$U] of var \$T: assert(bool: b,
                                         string: msg,
                                         array [\$U] of var \$T: x)
```

If `b` is true, return `x` , otherwise abort with message `msg` .

```
function array [\$U] of var opt \$T: assert(bool: b,
                                         string: msg,
                                         array [\$U] of var opt \$T: x)
```

If `b` is true, return `x` , otherwise abort with message `msg` .

```
test assert(bool: b, string: msg)
```

If `b` is true, return true, otherwise abort with message `msg` .

```
function \$T: trace(string: msg, \$T: x)
```

Return `x` , and print message `msg` .

```
function var \$T: trace(string: msg, var \$T: x)
```

Return `x` , and print message `msg` .

```
function var opt \$T: trace(string: msg, var opt \$T: x)
```

Return `x` , and print message `msg` .

```
test trace(string: msg)
```

Return true, and print message msg .

```
function $T: trace_stdout(string: msg, $T: x)
```

Return x , and print message msg .

```
function var $T: trace_stdout(string: msg, var $T: x)
```

Return x , and print message msg .

```
function var opt $T: trace_stdout(string: msg, var opt $T: x)
```

Return x , and print message msg .

```
test trace_stdout(string: msg)
```

Return true, and print message msg .

#### 4.2.5.13 Functions for enums

```
function $$E: enum_next(set of $$E: e, $$E: x)
```

Return next greater enum value of x in enum e

```
function var $$E: enum_next(set of $$E: e, var $$E: x)
```

Return next greater enum value of x in enum e

```
function $$E: enum_prev(set of $$E: e, $$E: x)
```

Return next smaller enum value of x in enum e

```
function var $$E: enum_prev(set of $$E: e, var $$E: x)
```

Return next smaller enum value of x in enum e

```
function $$E: to_enum(set of $$E: X, int: x)
```

Convert x to enum type X

```
function var $$E: to_enum(set of $$E: X, var int: x)
```

Convert x to enum type X

```
function array [$$U] of $$E: to_enum(set of $$E: X,  
array [$$U] of int: x)
```

Convert x to enum type X

```
function array [$$U] of var $$E: to_enum(set of $$E: X,  
array [$$U] of var int: x)
```

Convert x to enum type X

```
function set of $$E: to_enum(set of $$E: X, set of int: x)
```

Convert x to enum type X

#### 4.2.5.14 Random Number Generator builtins

These functions implement random number generators from different probability distributions.

```
test bernoulli(float: p)
```

Return a boolean sample from the Bernoulli distribution defined by probability **p**

```
function int: binomial(int: t, float: p)
```

Return a sample from the binomial distribution defined by sample number **t** and probability **p**

```
function float: cauchy(float: mean, float: scale)
```

Return a sample from the cauchy distribution defined by **mean**, **scale**

```
function float: cauchy(int: mean, float: scale)
```

Return a sample from the cauchy distribution defined by **mean**, **scale**

```
function float: chisquared(int: n)
```

Return a sample from the chi-squared distribution defined by the degree of freedom **n**

```
function float: chisquared(float: n)
```

Return a sample from the chi-squared distribution defined by the degree of freedom **n**

```
function int: discrete_distribution(array [int] of int: weights)
```

Return a sample from the discrete distribution defined by the array of weights **weights** that assigns a weight to each integer starting from zero

```
function float: exponential(int: lambda)
```

Return a sample from the exponential distribution defined by **lambda**

```
function float: exponential(float: lambda)
```

Return a sample from the exponential distribution defined by **lambda**

```
function float: fdistribution(float: d1, float: d2)
```

Return a sample from the Fisher-Snedecor F-distribution defined by the degrees of freedom **d1**, **d2**

```
function float: fdistribution(int: d1, int: d2)
```

Return a sample from the Fisher-Snedecor F-distribution defined by the degrees of freedom **d1**, **d2**

```
function float: gamma(float: alpha, float: beta)
```

Return a sample from the gamma distribution defined by **alpha, beta**

```
function float: gamma(int: alpha, float: beta)
```

Return a sample from the gamma distribution defined by **alpha, beta**

```
function float: lognormal(float: mean, float: std)
```

Return a sample from the lognormal distribution defined by **mean, std**

```
function float: lognormal(int: mean, float: std)
```

Return a sample from the lognormal distribution defined by **mean, std**

```
function float: normal(float: mean, float: std)
```

Return a sample from the normal distribution defined by **mean, std**

```
function float: normal(int: mean, float: std)
```

Return a sample from the normal distribution defined by **mean, std**

```
function int: poisson(float: mean)
```

Return a sample from the poisson distribution defined by **mean**

```
function int: poisson(int: mean)
```

Return a sample from the poisson distribution defined by an integer **mean**

```
function float: tdistribution(float: n)
```

Return a sample from the student' s t-distribution defined by the sample size **n**

```
function float: tdistribution(int: n)
```

Return a sample from the student' s t-distribution defined by the sample size **n**

```
function float: uniform(float: lowerbound, float: upperbound)
```

Return a sample from the uniform distribution defined by **lowerbound, upperbound**

```
function int: uniform(int: lowerbound, int: upperbound)
```

Return a sample from the uniform distribution defined by **lowerbound, upperbound**

```
function float: weibull(float: shape, float: scale)
```

Return a sample from the Weibull distribution defined by **shape, scale**

```
function float: weibull(int: shape, float: scale)
```

Return a sample from the Weibull distribution defined by **shape, scale**

#### 4.2.5.15 Special constraints

These predicates allow users to mark constraints as e.g. symmetry breaking or redundant, so that solvers can choose to implement them differently.

We cannot easily use annotations for this purpose, since annotations are propagated to all constraints in a decomposition, which may be incorrect for redundant or symmetry breaking constraints in the presence of common subexpression elimination (CSE).

```
predicate implied_constraint(var bool: b)
```

Mark b as an implied constraint (synonym for redundant\_constraint)

```
predicate redundant_constraint(var bool: b)
```

Mark b as a redundant constraint

```
predicate symmetry_breaking_constraint(var bool: b)
```

Mark b as a symmetry breaking constraint

#### 4.2.5.16 Language information

These functions return information about the MiniZinc system.

```
function int: mzn_compiler_version()
```

Return MiniZinc version encoded as an integer (major\*10000+minor\*1000+patch).

```
function string: mzn_version_to_string(int: v)
```

Return string representation of v given an integer major\*10000+minor\*1000+patch

### 4.2.6 FlatZinc builtins

These are the standard constraints that need to be supported by FlatZinc solvers (or redefined in the `redefinitions.mzn` file).

#### 4.2.6.1 Integer FlatZinc builtins

```
predicate array_int_element(var int: b,
                           array [int] of int: as,
                           var int: c)
```

Constrains as [ b ] = c

```
predicate array_int_maximum(var int: m, array [int] of var int: x)
```

Constrains m to be the maximum value of the (non-empty) array x

```
predicate array_int_minimum(var int: m, array [int] of var int: x)
```

Constrains  $m$  to be the minimum value of the (non-empty) array  $x$

```
predicate array_var_int_element(var int: b,
                                array [int] of var int: as,
                                var int: c)
```

Constrains  $as[b] = c$

```
predicate int_abs(var int: a, var int: b)
```

Constrains  $b$  to be the absolute value of  $a$

```
predicate int_div(var int: a, var int: b, var int: c)
```

Constrains  $a / b = c$

```
predicate int_eq(var int: a, var int: b)
```

Constrains  $a$  to be equal to  $b$

```
predicate int_eq_reif(var int: a, var int: b, var bool: r)
```

Constrains  $(a = b) \leftrightarrow r$

```
predicate int_le(var int: a, var int: b)
```

Constrains  $a$  to be less than or equal to  $b$

```
predicate int_le_reif(var int: a, var int: b, var bool: r)
```

Constrains  $(a \leq b) \leftrightarrow r$

```
predicate int_lin_eq(array [int] of int: as,
                     array [int] of var int: bs,
```

```
int: c)
```

Constrains  $\mathbf{c} = \sum_i \mathbf{as}[i] * \mathbf{bs}[i]$

```
predicate int_lin_eq_reif(array [int] of int: as,
                           array [int] of var int: bs,
                           int: c,
                           var bool: r)
```

Constrains  $\mathbf{r} \leftrightarrow (\mathbf{c} = \sum_i \mathbf{as}[i] * \mathbf{bs}[i])$

```
predicate int_lin_le(array [int] of int: as,
                     array [int] of var int: bs,
                     int: c)
```

Constrains  $\sum \mathbf{as}[i] * \mathbf{bs}[i] \leq \mathbf{c}$

```
predicate int_lin_le_reif(array [int] of int: as,
                           array [int] of var int: bs,
                           int: c,
                           var bool: r)
```

Constrains  $\mathbf{r} \leftrightarrow (\sum \mathbf{as}[i] * \mathbf{bs}[i] \leq \mathbf{c})$

```
predicate int_lin_ne(array [int] of int: as,
                     array [int] of var int: bs,
                     int: c)
```

Constrains  $\mathbf{c} \neq \sum_i \mathbf{as}[i] * \mathbf{bs}[i]$

```
predicate int_lin_ne_reif(array [int] of int: as,
                           array [int] of var int: bs,
                           int: c,
                           var bool: r)
```

Constrains  $\mathbf{r} \leftrightarrow (\mathbf{c} \neq \sum_i \mathbf{as}[i] * \mathbf{bs}[i])$

```
predicate int_lt(var int: a, var int: b)
```

Constrains  $a < b$

```
predicate int_lt_reif(var int: a, var int: b, var bool: r)
```

Constrains  $r \leftrightarrow (a < b)$

```
predicate int_max(var int: a, var int: b, var int: c)
```

Constrains  $\max(a, b) = c$

```
predicate int_min(var int: a, var int: b, var int: c)
```

Constrains  $\min(a, b) = c$

```
predicate int_mod(var int: a, var int: b, var int: c)
```

Constrains  $a \% b = c$

```
predicate int_ne(var int: a, var int: b)
```

Constrains  $a \neq b$

```
predicate int_ne_reif(var int: a, var int: b, var bool: r)
```

$r \leftrightarrow (a \neq b)$

```
predicate int_plus(var int: a, var int: b, var int: c)
```

Constrains  $a + b = c$

```
predicate int_pow(var int: x, var int: y, var int: z)
```

Constrains  $z = x^y$

```
predicate int_pow_fixed(var int: x, int: y, var int: z)
```

Constrains  $z = x^y$

```
predicate int_times(var int: a, var int: b, var int: c)
```

Constrains  $a * b = c$

#### 4.2.6.2 Bool FlatZinc builtins

```
predicate array_bool_and(array [int] of var bool: as, var bool: r)
```

Constrains  $r \leftrightarrow \bigwedge_i \text{as}[i]$

```
predicate array_bool_element(var int: b,
                             array [int] of bool: as,
                             var bool: c)
```

Constrains  $\text{as}[\text{b}] = \text{c}$

```
predicate array_bool_or(array [int] of var bool: as, var bool: r)
```

Constrains  $r \leftrightarrow \bigvee_i \text{as}[i]$

```
predicate array_bool_xor(array [int] of var bool: as)
```

Constrains  $r \leftrightarrow \bigoplus_i \text{as}[i]$

```
predicate array_var_bool_element(var int: b,
                                 array [int] of var bool: as,
                                 var bool: c)
```

Constrains  $\text{as}[\text{b}] = \text{c}$

```
predicate bool2int(var bool: a, var int: b)
```

Constrains  $\mathbf{b} \in \{0, 1\}$  and  $\mathbf{a} \leftrightarrow \mathbf{b} = 1$

```
predicate bool_and(var bool: a, var bool: b, var bool: r)
```

Constrains  $\mathbf{r} \leftrightarrow \mathbf{a} \wedge \mathbf{b}$

```
predicate bool_clause(array [int] of var bool: as,
                     array [int] of var bool: bs)
```

Constrains  $\bigvee_i \mathbf{as}[i] \vee \bigvee_j \neg \mathbf{bs}[j]$

```
predicate bool_eq(var bool: a, var bool: b)
```

Constrains  $\mathbf{a} = \mathbf{b}$

```
predicate bool_eq_reif(var bool: a, var bool: b, var bool: r)
```

Constrains  $\mathbf{r} \leftrightarrow (\mathbf{a} = \mathbf{b})$

```
predicate bool_le(var bool: a, var bool: b)
```

Constrains  $\mathbf{a} \leq \mathbf{b}$

```
predicate bool_le_reif(var bool: a, var bool: b, var bool: r)
```

Constrains  $\mathbf{r} \leftrightarrow (\mathbf{a} \leq \mathbf{b})$

```
predicate bool_lin_eq(array [int] of int: as,
                     array [int] of var bool: bs,
                     var int: c)
```

Constrains  $\mathbf{c} = \sum_i \mathbf{as}[i] * \mathbf{bs}[i]$

```
predicate bool_lin_le(array [int] of int: as,
                      array [int] of var bool: bs,
                      int: c)
```

Constrains  $c \leq \sum_i as[i] * bs[i]$

```
predicate bool_lt(var bool: a, var bool: b)
```

Constrains  $a < b$

```
predicate bool_lt_reif(var bool: a, var bool: b, var bool: r)
```

Constrains  $r \leftrightarrow (a < b)$

```
predicate bool_not(var bool: a, var bool: b)
```

Constrains  $a \neq b$

```
predicate bool_or(var bool: a, var bool: b, var bool: r)
```

Constrains  $r \leftrightarrow a \vee b$

```
predicate bool_xor(var bool: a, var bool: b, var bool: r)
```

Constrains  $r \leftrightarrow a \oplus b$

```
predicate bool_xor(var bool: a, var bool: b)
```

Constrains  $a \oplus b$

#### 4.2.6.3 Set FlatZinc builtins

```
predicate array_set_element(var int: b,
                           array [int] of set of int: as,
                           var set of int: c)
```

Constrains  $b = c$

```
predicate array_var_set_element(var int: b,
                                array [int] of var set of int: as,
                                var set of int: c)
```

Constrains  $b = c$

```
predicate set_card(var set of int: S, var int: x)
```

Constrains  $x = |S|$

```
predicate set_diff(var set of int: x,
                  var set of int: y,
                  var set of int: r)
```

Constrains  $r = x \setminus y$

```
predicate set_eq(var set of int: x, var set of int: y)
```

Constrains  $x = y$

```
predicate set_eq_reif(var set of int: x,
                      var set of int: y,
                      var bool: r)
```

Constrains  $r \leftrightarrow (x = y)$

```
predicate set_in(var int: x, set of int: S)
```

Constrains  $x \in S$

```
predicate set_in(var int: x, var set of int: S)
```

Constrains  $x \in S$

```
predicate set_in_reif(var int: x, set of int: S, var bool: r)
```

Constrains  $r \leftrightarrow (x \in S)$

```
predicate set_in_reif(var int: x, var set of int: S, var bool: r)
```

Constrains  $r \leftrightarrow (x \in S)$

```
predicate set_intersect(var set of int: x,
                      var set of int: y,
                      var set of int: r)
```

Constrains  $r = x \cap y$

```
predicate set_le(var set of int: x, var set of int: y)
```

Constrains  $x \leq y$  (lexicographic order)

```
predicate set_lt(var set of int: x, var set of int: y)
```

Constrains  $x < y$  (lexicographic order)

```
predicate set_ne(var set of int: x, var set of int: y)
```

Constrains  $x \neq y$

```
predicate set_ne_reif(var set of int: x,
                      var set of int: y,
                      var bool: r)
```

Constrains  $r \leftrightarrow (x \neq y)$

```
predicate set_subset(var set of int: x, var set of int: y)
```

Constrains  $x \subseteq y$

```
predicate set_subset_reif(var set of int: x,
                         var set of int: y,
                         var bool: r)
```

Constrains  $r \leftrightarrow (x \subseteq y)$

```
predicate set_superset(var set of int: x, var set of int: y)
```

Constrains  $x \supseteq y$

```
predicate set_symdiff(var set of int: x,
                      var set of int: y,
                      var set of int: r)
```

Constrains  $r$  to be the symmetric difference of  $x$  and  $y$

```
predicate set_union(var set of int: x,
                   var set of int: y,
                   var set of int: r)
```

Constrains  $r = x \cup y$

#### 4.2.6.4 Float FlatZinc builtins

```
predicate array_float_element(var int: b,
                            array [int] of float: as,
                            var float: c)
```

Constrains  $as[b] = c$

```
predicate array_float_maximum(var int: m, array [int] of var int: x)
```

Constrains  $m$  to be the maximum value of the (non-empty) array  $x$

```
predicate array_float_minimum(var int: m, array [int] of var int: x)
```

Constrains  $m$  to be the minimum value of the (non-empty) array  $x$

```
predicate array_var_float_element(var int: b,  
                                 array [int] of var float: as,  
                                 var float: c)
```

Constrains  $a = b$

```
predicate float_abs(var float: a, var float: b)
```

Constrains  $b$  to be the absolute value of  $a$

```
predicate float_acos(var float: a, var float: b)
```

Constrains  $b = \text{acos}(a)$

```
predicate float_acosh(var float: a, var float: b)
```

Constrains  $b = \text{acosh}(a)$

```
predicate float_asin(var float: a, var float: b)
```

Constrains  $b = \text{asin}(a)$

```
predicate float_asinh(var float: a, var float: b)
```

Constrains  $b = \text{asinh}(a)$

```
predicate float_atan(var float: a, var float: b)
```

Constrains  $b = \text{atan}(a)$

```
predicate float_atanh(var float: a, var float: b)
```

Constrains  $b = \text{atanh}(a)$

```
predicate float_cos(var float: a, var float: b)
```

Constrains  $b = \cos(a)$

```
predicate float_cosh(var float: a, var float: b)
```

Constrains  $b = \cosh(a)$

```
predicate float_div(var float: a, var float: b, var float: c)
```

Constrains  $a / b = c$

```
predicate float_dom(var float: x, array [int] of float: as)
```

Constrains the domain of  $x$  using the values in  $as$ , using each pair of values as  $[2^* i - 1]..[2^* i]$  for  $i$  in  $1..n/2$  as a possible range

```
predicate float_eq(var float: a, var float: b)
```

Constrains  $a = b$

```
predicate float_eq_reif(var float: a, var float: b, var bool: r)
```

Constrains  $r \leftrightarrow (a = b)$

```
predicate float_exp(var float: a, var float: b)
```

Constrains  $b = \exp(a)$

```
predicate float_in(var float: a, float: b, float: c)
```

Constrains  $a \in [b, c]$

```
predicate float_in_reif(var float: a, float: b, float: c, var bool: r)
```

Constrains  $r \leftrightarrow a \in [b, c]$

```
predicate float_le(var float: a, var float: b)
```

Constrains  $a \leq b$

```
predicate float_le_reif(var float: a, var float: b, var bool: r)
```

Constrains  $r \leftrightarrow (a \leq b)$

```
predicate float_lin_eq(array [int] of float: as,
                      array [int] of var float: bs,
                      float: c)
```

Constrains  $c = \sum_i \mathbf{as}[i] * \mathbf{bs}[i]$

```
predicate float_lin_eq_reif(array [int] of float: as,
                            array [int] of var float: bs,
                            float: c,
                            var bool: r)
```

Constrains  $r \leftrightarrow (c = \sum_i \mathbf{as}[i] * \mathbf{bs}[i])$

```
predicate float_lin_le(array [int] of float: as,
                      array [int] of var float: bs,
                      float: c)
```

Constrains  $c \leq \sum_i \mathbf{as}[i] * \mathbf{bs}[i]$

```
predicate float_lin_le_reif(array [int] of float: as,
                            array [int] of var float: bs,
                            float: c,
                            var bool: r)
```

Constrains  $r \leftrightarrow (c \leq \sum_i \mathbf{as}[i] * \mathbf{bs}[i])$

```
predicate float_lin_lt(array [int] of float: as,
                      array [int] of var float: bs,
```

```
float: c)
```

Constrains  $c < \sum_i \mathbf{as}[i] * \mathbf{bs}[i]$

```
predicate float_lin_lt_reif(array [int] of float: as,
                             array [int] of var float: bs,
                             float: c,
                             var bool: r)
```

Constrains  $r \leftrightarrow (c < \sum_i \mathbf{as}[i] * \mathbf{bs}[i])$

```
predicate float_lin_ne(array [int] of float: as,
                      array [int] of var float: bs,
                      float: c)
```

Constrains  $c \neq \sum_i \mathbf{as}[i] * \mathbf{bs}[i]$

```
predicate float_lin_ne_reif(array [int] of float: as,
                            array [int] of var float: bs,
                            float: c,
                            var bool: r)
```

Constrains  $r \leftrightarrow (c \neq \sum_i \mathbf{as}[i] * \mathbf{bs}[i])$

```
predicate float_ln(var float: a, var float: b)
```

Constrains  $b = \ln(a)$

```
predicate float_log10(var float: a, var float: b)
```

Constrains  $b = \log_{10}(a)$

```
predicate float_log2(var float: a, var float: b)
```

Constrains  $b = \log_2(a)$

```
predicate float_lt(var float: a, var float: b)
```

Constrains  $a < b$

```
predicate float_lt_reif(var float: a, var float: b, var bool: r)
```

Constrains  $r \leftrightarrow (a < b)$

```
predicate float_max(var float: a, var float: b, var float: c)
```

Constrains  $\max(a, b) = c$

```
predicate float_min(var float: a, var float: b, var float: c)
```

Constrains  $\min(a, b) = c$

```
predicate float_ne(var float: a, var float: b)
```

Constrains  $a \neq b$

```
predicate float_ne_reif(var float: a, var float: b, var bool: r)
```

Constrains  $r \leftrightarrow (a \neq b)$

```
predicate float_plus(var float: a, var float: b, var float: c)
```

Constrains  $a + b = c$

```
predicate float_pow(var float: x, var float: y, var float: z)
```

Constrains  $z = x^y$

```
predicate float_sin(var float: a, var float: b)
```

Constrains  $b = \sin(a)$

```
predicate float_sinh(var float: a, var float: b)
```

Constrains  $b = \sinh(a)$

```
predicate float_sqrt(var float: a, var float: b)
```

Constrains  $b = \sqrt{a}$

```
predicate float_tan(var float: a, var float: b)
```

Constrains  $b = \tan(a)$

```
predicate float_tanh(var float: a, var float: b)
```

Constrains  $b = \tanh(a)$

```
predicate float_times(var float: a, var float: b, var float: c)
```

Constrains  $a * b = c$

```
predicate int2float(var int: x, var float: y)
```

Constrains  $y = x$

#### 4.2.6.5 FlatZinc builtins added in MiniZinc 2.0.0.

These functions and predicates define built-in operations of the MiniZinc language that have been added in MiniZinc 2.0.0. Solvers that support these natively need to include a file called redefinitions-2.0.mzn in their solver library that redefines these predicates as builtins.

```
predicate array_float_maximum(var float: m,
                             array [int] of var float: x)
```

Constrains  $m$  to be the maximum value in array  $x$ .

```
predicate array_float_minimum(var float: m,
                             array [int] of var float: x)
```

Constrains  $m$  to be the minimum value in array  $x$ .

```
predicate array_int_maximum(var int: m, array [int] of var int: x)
```

Constrains  $m$  to be the maximum value in array  $x$ .

```
predicate array_int_minimum(var int: m, array [int] of var int: x)
```

Constrains  $m$  to be the minimum value in array  $x$ .

```
predicate bool_clause_reif(array [int] of var bool: as,
                           array [int] of var bool: bs,
                           var bool: b)
```

Reified clause constraint. Constrains  $\mathbf{b} \leftrightarrow \bigvee_i \mathbf{as}[i] \vee \bigvee_j \neg \mathbf{bs}[j]$

#### 4.2.6.6 FlatZinc builtins added in MiniZinc 2.0.2.

These functions and predicates define built-in operations of the MiniZinc language that have been added in MiniZinc 2.0.2. Solvers that support these natively need to include a file called redefinitions-2.0.2.mzn in their solver library that redefines these predicates as builtins.

```
predicate array_var_bool_element_nonshifted(var int: idx,
                                             array [int] of var bool: x,
                                             var bool: c)
```

Element constraint on array with MiniZinc index set, constrains  $x[\text{idx}] = c$  This can be overridden in a solver that can perform the index calculation more efficiently than using a MiniZinc decomposition.

```
predicate array_var_float_element_nonshifted(var int: idx,
                                              array [int] of var float: x,
                                              var float: c)
```

Element constraint on array with MiniZinc index set, constrains  $x[\text{idx}] = c$  This can be overridden in a solver that can perform the index calculation more efficiently than using a MiniZinc decomposition.

```
predicate array_var_int_element_nonshifted(var int: idx,
                                         array [int] of var int: x,
                                         var int: c)
```

Element constraint on array with MiniZinc index set, constrains  $x[\text{idx}] = c$  This can be overridden in a solver that can perform the index calculation more efficiently than using a MiniZinc decomposition.

```
predicate array_var_set_element_nonshifted(var int: idx,
                                         array [int] of var set of int: x,
                                         var set of int: c)
```

Element constraint on array with MiniZinc index set, constrains  $x[\text{idx}] = c$  This can be overridden in a solver that can perform the index calculation more efficiently than using a MiniZinc decomposition.

#### 4.2.6.7 FlatZinc builtins added in MiniZinc 2.1.0.

These functions and predicates define built-in operations of the MiniZinc language that have been added in MiniZinc 2.1.0. Solvers that support these natively need to include a file called redefinitions-2.1.0.mzn in their solver library that redefines these predicates as builtins.

```
predicate float_dom(var float: x, array [int] of float: as)
```

Constrains  $x$  to take one of the values in  $as$

```
predicate float_in(var float: x, float: a, float: b)
```

Constrains  $a \leq x \leq b$

#### 4.2.6.8 FlatZinc builtins added in MiniZinc 2.1.1.

These functions and predicates define built-in operations of the MiniZinc language that have been added in MiniZinc 2.1.1. Solvers that support these natively need to include a file called

redefinitions-2.1.1.mzn in their solver library that redefines these predicates as builtins.

```
function var $$E: max(var set of $$E: s)
```

Returns variable constrained to be equal to the maximum of the set  $s$ . An alternative implementation can be found in the comments of the source code.

```
function var $$E: min(var set of $$E: s)
```

Returns variable constrained to be equal to the minimum of the set  $s$ . An alternative implementation can be found in the comments of the source code.

# CHAPTER 4.3

---

## Interfacing Solvers to Flatzinc

---

This document describes the interface between the MiniZinc system and FlatZinc solvers.

### 4.3.1 Specification of FlatZinc

This document is the specification of the FlatZinc modelling language. It also includes a definition of the standard command line options a FlatZinc solver should support in order to work with the `minizinc` driver program (and the MiniZinc IDE).

FlatZinc is the target constraint modelling language into which MiniZinc models are translated. It is a very simple solver independent problem specification language, requiring minimal implementation effort to support.

Throughout this document:  $r_1, r_2$  denote float literals;  $x_1, x_2, \dots, x_k, i, j, k$  denote int literals;  $y_1, y_2, \dots, y_k, y_i$  denote literal array elements.

#### 4.3.1.1 Comments

Comments start with a percent sign % and extend to the end of the line. Comments can appear anywhere in a model.

### 4.3.1.2 Types

There are three varieties of types in FlatZinc.

- *Parameter* types apply to fixed values that are specified directly in the model.
- *Variable* types apply to values computed by the solver during search. Every parameter type has a corresponding variable type; the variable type being distinguished by a `var` keyword.
- *Annotations* and *strings*: annotations can appear on variable declarations, constraints, and on the solve goal. They provide information about how a variable or constraint should be treated by the solver (e.g., whether a variable should be output as part of the result or whether a particular constraint should implemented using domain consistency). Strings may appear as arguments to annotations, but nowhere else.

#### Parameter types

Parameters are fixed quantities explicitly specified in the model (see rule `<par-type>` in Section 4.3.6).

Type	Values
<code>bool</code>	true or false
<code>float</code>	float
<code>int</code>	int
<code>set of int</code>	subset of int
<code>array [1.. n ] of bool</code>	array of bools
<code>array [1.. n ] of float</code>	array of floats
<code>array [1.. n ] of int</code>	array of ints
<code>array [1.. n ] of set of int</code>	array of sets of ints

A parameter may be used where a variable is expected, but not vice versa.

In predicate declarations the following additional parameter types are allowed.

Type	Values
$r_a \dots r_b$	bounded float
$x_a \dots x_b$	int in range
{ $x_a, x_b, \dots, x_k$ }	int in set
set of $x_a \dots x_b$	subset of int range
set of { $x_a, x_b, \dots, x_k$ }	subset of int set
array [1.. n] of $r_a \dots r_b$	array of floats in range
array [1.. n] of $x_a \dots x_b$	array of ints in range
array [1.. n] of set of $x_a \dots x_b$	array of sets of ints in range
array [1.. n] of set of { $x_a, x_b, \dots, x_k$ }	array of subsets of set

A range  $x_a \dots x_b$  denotes a closed interval  $\{x | x_a \leq x \leq x_b\}$  (same for float ranges).

An array type appearing in a predicate declaration may use just int instead of 1.. n for the array index range in cases where the array argument can be of any length.

## Variable types

Variables are quantities decided by the solver (see rules [<basic-var-type>](#) and [<array-var-type>](#) in Section 4.3.6).

Variable type
var bool
var float
var $r_a \dots r_b$
var int
var $x_a \dots x_b$
var { $x_a, x_b, \dots, x_k$ }
var set of $x_a \dots x_b$
var set of { $x_a, x_b, \dots, x_k$ }
array [1.. n] of var bool
array [1.. n] of var float
array [1.. n] of var $r_a \dots r_b$
array [1.. n] of var int
array [1.. n] of var $x_a \dots x_b$
array [1.. n] of var set of $x_a \dots x_b$
array [1.. n] of var set of { $x_a, x_b, \dots, x_k$ }

In predicate declarations the following additional variable types are allowed.

Variable type
var set of int
array [1.. n ] of var set of int

An array type appearing in a predicate declaration may use just `int` instead of `1.. n` for the array index range in cases where the array argument can be of any length.

### The string type

String literals and literal arrays of string literals can appear as annotation arguments, but not elsewhere. Strings have the same syntax as in C programs (namely, they are delimited by double quotes and the backslash character is used for escape sequences).

Examples:

```
"" % The empty string.  
"Hello."  
"Hello,\nWorld\t\"quoted!\"\n" % A string with an embedded newline, tab and  
→quotes.
```

#### 4.3.1.3 Values and expressions

(See rule `<expr>` in Section 4.3.6)

Examples of literal values:

Type Literals `bool` `true`, `false` `float` `2.718`, `-1.0`, `3.0e8` `int` `-42`, `0`, `69` `set of int` `{}`, `{2, 3, 5}`, `1..10` `arrays` `[]`, `[ ya, ..., yk ]`

where each array element  $y_i$  is either: a non-array literal, or the name of a non-array parameter or variable,  $v$ . For example:

```
[1, 2, 3] % Just literals  
[x, y, z] % x, y, and z are variables or parameters.  
[x, 3] % Mix of identifiers and literals
```

Section 4.3.6 gives the regular expressions specifying the syntax for float and int literals.

#### 4.3.1.4 FlatZinc models

A FlatZinc model consists of:

1. zero or more external predicate declarations (i.e., a non-standard predicate that is supported directly by the target solver);
2. zero or more parameter declarations;
3. zero or more variable declarations;
4. zero or more constraints;
5. a solve goal

in that order.

FlatZinc uses the UTF-8 character set. Non-ASCII characters can only appear in string literals.

FlatZinc syntax is case sensitive (foo and Foo are different names). Identifiers start with a letter ([A-Za-z]) and are followed by any sequence of letters, digits, or underscores ([A-Za-z0-9\_]). Additionally, identifiers of variable or parameter names may start with an underscore. Identifiers that correspond to the names of predicates, predicate parameters and annotations cannot have leading underscores.

The following keywords are reserved and cannot be used as identifiers: annotation, any, array, bool, case, constraint, diff, div, else, elseif, endif, enum, false, float, function, if, in, include, int, intersect, let, list, maximize, minimize, mod, not, of, satisfy, subset, superset, output, par, predicate, record, set, solve, string, symdiff, test, then, true, tuple, union, type, var, where, xor. Note that some of these keywords are not used in FlatZinc. They are reserved because they are keywords in Zinc and MiniZinc.

FlatZinc syntax is insensitive to whitespace.

#### 4.3.1.5 Predicate declarations

(See rule `<predicate-item>` in Section 4.3.6)

Predicates used in the model that are not standard FlatZinc must be declared at the top of a FlatZinc model, before any other lexical items. Predicate declarations take the form

```
<predicate-item> ::= "predicate" <identifier> "(" [ <pred-param-type> :_
    ↵<identifier> "," ... ] ")" ";"
```

Annotations are not permitted anywhere in predicate declarations.

It is illegal to supply more than one predicate declaration for a given <identifier>.

Examples:

```
% m is the median value of {x, y, z}.
%
predicate median_of_3(var int: x, var int: y, var int: z, var int: m);

% all_different([x1, ..., xn]) iff
% for all i, j in 1..n: xi != xj.
%
predicate all_different(array [int] of var int: xs);

% exactly_one([x1, ..., xn]) iff
% there exists an i in 1..n: xi = true
% and for all j in 1..n: j != i -> xj = false.
%
predicate exactly_one(array [int] of var bool: xs);
```

#### 4.3.1.6 Parameter declarations

(See rule param\_decl in Section 4.3.6)

Parameters have fixed values and must be assigned values:

```
<par-decl-item> ::= <par-type> ":" <var-par-identifier> "=" <par-expr> ";"
```

where <par-type> is a parameter type, <var-par-identifier> is an identifier, and <par-expr> is a literal value (either a basic integer, float or bool literal, or a set or array of such literals).

Annotations are not permitted anywhere in parameter declarations.

Examples:

```
float: pi = 3.141;
array [1..7] of int: fib = [1, 1, 2, 3, 5, 8, 13];
bool: beer_is_good = true;
```

### 4.3.1.7 Variable declarations

(See rule `var_decl` in Section 4.3.6)

Variables have variable types and can be declared with optional assignments. The assignment can fix a variable to a literal value, or create an alias to another variable. Arrays of variables always have an assignment, defining them in terms of an array literal that can contain identifiers of variables or constant literals. Variables may be declared with zero or more annotations.

```
<var-decl-item> ::= <basic-var-type> ":" <var-par-identifier> <annotations>
  ↪ [ "=" <basic-expr> ] ";" 
    | <array-var-type> ":" <var-par-identifier> <annotations>
  ↪ "=" <array-literal> ";"
```

where `<basic-var-type>` and `<array-var-type>` are variable types, `<var-par-identifier>` is an identifier, `<annotations>` is a (possibly empty) set of annotations, `<basic-expr>` is an identifier or a literal, and `<array-literal>` is a literal array value.

Examples:

```
var 0..9: digit;
var bool: b;
var set of 1..3: s;
var 0.0..1.0: x;
var int: y :: mip;           % 'mip' annotation: y should be a MIP variable.
array [1..3] of var 1..10: b = [y, 3, digit];
```

### 4.3.1.8 Constraints

(See rule `<constraint-item>` in Section 4.3.6)

Constraints take the following form and may include zero or more annotations:

```
<constraint-item> ::= "constraint" <identifier> "(" [ <expr> "," ... ] ")" 
  ↪ <annotations> ";"
```

The arguments expressions (`<expr>`) can be literal values or identifiers.

Examples:

```
constraint int_le(0, x);      % 0 <= x
constraint int_lt(x, y);      % x < y
constraint int_le(y, 10);      % y <= 10
% 'domain': use domain consistency for this constraint:
% 2x + 3y = 10
constraint int_lin_eq([2, 3], [x, y], 10) :: domain;
```

### 4.3.1.9 Solve item

(See rule `<solve-item>` in Section 4.3.6)

A model finishes with a solve item, taking one of the following forms:

```
<solve-item> ::= "solve" <annotations> "satisfy" ;"
| "solve" <annotations> "minimize" <basic-expr> ;"
| "solve" <annotations> "maximize" <basic-expr> ;"
```

The first alternative searches for any satisfying assignment, the second one searches for an assignment minimizing the given expression, and the third one for an assignment maximizing the expression. The `<basic-expr>` can be either a variable identifier or a literal value (if the objective function is constant).

A solution consists of a complete assignment where all variables in the model have been given a fixed value.

Examples:

```
solve satisfy;      % Find any solution using the default strategy.

solve minimize w;   % Find a solution minimizing w, using the default_
                     ↴strategy.

% First label the variables in xs in the order x[1], x[2], ...
% trying values in ascending order.
solve :: int_search(xs, input_order, indomain_min, complete)
       satisfy;    % Find any solution.

% First use first-fail on these variables, splitting domains
```

```
% at each choice point.

solve :: int_search([x, y, z], first_fail, indomain_split, complete)
maximize x; % Find a solution maximizing x.
```

### 4.3.1.10 Annotations

Annotations are optional suggestions to the solver concerning how individual variables and constraints should be handled (e.g., a particular solver may have multiple representations for int variables) and how search should proceed. An implementation is free to ignore any annotations it does not recognise, although it should print a warning on the standard error stream if it does so. Annotations are unordered and idempotent: annotations can be reordered and duplicates can be removed without changing the meaning of the annotations.

An annotation is prefixed by `:::`, and either just an identifier or an expression that looks like a predicate call:

```
<annotations> ::= [ ":" <annotation> ]*
<annotation> ::= <identifier>
               | <identifier> "(" <ann-expr> "," ... ")"
<ann-expr>   ::= <expr>
               | <annotation>
```

The arguments of the second alternative can be any expression or other annotations (without the leading `:::`).

#### Search annotations

While an implementation is free to ignore any or all annotations in a model, it is recommended that implementations at least recognise the following standard annotations for solve items.

```
seq_search([<searchannotation>, ...])
```

allows more than one search annotation to be specified in a particular order (otherwise annotations can be handled in any order).

A `<searchannotation>` is one of the following:

```

int_search(<vars>, <varchoiceannotation>, <assignmentannotation>,
           ↳<strategyannotation>)

bool_search(<vars>, <varchoiceannotation>, <assignmentannotation>,
            ↳<strategyannotation>)

set_search(<vars>, <varchoiceannotation>, <assignmentannotation>,
            ↳<strategyannotation>)

```

where `<vars>` is the identifier of an array variable or an array literal specifying the variables to be assigned (ints, bools, or sets respectively). Note that these arrays may contain literal values.

`<varchoiceannotation>` specifies how the next variable to be assigned is chosen at each choice point. Possible choices are as follows (it is recommended that implementations support the starred options):

<code>input_order</code>	Choose variables in the order they appear in <code>vars</code> .
<code>first_fail</code>	Choose the variable with the smallest domain.
<code>anti_first_fail</code>	Choose the variable with the largest domain.
<code>smallest</code>	Choose the variable with the smallest value in its domain.
<code>largest</code>	Choose the variable with the largest value in its domain.
<code>occurrence</code>	Choose the variable with the largest number of attached constraints.
<code>most_constraints</code>	Choose the variable with the smallest domain, breaking ties using the number of constraints.
<code>max_regret</code>	Choose the variable with the largest difference between the two smallest values in its domain.
<code>dom_w_deg</code>	Choose the variable with the smallest value of domain size divided by weighted degree, where the weighted degree is the number of times the variables been in a constraint which failed

`<assignmentannotation>` specifies how the chosen variable should be constrained. Possible choices are as follows (it is recommended that implementations support at least the starred options):

indomain_min	★	Assign the smallest value in the variable's domain.
indomain_max	★	Assign the largest value in the variable's domain.
indomain_middle		Assign the value in the variable's domain closest to the mean of its current bounds.
indomain_median		Assign the middle value in the variable's domain.
indomain		Nondeterministically assign values to the variable in ascending order.
indomain_random		Assign a random value from the variable's domain.
indomain_split		Bisect the variable's domain, excluding the upper half first.
indomain_reverse	<del>indomain_split</del>	<del>Bisect</del> the variable's domain, excluding the lower half first.
indomain_interval		If the variable's domain consists of several contiguous intervals, reduce the domain to the first interval. Otherwise just split the variable's domain.

Of course, not all assignment strategies make sense for all search annotations (e.g., `bool_search` and `indomain_split`).

Finally, `<strategyannotation>` specifies a search strategy; implementations should at least support complete (i.e., exhaustive search).

## Output annotations

Model output is specified through variable annotations. Non-array output variables are annotated with `output_var`. Array output variables are annotated with `output_array([x1 .. x2, ...])` where  $x_1 \dots x_2$ , ... are the index set ranges of the original MiniZinc array (which may have had multiple dimensions and/or index sets that do not start at 1). See Section 4.3.2 for details on the output format.

## Variable definition annotations

To support solvers capable of exploiting functional relationships, a variable defined as a function of other variables may be annotated thus:

```
var int: x :: is_defined_var;
...
constraint int_plus(y, z, x) :: defines_var(x);
```

(The `defines_var` annotation should appear on exactly one constraint.) This allows a solver to represent `x` internally as a representation of  $y+z$  rather than as a separate constrained variable. The `is_defined_var` annotation on the declaration of `x` provides “early warning” to the solver that such an option is available.

## Intermediate variables

Intermediate variables introduced during conversion of a MiniZinc model to FlatZinc may be annotated thus:

```
var int: X_INTRODUCED_3 :: var_is_introduced;
```

This information is potentially useful to the solver’s search strategy.

## Constraint annotations

Annotations can be placed on constraints advising the solver how the constraint should be implemented. Here are some constraint annotations supported by some solvers:

<code>bounds</code> or <code>boundsZ</code>	Use integer bounds propagation.
<code>boundsR</code>	Use real bounds propagation.
<code>boundsD</code>	A tighter version of <code>boundsZ</code> where support for the bounds must exist.
<code>domain</code>	Use domain propagation.
<code>priority(k)</code>	where <code>k</code> is an integer constant indicating propagator priority.

## 4.3.2 Output

An implementation can produce three types of output: solutions, statistics, and errors.

### 4.3.2.1 Solution output

An implementation must output values for all and only the variables annotated with `output_var` or `output_array` (`output` annotations must not appear on parameters). Output must be printed to the standard output stream.

For example:

```

var 1..10: x :: output_var;
var 1..10: y;      % y is not output.
% Output zs as a "flat" representation of a 2D array:
array [1..4] of var int: zs :: output_array([1..2, 1..2]);

```

All non-error output must be sent to the standard output stream.

Output must take the following form:

```
<var-par-identifier> = <basic-literal-expr> ;
```

or, for array variables,

```
<var-par-identifier> = array<N>d(<a>..<b>, ..., [<y1>, <y2>, ... <yk>]);
```

where `<N>` is the number of index sets specified in the corresponding `output_array` annotation, `<a>..<b>`, ... are the index set ranges, and `<y1>`, `<y2>`, ... `<yk>` are literals of the element type.

Using this format, the output of a FlatZinc model solution is suitable for input to a MiniZinc model as a data file (this is why parameters are not included in the output).

Implementations must ensure that *all* model variables (not just the output variables) have satisfying assignments before printing a solution.

The output for a solution must be terminated with ten consecutive minus signs on a separate line: -----.

Multiple solutions may be output, one after the other, as search proceeds. How many solutions should be output depends on the mode the solver is run in as controlled by the `-a` command line flag (see [Section 4.3.4](#)).

If at least one solution has been found and search then terminates having explored the whole search space, then ten consecutive equals signs should be printed on a separate line: ======.

If no solutions have been found and search terminates having explored the whole search space, then =====UNSATISFIABLE===== should be printed on a separate line.

If the objective of an optimization problem is unbounded, then =====UNBOUNDED===== should be printed on a separate line.

If no solutions have been found and search terminates having *not* explored the whole search space, then =====UNKNOWN===== should be printed on a separate line.

Implementations may output further information about the solution(s), or lack thereof, in the form of FlatZinc comments.

Examples:

Asking for a single solution to this model:

```
var 1..3: x :: output_var;  
solve satisfy
```

might produce this output:

```
x = 1;  
-----
```

Asking for all solutions to this model:

```
array [1..2] of var 1..3: xs :: output_array([1..2]);  
constraint int_lt(xs[1], xs[2]);    % x[1] < x[2].  
solve satisfy
```

might produce this output:

```
xs = array1d(1..2, [1, 2]);  
-----  
xs = array1d(1..2, [1, 3]);  
-----  
xs = array1d(1..2, [2, 3]);  
-----  
=====
```

Asking for a single solution to this model:

```
var 1..10: x :: output_var;  
solve maximize x;
```

should produce this output:

```
x = 10;
-----
-----
```

The row of equals signs indicates that a complete search was performed and that the last result printed is the optimal solution.

Running a solver on this model with some termination condition (such as a very short time-out):

```
var 1..10: x :: output_var;
solve maximize x;
```

might produce this output:

```
x = 1;
-----
x = 2;
-----
x = 3;
-----
```

Because the output does not finish with =====, search did not finish, hence these results must be interpreted as approximate solutions to the optimization problem.

Asking for a solution to this model:

```
var 1..3: x :: output_var;
var 4..6: y :: output_var;
constraint int_lt(y, x);    % y < x.
solve satisfy;
```

should produce this output:

```
=====UNSATISFIABLE=====
```

indicating that a complete search was performed and no solutions were found (i.e., the problem is unsatisfiable).

### 4.3.2.2 Statistics output

FlatZinc solvers can output statistics in a standard format so that it can be read by scripts, for example, in order to run experiments and automatically aggregate the results. Statistics should be printed to the standard output stream in the form of FlatZinc comments that follow a specific format. Statistics can be output at any time during the solving, i.e., before the first solution, between solutions, and after the search has finished.

Each value should be output on a line of its own in the following format:

```
%%mzn-stat: <name>=<value>
```

Each block of statistics is terminated by a line of its own with the following format:

```
%%mzn-stat-end
```

The `<name>` describes the kind of statistics gathered, and the `<value>` can be any value of a MiniZinc type. The following names are considered standard statistics:

Name	Type	Explanation
nodes	int	Number of search nodes
failures	int	Number of leaf nodes that were failed
restarts	int	Number of times the solver restarted the search
variables	int	Number of variables
intVariables	int	Number of integer variables created
boolVariables	int	Number of bool variables created
floatVariables	int	Number of float variables created
setVariables	int	Number of set variables created
propagators	int	Number of propagators created
propagations	int	Number of propagator invocations
peakDepth	int	Peak depth of search tree
nogoods	int	Number of nogoods created
backjumps	int	Number of backjumps
peakMem	float	Peak memory (in Mbytes)
initTime	float	Initialisation time (in seconds)
solveTime	float	Solving time (in seconds)

### 4.3.2.3 Error and warning output

Errors and warnings must be output to the standard error stream. When an error occurs, the implementation should exit with a non-zero exit code, signaling failure.

## 4.3.3 Solver-specific Libraries

Constraints in FlatZinc can call standard predicates as well as solver-specific predicates. Standard predicates are the ones that the MiniZinc compiler assumes to be present in all solvers. Without further customisation, the compiler will try to compile the entire model into a set of these standard predicates.

Solvers can use custom predicates and *redefine* standard predicates by supplying a *solver specific library* of predicate declarations. Examples of such libraries can be found in the binary distribution of MiniZinc, inside the share/minizinc/gecode and share/minizinc/chuffed directories.

The solver-specific library needs to be made available to the MiniZinc compiler by specifying its location in the solver's configuration file, see [Section 4.3.5](#).

### 4.3.3.1 Standard predicates

FlatZinc solvers need to support the predicates listed as FlatZinc builtins in the library reference documentation, see [Section 4.2.6](#).

Any standard predicate that is not supported by a solver needs to be *redefined*. This can be achieved by placing a file called redefinitions.mzn in the solver's MiniZinc library, which can contain alternative definitions of predicates, or define them as unsupported using the abort predicate.

Example for a redefinitions.mzn:

```
% Redefine float_sinh function in terms of exp
predicate float_sinh(var float: a, var float: b) =
    b == (exp(a)-exp(-a))/2.0;

% Mark float_tanh as unsupported
predicate float_tanh(var float: a, var float: b) =
    abort("The builtin float_tanh is not supported by this solver.");
```

The redefinition can use the full MiniZinc language. Note, however, that redefining builtin predicates in terms of MiniZinc expressions can lead to problems if the MiniZinc compiler translates the high-level expression back to the redefined builtin.

The reference documentation ([Section 4.2.6](#)) also contains sections on builtins that were added in later versions of MiniZinc. In order to maintain backwards compatibility with solvers that don't support these, they are organised in redefinition files with a version number attached, such as `redefinitions-2.0.mzn`. In order to declare support for these builtins, the solver-specific library must contain the corresponding `redefinitions` file, with the predicates either redefined in terms of other predicates, or declared as supported natively by the solver by providing a predicate declaration without a body.

Example for a `redefinitions-2.0.mzn` that declares native support for the predicates added in MiniZinc 2.0:

```
predicate bool_clause_reif(array[int] of var bool: as,
                           array[int] of var bool: bs,
                           var bool: b);
predicate array_int_maximum(var int: m, array[int] of var int: x);
predicate array_float_maximum(var float: m, array[int] of var float: x);
predicate array_int_minimum(var int: m, array[int] of var int: x);
predicate array_float_minimum(var float: m, array[int] of var float: x);
```

#### 4.3.3.2 Solver-specific predicates

Many solvers have built-in support for some of the constraints in the MiniZinc standard library. But without declaring which constraints they support, MiniZinc will assume that they don't support any except for the standard FlatZinc builtins mentioned in the section above.

A solver can declare that it supports a non-standard constraint by overriding one of the files of the standard library in its own solver-specific library. For example, assume that a solver supports the `all_different` constraint on integer variables. In the standard library, this constraint is defined in the file `all_different_int.mzn`, with the following implementation:

```
predicate all_different_int(array[int] of var int: x) =
  forall(i,j in index_set(x) where i < j) ( x[i] != x[j] );
```

A solver, let's call it *OptiSolve*, that supports this constraint natively can place a file with the same name, `all_different_int.mzn`, in its library, and redefine it as follows:

```

precate optisolve_alldifferent(array[int] of var int: x);

predicate all_different_int(array[int] of var int: x) =
    optisolve_alldifferent(x);

```

When a MiniZinc model that contains the `all_different` constraint is now compiled with the *OptiSolve* library, it will contain calls to the newly defined predicate `optisolve_alldifferent`.

#### 4.3.4 Command Line Interface

In order to work with the `minizinc` command line driver, a FlatZinc solver must be an executable (which can include e.g. shell scripts) that can be invoked as follows:

```
$ <executable-name> [options] model.fzn
```

where `<executable-name>` is the name of the executable. Solvers may support the following standard options:

`-a`

Instructs the solver to report *all* solutions in the case of satisfaction problems, or print *intermediate* solutions of increasing quality in the case of optimisation problems.

`-n <i>`

Instructs the solver to stop after reporting *i* solutions (only used with satisfaction problems).

`-f`

Instructs the solver to conduct a “free search”, i.e., ignore any search annotations. The solver is not *required* to ignore the annotations, but it is *allowed* to do so.

`-s`

Print statistics during and/or after the search for solutions. Statistics should be printed as FlatZinc comments to the standard output stream. See below for a standard format for statistics.

`-v`

Print log messages (verbose solving) to the standard error stream. If solvers choose to print to standard output instead, all messages must be valid comments (i.e., start with a % character).

-p <i>

Run with i parallel threads (for multi-threaded solvers).

-r <i>

Use i as the random seed (for any random number generators the solver may be using).

### 4.3.5 Solver Configuration Files

In order for a solver to be available to MiniZinc, it has to be described in a *solver configuration file*. This is a simple file, in JSON or .dzn format, that contains some basic information such as the solver's name, version, where its library of global constraints can be found, and a path to its executable.

A solver configuration file must have file extension .msc (for MiniZinc Solver Configuration), and can be placed in any of the following locations:

- In the minizinc/solvers/ directory of the MiniZinc installation. If you install MiniZinc from the binary distribution, this directory can be found at /usr/share/minizinc/solvers on Linux systems, inside the MiniZincIDE application on macOS system, and in the Program Files\\MiniZinc IDE (bundled) folder on Windows.
- In the directory \$HOME/.minizinc/solvers on Linux and macOS systems, and the Application Data directory on Windows systems.
- In any directory listed on the MZN\_SOLVER\_PATH environment variable (directories are separated by : on Linux and macOS, and by ; on Windows systems).
- In any directory listed in the mzn\_solver\_path option of the global or user-specific configuration file (see [Section 3.1.4](#))
- Alternatively, you can use the MiniZinc IDE to create solver configuration files, see [Section 3.2.5.2](#) for details.

Solver configuration files must be valid JSON or .dzn files. As a JSON file, it must be an object with certain fields. As a .dzn file, it must consist of assignment items.

For example, a simple solver configuration in JSON format could look like this:

```
{  
  "name" : "My Solver",  
  "version": "1.0",  
  "id": "org.myorg.my_solver",
```

```

"executable": "fzn-mysolver"
}

```

The same configuration in .dzn format would look like this:

```

name = "My Solver";
version = "1.0";
id = "org.myorg.my_solver";
executable = "fzn-mysolver";

```

Here is a list of all configuration options recognised by the configuration file parser. Any valid configuration file must at least contain the fields name, version, id, and executable.

- name (string, required): The name of the solver (displayed, together with the version, when you call `minizinc --solvers`, and in the MiniZinc IDE).
- version (string, required): The version of the solver.
- id (string, required): A unique identifier for the solver, “reverse domain name” notation.
- executable (string, required): The executable for this solver that can run FlatZinc files. This can be just a file name (in which case the solver has to be on the current PATH), or an absolute path to the executable, or a relative path (which is interpreted relative to the location of the configuration file).
- mznlib (string, default ""): The solver-specific library of global constraints and redefinitions. This should be the name of a directory (either an absolute path or a relative path, interpreted relative to the location of the configuration file). For solvers whose libraries are installed in the same location as the MiniZinc standard library, this can also take the form -G<solverlib>, e.g., -Ggecode (this is mostly the case for solvers that ship with the MiniZinc binary distribution).
- tags (list of strings, default empty): Each solver can have one or more tags that describe its features in an abstract way. Tags can be used for selecting a solver using the `--solver` option. There is no fixed list of tags, however we recommend using the following tags if they match the solver’s behaviour:
  - “cp”: for Constraint Programming solvers
  - “mip”: for Mixed Integer Programming solvers
  - “float”: for solvers that support float variables
  - “api”: for solvers that use the internal C++ API

- `stdFlags` (list of strings, default empty): Which of the standard solver command line flags are supported by this solver. The standard flags are `-a`, `-n`, `-s`, `-v`, `-p`, `-r`, `-f`.
- `extraFlags` (list of list of strings, default empty): Extra command line flags supported by the solver. Each entry must be a list of four strings. The first string is the name of the option (e.g. `--special-algorithm`). The second string is a description that can be used to generate help output (e.g. `"which special algorithm to use"`). The third string specifies the type of the argument (`"int"`, `"bool"`, `"float"` or `"string"`). The fourth string is the default value.
- `supportsMzn` (bool, default `false`): Whether the solver can run MiniZinc directly (i.e., it implements its own compilation or interpretation of the model).
- `supportsFzn` (bool, default `true`): Whether the solver can run FlatZinc. This should be the case for most solvers
- `needsSolns20ut` (bool, default `true`): Whether the output of the solver needs to be passed through the MiniZinc output processor.
- `needsMznExecutable` (bool, default `false`): Whether the solver needs to know the location of the MiniZinc executable. If true, it will be passed to the solver using the `mzn-executable` option.
- `needsStdlibDir` (bool, default `false`): Whether the solver needs to know the location of the MiniZinc standard library directory. If true, it will be passed to the solver using the `stdlib-dir` option.
- `isGUIApplication` (bool, default `false`): Whether the solver has its own graphical user interface, which means that MiniZinc will detach from the process and not wait for it to finish or to produce any output.

### 4.3.6 Grammar

This is the full grammar for FlatZinc. It is a proper subset of the MiniZinc grammar (see Section 4.1.14). However, instead of specifying all the cases in the MiniZinc grammar that do *not* apply to FlatZinc, the BNF syntax below contains only the relevant syntactic constructs. It uses the same notation as in Section 4.1.2.

```
% A FlatZinc model
<model> ::= [
  <predicate-item> ]*
  [ <par-decl-item> ]*
```

```

[ <var-decl-item> ]*
[ <constraint-item> ]*
<solve-item>

% Predicate items
<predicate-item> ::= "predicate" <identifier> "(" [ <pred-param-type> :_
→<identifier> "," ... ] ")" ";"

% Identifiers
<identifier> ::= [A-Za-z][A-Za-z0-9_]*

<basic-par-type> ::= "bool"
| "int"
| "float"
| "set of int"

<par-type> ::= <basic-par-type>
| "array" "[" <index-set> "]" "of" <basic-par-type>

<basic-var-type> ::= "var" "bool"
| "var" "int"
| "var" <int-literal> ".." <int-literal>
| "var" "{" <int-literal> "," ... "}"
| "var" "float"
| "var" <float-literal> ".." <float-literal>
| "var" "set" "of" <int-literal> ".." <int-literal>
| "var" "set" "of" "{" [ <int-literal> "," ... ] "}"

<array-var-type> ::= "array" "[" <index-set> "]" "of" <basic-var-type>

<index-set> ::= "1" ".." <int-literal>

<basic-pred-param-type> ::= <basic-par-type>
| <basic-var-type>
| <int-literal> ".." <int-literal>
| <float-literal> ".." <float-literal>
| "{" <int-literal> "," ... "}"
| "set" "of" <int-literal> .. <int-literal>

```

```

| "set" "of" "{" [ <int-literal> "," ... ] "}"
| "var" "set" "of" "int"

<pred-param-type> ::= <basic-pred-param-type>
                     | "array" "[" <pred-index-set> "]" "of"_
                     ↵<basic-pred-param-type>

<pred-index-set> ::= <index-set>
                     | "int"

<basic-literal-expr> ::= <bool-literal>
                         | <int-literal>
                         | <float-literal>
                         | <set-literal>

<basic-expr> ::= <basic-literal-expr>
                  | <var-par-identifier>

<expr> ::= <basic-expr>
           | <array-literal>

<par-expr> ::= <basic-literal-expr>
               | <par-array-literal>

<var-par-identifier> ::= [A-Za-z_][A-Za-z0-9_]*

% Boolean literals
<bool-literal> ::= "false"
                   | "true"

% Integer literals
<int-literal> ::= [0-9]*
                  | 0x[0-9A-Fa-f]*
                  | 0o[0-7]*

% Float literals
<float-literal> ::= [0-9]+.[0-9]*
                      | [0-9]+.[0-9]+[Ee][-+]?[0-9]*

```

```

| [0-9]+[Ee][-+]?[0-9]++

% Set literals

<set-literal> ::= "{" [ <int-literal> "," ... ] "}"
    | <int-literal> ".." <int-literal>
    | "{" [ <float-literal> "," ... ] "}"
    | <float-literal> ".." <float-literal>

<array-literal> ::= "[" [ <basic-expr> "," ... ] "]"

<par-array-literal> ::= "[" [ <basic-literal-expr> "," ... ] "]"

% Parameter declarations

<par-decl-item> ::= <par-type> ":" <var-par-identifier> "=" <par-expr> ";"

% Variable declarations

<var-decl-item> ::= <basic-var-type> ":" <var-par-identifier> <annotations>
    ↵ [ "=" <basic-expr> ] ";"
        | <array-var-type> ":" <var-par-identifier> <annotations>
    ↵ "=" <array-literal> ";"

% Constraint items

<constraint-item> ::= "constraint" <identifier> "(" [ <expr> "," ... ] ")"
    ↵ <annotations> ";"

% Solve item

<solve-item> ::= "solve" <annotations> "satisfy" ;"
    | "solve" <annotations> "minimize" <basic-expr> ;"
    | "solve" <annotations> "maximize" <basic-expr> ;"

% Annotations

<annotations> ::= [ ":" <annotation> ]*

```

```
<annotation> ::= <identifier>
               | <identifier> "(" <ann-expr> "," ... ")"
 
<ann-expr>   ::= <expr>
               | <annotation>

% End of FlatZinc grammar
```

---

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---

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