Energy and Flocking Effects in Predator-Prey Population Models

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Abstract

This paper consists of the dynamics of a simulated predator-prey ecosystem, which consists of foxes and rabbits. The interactions between these two types of agents, the parameters used for modeling their behavior, and the reaction of the ecosystem to these behaviors will be investigated further. We implemented four different simulations: (1) without flocking and without energy levels, (2) without flocking but with energy levels, (3) with flocking and without energy levels, and (4) with both flocking and energy levels." From these simulations, we analyzed the impact that different situations have on the agent's populations.

1 Introduction

The Lotka-Volterra predator-prey model, also known as the Lotka-Volterra equations, is a fundamental mathematical model used to describe the dynamics of biological systems in which two species interact: a predator and its prey. The model is named after Alfred J. Lotka and Vito Volterra, who independently developed the equations in the early 20th century [Das and Gupta(2011)].

Lotka-Volterra's predator-prey models are commonly used for a variety of purposes in different fields, such as biology and ecology, as well as predicting stock market behavior. It is used to model the evolution of N discrete dynamic variables. In predator-prey competition modeling, there are typically two co-dependent populations governed by formulae which specify how each changes with time [Ofer Malcai(2002)].

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad \frac{dy}{dt} = \delta xy - \gamma y$$

Here, x represents the prey population, y represents the predator population, α is the prey's birth rate, β is the predation rate coefficient, γ is the predator's death rate, and δ is the rate at which predators increase by consuming prey [Das and Gupta(2011)].

1.1 Problem Definition and Scope

This paper considers a Lotka-Volterra predator-prey model with rabbits, denoted by green triangles, as prey, and foxes, denoted by red triangles, as predators. Various implementations are considered, including ones with and without the concept of energy for foxes. Implementations with and without flocking behavior for rabbits have been included as well, which reflect a collective behavior observed in nature. The experiments conducted assess the utility of energy to establish a baseline. After this, flocking is assessed as an additional behavior in the simulation, with specific relation to the programs' performance. The results and discussion have been presented below.

1.2 Motivation

The purpose of the experiments presented by this paper revolve around gaining a deeper understanding of both Lotka-Volterra models and flocking behavior, as biological processes. Flocking is another concept important to both biology and mathematics, with applications in swarm robotics [Beaver and Malikopoulos(2021)]. It describes the behavior often seen in large groups of birds or fish, moving as distinctly separate agents yet also approximately as one organism. This emergent behavior was first translated to a simulation by Craig Reynolds, who developed heuristic rules to model flocking through a computer program [Reynolds(1987)]. These rules were adapted and presented as an optimal learning-based approach to flocking by [Koichiro Morihiro(2006)], and are considered here. The most integral concepts to flocking behavior are alignment, separation, and cohesion. Alignment rewards agents for maintaining approximately equal velocities to their neighbors. Separation refers to the avoidance of collisions between agents. Cohesion refers to the state of the flock as one being. In other words, it encourages movement towards the local centers of mass within the flock. They are calculated through the following equations:

$$V_N = \frac{1}{N} \sum_{i \in N} V_i \tag{1}$$

$$a = V_N - V_{boid} \tag{2}$$

$$s = \frac{1}{N} \sum_{i \in N} (X_{boid} - X_i) \tag{3}$$

$$\overline{X_N} = \frac{1}{N} \sum_{i \in N} X_i \tag{4}$$

$$f_c = \overline{X_N - X_{boid}} \tag{5}$$

$$c = f_c - V_{boid} \tag{6}$$

Where V represents velocity, N the number of agents, a the alignment, s the separation, c the cohesion, f_c the cohesion force, and X the position of (a) boid(s) [Koichiro Morihiro(2006)]. In this implementation, these (being a, c, and s in the formulae) are added together to form a flocking vector which is added to the movement.

The Lotka-Volterra model is in itself very interesting and useful for behavioral research. A model which combines the Lotka-Volterra simulation with flocking behavior could help in further analyzing emergent behaviors in wildlife ecosystems. Specifically, it could aid in analyzing predator-prey dynamics in a population with flocking. For this purpose, a primary aim of this work is to assess the relationship between flocking and the Lotka-Volterra model. In our purpose to explore the model, four different scenarios and their respective performances are considered: without energy for predators or flocking in prey, with energy for predators but no flocking in prey, with flocking in prey but no energy for predators, and with both energy and flocking present.

1.3 Research Question

The purposes of this research require a focus on the flocking behavior as an additional measure to the Lotka-Volterra model. The exact impact of this feature is difficult to anticipate. Therefore, the research question in consideration is as follows.

RQ: To what extent does flocking behavior in prey influence the performance of a competitive predator-prey Lotka-Volterra model?

The question leads to the following hypotheses.

H0: Flocking behavior has no impact on the model's overall performance

HA: Flocking influences the model's overall performance in some way

2 Methodology

The programs were developed using Python 3.11 as a base language, as well as PyGame with Violet simulator as an additional module to handle the visual aspects of the simulation. Each program has a Configuration class to store all the parameters of the simulation, such as movement speed or initial populations of rabbits and foxes. The exact parameters change depending on the implementation, as described below. Several parameters correspond to the constant variables seen in the equations above. These are variables governing concepts like the reproduction rate of either species or the death rate of foxes, as well as the predation rate. While these concepts are closely related to the theoretical formulae, the implementation treats agents on the individual level. Therefore, unlike with different approaches such as evolutionary algorithms, there is no straightforward method of controlling the whole population directly at each time interval. For this reason, the formulae have been used as a guide for the models rather than as blueprint behaviors to be followed.

2.1 Configuration and Parameters

Parameter configurations are specific to each individual implementation. Overlapping parameters have been kept identical so as to allow for fair assessment of each program's performance. Initial populations are set such that there are considerably more rabbits than foxes, with 50 and 25, respectively. This is to prevent early extinction of the rabbits (in the first cycle of the simulation). Each implementation includes variables to track the fox birth and natural death rate, rabbit birth rate, and the rate of predation. Beyond this, parameters vary depending on whether they use energy or flocking. Energy implementations include energy-specific concepts: initial energy, energy gained by eating a rabbit, and the energy cost of reproduction. Flocking-specific parameters are cohesion, separation, and alignment weights, as per the central concepts behind the mechanics of flocking [Beaver and Malikopoulos(2021)]. The overlapping parameters of each implementation were set to equal values to prevent interference from confounding variables.

2.2 Non-Flocking Experiment Setup

The differences between energy and energy-free versions are retained within the flocking implementation as well. This subsection therefore discusses these differences as well as the experiment setup for the flocking-free models.

A Configuration class keeps track of all the configuration parameters used. The most significant parameters are the aforementioned storing of the initial populations, as well as the rabbit birth rate, fox reproduction and death rate, and the predation rate. These manipulate the dynamics of the populations, by controlling how often reproduction or death happens, as well as how often predators hunt successfully. The energy implementations also contain parameters to control for energy changes. These are specifically the initial energy a fox has, the energy gained from eating a rabbit, and the reproduction energy cost.

Each agent type has its own class. The fox class specifies fox behavior. On spawn, agents move randomly. They have a chance of natural death according to the fox death rate, that is also proportional to the time that has elapsed. Thus, the longer an agent lives, the more likely it is for it to die. When a rabbit is encountered, it is killed according to the probability specified by the predation rate. A fox can reproduce according to the reproduction probability. In the energy version, energy depletes as time passes. Thus, a fox dies when its energy drops to zero as well as from natural death. When a rabbit is eaten, the fox gains a fixed amount of energy from it, and when the predator's energy is higher than the reproduction cost threshold, it reproduces. In this way, energy functions as a type of currency in these implementations.

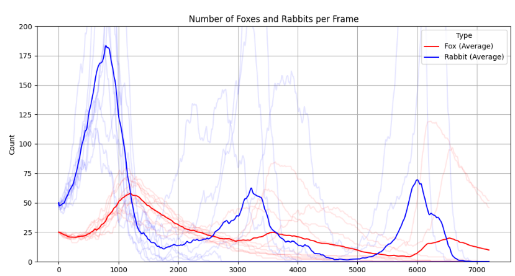
Rabbits begin random movement on spawn. They reproduce according to their reproduction rate. In the energy version, every 10 seconds that a rabbit has not reproduced, the probability changes to 0.5 and reproduction is attempted again. This measure was implemented to prevent early extinction in the energy models.

2.3 Flocking Experiment Setup

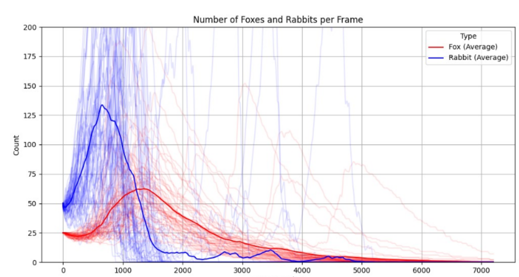
The flocking configuration contains small differences discussed here. Primarily, three new parameters are added to the Configuration class, namely the separation, alignment, and cohesion weights, respectively. These mirror the weights of each integral flocking concept shown in the formulae below. The fox class is identical to the non-flocking versions, considering that the behavior is only applied to rabbits. The rabbit class encourages random movement as before, however, if neighboring agents of the same type are detected, they will begin flocking. This is achieved by setting values for the alignment, separation, and cohesion according to their weights and respective equations, as depicted below. These are then combined into a single flocking vector which influences the movement of the rabbit. The equations that govern flocking behavior and were implemented in the code are as follows:

3 Results

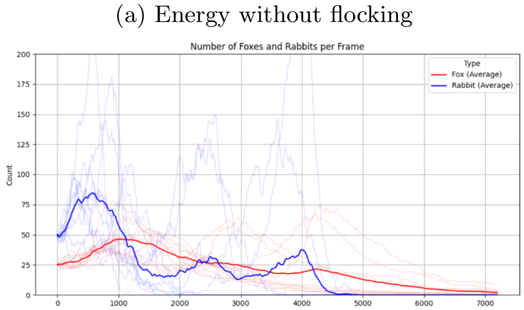
The graph of the simulation that uses an energy attribute and has no flocking shows a decent Lotka-Volterra predator-prey model with three peaks from the rabbits, followed by smaller peaks of the prey. These rabbit peaks occur due to their growth, while fox peaks occur due to the large number of rabbits, making catching one easier. The graph of the simulation that uses an energy attribute and also flocking for the rabbits shows a different model. The graph only has one big peak for the rabbits, followed by a peak of the foxes, but the rabbits do not produce another typical peak. This likely happens because when flocking is introduced, predators have too great an advantage. The fox then eats the entire flock, reproducing and causing all the rabbits to die out. This is why the graph has one peak and decreases afterward for both the rabbits and the foxes. The graph without the energy attribute and without flocking also shows signs of a Lotka-Volterra model, with three distinct rabbit peaks followed by smaller fox peaks. The first fox peak is clearly seen, but the second and third are much harder to see. The same simulation but with flocking shows limited signs of a Lotka-Volterra model. Rabbit peaks begin significantly but then die out, retaining only small peaks. The foxes in this model show a big peak after the first rabbit peak but then decline, with small peaks following the rabbit peaks. This significant first peak is likely because the rabbits are in flocks and all die when encountered by one fox, who can then reproduce greatly.



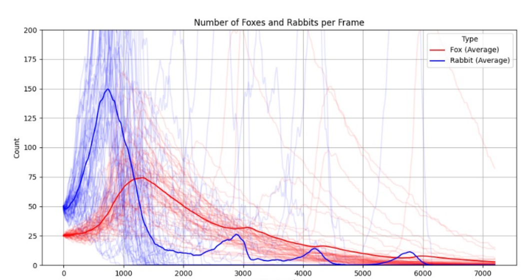
(a) Energy without flocking



(b) Energy with flocking



(c) Energy-Free without flocking



(d) Energy-Free with flocking

Figure 1: Simulation results for different configurations.

3.1 Statistical Analysis

We have opted to use a T-distribution method in order to determine whether flocking behavior in prey influences the performance of a competitive Predator-Prey Lotka-Volterra model. We conducted a series of simulations and applied statistical testing, which was used to evaluate the differences in population dynamics between different implementations. Two comparisons were made to determine statistical significance.

(1) Simulation with energy reduction with flocking VS (2) Simulation with energy reduction without flocking

(3) Simulation without energy reduction with flocking VS (4) Simulation without energy reduction without flocking

To statistically compare the population means between the flocking and no flocking scenarios, we utilized the independent two-sample t-test. This method allows us to determine if there is a significance difference in the average populations of foxes

For the statistical comparison of the models we will use an independent t-test. The t-test is denoted as:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{7}$$

where:

- $\bar{X}_1$ and $\bar{X}_2$ are the sample means,
- s_1^2 and s_2^2 are the sample variances,
- n_1 and n_2 are the sample sizes.

1. **Calculate Means and Standard Deviations:** Compute the mean and standard deviation of the population values for each scenario.
2. **Calculate T-statistic:** Use the formula given previously.
3. **Calculate Degrees of Freedom:** Using python built-in methods.
4. **Determine P-value:** Using the t-statistic and degrees of freedom, the p-value is calculated to determine the statistical significance of the results.

We used a significance level of $\alpha = 0.05$ for our tests. This means that we are willing to accept a 5% chance of rejecting the null hypothesis when it is actually true. The choice of $\alpha = 0.05$ due to it being a common method used to balance risking false negatives and false positives.

3.2 Statistical Results

3.2.1 Energy-Free Versions

Fox Population

- T-statistic: 25.94
- P-value: 0.0000
- Degrees of Freedom: 10483.12

Rabbit Population

- T-statistic: 51.22
- P-value: 0.0000

- Degrees of Freedom: 7682.72

The results indicate a significant statistical difference in the populations of foxes and rabbits between the flocking and no flocking scenarios in the energy-free versions. The extremely low p-values (0.0000) suggest that the differences in population means are not due to random chance.

For fox populations, the t-statistic of 25.94 shows a deviation between the two scenarios, which indicates a significant impact of flocking behavior. Similarly, the rabbit populations' t-statistic of 51.22 highlights a pronounced effect of flocking on rabbit survival and dynamics.

3.2.2 Energy Versions

Fox Population

- T-statistic: 12.00
- P-value: 0.0000
- Degrees of Freedom: 12629.97

Rabbit Population

- T-statistic: -3.38
- P-value: 0.0007
- Degrees of Freedom: 14257.72

The results for the energy versions also show significant differences in the populations of foxes and rabbits between the flocking and no flocking scenarios. The low p-values (0.0000 for foxes and 0.0007 for rabbits) indicate that the differences are statistically significant.

For fox populations, a t-statistic of 12.00 suggests a noticeable effect of flocking behavior under energy constraints. The negative t-statistic for rabbits (-3.38) reflects that while there is a statistically significant difference, the direction of the effect is opposite, this could mean that there is a negative effect of flocking under energy constraints for rabbits.

4 Conclusion

In this study, we investigated the impact of flocking behavior on the predator-prey dynamics in a Lotka-Volterra model. By conducting various simulations with and without flocking, and with and without energy constraints, we aimed to understand how these factors influence the populations of foxes and rabbits. Our experiments demonstrated that flocking behavior significantly affects the population dynamics of both species. In scenarios without energy constraints, both foxes and rabbits showed many differences in population means with extremely low p-values, indicating the significance of the observed differences. With energy constraints, the significant t-statistics for foxes and rabbits further highlighted the role of flocking behavior, albeit with a notable directional change for rabbits.

The statistical analysis confirmed that flocking behavior introduces complexity to the predator-prey interactions, impacting the survivability and reproduction rates of both species. These results highlight the importance of behavioral factors, such as flocking, in ecological models to achieve a more comprehensive understanding of ecosystem dynamics. Future work could explore the interplay between other behaviors and environmental factors, providing deeper insights into the intricacies of predator-prey relationships.

4.1 Future Research

Future work can build upon this study in several ways. One of the many methods is to develop additional behavioral phenomena, like energy for rabbits, or active fleeing/chasing behavior between Rabbits and Foxes. These could serve to simulate a more realistic environment, comparable to real-life scenarios.

Author Contribution

All authors contributed equally to this work

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