GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 4: NAIVE BAYESIAN CLASSIFICATION

LAST TIME:

- CLASSIFICATION PROBLEMS
- TRAINING/TEST SETS & CROSS-VALIDATION
- KNN CLASSIFICATION

QUESTIONS?

I. INTRO TO PROBABILITY

II. NAÏVE BAYESIAN CLASSIFICATION

EXERCISES:

III. IMPLEMENTING A SPAM FILTER

0. DATA SCIENCE IN THE NEWS

I. INTRO TO PROBABILITY

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The probability of event A *is denoted* $P(A)$.

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The probability of the sample space $P(\Omega)$ *is 1.*

Q: Consider two events A & B. How can we characterize the **intersection** *of these events?*

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A: With the **joint probability** *of A and B, written $P(AB)$.*

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This information about B *transforms* the sample space.

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This is called the **conditional probability** *of A given B, written $P(A|B) = P(AB) / P(B)$.*

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Notice, with this we can also write $P(AB) = P(A|B) * P(B)$.

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A: Information about one does not affect the probability of the other.

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Using the definition of the conditional probability, we can also write:

$$P(A|B) = P(AB) / P(B) = P(A) \rightarrow P(AB) = P(A) * P(B)$$

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→ $P(A|B) = P(B|A) * P(A) / P(B)$ *by rearranging last step*

This result is called **Bayes' theorem**. *Here it is again:*

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This result is called **Bayes' theorem**. *Here it is again:*

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Some facts:
- This is a simple algebraic relationship using elementary definitions.
- It's interesting because it's kind of a "wormhole" between two different "interpretations" of probability.
- It's a very powerful computational tool.

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The ***frequentist interpretation*** *regards an event's probability as its limiting frequency across a very large number of trials.*

The ***Bayesian interpretation*** *regards an event's probability as a "degree of belief," which can apply even to events that have not yet occurred.*

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This a good direction to head if you like math and/or if you're interested in learning about cutting-edge data science techniques.

II. NAÏVE BAYESIAN CLASSIFICATION

Suppose we have a dataset with features $x_1, \ldots, x_n$ and a class label c. What can we say about classification using Bayes' theorem?

Suppose we have a dataset with features $x_1, \ldots, x_n$ and a class label C. What can we say about classification using Bayes' theorem?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, given the data we observe.

source: Data Analysis with Open Source Tools, by Philipp K. Janert. O'Reilly Media, 2011.

Each term in this relationship has a name, and each plays a distinct role in any Bayesian calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **likelihood function**. *It represents the joint probability of observing features* $\{x_i\}$ *given that that record belongs to class* C.

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We can observe the value of the likelihood function from the training data.

This term is the **prior probability** *of* C*. It represents the probability of a record belonging to class* C *before the data is taken into account.*

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The value of the prior is also observed from the data.

This term is the **normalization constant.** *It doesn't depend on* C, *and is generally ignored until the end of the computation.*

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The normalization constant doesn't tell us much.

This term is the **posterior probability** *of* C*. It represents the probability of a record belonging to class* C *after the data is taken into account.*

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The goal of any Bayesian computation is to find ("learn") the posterior distribution of a particular variable.

The idea of Bayesian inference, then, is to **update** *our beliefs about the distribution of C using the data ("evidence") at our disposal.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Then we can use the posterior for prediction.

Methods	*Predictions*
“classical” (frequentist)	*point estimates*
Bayesian	*distributions*

Example: Spam classification

You have a database of emails.

60% of those emails are spam

80% of those emails that are spam have the word "buy"

20% of those emails that are spam don't have the word "buy"

40% of those emails aren't spam

10% of those emails that aren't spam have the word "buy"

90% of those emails that aren't spam don't have the word "buy”

What is the probability that an email is spam if it has the word "buy"?

Example: Spam classification

P(spam) = the probability that an email is spam

P(not spam) = the probability that an email isn't spam

P("buy"|spam) = the probability that an email that it is spam has the word "buy"

P("buy"|not spam) = the probability that an email that it isn't spam has the word "buy"

P(spam|"buy") = the probability that an email that has the word "buy" is spam

Example: Spam classification

*P("buy"|spam) * P(spam) counts all the emails that are spam and have the word "buy"*
*P("buy"|not spam) * P(not spam) counts all the emails that aren't spam and have the word "buy"*

*Summing the previous two P("buy"|spam) * P(spam) + P("buy"|not spam) * P(not spam) we count all the emails that have the word "buy"*

So our answer will be:

*P(spam|"buy") = P("buy"|spam) * P(spam) / (P("buy"|spam) * P(spam) + P("buy"|not spam) * P(not spam))*

III. SPAM FILTER

EXERCISE – SPAM FILTER (DOCUMENT CLASSIFICATION)

KEY OBJECTIVES

- preprocess data
- perform naïve Bayes classification

TOOLS

- e1071, tm