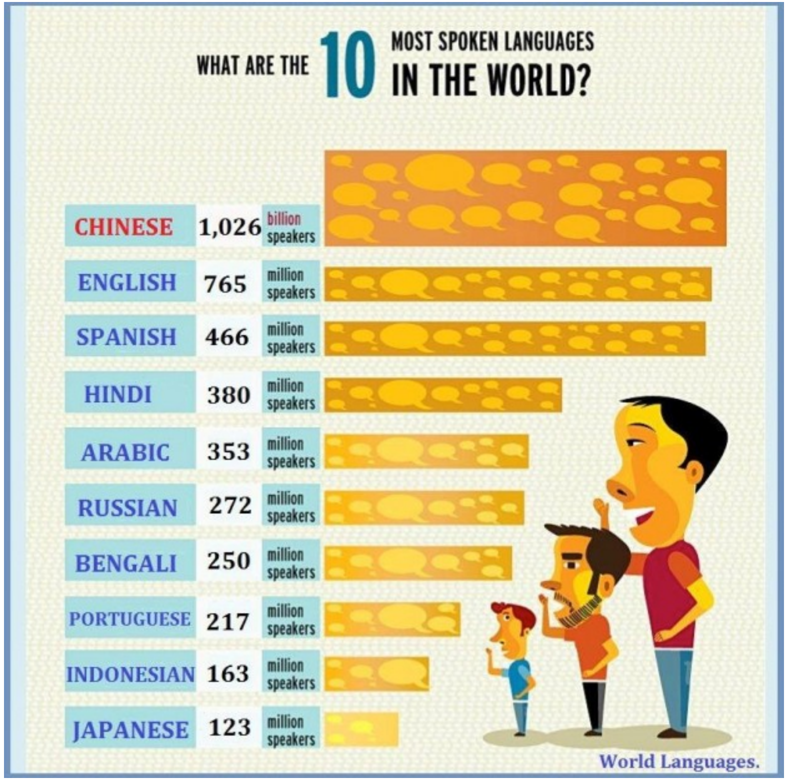
GENERAL ASSEMBLY

INTRO TO DATA SCIENCE
LECTURE 6: LOGISTIC REGRESSION

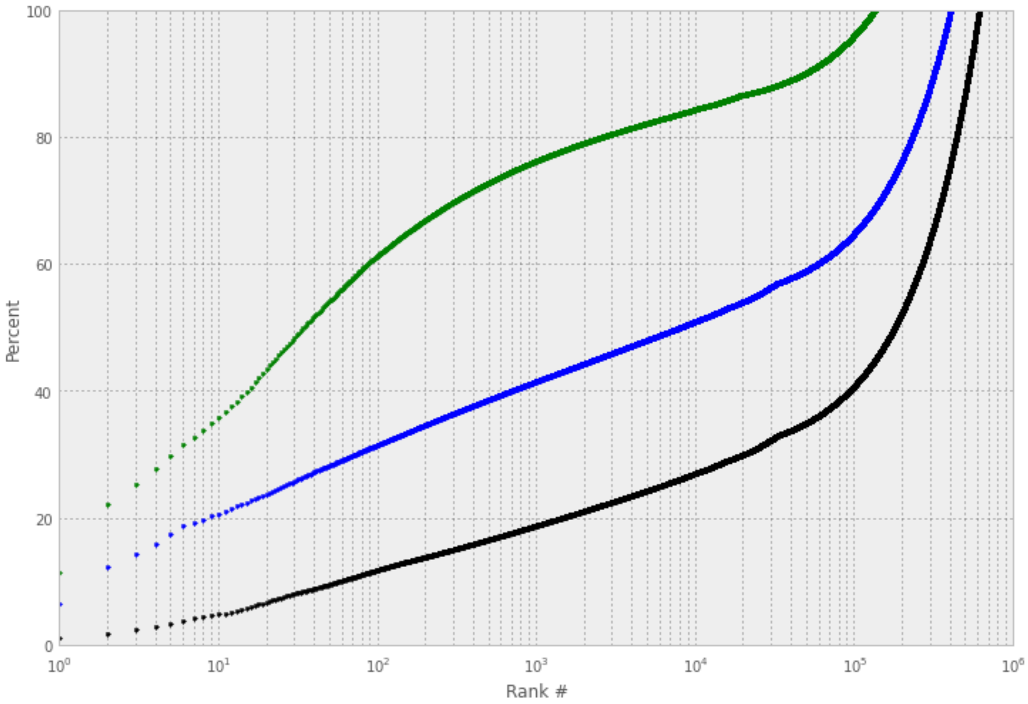
I. LOGISTIC REGRESSION
II. OUTCOME VARIABLES
III. ERROR TERMS
IV. INTERPRETING RESULTS

EXERCISES:
IMPLEMENTING A LOGISTIC FIT IN R

DATA SCIENCE IN THE NEWS

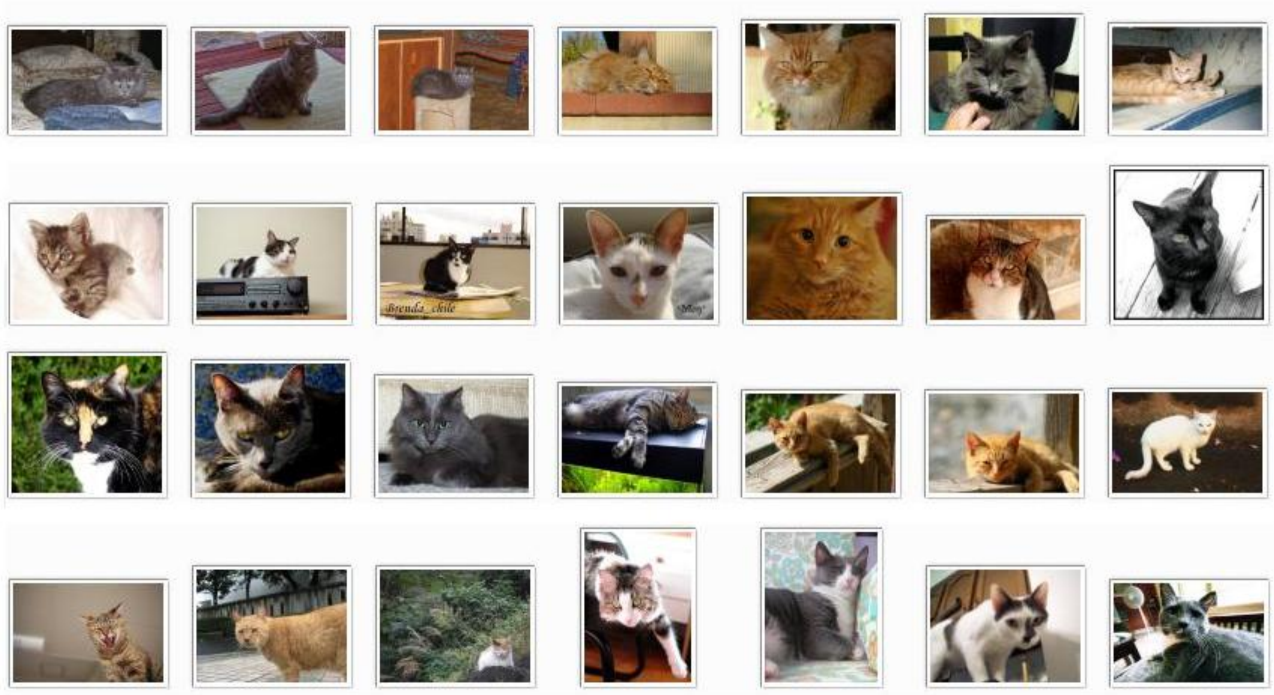


http://flowingdata.com/2013/07/15/open-thread-what-is-wrong-with-these-charts/



http://engineering.quora.com/Quora-Machine-Learning-CodeSprint

DATA SCIENCE IN THE NEWS



http://137.189.35.203/WebUI/CatDatabase/catData.html

I. LOGISTIC REGRESSION

	continuous	*categorical*
supervised	???	???
unsupervised	???	???

	continuous	categorical
supervised	regression	classification
unsupervised	dimension reduction	clustering

Q: What is **logistic regression***?*

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A: A generalization of the linear regression model to classification problems.

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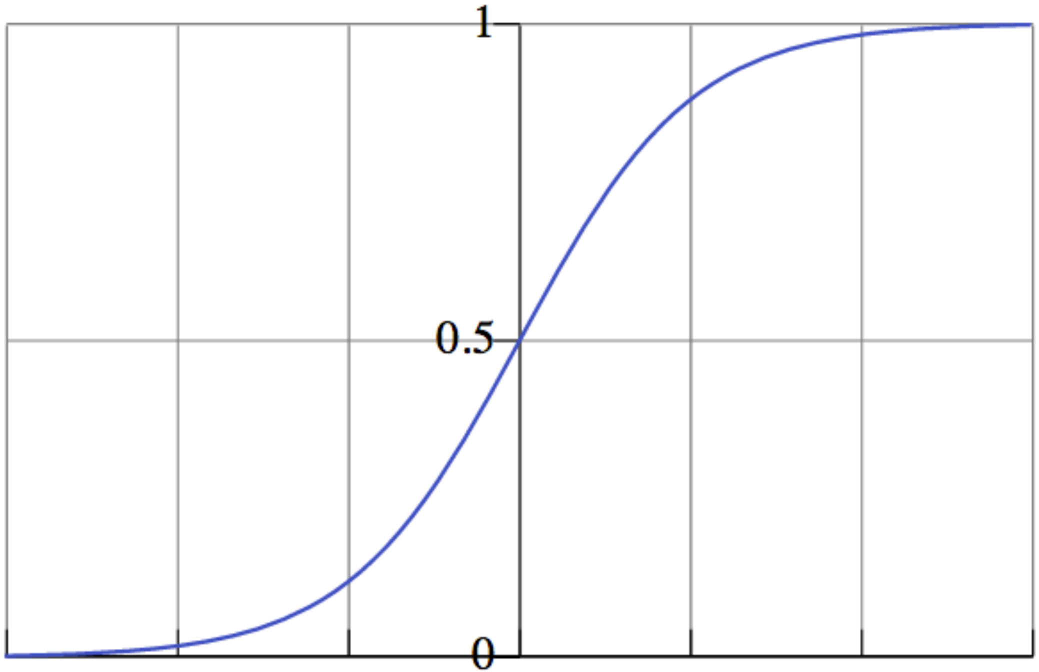
In logistic regression, we use a set of covariates to predict probabilities *of (binary) class membership.*

These probabilities are then mapped to class labels*, thus solving the classification problem.*

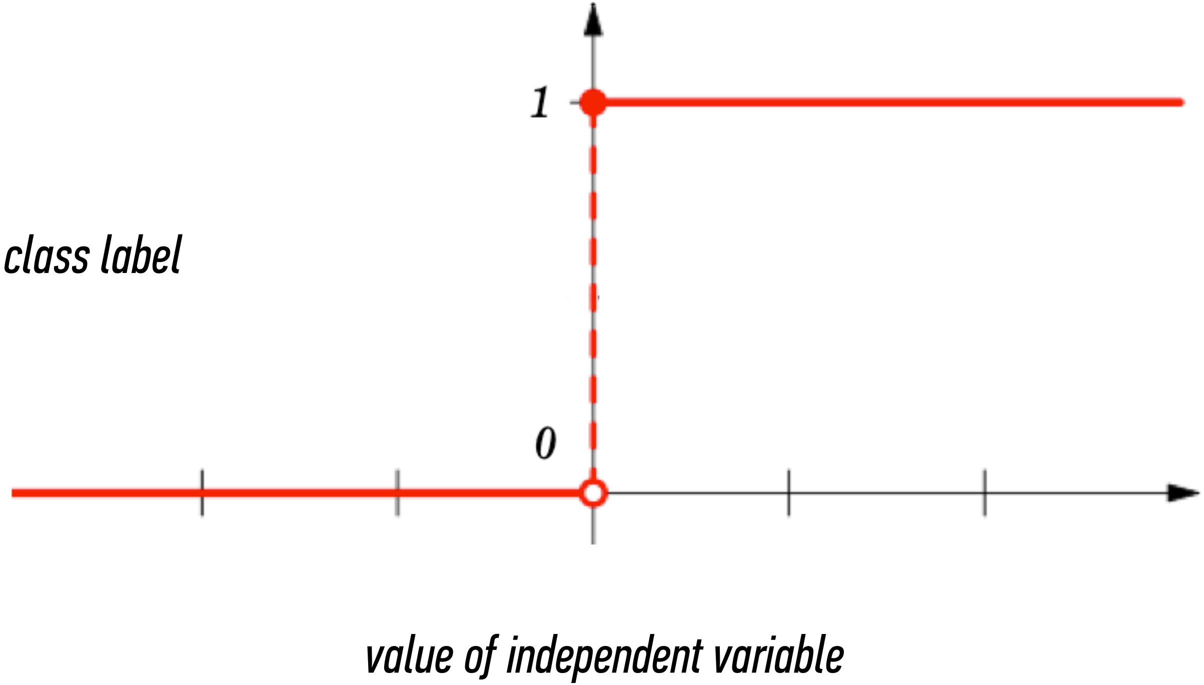
probability of belonging to class



value of independent variable

NOTE

Probability predictions look like this.



NOTE

Probabilities are “snapped” to class labels (eg by threshholding at 50%).

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The first difference is in the outcome variable.

The second difference is in the error term.

II. OUTCOME VARIABLES

The key variable in any regression problem is the **conditional mean** *of the outcome variable* y *given the value of the covariate* x*:*

$$E(y|x)$$

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$$E(y|x)$$

In linear regression, we assume that this conditional mean is a linear function taking values in $(-\infty, +\infty)$:

$$E(y|x) = \alpha + \beta x$$

In logistic regression, we've seen that the conditional mean of the outcome variable takes values only in the unit interval $[0, 1]$.

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The first step in extending the linear regression model to logistic regression is to map the outcome variable $\mathbb{E}(y|x)$ *into the unit interval.*

Q: How do we do this?

A: By using a transformation called the **logistic function***:*

$$E(y|x) = \pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

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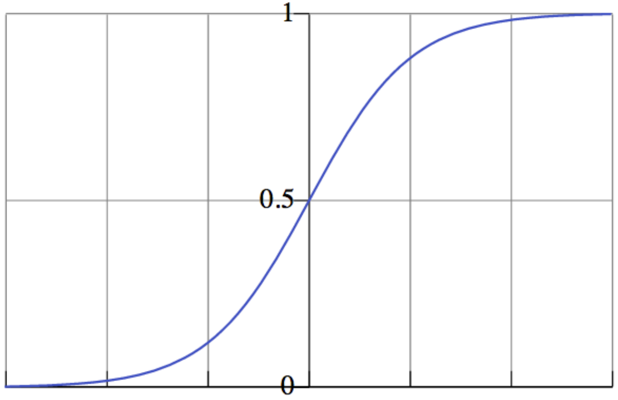
We've already seen what this looks like:



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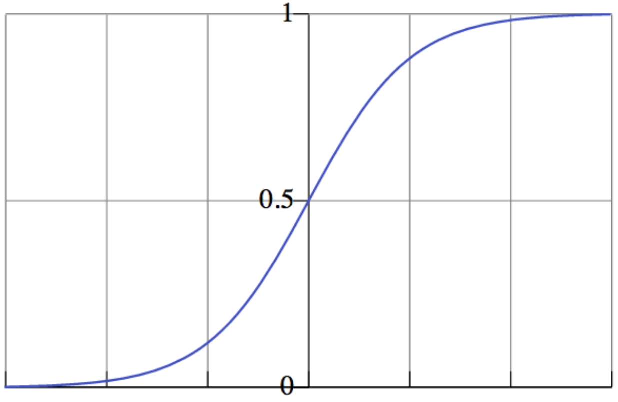

NOTE

For any value of x, y is in the interval [0, 1]

This is a nonlinear transformation!

The **logit function** *is an important transformation of the logistic function. Notice that it returns the linear model!*

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NOTE

This name hints at its usefulness in interpreting our results.

We will see why shortly.

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III. ERROR TERMS

The second difference between linear regression and the logistic regression model is in the error term.

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One of the key assumptions of linear regression is that the error terms follow independent Gaussian distributions with zero mean and constant variance:

$$\epsilon \sim N(0, \sigma^2)$$

In logistic regression, the outcome variable can take only two values: 0 or 1.

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It's easy to show from this that instead of following a Gaussian distribution, the error term in logistic regression follows a Bernoulli distribution:

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NOTE

This is the same distribution followed by a coin toss.

Think about why this makes sense!

These two key differences define the logistic regression model, and they also lead us to a kind of unification of regression techniques called **generalized linear models**.

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Briefly, GLMs generalize the distribution of the error term, and allow the conditional mean of the response variable to be related to the linear model by a **link function***.*

In the present case, the error term follows a Bernoulli distribution, and the logit is the link function that connects us to the linear predictor.

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Since the Bernoulli distribution and the logit function share a common parameter π, we say that the logit is the **canonical link function** *for the Bernoulli distribution.*

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NOTE

This terminology is just FYI!

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IV. INTERPRETING RESULTS

In linear regression, the parameter β represents the change in the response variable for a unit change in the covariate.

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In logistic regression, β represents the change in the logit function for a unit change in the covariate.

Interpreting this change in the logit function requires another definition first.

The **odds** *of an event are given by the ratio of the probability of the event by its complement:*

$$O(x = 1) = \frac{\pi(1)}{(1 - \pi(1))}$$

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The **odds ratio** *of a binary event is given by the odds of the event divided by the odds of its complement:*

$$OR = \frac{O(x=1)}{O(x=0)} = \frac{\pi(1)/[1-\pi(1)]}{\pi(0)/[1-\pi(0)]}$$

Substituting the definition of $\pi(x)$ into this equation yields (after some algebra),

$$OR = e^{\beta}$$

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This simple relationship between the odds ratio and the parameter β is what makes logistic regression such a powerful tool.

Q: So how do we interpret this?

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A: The odds ratio of a binary event gives the increase in likelihood of an outcome if the event occurs.

Suppose we are interested in mobile purchase behavior. Let y be a class label denoting purchase/no purchase, and let x denote a mobile OS (for example, iOS).

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In this case, an odds ratio of 2 (eg, $\beta = \log(2)$) indicates that a purchase is twice as likely for an iOS user as for a non-iOS user.

EX: LOGISTIC REGRESSION

EXERCISE 1 – LINEAR REGRESSION

KEY OBJECTIVES

- perform a logistic fit

TOOLS

- glm {stat}