

# Adaptive Controllers For Battery Management System

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**Abstract**—In this century, electronic devices are changing the world. Also, a battery is a widely used electronic component in this era. It can be found everywhere that needs to be supplied by electronic power. Like in figure 1, it is used to connect the battery and outside components. In people's daily life, televisions, laptops and even mobile phones are highly demanded on the battery. The battery provides and stabilizes power. As it is such highly important stuff, it needs to be figured out that what will affect the performance of batteries.

**Index Terms**—System Modeling, Control, Battery Management System

## I. INTRODUCTION

In this paper, the main idea is from the battery management system (BMS). For a battery management system, it is necessary to build a mathematical model in order to get a more detailed and useful approach towards further research and development. An accurate model can represent a much more powerful understanding of the application of further usage. The following paragraph states some applications of the necessary mathematical modelling of the battery electronic circuit.

When the battery is used, it is essential for people to get an insight of what is the state of the battery currently. The state is an important indicator of how well the battery performed in the following tasks. Based on these indicators, the battery may be treated and maintained in a more scientific manner.

When predicting and evaluating the state of charge, it is still a complex problem that nonlinear electrochemical processes are hard to model. It is a dynamic process that needs to be accurately simulated. If the state of charge can be monitored in real-time, it will make the battery performance better, lifetime longer and protect the battery from damage by inappropriate manipulations. Thus, an equation model can provide such a reference to a certain degree.

State of health is a common-used factor for evaluating the state of the battery. It is known to all that the lifetime of a battery is always decreasing during working. The capacity of charging or discharging decreases over time is called capacity fading, while the power loss during absorbing and delivering is called power fading [2]. Also, there are other factories that may influence the state of health. For example, in an appropriate temperature between -30 and 52 degrees centigrade, the Lithium battery can usually perform ideally

and efficiency [3]. While the temperature is much higher or lower than this interval, the battery will have a significant capacity loss which makes it easy to dry up. Moreover, if the temperature is not controlled well, the battery may be damaged at last.

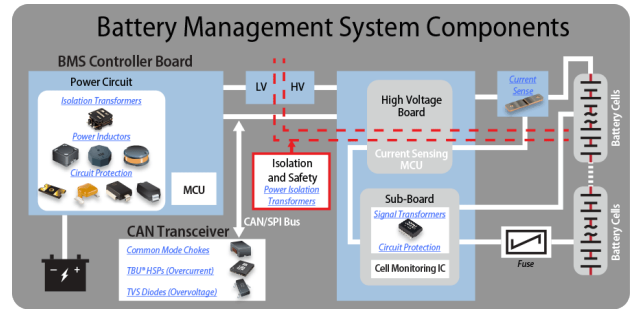


Fig. 1. Diagram of BMS in the circuit [1]

The remainder of this paper is organised as follows: Section II introduced the parameters essential in the analysis, then built the circuit diagram and established the numerical equation for the model. Section III designed and provided the controller that can control the dynamical properties based on the state-space model built in the previous section. And finally validated the effectiveness and robustness of the controller with some different real-case scenarios.

## II. SYSTEM MODEL AND PROBLEM DEFINITION

### A. Electronic Circuit Modeling

When predicting the state of charge and the state of health, a dynamic model of the battery should be built. In this part, a model referenced from [4] has introduced a traditional Lithium-ion battery model that is applicable for this study. For mechanism, a Lithium-ion battery has electrochemical reactions in its cell. During discharge, There is an oxidation half-reaction at the anode which produces a positive charge and negative electrons at the anode and cathode [5]. There are many electronic components that can represent batteries to a certain degree. For simplest, a capacitor can be charged and discharged to analog a small 'battery'. Moreover, in this report, a relatively proper model of battery is the Saft supplied NREL [3] that can model a 12Ah Lithium-ion cell. Further, it can be treated as a resistor-capacitor (RC) model.

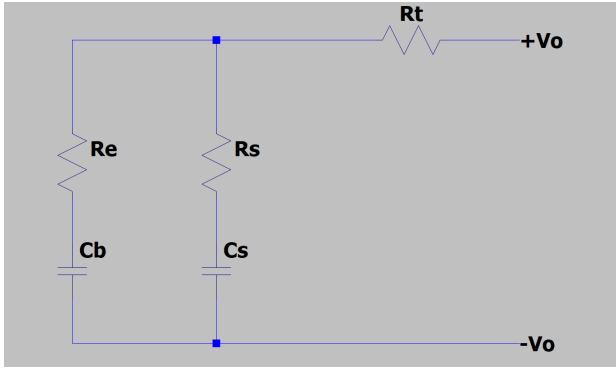


Fig. 2. Circuit diagram for Lithium-ion battery model

As in figure 2, it can be simplified as capacitors and resistors.  $R_t$  is the terminal resistor,  $R_s$  is the surface resistor and  $R_e$  is the end resistor.  $C_s$  and  $C_b$  represents surface and bulk capacitors respectively. The whole circuit is equivalent to a power supplier that has the anode and cathode labeled with  $+V_o$  and  $-V_o$ . Also, denote  $I$  as the total current while  $I_s$  and  $I_b$  are current goes though capacitor  $C_s$  and  $C_b$ .

### B. Problem definition

Through a research, the model that is used by National Renewable Energy Laboratory (US) gave a reasonable assumption of the parameters listed [4].

TABLE I  
ASSUMED PARAMETERS

Parameter name	Value	Unit	Reference
$R_t$	1.2	$\Omega$	[4]
$R_e$	1.1	$\Omega$	[4]
$R_s$	0.4	$\Omega$	[4]
$C_b$	82	F	[4]
$C_s$	4	F	[4]

Batteries are widely used in electronic devices. Each device needs a nominated voltage to satisfy its working load. If the voltage is lower, it may affect the efficiency of the device. If the voltage is high, there is a possibility that the device will be destroyed by the large power.

The purpose of the battery management system is to provide an ideal voltage that satisfies the requirement of external peripherals. By using a similar battery structure, the management system can always control an ideal output. Thus, the output voltage is the output that we want to control by using different input currents  $I$ . For example, take a stochastic increase of both capacitors, the voltage supplied to the smartphone is 3.7V with a tolerance of 5% [7]. In most cases, there should be a fuse that protects the other components. However, the battery management system should take its best to provide a stable nominated voltage by taking a well-performed controller.

In this report, the criterion for the controller should be following the next two points. First is the limitation of overshoot. Generally, the tolerance can be 1.1% [6]. It is a small amount of overshoot, however, this is necessary especially for

the precise instruments. Secondly, the rise time is less than 2 seconds. As the battery is the power supplier for devices, a rapid response represents better performance.

### C. Dynamical Equation

In this part, the mathematical model of the above circuit will be constructed in detail.

By analyzing the fundamental electronic circuit with Kirchhoff's circuit laws, the total voltage in a circuit loop should be zero:

$$V_o = I * R_t + I_b * R_e + V_{Cb} \quad (1)$$

$$V_o = I * R_t + I_s * R_s + V_{Cs} \quad (2)$$

Combining and simplifying equation (1) and (3):

$$I_s * R_s + V_{Cs} = I_b * R_e + V_{Cb} \quad (3)$$

The voltage though the capacitor is proportional to the integration of the passing current ( $i$ ). Its voltage will never be sudden changed. Thus, it has the representation by:

$$V_c = \frac{1}{C} \int i dt$$

$$I_c = C * \dot{V}_c \quad (4)$$

Where  $C$  is the capacitance of the capacitor depending on the property of particular capacitor. Substituting equation (4) into equation (3), two representation of voltage of capacitor can be concluded:

$$\dot{V}_{Cb} = \frac{1}{C_b * (R_e + R_s)} * V_{Cs} - \frac{1}{C_b * (R_e + R_s)} * V_{Cb} + \frac{R_s}{C_b * (R_e + R_s)} * I \quad (5)$$

$$\dot{V}_{Cs} = -\frac{1}{C_s * (R_e + R_s)} * V_{Cs} + \frac{1}{C_s * (R_e + R_s)} * V_{Cb} + \frac{R_e}{C_s * (R_e + R_s)} * I \quad (6)$$

By combining equation (1) and (2), we can get a relationship between  $V_o$  and  $V_{Cb} + V_{Cs}$ .

$$V_o = \frac{V_{Cb} + V_{Cs}}{+} (R_t + \frac{R_e R_s}{R_e + R_s})$$

$$\dot{V}_o = \frac{\dot{V}_{Cb} + \dot{V}_{Cs}}{2} \quad (7)$$

Finally, plug the equation (5) and (6) into it, the third equation can be get:

$$\dot{V}_o = (-\frac{1}{C_b * (R_e + R_s)} + \frac{1}{C_s * (R_e + R_s)}) * V_{Cb} + (\frac{1}{C_b * (R_e + R_s)} - \frac{1}{C_s * (R_e + R_s)}) * V_o + [\frac{1}{C_b * (R_e + R_s)} * (0.5R_s + R_t + \frac{R_e R_s}{R_e + R_s}) + \frac{1}{C_s * (R_e + R_s)} * (0.5R_e - R_t - \frac{R_e R_s}{R_e + R_s})] * I \quad (8)$$

#### D. State-space model

As the dynamical equation shown, the relationships between current, voltage, capacitance and resistance are quite complex. It is not a simple linear or non-linear system. The state-space model provides an efficient representation for a group of equations by using matrices.

Dynamic equations derived in the previous section can be gathered into a state-space system. For a standard state-space form, it can be written as:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

Plug the specific parameter values into the equation (5) (6) (7). Then arrange them in to the matrix A, B and C.

$$\begin{bmatrix} \dot{V}_{Cb} \\ \dot{V}_{Cs} \\ \dot{V}_o \end{bmatrix} = \begin{bmatrix} -\frac{1}{123} & \frac{1}{123} & 0 \\ \frac{1}{6} & -\frac{1}{6} & 0 \\ \frac{13}{82} & 0 & -\frac{13}{82} \end{bmatrix} \begin{bmatrix} V_{Cb} \\ V_{Cs} \\ V_o \end{bmatrix} + \begin{bmatrix} \frac{9}{10} \\ \frac{3}{5} \\ \frac{1}{500} \end{bmatrix} I$$

$$V_o = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{Cb} \\ V_{Cs} \\ V_o \end{bmatrix}$$

#### E. Stability

This is a dynamical system where each voltage variable is both interacting with itself and others. When it is viewed as a plant without the controller, the eigenvalues of matrix A can reflect the stability of the original system. Therefore, the first step when considering the stability is deriving eigenvalues  $\lambda$ :

$$\lambda_1 = 0 \quad \lambda_2 = -\frac{13}{82} \quad \lambda_3 = -\frac{43}{246} \quad (9)$$

Since there is a  $\lambda$  that does not have the negative real part, the system is not stable. In the following part, controllers will be designed and applied to make the system stable and have the ability to track any reference input.

### III. SOLUTIONS AND NUMERICAL VALIDATIONS

#### A. PID Control

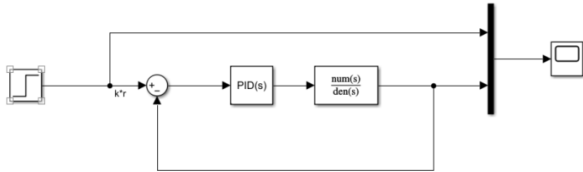


Fig. 3. PID control system layout in Simulink

Proportional, Integral and derivative (PID) is a classical control method that is widely used in many fields. The first controller we are going to use is the PID controller. Firstly the property of the system needs to be known. By using the MatLab function 'ss2tf', the higher-order system with a complex transfer function can be got:

$$G(s) = \frac{0.002s^2 + 0.143s + 0.0246}{s^3 + 0.3s^2 + 0.0277s} \quad (10)$$

Thus, it is harder to tune PID parameters using a traditional method compared with the first and second-order systems. In this case, the Ziegler-Nichols method provides a general and powerful approach for tuning. Since determining the ultimate gain and period using the experimental method is time-consuming [8]. Gain margin (GM) is a good factor for predicting ultimate gain.

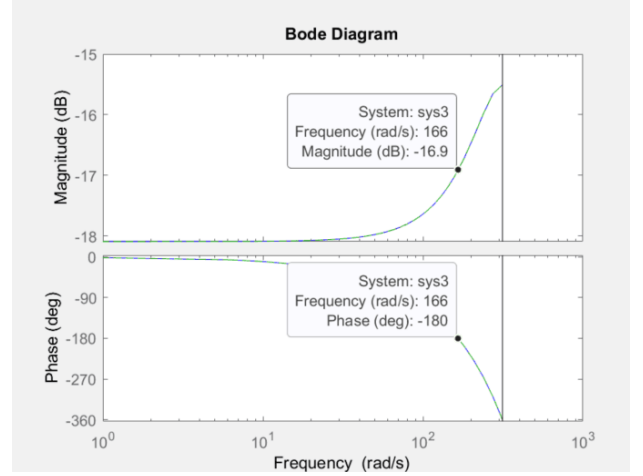


Fig. 4. Bode plot of the system

From figure 4, GM can be calculated to be  $-0.059$ . Taking initial  $K_p = 10 \frac{GM}{20} = 0.993$ , from figure 5 the response trajectory can reflect the time period of  $T_u = 17.5$ .

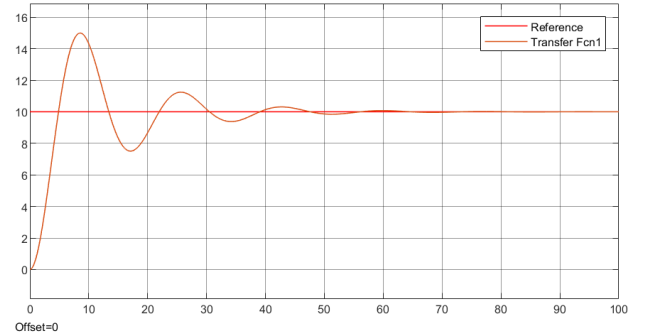


Fig. 5. Initial system response

TABLE II  
ZIEGLER-NICHOLS METHOD

Control Type	$K_p$	$K_i$	$K_d$	Reference
P	$0.5K_u$			[9]
PI	$0.45K_u$	$0.54K_u/T_u$		[9]
PD	$0.8K_u$		$0.1K_u/T_u$	[9]
classic PID	$0.6K_u$	$1.2K_u/T_u$	$0.075K_u/T_u$	[9]

Following table II, the gains for a classic PID can be calculated as:

$$K_p = 0.4 \quad K_i = 0.068 \quad K_d = 1.3 \quad (11)$$

where  $K_p$  is the proportional gain,  $K_i$  is the integral gain and  $K_d$  is the derivative gain.

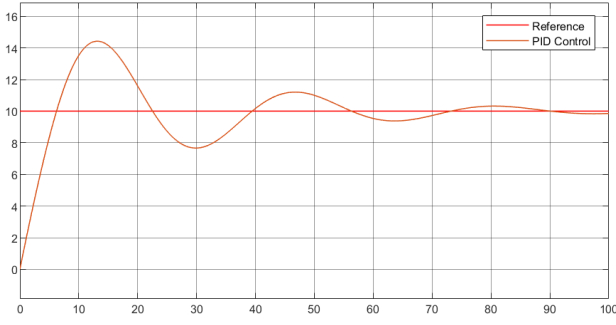


Fig. 6. Applied Ziegler–Nichols method

However, it is obvious that the response characteristic is not ideal. The overshoot is too large and the rise time is too long. The next tuning intuition is raising the value of  $K_p$  and  $K_d$  to shorten the rising period as well as limit the overshoot. In the finalized figure 7, the parameters are adjusted to:

$$K_p = 10 \quad K_i = 0.068 \quad K_d = 16 \quad (12)$$

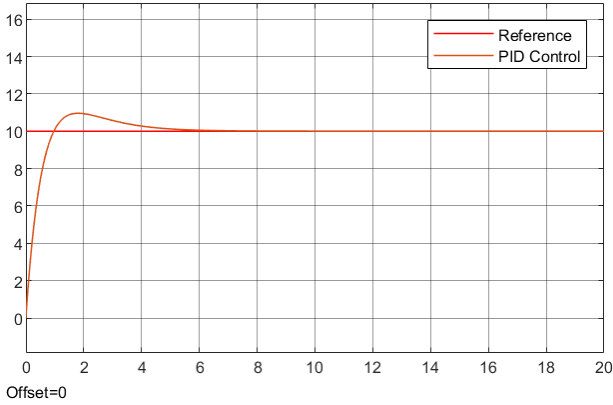


Fig. 7. Finalized PID control response with  $r = 10$

The resulting plot of PID control is reasonable, however, this is an experimental method that is not accurate when the specific response properties are required. Hence, there is another controller designed based on the theoretical approach in the next section.

### B. State-feedback Control

Currently, we are considering the case that the state of the system is known. Thus, a state feedback controller can be applied to make the output be what the reference is. The general controller can be represented as:

$$u = -Kx + kr \quad (13)$$

where  $K$  is the feedback and  $k$  is the input reference gain.

For the first step, the reachability (controllability) should be checked.

$$W = [A \quad AB \quad A^2B] \quad (14)$$

$W$  is the reachability matrix. When  $\det(W) \neq 0$ , the system tends to be controllable. Under this circumstance, the battery management system is controllable. Then, the reachable canonical form needs to be found to get a standard forms which are represented by  $\tilde{A}$  and  $\tilde{B}$ . The vector  $a$  has 3 components which are the coefficients of the  $W$  characteristic equation. Compute the characteristic polynomial  $\det(sI - A)$  to get the reachable conical form  $\dot{z} = \tilde{A}z + \tilde{B}u$ :

$$a1 = \frac{1}{3} \quad a2 = \frac{559}{20172} \quad a3 = 0; \quad (15)$$

$$\tilde{A} = \begin{bmatrix} -a1 & -a2 & -a3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

Uniformly:

$$\tilde{W} = [\tilde{A} \quad \tilde{A}\tilde{B} \quad \tilde{A}^2\tilde{B}] \quad (17)$$

So we can know that the transformation matrix between  $\dot{x}$  and  $\dot{z}$  is

$$T = \frac{\tilde{W}}{W} \quad (18)$$

As mentioned, the overshoot  $M \approx 1\%$  and rise time  $t_r \approx 2s$ . Since it is a third order syste, assume the characteristic equation is  $(s + \zeta\omega_n)(s^2 + 2\zeta\omega_n s + \omega_n^2)$ . The relationship between characteristic equation and requirement is:

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} < 5\% \quad \zeta > 0.82 \quad (19)$$

$$t_r = \frac{3.93}{\zeta\omega_n} < 2 \quad \omega_n > 11 \quad (20)$$

Plug the approximated value of  $\zeta$  and  $\omega_n$  into the characteristic equation, the vector  $P$  can be get where:

$$p_1 = 11.79 \quad p_2 = 53.83 \quad p_3 = 9.17 \quad (21)$$

The feedback matrix  $K$  is:

$$K = \tilde{K} * T = [250 \quad -359 \quad 481] \quad (22)$$

The input reference gain  $k$  is:

$$k = \frac{-1}{(C/(A - B * K) * B)} = 373 \quad (23)$$

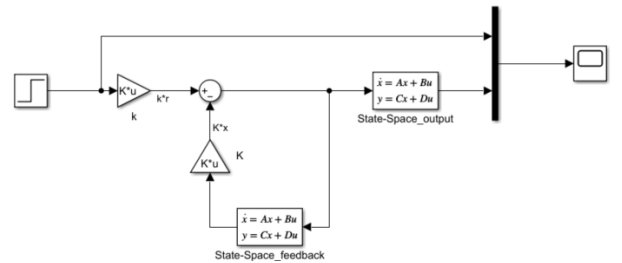


Fig. 8. Simulink model for state-feedback

The controller and system diagram is modelled in the Simulation. With each component labelled, it is clear to see how the controller works by using the state of the system. In figure 8, there are two system blocks. They are the same except for the output matrix  $C$ . The state-space output block is using the output  $y = V_o$ , while the state-space feedback block gives all available states back to feed the controller. (This has the same result as using a multiplex.)

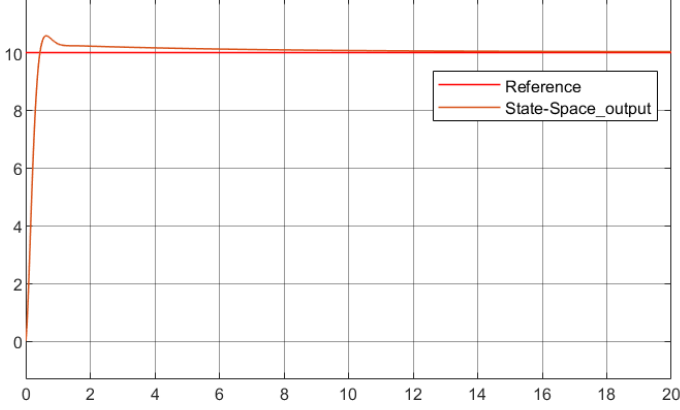


Fig. 9. State-feedback with  $r = 10$

By using the Simulink model, we can get a general concept of how the controller performs. The system converges fast with a little overshoot which is just as we expected. Figure 9 demonstrates an ideal step response of the whole controlled system.

### C. Validations

After several controlling approaches, the performance of each is going to be validated and compared through several situations. In this section, four challenges will be conducted to test the designed controllers. They are following the sequence of multi-step input, uncertainty, disturbance and noises of input.

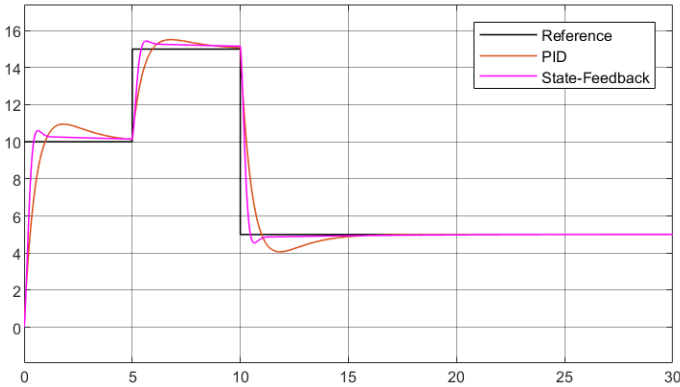


Fig. 10. Multi step input reference

The first test case is the multi-step input references. As shown in figure 10, the input reference changed 2 times at 5 second and 10 second. PID controller and state feedback controller are both applied for this (and following) task. The trajectories of them are all clearly labeled in different colours. In this case, state feedback displays the excellent performance, while PID does not very good. The overshoot of PID controller is larger than the requirement of 1% in the last response which is not we want. But overall, this can actually be further tuned to get a lower overshoot.

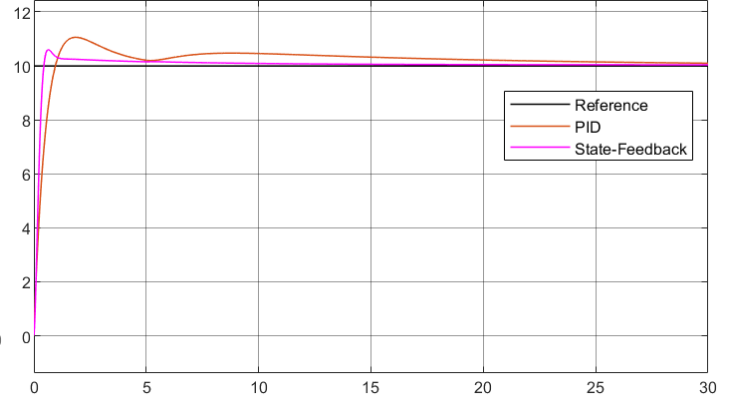


Fig. 11. Disturbance

The second test for controllers is disturbance. In real-life, the connections between each component are not ideally zero conductance. When there is dirt or dopant, they may be heated by the passing current and make the working environment of the system hot. The temperature is a factor that may affect many properties of electronic components. Thus when the heat is accumulated to a certain degree, it is like a disturbance impacting the whole system. The disturbance is added between the controller and the plant. It is set to be triggered at 5 seconds. In figure 11, PID has an obvious impact towards the input disturbance. But it recovers gradually to the original reference in about 10 to 15 seconds. On the other hand, state feedback performs perfectly in this test case.

In reality, the capacitor may fatigue which causes the capacitance lower down. Thus, as time passes, the capacitance  $C_b$  and  $C_s$  may gradually smaller which makes the entities in matrix  $A$  and the transfer function change. For example, the system equation is reconstructed to be:

$$\begin{bmatrix} \dot{V}_{Cb} \\ \dot{V}_{Cs} \\ \dot{V}_o \end{bmatrix} = \begin{bmatrix} -\frac{1}{12} & \frac{1}{12} & 0 \\ \frac{1}{13} & -\frac{1}{5} & 0 \\ \frac{13}{41} & 0 & -\frac{13}{41} \end{bmatrix} \begin{bmatrix} V_{Cb} \\ V_{Cs} \\ V_o \end{bmatrix} + \begin{bmatrix} \frac{9}{10} \\ \frac{2}{5} \\ \frac{1}{500} \end{bmatrix} I \quad (24)$$

The transfer function also changes to:

$$G(s) = \frac{0.002s^2 + 0.286s + 0.0634}{s^3 + 0.0567s^2 + 0.0793s} \quad (25)$$

This is the uncertainty test. No-one knows how the system will change. It can be arbitrary changed according to the established dynamic model. Although the system changes, the

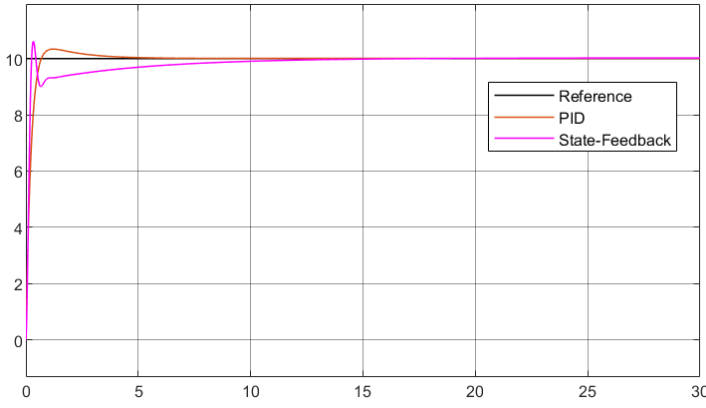


Fig. 12. Uncertainty

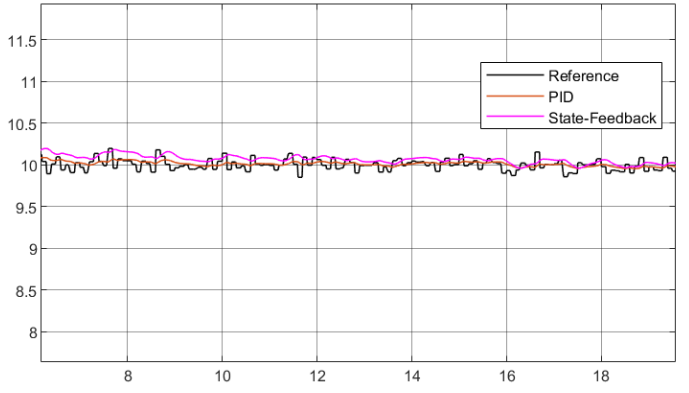


Fig. 14. Zoomed in figure of input noises

controllers remain the same. From figure 12, the trajectory of state feedback control changes significantly. It has a rapid oscillation when reaching the reference. The PID response ideally this time is very stable and robust to the uncertainty. This modelling result can provide an evidence that PID is more robustness when dealing with the fatigue of components.

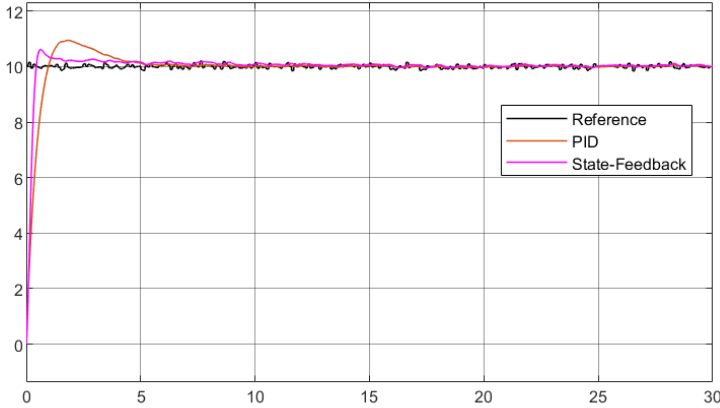


Fig. 13. Input noises

However, if the noise is larger, the range of output voltage will be unsafe. Because the range will exceed 1% limitation. This gives a remainder that the input signal needs to be stable in steady-state so that the output can be in an ideal manner.

From the above four validations, we can see that in most cases, both systems can output a good trajectory within requirements. Although some unexpected cases may affect the output voltage, they are still acceptable for powerful controllers.

The Nyquist plot of the loop function is shown in figure 15. It does not enclose the point  $(-1, 0)$  which means  $N = 0$ . Also, there is no unstable pole lies on the right half of the plane. Thus,  $P = 0$ . Combining these two conditions, it can be concluded that the system is stable after control.

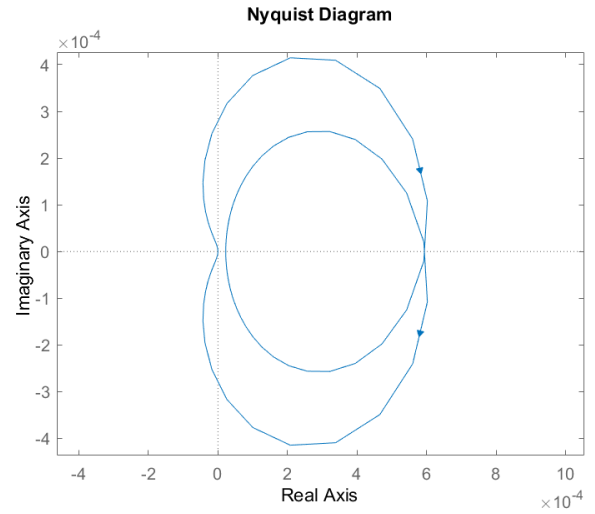


Fig. 15. Nyquist plot of the loop function

#### IV. CONCLUSIONS

In this paper, a battery management system is used as the background of the design. By selecting a battery equivalent model, suitable controllers are then designed and validated in the numerical result part.

Firstly the classic PID controller is applied by tuning with the Ziegler-Nichols method. It can give a quick sense of the shape and the requirement of the response. However, it cannot be accurately tuned according to the overshoot and rise criterion. Therefore, sometimes it may exceed the requirement we set in the problem definition part. On the other hand, it works perfectly in the uncertainty test which is a simulation of burn-in.

Then a state feedback is applied to the state-space model. This is an advanced controller which can take every state in the model into consideration. During the calculation of criteria, it is fussy. Nevertheless, it acts accurately in the most of following validation processes. Even if the curve is not ideal in the uncertainty test, it is still under the requirement of

overshoot. This controller can protect the outside peripherals powerfully.

Overall, both state feedback and PID controllers work well. By applying a battery management system, the battery can output stable power to the outside. The issues of the battery can be managed and found by people immediately if the output voltage is not ideal. In future work, the output feedback control can also be taken into consideration. It can let people observe the state more directly. Also, the machine learning approach can help control better the slight change in the battery (burn-in, disturbances). It is a more advanced control which will be suitable for more aspects of the area.

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