ECON 220C PROBLEM SET # 1

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1. Questions from textbook

1.1. Romer 5.8.

(a). We can start with the full Lagrangian, after substituting

$$C_t = K_t + Y_t - K_{t+1} = K_t + AK_t + e_t - K_{t+1} = (1 + A)K_t + e_t - K_{t+1}$$

to get

$$\mathcal{L} = E\left[\sum_{t=0}^{\infty} \frac{u(C_t) + \lambda_t((1 + A)K_t + e_t - K_{t+1})}{(1 + \rho)^t}\right]$$

which gives first order conditions with respect to C_t and K_{t+1} (which are chosen each period) of

$$u'(C_t) = \lambda_t$$

and

$$\lambda_t = \frac{(1 + A)E[\lambda_{t+1}]}{(1 + \rho)}$$

and combining gives

$$u'(C_t) = \frac{(1 + A)E[u'(C_{t+1})]}{(1 + \rho)}$$

but since $A = \rho$, we are left with a more standard

$$u'(C_t) = E[u'(C_{t+1})]$$

and we can substitute for $u'(C_t)$ since we are given the form of the utility function to get

$$u'(C_t) = 1 - 2\theta C_t$$

and we now have

$$1 - 2\theta C_t = E[1 - 2\theta C_{t+1}]$$

and from linearity of expectation we can cancel terms to get

$$C_t = E[C_{t+1}]$$

as our Euler equation.

(b). Substituting the guessed form into the resource constraint

$$C_t = (1 + A)K_t + e_t - K_{t+1}$$

gets us

$$\alpha + \beta K_t + \gamma e_t = (1 + A)K_t + e_t - K_{t+1}$$

and upon rearranging we have

$$K_{t+1} = (1 + A - \beta)K_t + (1 - \gamma)e_t - \alpha$$

as our function for future capital.

(c). We have given that

$$C_t = \alpha + \beta K_t + \gamma e_t$$

so we merely need to find

$$E[C_{t+1}] = E[\alpha + \beta K_{t+1} + \gamma e_{t+1}]$$

By linearity and substituting our earlier result, we have

$$E[C_{t+1}] = \alpha + \beta((1 + A - \beta)K_t + (1 - \gamma)e_t - \alpha) + \gamma E[e_{t+1}]$$

Since we are given that ε_t has expectation zero, we can substitute

$$E[e_{t+1}] = \phi e_t$$

(since we are merely applying the law of motion for e in period $t + 1$ rather than period t) and so we have

$$E[C_{t+1}] = \alpha + \beta((1 + A - \beta)K_t + (1 - \gamma)e_t - \alpha) + \gamma\phi e_t$$

which we can simplify as

$$E[C_{t+1}] = (1 - \beta)\alpha + \beta(1 + A - \beta)K_t + (\beta - \gamma\beta + \gamma\phi)e_t$$

and from the Euler equation we have

$$\alpha + \beta K_t + \gamma e_t = (1 - \beta)\alpha + \beta(1 + A - \beta)K_t + (\beta - \gamma\beta + \gamma\phi)e_t$$

and so we will have

$$\begin{aligned} \alpha &= (1 - \beta)\alpha \\ \beta &= \beta(1 + A - \beta) \\ \gamma &= (\beta - \gamma\beta + \gamma\phi) \end{aligned}$$

which upon solving this system (with 3 equations and 3 unknowns since A and ϕ are given) yield

$$\begin{aligned} \alpha &= 0 \\ \beta &= A \\ \gamma &= \frac{A}{1 + A - \phi} \end{aligned}$$

for the parameters.

(d).

1.2. **Romer 5.9.**

(a). We can start with the full Lagrangian, after substituting

$$C_t = K_t + Y_t - K_{t+1} = K_t + AK_t + -K_{t+1} = (1+A)K_t - K_{t+1}$$

to get

$$\mathcal{L} = E\left[\sum_{t=0}^{\infty} \frac{u(C_t) + \lambda_t((1+A)K_t - K_{t+1})}{(1+\rho)^t}\right]$$

which gives first order conditions with respect to C_t and K_{t+1} (which are chosen each period) of

$$u'(C_t) = \lambda_t$$

and

$$\lambda_t = \frac{(1+A)E[\lambda_{t+1}]}{(1+\rho)}$$

and combining gives

$$u'(C_t) = \frac{(1+A)E[u'(C_{t+1})]}{(1+\rho)}$$

but since $A = \rho$, we are left with a more standard

$$u'(C_t) = E[u'(C_{t+1})]$$

and we can substitute for $u'(C_t)$ since we are given the form of the utility function to get

$$1 - 2\theta(C_t + v_t) = 1 - 2\theta(E[C_{t+1}] + E[v_t]) \implies C_t + v_t = E[C_{t+1}]$$

from linearity of expectation and the zero mean shock.

(b). Substituting the guessed form into the resource constraint

$$C_t = (1+A)K_t - K_{t+1}$$

gets us

$$\alpha + \beta K_t + \gamma v_t = (1+A)K_t - K_{t+1}$$

and upon rearranging we have

$$K_{t+1} = (1 + A - \beta)K_t - \gamma v_t - \alpha$$

as our function for future capital.

(c). We have given that

$$C_t = \alpha + \beta K_t + \gamma v_t$$

so we merely need to find

$$E[C_{t+1}] = E[\alpha + \beta K_{t+1} + \gamma v_{t+1}]$$

By linearity and substituting our earlier result, we have

$$E[C_{t+1}] = \alpha + \beta((1 + A - \beta)K_t - \gamma v_t - \alpha) + \gamma E[v_{t+1}]$$

Since we are given that v_t has expectation zero, we can substitute to obtain

$$E[C_{t+1}] = \alpha + \beta((1 + A - \beta)K_t - \gamma v_t - \alpha)$$

which we can simplify as

$$E[C_{t+1}] = (1 - \beta)\alpha + \beta(1 + A - \beta)K_t - \gamma\beta v_t$$

and from the Euler equation we have

$$\alpha + \beta K_t + (\gamma + 1)v_t = (1 - \beta)\alpha + \beta(1 + A - \beta)K_t - \gamma\beta v_t$$

and so we will have

$$\begin{aligned} \alpha &= (1 - \beta)\alpha \\ \beta &= \beta(1 + A - \beta) \\ \gamma + 1 &= \gamma\beta \end{aligned}$$

which upon solving this system (with 3 equations and 3 unknowns since A is given) yield

$$\begin{aligned} \alpha &= 0 \\ \beta &= A \\ \gamma &= -\frac{1}{1 + A} \end{aligned}$$

for the parameters.

(d). Solving for consumption gives

$$C_t = AK_t - \frac{1}{1 + A} \times v_t$$

while saving is

$$K_{t+1} = K_t + \frac{1}{1 + A} \times v_t$$

and so a one time positive shock means that the capital stock is higher forever (since there is persistence), and this higher capital stock will mean both output and consumption are higher.

1.3. **Romer 5.11.**

2. Permanent income hypothesis and the "excess smoothness" puzzle

2.1. Saving responses to shocks.

3. Estimation of adjustment costs

4. Practice log-linearization

4.1.

4.2.

4.3.

4.4.

4.5.

4.6.

4.7.