ECON 210C PROBLEM SET # 3

MINKI KIM

1. Variable labor supply in the RBC model

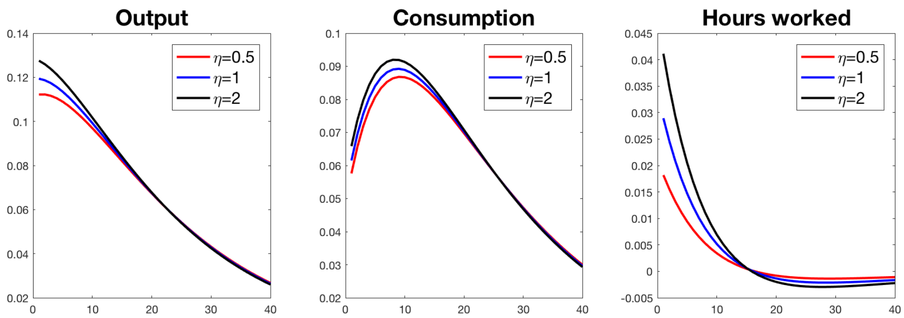

FIGURE 1. Impulse responses with varying η

	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	Data
$Stdev(Y)$	1.54	1.64	1.74	1.72
$Stdev(C)$	0.97	1.02	1.08	1.27
$Stdev(L)$	0.23	0.37	0.53	1.59

TABLE 1. Response to a transitory discount factor shock

As one would expect, the fits get better as we calibrate the Frisch elasticity to bigger values. A large Frisch elasticity generates stronger intertemporal substitution of labor suppply, and hence amplifies the effect of shocks. However, even with a large Frisch elasticity, consumption is too smooth, and the volatility of hours generated from the model falls short of the empirical counterpart.

2. Variable capital utilization in an RBC model

(a) The Lagrangian of the firm's profit maximization problem is:

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_t \sum_s \left(\prod_{k=1}^{s} (1 + R_{t+k}) \right)^{-1} \\ &\times \left[(U_{t+s}K_{t+s-1})^{\alpha} (Z_{t+s}N_{t+s})^{1-\alpha} - W_{t+s}N_{t+s} - I_{t+s} + q_{t+s} \left(-K_{t+s} + (1 - \delta(U_t))K_{t+s-1} + I_{t+s} \right) \right]\end{aligned}$$

The first order conditions are:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial N_t} &: \quad W_t = (1-\alpha)\left(U_t K_{t-1}\right)^{\alpha}\left(Z_t N_t\right)^{-\alpha} \\
\frac{\partial \mathcal{L}}{\partial I_t} &: \quad q_t = 1 \\
\frac{\partial \mathcal{L}}{\partial K_t} &: \quad q_t = \mathbb{E}_t \frac{1}{1+R_{t+1}}\left[\alpha\left(U_{t+1}K_t\right)^{\alpha}\left(Z_{t+1}N_{t+1}\right)^{1-\alpha} + q_{t+1}\left(1-\delta(U_{t+1})\right)\right] \\
\frac{\partial \mathcal{L}}{\partial U_t} &: \quad q_t \delta^{'}(U_t) K_{t-1} = \alpha\left(U_t K_{t-1}\right)^{\alpha-1} K_{t-1}\left(Z_t N_t\right)^{1-\alpha}
\end{aligned}
$$

Combining the second and the third equations, we get the expression for the rental rate of capital.

$$R_{t+1} = \alpha\left(U_{t+1}K_t\right)^{\alpha}\left(Z_{t+1}N_{t+1}\right)^{1-\alpha} - \delta(U_{t+1})$$

The rental rate depends on U_t because both MPK and depreciation rates depend on U_t.

(b) Log linearized version of $q_t\delta^{'}(U_t)K_{t-1} = \alpha U_t^{\alpha-1} K_{t-1}^{\alpha}\left(Z_t N_t\right)^{1-\alpha}$ is

$$\check{q}_t + \frac{\delta^{''}(\bar{U})\bar{U}}{\delta^{'}(\bar{U})}\check{U}_t + \check{K}_{t-1} = (\alpha-1)\check{U}_t + \alpha \check{K}_{t-1} + (1-\alpha)\left(\check{Z}_t + \check{N}_t\right)$$

Using $\check{q}_t = 0$ and $\check{Y}_t = \alpha\left(\check{U}_t + \check{K}_{t-1}\right) + (1-\alpha)\left(\check{Z}_t + \check{N}_t\right)$, we can express $\check{U}_t$ in terms of $\check{Y}_t, \check{K}_t$, and Δ.

$$\check{U}_t = \frac{1}{1+\Delta}\left(\check{Y}_t - \check{K}_{t-1}\right)$$

(c) The production function in a log-linear form is:

$$
\begin{aligned}
\check{Y}_t &= \alpha\left(\check{U}_t + \check{K}_{t-1}\right) + (1-\alpha)\left(\check{Z}_t + \check{N}_t\right) \\
&= \frac{\alpha}{1+\Delta}\left(\check{Y}_t - \check{K}_{t-1}\right) + \alpha \check{K}_{t-1} + (1-\alpha)\left(\check{Z}_t + \check{N}_t\right)
\end{aligned}
$$

Isolate $\check{Y}_t$:

$$
\begin{aligned}
\check{Y}_t &= \frac{\Delta\alpha}{1+\Delta-\alpha}\check{K}_{t-1} + \frac{(1+\Delta)(1-\alpha)}{1+\Delta-\alpha}\left(\check{Z}_t + \check{N}_t\right) \\
&= \check{Z}_t + \check{N}_t \quad (\text{when } \Delta = 0) \\
&= \alpha \check{K}_{t-1} + (1-\alpha)\left(\check{Z}_t + \check{N}_t\right) \quad (\text{when } \Delta = \infty)
\end{aligned}
$$

(i) $\Delta = 0$ means that the steady state capital utilization rate is zero. Hence no matter how big the capital stock is, it does not contribute to the output. Therefore, deviations of output from its steady state solely depend on technology and labor.

(ii) $\Delta = \infty$ means that steady state capital utilization rate is one. In this case, this model boils down to a model without capital utilization, since 100% of capital stock is always used in production. Therefore, deviations of output from its steady state depend on all three inputs of the production function, with weights corresponding to the inputs same as the Cobb-Douglas coefficients.

(iii) Consider the case when $0 < \Delta < \infty$. In this case, the log linearized production function is written as:

$$\check{Y}_t = \frac{\Delta\alpha}{1+\Delta-\alpha}\check{K_{t-1}} + (1-\alpha)\left(\check{Z}_t + \check{N}_t\right) + \frac{\alpha(1-\alpha)}{1+\Delta-\alpha}\left(\check{Z}_t + \check{N}_t\right)$$

Since capital stock is not fully used in production, the contributions of Z_t and N_t in Y_t is higher than when $\Delta = \infty$.

(d)

3. Homework in macroeconomics