

# ECON 210C PROBLEM SET # 3

NATHANIEL BECHHOFFER

## 1. VARIABLE LABOR SUPPLY IN THE RBC MODEL

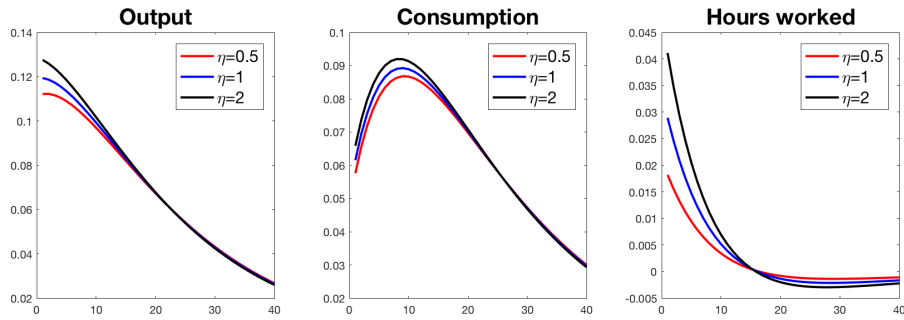


FIGURE 1. Impulse responses with varying  $\eta$

|            | $\eta = 0.5$ | $\eta = 1$ | $\eta = 2$ | Data |
|------------|--------------|------------|------------|------|
| $\sigma_Y$ | 1.54         | 1.64       | 1.74       | 1.72 |
| $\sigma_C$ | 0.97         | 1.02       | 1.08       | 1.27 |
| $\sigma_L$ | 0.23         | 0.37       | 0.53       | 1.59 |

TABLE 1. Response to a transitory discount factor shock

Larger Frisch elasticity values imply a better fit, as they generate stronger inter-temporal substitution of labor supply, amplifying the effect of shocks. Consumption is still too smooth, and the volatility of hours is too low.

## 2. VARIABLE CAPITAL UTILIZATION IN AN RBC MODEL

(a). Firms choose capital utilization  $U$ , capital  $K$ , and labor demand  $N$ .

The production function that we can use directly (since the output is the numeraire) is

$$Y_t = (U_t K_{t-1})^\alpha (Z_t N_t)^{1-\alpha}$$

and since the firms own capital, they face the constraint

$$K_t = I_t + (1 - \delta(U_t))K_{t-1}$$

but they also have to pay wages  $W_t N_t$  and invest  $I_t$  so we can set up the Lagrangian

$$\mathcal{L} = E \sum_s \left( \prod_{k=1}^s (1+r_{t+k})^{-1} \right) \left( (U_{t+s} K_{t+s-1})^\alpha (Z_{t+s} N_{t+s})^{1-\alpha} - W_{t+s} N_{t+s} - I_{t+s} + q_{t+s} (-K_{t+s} + I_{t+s} + (1 - \delta(U_{t+s}))K_{t+s-1}) \right)$$

so we have first order conditions:

for labor we have

$$W_t = (1 - \alpha)(U_t K_{t-1})^\alpha Z_t^{1-\alpha} N_t^{-\alpha}$$

for investment we have

$$q_t = 1$$

for capital at time  $t$  we have

$$q_t = E \left[ \frac{1}{1+r_{t+1}} \left( \alpha U_{t+1}^\alpha K_t^{\alpha-1} (Z_{t+1} N_{t+1})^{1-\alpha} + q_{t+1} (1 - \delta(U_{t+1})) \right) \right]$$

and finally we have the condition for utilization

$$\alpha U_t^{\alpha-1} K_{t-1}^{\alpha} (Z_t N_t)^{1-\alpha} = q_t K_{t-1} \delta'(U_t)$$

Combining the investment and capital optimality conditions yields the expression for the rental rate of capital.

$$R_{t+1} = \alpha U_{t+1}^\alpha K_t^{\alpha-1} (Z_{t+1} N_{t+1})^{1-\alpha} - \delta(U_{t+1})$$

The rental rate depends on utilization because the marginal product of capital and its depreciation rate depend on utilization.

## 3. HOMEWORK IN MACROECONOMICS

(a). The Lagrangian for the household's maximization problem is:

$$\mathcal{L} = (C_m^\rho + C_h^\rho)^{\frac{1}{\rho}} - \left( \frac{1}{\eta} + 1 \right)^{-1} (L_h + L_m)^{\frac{1}{\eta}+1} + \lambda (W L_m - C_m) + \xi (L_h - C_h)$$

The first order conditions for the interior solutions are:

$$\begin{aligned}\frac{1}{\rho} (C_m^\rho + C_h^\rho)^{\frac{1}{\rho}-1} \rho C_m^{\rho-1} &= \lambda \\ \frac{1}{\rho} (C_m^\rho + C_h^\rho)^{\frac{1}{\rho}-1} \rho C_h^{\rho-1} &= \xi \\ (L_h + L_m)^{\frac{1}{\eta}} &= \lambda W \\ (L_h + L_m)^{\frac{1}{\eta}} &= \xi\end{aligned}$$

(b). From the two first order conditions for labor, we have

$$\xi = \lambda W$$

(c). From the two first order conditions for consumption, we have

$$\xi = \lambda \left( \frac{C_h}{C_m} \right)^{\rho-1}$$

(d). With the budget constraints binding, we have

$$C_h = L_h$$

and from above we get

$$C_h = C_m W^{\frac{1}{\rho-1}}$$

(e). We now have

$$L_h = C_m W^{\frac{1}{\rho-1}}$$

and we can assume the budget constraint holds for formal markets to make the substitution

$$C_m = W L_m$$

getting us

$$L_h = W L_m W^{\frac{1}{\rho-1}}$$

equivalent to

$$L_h = L_m W^{\frac{\rho}{\rho-1}}$$

and from our first order conditions we have

$$L_h + L_m = (\lambda W)^\eta$$

so we can substitute for  $L_m$  to get

$$L_h = [(\lambda W)^\eta - L_h] W^{\frac{\rho}{\rho-1}}$$

so we have

$$L_h(1 + W^{\frac{\rho}{\rho-1}}) = (\lambda W)^\eta W^{\frac{\rho}{\rho-1}}$$

and finally we get

$$L_h = \frac{(\lambda W)^\eta W^{\frac{\rho}{\rho-1}}}{(1 + W^{\frac{\rho}{\rho-1}})}$$

(f). We can simplify to write

$$L_h = \frac{(\lambda)^\eta W^{\frac{\rho+\rho\eta-\eta}{\rho-1}}}{(1 + W^{\frac{\rho}{\rho-1}})}$$

and then we have

$$\frac{\partial L_h}{\partial W} = \frac{(1 + W^{\frac{\rho}{\rho-1}})(\lambda)^\eta (\frac{\rho+\rho\eta-\eta}{\rho-1}) W^{\frac{\rho+\rho\eta-\eta}{\rho-1}-1} - (\lambda)^\eta W^{\frac{\rho+\rho\eta-\eta}{\rho-1}} (\frac{\rho}{\rho-1}) W^{\frac{\rho}{\rho-1}-1}}{(1 + W^{\frac{\rho}{\rho-1}})^2}$$

with

$$\frac{\partial L_h}{\partial W} \cdot \frac{W}{L_h} = \frac{(1 + W^{\frac{\rho}{\rho-1}})(\frac{\rho+\rho\eta-\eta}{\rho-1}) - (\frac{\rho}{\rho-1}) W^{\frac{\rho}{\rho-1}}}{(1 + W^{\frac{\rho}{\rho-1}})}$$

as the elasticity of  $L_h$  with respect to  $W$ .