ECON 210C PROBLEM SET # 3

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1. Variable labor supply in the RBC model

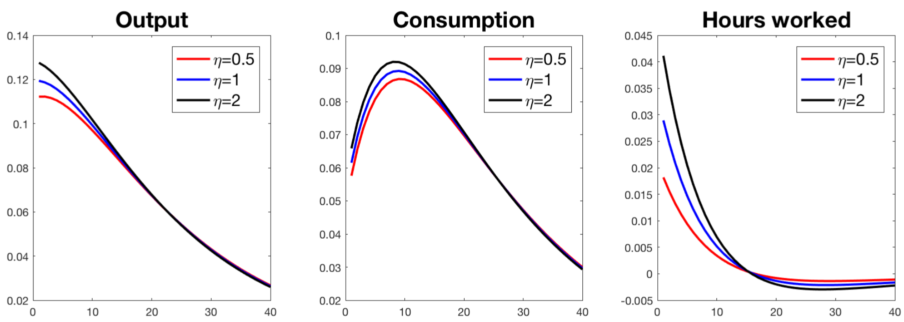

Figure 1. Impulse responses with varying η

	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	Data
σ_Y	1.54	1.64	1.74	1.72
σ_C	0.97	1.02	1.08	1.27
σ_L	0.23	0.37	0.53	1.59

Table 1. Response to a transitory discount factor shock

Larger Frisch elasticity values imply a better fit, as they generate stronger inter-temporal substitution of labor supply, amplifying the effect of shocks. Consumption is still too smooth, and the volatility of hours is too low.

2. Variable capital utilization in an RBC model

(a). Firms choose capital utilization U, capital K, and labor demand N.

The production function that we can use directly (since the output is the numeraire) is

$$Y_t = (U_t K_{t-1})^{\alpha} (Z_t N_t)^{1-\alpha}$$

and since the firms own capital, they face the constraint

$$K_t = I_t + (1 - \delta(U_t))K_{t-1}$$

but they also have to pay wages $W_t N_t$ and invest I_t so we can set up the Lagrangian

$$\mathcal{L} = E\sum_s (\prod_{k=1}^{s}(1+r_{t+k})^{-1})\Big((U_{t+s}K_{t+s-1})^{\alpha}(Z_{t+s}N_{t+s})^{1-\alpha} - W_{t+s}N_{t+s} - I_{t+s} + q_{t+s}(-K_{t+s} + I_{t+s} + (1-\delta(U_{t+s}))K_{t+s-1})\Big)$$

so we have first order conditions:

for labor we have

$$W_t = (1-\alpha)(U_t K_{t-1})^{\alpha} Z_t^{1-\alpha} N_t^{-\alpha}$$

for investment we have

$$q_t = 1$$

for capital at time t we have

$$q_t = E\left[\frac{1}{1+r_{t+1}}\left(\alpha U_{t+1}^{\alpha} K_t^{\alpha-1}(Z_{t+1}N_{t+1})^{1-\alpha} + q_{t+1}(1-\delta(U_{t+1})\right)\right]$$

and finally we have the condition for utilization

$$\alpha U_t^{\alpha-1} K_{t-1}^{\alpha}(Z_t N_t)^{1-\alpha} = q_t K_{t-1}\delta'(U_t)$$

Combining the investment and capital optimality conditions yields the expression for the rental rate of capital.

$$R_{t+1} = \alpha U_{t+1}^{\alpha} K_t^{\alpha-1}\left(Z_{t+1}N_{t+1}\right)^{1-\alpha} - \delta(U_{t+1})$$

The rental rate depends on utilization because the marginal product of capital and its depreciation rate depend on utilization.

3. Homework in macroeconomics

(a). The Lagrangian for the household's maximization problem is:

$$\mathcal{L} = \left(C_m^{\rho} + C_h^{\rho}\right)^{\frac{1}{\rho}} - \left(\frac{1}{\eta} + 1\right)^{-1}(L_h + L_m)^{\frac{1}{\eta}+1} + \lambda\left(WL_m - C_m\right) + \xi\left(L_h - C_h\right)$$

The first order conditions for the interior solutions are:

$$\frac{\cancel{1}}{\cancel{\rho}}\left(C_m^\rho + C_h^\rho\right)^{\frac{1}{\rho}-1} \cancel{\rho} C_m^{\rho-1} = \lambda$$

$$\frac{\cancel{1}}{\cancel{\rho}}\left(C_m^\rho + C_h^\rho\right)^{\frac{1}{\rho}-1} \cancel{\rho} C_h^{\rho-1} = \xi$$

$$(L_h + L_m)^{\frac{1}{\eta}} = \lambda W$$

$$(L_h + L_m)^{\frac{1}{\eta}} = \xi$$

(b). From the two first order conditions for labor, we have

$$\xi = \lambda W$$

(c). From the two first order conditions for consumption, we have

$$\xi = \lambda \left(\frac{C_h}{C_m}\right)^{\rho-1}$$

(d). With the budget constraints binding, we have

$$C_h = L_h$$

and from above we get

$$C_h = C_m W^{\frac{1}{\rho-1}}$$

(e). We now have

$$L_h = C_m W^{\frac{1}{\rho-1}}$$

and we can assume the budget constraint holds for formal markets to make the substitution

$$C_m = W L_m$$

getting us

$$L_h = W L_m W^{\frac{1}{\rho-1}}$$

equivalent to

$$L_h = L_m W^{\frac{\rho}{\rho-1}}$$

and from our first order conditions we have

$$L_h + L_m = (\lambda W)^\eta$$

so we can substitute for L_h to get

$$(\lambda W)^\eta - L_m = L_m W^{\frac{\rho}{\rho-1}}$$

so we have

$$L_m(1 + W^{\frac{\rho}{\rho-1}}) = (\lambda W)^\eta$$

and thus

$$L_m = \frac{(\lambda W)^\eta}{(1 + W^{\frac{\rho}{\rho-1}})}$$

(f). We now have

$$\frac{\partial L_h}{\partial W} = \frac{(1+W^{\frac{\rho}{\rho-1}})\lambda^\eta \eta W^{\eta-1} - (\lambda W)^\eta (\frac{\rho}{\rho-1}) W^{\frac{\rho}{\rho-1}-1}}{(1+W^{\frac{\rho}{\rho-1}})^2}$$

with

$$\frac{\partial L_m}{\partial W} \cdot \frac{W}{L_m} = \frac{(1+W^{\frac{\rho}{\rho-1}})\eta - (\frac{\rho}{\rho-1}) W^{\frac{\rho}{\rho-1}}}{(1+W^{\frac{\rho}{\rho-1}})}$$

as the elasticity of L_h with respect to W.

(g).

(h). We had

$$L_m = \frac{(\lambda W)^\eta}{(1+W^{\frac{\rho}{\rho-1}})}$$

and from part c we had

$$\xi = \lambda \left(\frac{C_h}{C_m}\right)^{\rho-1}$$

which we can substitute from the first order conditions to get

$$(L_h + L_m)^{\frac{1}{\eta}} = \lambda \left(\frac{C_h}{C_m}\right)^{\rho-1}$$

with the budget constraint giving us

$$(L_h + L_m)^{\frac{1}{\eta}} = \lambda \left(\frac{L_h}{W L_m}\right)^{\rho-1}$$

so we have

$$\lambda = (L_h + L_m)^{\frac{1}{\eta}} \left(\frac{L_h}{W L_m}\right)^{1-\rho}$$

and now We had

$$L_m = \frac{(L_h + L_m)\left(\frac{L_h}{W L_m}\right)^{\eta - \eta\rho} W^\eta}{(1+W^{\frac{\rho}{\rho-1}})}$$

and this gets us

$$L_m = \frac{(L_h + L_m)\left(\frac{L_h}{L_m}\right)^{\eta - \eta\rho} W^{\eta\rho}}{(1+W^{\frac{\rho}{\rho-1}})}$$

and we know

$$L_h = L_m W^{\frac{\rho}{\rho-1}}$$

so we have

$$L_m = \frac{\cancel{\left(L_m W^{\frac{\rho}{\rho-1}} + L_m\right)} \left(\frac{\cancel{L_m} W^{\frac{\rho}{\rho-1}}}{\cancel{L_m}}\right)^{\eta-\eta\rho} W^{\eta\rho}}{\cancel{(1+W^{\frac{\rho}{\rho-1}})}}$$

so that gets us to

$$L_m = W^{\frac{\rho}{\rho-1}\times(\eta-\eta\rho)} \times W^{\eta\rho}$$

equivalently

$$L_m = W^{\frac{\rho}{\rho-1}\times(\eta-\eta\rho)+\eta\rho}$$

(i). We now have

$$\frac{\partial L_h}{\partial W} = \left(\frac{\rho}{\rho-1} \times (\eta-\eta\rho) + \eta\rho\right) W^{\frac{\rho}{\rho-1}\times(\eta-\eta\rho)+\eta\rho-1}$$

with

$$\frac{\partial L_m}{\partial W} \cdot \frac{W}{L_m} = \left(\frac{\rho}{\rho-1} \times (\eta-\eta\rho) + \eta\rho\right)$$