

I. RBC with Variable Labor Supply

Table 1 compares volatilities across different parameterizations of η with what is observed in the data. Larger values of η improve the fit of the model. There is greater persistence since more elastic labor supply means shocks affect hours and wages to a greater extent. Consumption and labor supply, however, remain excessively smooth. The classical RBC model implies a greater Frisch elasticity based on the data.

Table 1: Comparing second moments of output, consumption, and labor supply

	Data	$\eta = 0.5$	$\eta = 1$	$\eta = 2$
Consumption	1.27	0.97	1.03	1.09
Output	1.72	1.56	1.67	1.79
Hours	1.59	0.24	0.41	0.60

II. RBC with Variable Capital Utilization

The firm solves the profit maximization problem.

$$\max_{\{N_t, I_t, U_t\}} E \sum_{t=0}^{\infty} \prod_{s=0}^t (1 + r_t)^{-1} (Y_t - N_t w_t - I_t)$$

$$Y_t = (U_t K_t)^\alpha (Z_t N_t)^{1-\alpha}$$

$$K_{t+1} = (1 - \delta(U_t)) K_t + I_t$$

We equalize prices of capital and the consumption good as in equilibrium.

1. The first order conditions are given by

$$\text{Labor demand: } w_t = (1 - \alpha) Z_t^{1-\alpha} \left(\frac{U_t K_t}{N_t} \right)^\alpha$$

Shadow value of capital: $q_t = 1$

$$\text{Euler equation: } E(1 + r_t)^{-1} \left(\alpha U_{t+1}^\alpha \left(\frac{K_{t+1}}{Z_{t+1} N_{t+1}} \right)^{\alpha-1} + q_{t+1} (1 - \delta_{t+1}) \right)$$

$$\text{Utilization: } \alpha K_t^\alpha \left(\frac{U_t}{Z_t N_t} \right)^{\alpha-1} = q_t \delta'(U_t) K_t$$

$$1 + r_t = \alpha U_{t+1}^\alpha \left(\frac{K_{t+1}}{Z_{t+1} N_{t+1}} \right)^{\alpha-1} + 1 - \delta(U_t)$$

Rental rate depends on the marginal product of capital and depreciation which are themselves both dependant on U_t .

2. Utilization satisfies $\alpha K_t^\alpha \left(\frac{U_t}{Z_t N_t} \right)^{\alpha-1} = q_t \delta'(U_t) K_t$.

$$\ln \alpha + \alpha \ln K_t + (\alpha - 1)(\ln U_t - \ln Z_t - \ln N_t) = \ln q_t + \ln \delta'(U_t) + \ln K_t$$

$$\ln \alpha + (\alpha - 1)(\ln K_t + \ln U_t - \ln Z_t - \ln N_t) = \ln q_t + \ln \delta'(U_t)$$

$$(\alpha - 1)(\check{K}_t + \check{U}_t - \check{Z}_t - \check{N}_t) = \check{q}_t + \frac{\delta''(\bar{U})}{\delta'(\bar{U})}(U_t - \bar{U})$$

$$(\alpha - 1)(\check{K}_t + \check{U}_t - \check{Z}_t - \check{N}_t) = \check{q}_t + \frac{\delta''(\bar{U})\bar{U}}{\delta'(\bar{U})}\check{U}_t = \check{q}_t + \Delta$$

$$\check{U}_t = \frac{\check{q}_t + \Delta}{\alpha - 1}(\check{Z}_t + \check{N}_t - \check{K}_t) = \frac{1}{1 + \Delta}(\check{Y}_t - \check{K}_t)$$

The final equality uses $\check{q}_t = 0$ from the first order condition and log linearized production.

3. We can use the previous derivation to reduce \check{Y}_t as a function of technology and inputs.

$$\begin{aligned}\check{Y}_t &= \alpha(\check{U}_t + \check{K}_t) + (1 - \alpha)(\check{Z}_t + \check{N}_t) \\ &= \frac{\alpha}{1 + \Delta}(\check{Y}_t - \check{K}_t) + \alpha\check{K}_t + (1 - \alpha)(\check{Z}_t + \check{N}_t) \\ \frac{1 - \alpha + \Delta}{1 + \Delta}\check{Y}_t &= \frac{\alpha\Delta}{1 + \Delta}\check{K}_t + (1 - \alpha)(\check{Z}_t + \check{N}_t) \\ \check{Y}_t &= \frac{\alpha\Delta}{1 - \alpha + \Delta}\check{K}_t + \frac{(1 - \alpha)(1 + \Delta)}{1 - \alpha + \Delta}(\check{Z}_t + \check{N}_t)\end{aligned}$$

Δ governs the sensitivity of \check{U}_t to the marginal rate of capital. The limiting case where $\Delta \rightarrow \infty$ is the standard (linearized) model and \check{U}_t is fixed at full utilization. The limit $\delta \rightarrow 0$ represents the case of no utilization so that \check{Y}_t depend solely on technology and labor.

4. The linearized labor demand function is $\check{w}_t = \check{Y}_t - \check{N}_t$. Substituting the expression for \check{Y}_t ,

$$\check{w}_t = \frac{\alpha\Delta}{1 - \alpha + \Delta}\check{K}_t + \frac{(1 - \alpha)(1 + \Delta)}{1 - \alpha + \Delta}(\check{Z}_t + \check{N}_t) - N_t$$

We can obtain an upward sloping demand function if labor exhibits increasing returns to scale.

$$\begin{aligned}\frac{(1 - \alpha)(1 + \Delta)}{1 - \alpha + \Delta} &> 1 \\ (1 - \alpha)(1 + \Delta) &> 1 - \alpha + \Delta \\ 1 - \alpha + \Delta - \Delta\alpha &> 1 - \alpha + \Delta \\ -\Delta\alpha &> 1\end{aligned}$$

Since $\Delta, \alpha > 0$, indeterminacy is impossible in this model. One way to achieve indeterminacy is to incorporate positive production externalities so that aggregate labor has increasing returns to scale. A model with endogenous capital utilization is more likely to exhibit indeterminacy because it amplifies the importance of labor in production.

III. Macroeconomics of Home Production

The household solves

$$\begin{aligned} \max_{c_m, c_h, L_h, L_m} & (C_m^\rho + C_h^\rho)^{\frac{1}{\rho}} - \left(\frac{1}{\eta} + 1\right)^{-1} (L_h + L_m)^{\frac{1}{\eta}+1} \\ \text{subject to } & C_m \leq wL_m, C_h \leq L_h \end{aligned}$$

The prices of consumption goods are equivalent.

1. The first order conditions are

$$\begin{aligned} \mathcal{L} &= (C_m^\rho + C_h^\rho)^{\frac{1}{\rho}} - \left(\frac{1}{\eta} + 1\right)^{-1} (L_h + L_m)^{\frac{1}{\eta}+1} + \lambda(wL_m - C_m) + \xi(L_h - C_h) \\ \lambda &= (C_m^\rho + C_h^\rho)^{\frac{1-\rho}{\rho}} C_m^{\rho-1} \\ \xi &= (C_m^\rho + C_h^\rho)^{\frac{1-\rho}{\rho}} C_h^{\rho-1} \\ (L_h + L_m)^{\frac{1}{\eta}} &= \lambda w = \xi \end{aligned}$$

2. $w = \frac{\xi}{\lambda}$. Wage is the ratio of marginal utilities of market and home goods.

3. $\lambda \frac{C_m}{C_h}^{1-\rho} = \xi$. The ratio of marginal utilities depends on the share of consumption.

4. $C_h = L_h = C_m w^{\frac{-1}{1-\rho}}$. The share of consumption depends on the relative wage.

5. Begin with $(L_h + L_m)^{\frac{1}{\eta}} = \lambda w$.

$$\begin{aligned} L_m &= (\lambda w)^\eta - L_h \\ &= (\lambda w)^\eta - C_m w^{\frac{-1}{1-\rho}} \\ &= (\lambda w)^\eta - L_m w^{\frac{-\rho}{1-\rho}} \\ &= (\lambda w)^\eta (1 + w^{\frac{-\rho}{1-\rho}})^{-1} \end{aligned}$$

6. Differentiate the above expression.

$$\begin{aligned} \frac{\partial L_m}{\partial w} &= \frac{(1 + W^{\frac{-\rho}{1-\rho}}) \lambda^\eta \eta W^{\eta-1} - (\lambda W)^\eta (\frac{-\rho}{1-\rho}) W^{\frac{-\rho}{1-\rho}-1}}{(1 + W^{\frac{-\rho}{1-\rho}})^2} \\ \varepsilon_m &= \frac{\partial L_m}{\partial w} \frac{w}{L_m} = \frac{(1 + W^{\frac{-\rho}{1-\rho}}) \eta - (\frac{-\rho}{1-\rho}) W^{\frac{-\rho}{1-\rho}}}{(1 + W^{\frac{-\rho}{1-\rho}})} \\ &= \eta + \left(\frac{\rho}{1-\rho}\right) \left(\frac{W^{\frac{-\rho}{1-\rho}}}{1 + W^{\frac{-\rho}{1-\rho}}}\right) \end{aligned}$$

7. Observing labor supply volatilities, the model with home production implies η weakly smaller than in the classical RBC because that model does not account for shifting “employment” in home production. With high elasticity of substitution, $\varepsilon_m \rightarrow \infty$ and households will more readily substitute market employment with home production. If goods are complements, ε_m approaches the Frisch elasticity η .
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10. Higher elasticity of substitution ($\rho \rightarrow 1$) results in higher “total” elasticity which can help explain labor supply volatilities without an infeasibly high Frisch elasticity.

IV. q -Theory with Variable Capital Utilization

V. Fiscal Multipliers in the RBC Model