Econ 210C Homework 5

Instructor: Johannes Wieland

Due: Wednesday 5/17/2018

1. Problems from Romer

1. Romer, Problem 6.13.
2. Romer, Problem 7.10.

2. Quadratic cost of adjusting prices and effect of money (Rotemberg 1982)

Consider a firm i that minimizes at time t:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[(p_{i,t+j} - p^*_{i,t+j})^2 + c(p_{i,t+j} - p_{i,t+j-1})^2 \right], \quad c > 0$$

where $p_{i,t+j}$ is the logarithm of the nominal price of firm i in period $t+j$, $p^*_{i,t+j}$ is the logarithm of the nominal price that the firm would choose in period $t+j$ in the absence of costs of changing prices. Costs of changing nominal prices are captured by the second term in the objective function. The information set at time t includes current and lagged $p_{i,t}$ and $p^*_{i,t}$.

(a) Are quadratic costs of adjustment—compared to, say, the fixed costs model—in any way plausible? What stylized facts about price adjustment are consistent or inconsistent with this model?

(b) Derive the first-order condition of the above minimization problem, giving the price p_{it} as a function of itself lagged, of its expectation next period, and of the optimal price. Solve the model using the method of undetermined coefficients. Assume that the solution takes the form $p_{i,t+j} = \nu_{-1} p_{i,t+j-1} + \sum_{l=0}^{\infty} v_l \mathbb{E}_t[p^*_{i,t+j+l}]$.

(c) Assume that $p^*_{i,t} = (1-\phi)p_t + \phi m_t$ with $\phi \in (0,1)$ where p_t is the logarithm of the price level and m_t is the logarithm of nominal money. The current value of m_t is in the information set at time t. Replacing $p^*_{i,t}$ in the first-order condition above and assuming symmetry, solve for the price level as a function of itself lagged and of current and expected values of nominal money.

(d) Assume that m_t follows a random walk, i.e. $m_t = m_{t-1} + \epsilon_t$ where ϵ_t is white noise. Solve for p_t as a function of p_{t-1} and m_t. Assume further that $y_t = m_t - p_t$ and solve for the dynamic effect of ϵ_t on y_t. Explain your results.

(e) Compare the results in (d) with the similar equations in the Calvo model. Is there a difference in aggregate dynamics between Calvo and quadratic adjustment model?

3. New Keynesian model in Dynare

Implement the new Keynesian model in Dynare.

(a) Write the nonlinear optimal reset price equation in the lecture notes in recursive form. You need to write down two recursive equations, one for the numerator and one for the denominator. They should look like $Z_{1,t} = a_t + b_t \mathbb{E}_t Z_{1,t+1}$ and $Z_{2,t} = c_t + d_t \mathbb{E}_t Z_{2,t+1}$. Then the real reset price is equal to $Z_{1,t}/Z_{2,t}$ and the system is recursive.

If you cannot solve for the recursive form, then just use the linearized NKPC we derived in class for the remainder of the question.

(b) Solve the model in Dynare. It should have an Euler equation, the national income identity, the monetary policy rule, inflation as a function of the real reset price, the evolution of price dispersion, and your system from part (a). All equations should be non-linear.

(c) Compute impulse response functions to a technology shock and a monetary policy shock. Compare your results with the method of undetermined coefficients in the lecture notes.

(d) OPTIONAL Add capital to your model. Compare the impulse response functions to a technology shock and a monetary policy shock with those of the RBC model. (Recall, $\kappa^* \to \infty$ means prices are flexible.) How do they differ? Explain intuitively.

4. Government spending multipliers in the new Keynesian model (Christiano, Eichenbaum and Evans, 2012)

This economy is characterized by the following log-linearized equations:

$$\check{C}_t = E_t \check{C}_{t+1} - \frac{1}{\psi}(i_t - E_t \pi_{t+1}) \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(\frac{\check{W}}{P} \right)_t, \quad \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \tag{2}$$

$$\left(\frac{\check{W}}{P} \right)_t = \psi \check{C}_t + \frac{1}{\eta} \check{L}_t \tag{3}$$

$$\check{Y}_t = \check{L}_t \tag{4}$$

$$\check{Y}_t = s_g \check{G}_t + (1 - s_g) \check{C}_t \tag{5}$$

$$i_t = \phi_\pi \pi_t, \quad \phi_\pi > 1 \tag{6}$$

(a) Interpret each of the equations (1)-(6)

(b) Rewrite the system as two equations that only contained current and future values of $\check{C}_t, \breve{\pi}_t, \check{G}_t$.

(c) Assume that government spending has the following dynamics:

$$\check{G}_t = \rho \check{G}_{t-1} + \epsilon_t, \quad \epsilon_t \sim i.i.d.(0, \sigma^2)$$

Since both $\check{C}_t$ and π_t are jump-variables, the only state variable of the system is $\check{G}_t$.

(d) Use the minimum-state variable criterion and the fact that the system is linear to solve for $\check{C}_t$ and π_t as a function of $\check{G}_t$. (I.e., guess that $\check{C}_t = c_g \check{G}_t$, $E_t \check{C}_{t+1} = c_g E_t \check{G}_{t+1}$, $\pi_t = \pi_g \check{G}_t$ and $E_t \pi_{t+1} = \pi_g E_t \check{G}_{t+1}$ and solve for c_g, π_g). In particular, show that

$$\pi_g = \frac{\frac{\kappa}{\eta}(1-\rho)s_g}{(1-\beta\rho)(1-\rho) + \frac{\kappa}{\psi}[\psi + \frac{1}{\eta}(1-s_g)](\phi_\pi - p)},$$
$$c_g = \frac{-\frac{\kappa}{\eta\psi}(\phi_\pi - \rho)s_g}{(1-\beta\rho)(1-\rho) + \frac{\kappa}{\psi}[\psi + \frac{1}{\eta}(1-s_g)](\phi_\pi - p)}.$$

(e) The government spending multiplier is $\frac{dY}{dG} = 1 + \frac{dC}{dG} = 1 + \frac{1-s_g}{s_g}\frac{d\check{C}}{d\check{G}}$. Determine the size of the government spending multiplier in the model.

(f) What range does the government spending multiplier fall into? Explain intuitively why the government spending multiplier falls in that range.

(g) Assume

$$(1-\beta\rho)(1-\rho) - \frac{\kappa}{\psi}[\psi + \frac{1}{\eta}(1-s_g)]\rho > 0,$$

What is the government spending multipliers when $\phi_\pi = 0$? Is it larger than 1? Explain your result intuitively.

(h) **Briefly**, why is analyzing the comparative static $\phi_\pi = 0$ potentially problematic?

We now consider the case of the zero lower bound (ZLB). The interest rate rule is now:

$$i_t = \max\{\phi_\pi \pi_t, -\bar{i}\},$$

where $-\bar{i}$ comes from the linearization. (Remember that i_t is the deviation of the nominal interest rate from the steady-state $\bar{i}$.)

Thus, when inflation falls below a certain value ($\pi_t < -\frac{\bar{i}}{\phi_\pi}$), then the ZLB binds, the nominal interest rate is constant at zero, and $i_t = -\bar{i}$.

To analyze this case we suppose that the economy starts at the ZLB:

$$i_t = -\bar{i},$$

and government spending takes some non-negative value, $\check{G}_t = \tilde{g} > 0$. The state is:

$$s_t = s^{ZLB} \equiv \{\text{ZLB binds}, \check{G}_t = \tilde{g}\},$$

Next period we either remain in the state s^{ZLB} with probability $1-p$ or we return to the steady-state, $s^{SS} = \{\text{ZLB does not bind}, \check{G}_t = 0\}$. The latter is an absorbing state. This set-up has the advantage that government spending is only positive when the ZLB binds, so we can meaningfully talk about the spending multiplier at the ZLB.

We guess that π_t and $\check{G}_t$ are linear functions of $\tilde{g}$ at the ZLB:

$$\begin{aligned}\pi_t &= cons + \pi_g^{ZLB}\tilde{g} \\ \check{C}_t &= cons + c_g^{ZLB}\tilde{g}\end{aligned}$$

and note that since the economy returns to steady-state (0 deviations) with probability 1-p:

$$\begin{aligned}E_t\pi_{t+1} &= p(cons + \pi_g^{ZLB}\tilde{g}) + (1-p) * 0 \\ E_t\check{C}_{t+1} &= \underbrace{p(cons + c_g^{ZLB}\tilde{g})}_{\text{stay at ZLB}} + \underbrace{(1-p) * 0}_{\text{exit from ZLB}}\end{aligned}$$

(i) Solve for π_g^{ZLB} and c_g^{ZLB}, noting that $i_t = -\bar{i}$ at the ZLB. (Hint: since we are only interested in π_g^{ZLB} and c_g^{ZLB} we can ignore the constant terms!) In particular, show that

$$\begin{aligned}\pi_g &= \frac{\frac{\kappa}{\eta}(1-p)s_g}{(1-\beta p)(1-p) - \frac{\kappa}{\psi}[\psi + \frac{1}{\eta}(1-s_g)]p}, \\ c_g &= \frac{\frac{\kappa}{\eta\psi}ps_g}{(1-\beta p)(1-p) - \frac{\kappa}{\psi}[\psi + \frac{1}{\eta}(1-s_g)]p}.\end{aligned}$$

(j) Calculate the government spending multiplier at the ZLB with $\frac{dY}{dG} = 1 + \frac{dC}{dG} = 1 + \frac{1-s_g}{s_g}\frac{d\check{C}}{d\tilde{g}}$.

(k) The following condition is necessary and sufficient for a unique, stable solution:

$$(1-\beta p)(1-p) - \frac{\kappa}{\psi}[\psi + \frac{1}{\eta}(1-s_g)]p > 0$$

Given that condition, what range does the government spending multiplier fall into? Explain intuitively why the government spending multiplier falls in that range.

(l) Is your answer to part (g) different to your answer to part (k)? Why or why not?