Econ 210C
Spring 2015
Midterm
5/4/2014

Name (Print): ____________________

This exam contains 12 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam.

Problem	Points	Score
1	12	
2	24	
3	46	
4	18	
Total:	100	

Readings (12 points)

1. Briefly (in approx. two sentences) comment on the following statements/questions.

   (a) (4 points) Cogley and Nason (1995) argue that the canonical RBC model cannot match two empirical facts. Name **one** of these facts.

   Solution:

   - Output growth is serially correlated.
   - Output has a hump-shaped impulse response to transitory shocks.

   (b) (4 points) What are the shocks that Greenwood, Hercowitz and Huffman (1988) emphasize in their paper?

   Solution: Shocks to the marginal efficiency of investment.

   (c) (4 points) What level of returns to scale do Basu and Fernald (1997) estimate at the 2-digit industry level: strongly increasing, slightly increasing, constant, slightly decreasing or strongly decreasing?

   Solution: Constant or slightly decreasing.

Can Tax Shocks Drive the Business Cycle? (88 points)

2. Consider the canonical RBC model with labor taxes τ_t (notation is as in our class): A consumer maximizes

$$\max \sum_t^{\infty} \beta^t \left(\ln C_t - \frac{(L_t^s)^{1+1/\eta}}{1+1/\eta} \right)$$
$$\text{s.t. } A_t + C_t = (1+R_t)A_{t-1} + W_t L_t^s + \pi_t + T_t$$
$$lim_{t\to+\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} A_t = 0$$

taking prices as given.

A representative firm maximizes profits **given labor tax** $\tau_t \geq 0$,

$$\max \sum_t^{\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} [Y_t - (1+\tau_t) W_t L_t^d - I_t]$$
$$\text{s.t. } Y_t = K_{t-1}^{\alpha} (L_t^d)^{1-\alpha}$$
$$K_t = (1-\delta) K_{t-1} + I_t.$$

taking prices as given.

The market clearing conditions are:

$$L_t^s = L_t^d = L_t$$
$$K_t = A_t$$
$$Y_t = C_t + I_t$$
$$T_t = \tau_t W_t L_t$$

So tax revenues are rebated lump-sum to households.

Standard parameter values apply:

$$0 < \alpha < 1$$
$$0 < \beta < 1$$
$$0 < \delta < 1$$
$$\eta > 0$$
$$\bar{\tau} > 0$$

where $\bar{\tau}$ is the steady-state level of taxes.

Question continues on next page.

(a) (12 points) Set up the Lagrangian and derive the first order conditions for the household w.r.t. C_t, L_t^s, A_t. Use λ_t as the Lagrange multiplier on the budget constraint. Briefly interpret each equation (one sentence each).

Solution: The Lagrangian is

$$\mathcal{L}_0 = \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \frac{(L_t^s)^{1+1/\eta}}{1+1/\eta} + \lambda_t \left(-A_t - C_t + (1+R_t)A_{t-1} + W_t L_t^s + \pi_t + T_t \right) \right] \tag{1}$$

The FOC are:

$$\begin{aligned} \frac{\partial \mathcal{L}_0}{\partial L_t^s} &= -(L_t^s)^{1/\eta} + \lambda_t W_t = 0 \\ \frac{\partial \mathcal{L}_0}{\partial C_t} &= \frac{1}{C_t} - \lambda_t = 0 \\ \frac{\partial \mathcal{L}_0}{\partial A_t} &= -\lambda_t + \beta(1+R_{t+1})\lambda_{t+1} = 0 \end{aligned}$$

The first equation states that the marginal disutility utility of labor must equal the marginal gain from supplying labor (marginal utility times the after-tax wage). The second equation states that marginal utility of consumption is equal to the Lagrange multiplier. The third equation states that the consumer must be indifferent between consuming one more unit of output today and saving and consuming tomorrow.

(b) (12 points) Set up the Lagrangian and derive the first order conditions for the firm w.r.t I_t, L_t^d, K_t. (Substitute for Y_t first.) Use q_t as the Lagrange multiplier on capital accumulation. Briefly interpret each equation (one sentence each).

Solution: The Lagrangian is

$$\mathcal{L}_0 = \sum_{t=0}^{\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} \left[K_{t-1}^{\alpha} (L_t^d)^{1-\alpha} - (1+\tau_t) W_t L_t^d - I_t + q_t(-K_t + (1-\delta)K_{t-1} + I_t) \right] \tag{2}$$

The FOC are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial L_t^d} &= (1-\alpha) K_{t-1}^{\alpha} (L_t^d)^{-\alpha} - (1+\tau_t) W_t = 0 \\ \frac{\partial \mathcal{L}}{\partial I_t} &= -1 + q_t = 0 \\ \frac{\partial \mathcal{L}}{\partial K_t} &= \alpha K_t^{\alpha-1} (L_t^d)^{1-\alpha} + q_t(1+R_{t+1}) - q_{t+1}(1-\delta) = 0 \end{aligned}$$

The first equation states that the marginal product of labor must equal its marginal cost (the tax-inclusive real wage). The second equation states that the marginal benefit of investment q_t must equal its marginal cost (= 1). Third equation states that the marginal product of capital must equal its user cost (marginal cost of using capital), which in this case is forgone interest plus depreciation $R_{t+1} + \delta$.

3. The log-linearized equations of the RBC model in question 2 give rise to the following dynamic equations:

$$\Delta \check{K}_t = -\frac{\bar{Y}}{\bar{K}}\left(\frac{1-\alpha}{\alpha+1/\eta}\right)\frac{\bar{\tau}}{1+\bar{\tau}}\check{\tau}_t + \left(\frac{\alpha(1-\alpha)}{\alpha+1/\eta}\frac{\bar{Y}}{\bar{K}} + \beta^{-1} - 1\right)\check{K}_{t-1} + \left(\frac{\bar{C}}{\bar{K}} + \frac{\bar{Y}}{\bar{K}}\frac{(1-\alpha)}{\alpha+1/\eta}\right)\check{\lambda}_t$$

$$\Delta \check{\lambda}_{t+1} = -\frac{\alpha\frac{\bar{Y}}{\bar{K}}}{\alpha\frac{\bar{Y}}{\bar{K}} + 1 - \delta}\left[-\left(\frac{1-\alpha}{1/\eta+\alpha}\right)\frac{\bar{\tau}}{1+\bar{\tau}}\check{\tau}_{t+1} + (1-\alpha)\left(\frac{\alpha}{1/\eta+\alpha} - 1\right)\check{K}_t + \frac{1-\alpha}{1/\eta+\alpha}\check{\lambda}_{t+1}\right]$$

(a) (10 points) Plot the dynamics of this system in a phase diagram as we did in class. I.e., plot the $\Delta K = 0$ and $\Delta\lambda = 0$ locus (K_t on x-axis, λ_t on y-axis). **Include all of the following**: axis labels, curve labels, steady-state (labeled), stable arm (labeled), and arrows showing the dynamics in each quadrant.

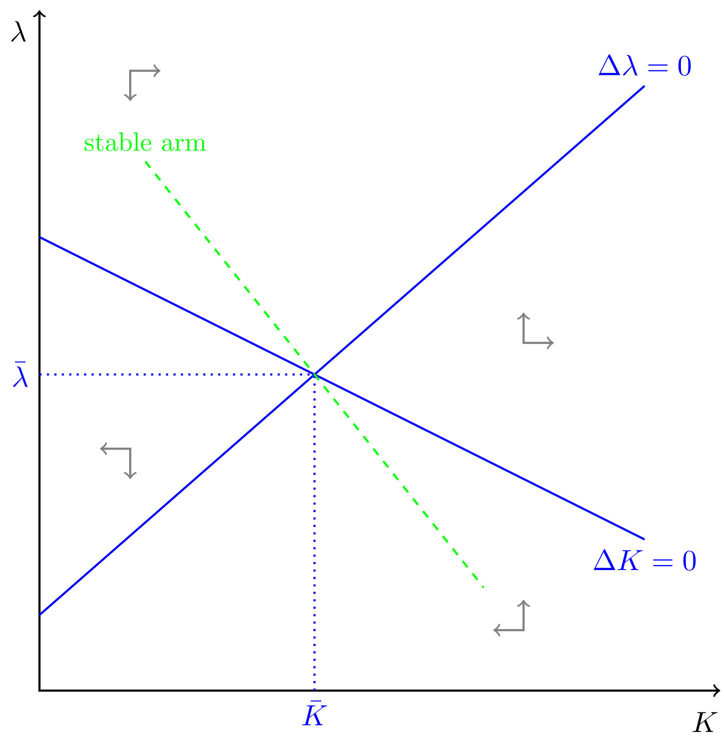

Figure 1: Phase Diagram in Steady-State.

(b) (21 points) Suppose that labor taxes τ_t rise permanently to a higher level.

Using the phase diagram in the (λ, K) space as well as the labor market diagram, plot the response of the economy to the shock. **Label all curves, axis and points.** Summarize the impact, transition, and steady-state responses for all variables in the table below (like we did in class) **and explain the intuition behind the response.**

Make the following assumptions when shifting the $\Delta K = 0$ and $\Delta\lambda = 0$ locus:
1. The new steady-state lies **above and to the left** of the old steady-state.
2. The new stable arm (saddle path) lies **above** the old stable arm (saddle path).

	Impact $t = t_0$	Transition $t \in (t_0, \infty)$	Steady State $t \to \infty$
λ	$\uparrow$	$\uparrow$	$\uparrow$
K	0	$\downarrow$	$\downarrow$
C	$\downarrow$	$\downarrow$	$\downarrow$
L	$\downarrow$	?	$\downarrow$
Y	$\downarrow$	?	$\downarrow$
I	$\downarrow$	$\uparrow$	$\downarrow$
W	$\downarrow$	$\downarrow$	$\downarrow$
R	$\downarrow$	$\uparrow$	0

Table 1: RBC model response to a permanent labor tax shock.

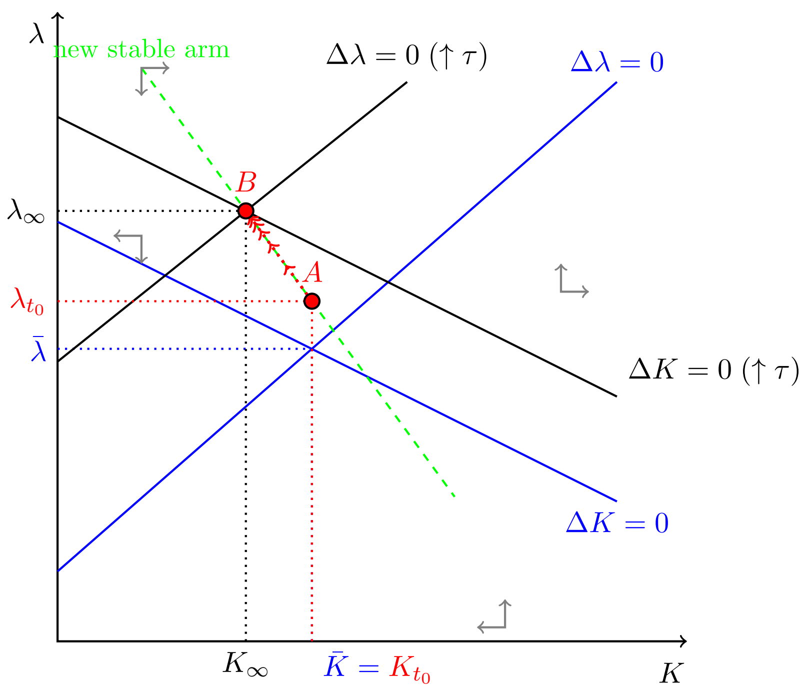

Figure 2: Response to permanent tax shock.

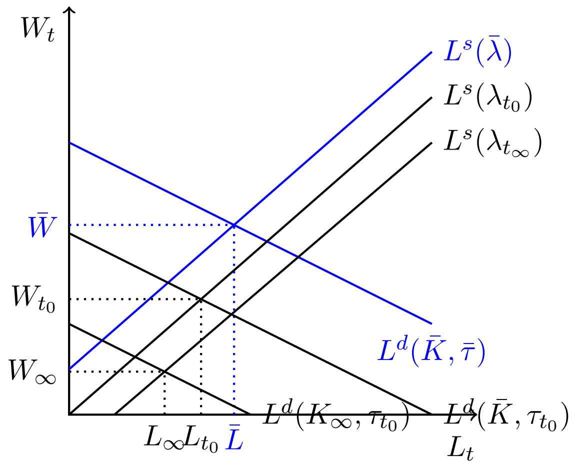

Figure 3: Response to permanent tax shock: labor market.

Solution: Based on our assumed (quantitatively reasonable) shifts of the loci we know that marginal utility and capital will be lower in the new steady-state. Intuitively, the increase in labor taxes distorts the equilibrium allocation, which (in this case) will translate into lower consumption and lower capital.

On impact we know the economy jumps up to the new stable arm, so marginal utility rises and consumption falls. Then gradually the economy gradually eats is capital and marginal utility increases further as capital declines.

Investment first declines on impact. This has to be the case since capital is falling. On the stable arm, investment will be lowest at t_0 and then gradually increase to its value at $t \to \infty$. In the long-run investment will be lower because the capital stock declined.

In the labor market there is both a shift-out in labor supply as consumers are poorer and want to compensate by working more, while firms reduce their demand as the after-tax wage has risen. Real wages fall, whereas the response of labor looks (for now) to be ambiguous.

Over the transition the labor supply increases further as marginal utility rises, while labor demand declines because of lower capital. We know in the long-run that labor must fall since the L/K ratio is pinned down by the constant long-run real interest rate. Real wages are also lower in the long-run because we only ever encounter increases in labor supply and reductions in labor demand.

We also know that output falls at t_0 and $t \to \infty$ since both investment and consumption decline. Over the transition the output movements are ambiguous since investment and consumption move in opposite directions and labor dynamics are ambiguous.

It remains to sign the real interest rate and labor on impact. The real interest rate must be below normal over the transition since consumption growth is negative. (Remember that consumption growth is proportional to $1 + R$). So we know that R falls on impact and then gradually rises along the stable arm.

The decline in R on impact then tells us that L must have fallen on impact since only a decline in L/K generates a decline in R in this set-up and K is fixed at t_0.

(c) (5 points) Are the impact responses ($t = t_0$) you derived consistent with the co-movement of output, labor, consumption and investment over the business cycle? **Explain your answer.** What does that tell you about the plausibility of attributing the business cycle to tax shocks?

Solution: Output, consumption, labor and investment all fall following a tax shock. So in principle, the co-movement of all these variables is consistent with what we observe over the business cycle, suggesting that tax shocks are not an implausible source of business cycles.

(d) (10 points) List **two** dimensions on which the standard RBC model with technology shocks did not perform well quantitatively. **Briefly,** discuss whether the RBC model with tax shocks would perform better on these dimensions. ($\approx$ 1 paragraph per dimension.)

Solution: All of the following are possible answers:

1. The RBC model generated little endogenous persistence.
2. The RBC model generated little endogenous amplification.
3. Real wages are too procyclical.
4. Technology shocks are so large that they imply (perhaps implausible) technological regress.
5. Measured productivity shocks are correlated with other demand shocks such as military spending and monetary policy shocks. It is therefore unclear whether the shocks we feed into the model are in fact technology shocks.

Tax shocks will not help with either (1), (2) or (3). For (1) and (2) changing the exogenous process is not going to help the endogenous properties of the model. Real wages are also going to be very procyclical following tax shocks: We see from the labor market diagram that real wages fall unambiguously. Tax shocks may provide a more plausible source of exogenous variation which can help with (4) and (5).

4. Some economists attribute the 2007-2009 recession at least in part to an *anticipation* of higher labor taxes following the election of President Obama.

   (a) (10 points) Redraw your phase diagram from part 3(a). Now suppose that economic agents expect a permanently higher level of labor taxes τ_t after t_1 periods from now ($t_0 = 0$). At t_1 these expectations turn out to be correct.

   Using the phase diagram in the (λ, K) space (**no need for labor market diagrams here!**), plot the response of the economy to the shock. **Label all curves, axis and points.** Summarize the impact, transition (before and after t_1), inflection, and steady-state responses for all variables in the table below (like we did in class).

   Make the following assumptions when shifting the $\Delta K = 0$ and $\Delta\lambda = 0$ locus for $t \geq t_1$:
   1. The new steady-state lies **above and to the left** of the old steady-state.
   2. The new stable arm (saddle path) lies **above** the old stable arm (saddle path).

	Impact $t = t_0$	Transition I $t \in (t_0, t_1)$	Inflection $t = t_1$	Transition II $t \in (t_1, \infty)$	Steady State $t \to \infty$
λ	$\uparrow$	$\downarrow$	0	$\uparrow$	$\uparrow$
K	0	$\uparrow$	0	$\downarrow$	$\downarrow$

Table 2: RBC model response to news of higher labor taxes.

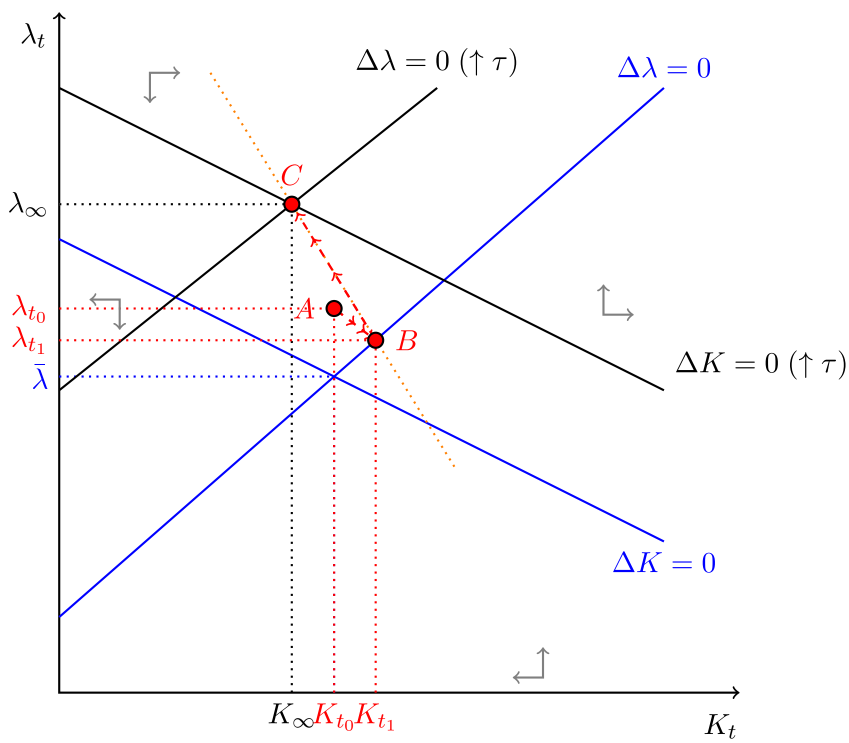

Figure 4: Phase Diagram: news of higher future taxes.

(b) (4 points) Draw the labor market diagram for t_0 (i.e., when news of higher taxes arrives). Explain **intuitively** why it is different to your answer in part 3(b).

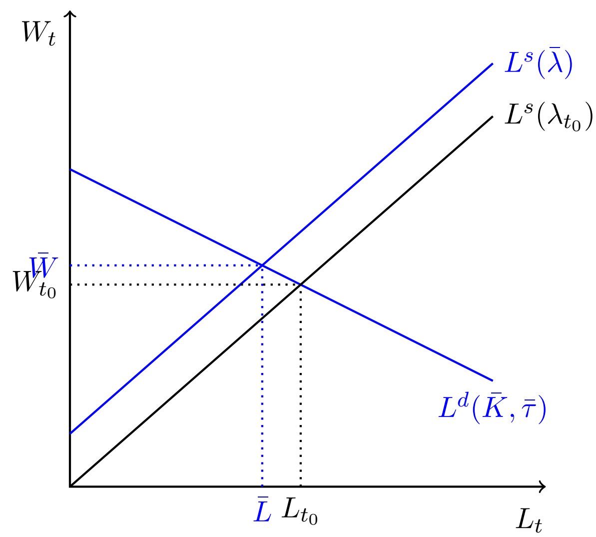

Figure 5: Response to permanent tax news: labor market.

Solution: Because taxes are unchanged at t_0 there is no change in labor demand. But consumers still increase labor supply because they know higher taxes will make them poorer in the future (in the sense of higher λ). Hence, we know that labor increases unambiguously and the real wage falls.

(c) (4 points) Based on part 4(b) determine the output response on impact. According to this model, are *news* of higher future labor taxes a plausible source of recessions?

Solution: Because labor supply rises and capital is fixed, we know from the production function that output rises on impact. So this news shock does not actually produce a recession before the increase in taxes is implemented.