I. Labor Supply Problem

Households maximize lifetime utility.

$$\max_{\{C_t, L_t\}} = \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

$$\sum_{t=0}^{\infty} (1+r)^{-t}(C_t - w_t L_t) = 0$$

Assume $\beta = (1+r)^{-1}$ and wages follow a law of motion.

$$w_t = \begin{cases} w^H, & t = 1, 3, 5, \dots \\ w^L, & t = 2, 4, 6, \dots \end{cases}$$

We derive the labor supply functions for two variants of the utility function. First, let $U(C_t, L_t) = \log C_t + \log(1 - L_t)$. The consumption-leisure condition is given by

$$\mathcal{L} = \sum_{t=0}^{\infty} (1+r)^{-t}[\log C_t + \log(1 - L_t) - \lambda_t(C_t - w_t L_t)]$$

$$\frac{1}{C_t} w_t = \frac{1}{1 - L_t}$$

$$\hat{L}_t(w_t) = \frac{w_t - C_t}{w_t}$$

Labor supply responds to shocks according to $\frac{\partial \hat{L}_t}{\partial w_t} = C_t w_t^{-2}$ and an elasticity of $\varepsilon_{\hat{L}} = C_t(L_t w_t)^{-1}$. Now let $U(C_t, L_t) = \log C_t + \log(1 - 0.5(L_t + L_{t-1}))$.

$$\mathcal{L} = \sum_{t=1}^{\infty} (1+r)^{-t}[\log C_t + \log(1 - 0.5(L_t + L_{t-1})) - \lambda_t(C_t - w_t L_t)]$$

$$\lambda_t = \frac{1}{C_t}$$

$$\lambda_t w_t = \frac{0.5}{1 - L_t} + \frac{0.5}{1 - L_t}$$

II. Demand Shocks

An economy contains a continuum of identical consumers who solve

$$\max_{\{C_t, L_t\}} \mathrm{E} \sum_{t=0}^{\infty} \beta^t \left(\log C_t - v_t \frac{L_t^{1+\chi}}{1+\chi} \right)$$

$$P_t K_{t+1} = W_t L_t + (P_t + d_t) K_t + \Pi_t - C_t$$

$$v_t \geq 0, \chi > 0$$

The capital stock is fixed at $\bar{K}$ and neither depreciates nor accumulates.

1. The consumption-leisure condition at time t is $\frac{W_t}{C_t} = v_t L_t^{\chi}$.

2. The consumer's intertemporal substitution satisfies $\frac{1}{C_t} = \frac{\beta}{C_{t+1}P_t}(P_{t+1} + d_{t+1})$.

3.

4. Labor supply is given by $L_t = \frac{W_t \lambda_t}{v_t}^{\frac{1}{\chi}}$ with elasticity $\frac{1}{\chi}$. A higher value of v_t is associated with a higher marginal utility of leisure and implies a steeper labor supply curve.

5. In this model, cyclicality is driven by demand shocks v_t.

III. Business Cycle and External Returns to Scale

A continuum of competitive firms operate the production technology $Y_{it} = E_t K_{it}^{\alpha}(Z_t L_{it})^{1-\alpha}$. Firms take $E_t = Y_t^{\frac{\gamma-1}{\gamma}}$ exogenously. The market clearing condition for consumption goods is $Y_t = C_t$.

1. The firm first order conditions yields the labor demand function.

$$\max_{K_{it}, L_{it}} E_t K_{it}^{\alpha}(Z_t L_{it})^{1-\alpha} - w_t L_t - d_t K_t$$

IV. Textbook Problems