I.   Investment and the Housing Market

Consider the following model of the housing market $I = \psi(P)$ where $\psi' > 0$ is demand for investment, $R(H)$ where $R' < 0$ is the rental cost of the house as a function of housing stock, $r + \delta = P^{-1}(R + \dot{P})$ equates the user cost and the rental rate, and the law of motion for housing is $\dot{H} = I - \delta H$.

1. It is reasonable to expect that $\psi' > 0$ and $R' < 0$ so that housing follows the law of demand in the rental and asset markets. The condition $r + \delta = P^{-1}(R + \dot{P})$ equates the value of real estate with its opportunity cost. Lastly, the model is closed with 4 endogenous variables and 4 equations.

2. We can reduce the system to two dynamic equations.

$$\begin{cases} \dot{H} & = \psi(P) - \delta H \\ \dot{P} & = P(r + \delta) - R(H) \end{cases}$$

3. Imposing the steady state yields $H = \delta^{-1}\psi(P)$ and $P = R(H)(r + \delta)^{-1}$. Without knowing $\psi(P)$ or $R(H)$ we at least know that $\frac{dH}{dP} > 0$ on the $\dot{H} = 0$ locus and $\frac{dH}{dP} < 0$ on the $\dot{P} = 0$ locus. H converges toward the $\dot{H} = 0$ locus when out of steady state while P diverges out.

4. A higher r is associated with lower prices and housing stock in the steady state. This occurs through a rise in R and a decline in I.

5. An unexpected, permanent increase in r will shift the $\dot{P} = 0$ locus towards the origin. Prices will immediately jump down to the new saddle path but at the same level of housing. Housing stock will decrease gradually to the new steady state. Investment will follow the transition of prices and rental rates will follow the transition of the housing stock.

6. An unexpected, temporary increase in r will shift the $\dot{P} = 0$ locus towards the origin as in the permanent case. Prices will immediately decrease after the announcement but to a lesser extent than with a permanent shock. Once the new r takes effect, the dynamics will decumulate capital and increment P. Once the transitory shock subsides, P, H will be on the stable path and housing will gradually increase to the original steady state.

7. When a permanent rise in r is known ahead of time, prices will decrease in anticipation of the shock. As the shock occurs housing stock will adjust to a lower steady state as before but the change will not be as large due to anticipatory behavior.

8. Suppose consumers have static expectations about the price of housing so $\mathrm{E}(\dot{P}) = 0$.

9.

II.   Discount Factor Shocks

Consider the model developed in class and allow the discount factor β to exogenously vary over time. This variation captures the idea that impatience to consume can fluctuate and it can be interpreted as a demand shock. This modification changes the consumer's Euler equation and rental rate determination

$$\frac{C_{t+1}}{C_t} = \beta_t(1 + R_{t+1}) = \beta_t(\alpha Y_{t+1}K_t^{-1} + (1 - \delta))$$

A linear approximation is given by log-linearizing.

$$\ln C_{t+1} - \ln C_t = \ln \beta_t + \ln(\alpha Y_{t+1}K_t^{-1} + (1 - \delta))$$

$$E\,\check{C}_{t+1} - \check{C}_t = \check{\beta}_t + \frac{\alpha \bar{Y} \bar{K}}{\alpha \bar{Y} \bar{K} + (1 - \delta)}(E\,\check{Y}_{t+1} - \check{K}_t)$$

$\check{\beta}_t$ only enters the costate dynamic equation, which becomes

$$\Delta \check{\lambda}_{t+1} = -\check{\beta}_t - \frac{\alpha \bar{Y} \bar{K}}{\alpha \bar{Y} \bar{K} + (1 - \delta)}\left[\left(1 + \frac{1 - \alpha}{\alpha + \eta^{-1}}\right)\check{Z}_{t+1} + (1 - \alpha)\left(\frac{\alpha}{\alpha + \eta^{-1}} - 1\right)\check{K}_t + \frac{1 - \alpha}{\alpha + \eta^{-1}}\check{\lambda}_{t+1}\right]$$

A positive discounting shock ($\beta < \beta'$) must be accompanied by a decrease in the marginal product of capital to maintain steady state consumption and investment. In the case of a permanent shock, $\check{\lambda}$ immediately jumps up to the new stable arm while $\check{K}$ transitions to a higher steady state following this path. Households have a greater motive to save in the short term but the marginal product of capital will decrease in the long term.

In the case of a transitory shock, $\check{\lambda}$ will likewise jump up but fall short of the new stable arm. $\check{K}$ will gradually increase and $\check{\lambda}$ will decrease during the transition phase during the shock. The path will inflect towards the original stable arm and reach it after the shock subsides. Tables 1 and 2 summarize the effect on the endogenous variables from a permanent and transitory shock, respectively.

Table 1: Comparative statics of a permanent demand shock

	Impact	Transition	Steady State
Consumption	–	+	+
Output	+	?	+
Investment	+	–	+
Employment	+	?	?
Wages	–	+	+
Interest Rate	+	–	–

Table 2: Comparative statics of a transitory demand shock

	Impact	Transition	Inflection	Transition	Steady State
Consumption	–	+	0	–	0
Output	+	?	0	?	0
Investment	+	–	–	+	0
Employment	+	?	0	–	0
Wages	–	+	0	–	0
Interest Rate	+	–	0	+	0

III.   Fake News

Returning to the classical RBC model as before, consider a positive productivity shock announced at t_0 and expected to occur in the future t_1 but is not realized. Suppose that the shock is such that $\bar{\lambda} < \bar{\lambda}_0$ and $\bar{K} > \bar{K}_0$. Initially, variables will transition as if facing an anticipated productivity shock. $\check{\lambda}$ will jump down just shy of the new stable arm so that $\check{\lambda}, \check{K}$ would have moved onto the new stable arm once the permanent shock occurs. Table 3 summarizes the effect on the endogenous variables from a permanent shock.

Table 3: Comparative statics of a permanent news shock

	Impact	Transition	Steady State
Consumption	−	+	+
Output	+	?	+
Investment	+	−	+
Employment	+	?	?
Wages	−	+	+
Interest Rate	+	−	−

IV.   Labor Supply

Suppose an individual with a T period horizon wants to consume a constant level of consumption C per period. She earns w per hour, she must decide how many hours N to work per period. The path of the wage rate w_t is known at $t = 0$. The rate of time preference and of interest are zero. The individual's problem is $\min_{\{N_t\}} \sum_{t=0}^{T} \ln(1 + N_t)$ subject to $CT = \sum_{t=0}^{T} w_t N_t$ and $N_t \geq 0$.

1. The Euler equation for intertemporal labor supply is $w_t(1 + N_t) = w_s(1 + N_s) \, \forall \, s, t \leq T$.

2. The individual works to equalize the ratio of marginal utility and marginal product of labor across periods. In an environment with a constant wage, no discounting, and perfect foresight, the individual will work the same amount each period.

3. This theory produces several testable implications. Under the proposed model, labor supply should have a positive relationship with contemporaneous wage changes and a negative relationship with announced, future changes.

4. Households will work more in the periods after an unexpected, permanent increase in wage in order to equalize the ratio of marginal utility and marginal product (a substitution effect).

5. If a delayed but expected, permanent wage increase were to occur, household will work less today and compensate by working more after the wage hike.

6. Labor supply elasticity is positive with respect to contemporaneous wages ($\varepsilon_L = 1$) but negative with respect to future wages due to intertemporal substitution.
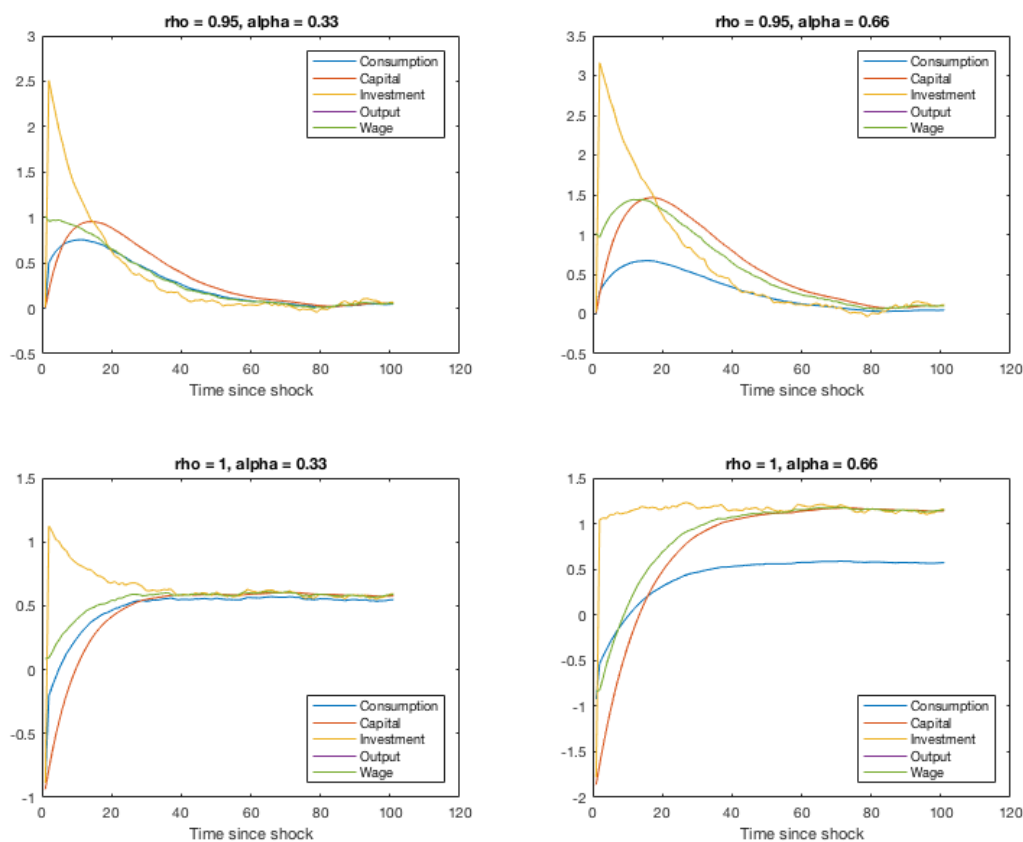
V.   Impulse Responses by Simulation

Assume $\check{G} = 0$ and $L = 1$ inelastically.

1. Check moments.

2. Figure 1 plots the impulse response of the linearized model. Since capital is a predetermined stock, it increases continuously in response to the shock and returns to the steady state gradually. Consumption and investment, as choice variables, jump immediately after the shock with investment being more senitive due to consumption smoothing. Wages and output respond immediately as functions of $\check{C}, \check{I}$ but are slowed down by $\check{K}$.

3. Suppose that $\rho = 1$ so that the technology shock persists indefinitely.

Figure 1: Impulse response for a unit productivity shock

VI. Impulse Responses with Dynare

A.   RBC Simulation

```
% Title: Econ 210C Homework 2
% Author: Justin Abraham
% Notes: Log-linearized RBC model

clear; clc;
tic

%%%%%%%%%
% Setup %
%%%%%%%%%

reps = 5000;
shockt = round(reps*0.98);

alpha = 0.33;
beta = 0.99;
delta = 0.076;
rho = 0.95;
sigma = 0.007;

KYbar = 3.32;
CKbar = (1/KYbar) - delta;
YCbar = (1/KYbar) / CKbar;

gamma = 1/beta + 1 - CKbar*(1 - beta * (1 - delta))*(alpha - 1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Reduced form solutions %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%

nu_kk = (gamma - sqrt(gamma^2 - (4/beta))) / 2;
nu_ck = (CKbar^(-1))*((1/beta) - nu_kk);
nu_kz = (YCbar * (1-rho) + (1-beta*(1-delta))*rho) / ((CKbar^(-1))*(1-rho) + (CKbar^(-1))*((1/beta) - nu_kk) - (1-beta*(1
nu_cz = YCbar - ((CKbar^(-1)) * nu_kz);

%%%%%%%%%%%%%%%%%%%%%%%%
% Initialize vectors %
%%%%%%%%%%%%%%%%%%%%%%%%

epsilon = sigma*randn(reps, 1);

Z = zeros(reps, 1);
C = zeros(reps, 1);
K = zeros(reps, 1);
I = zeros(reps, 1);
Y = zeros(reps, 1);
w = zeros(reps, 1);
r = zeros(reps, 1);

%%%%%%%%%%%%%%%
% Simulation %
%%%%%%%%%%%%%%%

for t = 2:reps

    if t == shockt
```

```
        Z(t) = Z(t-1) + 1;
    else
        Z(t) = rho*Z(t-1) + epsilon(t);
    end

    C(t) = nu_ck*K(t-1) + nu_cz*Z(t-1);
    K(t) = nu_kk*K(t-1) + nu_kz*Z(t-1);
    I(t) = (1/delta)*(K(t) - (1-delta)*K(t-1));
    Y(t) = Z(t) + alpha*K(t-1);
    w(t) = Z(t) + alpha*K(t-1);

end

subplot(2, 2, 1), plot(C(shockt:end, 1))

hold on
subplot(2, 2, 1), plot(K(shockt:end, 1))
subplot(2, 2, 1), plot(I(shockt:end, 1))
subplot(2, 2, 1), plot(Y(shockt:end, 1))
subplot(2, 2, 1), plot(w(shockt:end, 1))
hold off

title('rho = 0.95, alpha = 0.33'); xlabel('Time since shock'); legend('Consumption', 'Capital', 'Investment', 'Output', '

%%%%%%%%%%%%%%%%%%%%%%%%%%%
% rho = 0.95, alpha = 0.66 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%

alpha = 0.66;
beta = 0.99;
delta = 0.076;
rho = 0.95;
sigma = 0.007;

KYbar = 3.32;
CKbar = (1/KYbar) - delta;
YCbar = (1/KYbar) / CKbar;

gamma = 1/beta + 1 - CKbar*(1 - beta * (1 - delta))*(alpha - 1);

nu_kk = (gamma - sqrt(gamma^2 - (4/beta))) / 2;
nu_ck = (CKbar^(-1))*((1/beta) - nu_kk);
nu_kz = (YCbar * (1-rho) + (1-beta*(1-delta))*rho) / ((CKbar^(-1))*(1-rho) + CKbar^(-1))*((1/beta) - nu_kk) - (1-beta*(1
nu_cz = YCbar - ((CKbar^(-1)) * nu_kz);

epsilon = sigma*randn(reps, 1);

C = zeros(reps, 1);
K = zeros(reps, 1);
I = zeros(reps, 1);
Y = zeros(reps, 1);
w = zeros(reps, 1);
r = zeros(reps, 1);

for t = 2:reps

    C(t) = nu_ck*K(t-1) + nu_cz*Z(t-1);
    K(t) = nu_kk*K(t-1) + nu_kz*Z(t-1);
```

```
    I(t) = (1/delta)*(K(t) - (1-delta)*K(t-1));
    Y(t) = Z(t) + alpha*K(t-1);
    w(t) = Z(t) + alpha*K(t-1);

end

subplot(2, 2, 2), plot(C(shockt:end, 1))

hold on
subplot(2, 2, 2), plot(K(shockt:end, 1))
subplot(2, 2, 2), plot(I(shockt:end, 1))
subplot(2, 2, 2), plot(Y(shockt:end, 1))
subplot(2, 2, 2), plot(w(shockt:end, 1))
hold off

title('rho = 0.95, alpha = 0.66'); xlabel('Time since shock'); legend('Consumption', 'Capital', 'Investment', 'Output', '

%%%%%%%%%%%%%%%%%%%%%%%%
% rho = 1, alpha = 0.33 %
%%%%%%%%%%%%%%%%%%%%%%%%%

alpha = 0.33;
beta = 0.99;
delta = 0.076;
rho = 1;
sigma = 0.007;

KYbar = 3.32;
CKbar = (1/KYbar) - delta;
YCbar = (1/KYbar) / CKbar;

gamma = 1/beta + 1 - CKbar*(1 - beta * (1 - delta))*(alpha - 1);

nu_kk = (gamma - sqrt(gamma^2 - (4/beta))) / 2;
nu_ck = (CKbar^(-1))*((1/beta) - nu_kk);
nu_kz = (YCbar * (1-rho) + (1-beta*(1-delta))*rho) / ((CKbar^(-1))*(1-rho) + (CKbar^(-1))*((1/beta) - nu_kk) - (1-beta*(1
nu_cz = YCbar - ((CKbar^(-1)) * nu_kz);

epsilon = sigma*randn(reps, 1);

Z = zeros(reps, 1);
C = zeros(reps, 1);
K = zeros(reps, 1);
I = zeros(reps, 1);
Y = zeros(reps, 1);
w = zeros(reps, 1);
r = zeros(reps, 1);

for t = 2:reps

    if t == shockt
        Z(t) = Z(t-1) + 1;
    else
        Z(t) = rho*Z(t-1) + epsilon(t);
    end

    C(t) = nu_ck*K(t-1) + nu_cz*Z(t-1);
    K(t) = nu_kk*K(t-1) + nu_kz*Z(t-1);
    I(t) = (1/delta)*(K(t) - (1-delta)*K(t-1));
```

```
    Y(t) = Z(t) + alpha*K(t-1);
    w(t) = Z(t) + alpha*K(t-1);

end

subplot(2, 2, 3), plot(C(shockt:end, 1))

hold on
subplot(2, 2, 3), plot(K(shockt:end, 1))
subplot(2, 2, 3), plot(I(shockt:end, 1))
subplot(2, 2, 3), plot(Y(shockt:end, 1))
subplot(2, 2, 3), plot(w(shockt:end, 1))
hold off

title('rho = 1, alpha = 0.33'); xlabel('Time since shock'); legend('Consumption', 'Capital', 'Investment', 'Output', 'Wag

%%%%%%%%%%%%%%%%%%%%%%%%
% rho = 1, alpha = 0.66 %
%%%%%%%%%%%%%%%%%%%%%%%%%

alpha = 0.66;
beta = 0.99;
delta = 0.076;
rho = 1;
sigma = 0.007;

KYbar = 3.32;
CKbar = (1/KYbar) - delta;
YCbar = (1/KYbar) / CKbar;

gamma = 1/beta + 1 - CKbar*(1 - beta * (1 - delta))*(alpha - 1);

nu_kk = (gamma - sqrt(gamma^2 - (4/beta))) / 2;
nu_ck = (CKbar^(-1))*((1/beta) - nu_kk);
nu_kz = (YCbar * (1-rho) + (1-beta*(1-delta))*rho) / ((CKbar^(-1))*(1-rho) + (CKbar^(-1))*((1/beta) - nu_kk) - (1-beta*(1
nu_cz = YCbar - ((CKbar^(-1)) * nu_kz);

epsilon = sigma*randn(reps, 1);

C = zeros(reps, 1);
K = zeros(reps, 1);
I = zeros(reps, 1);
Y = zeros(reps, 1);
w = zeros(reps, 1);
r = zeros(reps, 1);

for t = 2:reps

    C(t) = nu_ck*K(t-1) + nu_cz*Z(t-1);
    K(t) = nu_kk*K(t-1) + nu_kz*Z(t-1);
    I(t) = (1/delta)*(K(t) - (1-delta)*K(t-1));
    Y(t) = Z(t) + alpha*K(t-1);
    w(t) = Z(t) + alpha*K(t-1);

end

subplot(2, 2, 4), plot(C(shockt:end, 1))

hold on
```

```
subplot(2, 2, 4), plot(K(shockt:end, 1))
subplot(2, 2, 4), plot(I(shockt:end, 1))
subplot(2, 2, 4), plot(Y(shockt:end, 1))
subplot(2, 2, 4), plot(w(shockt:end, 1))
hold off

title('rho = 1, alpha = 0.66'); xlabel('Time since shock'); legend('Consumption', 'Capital', 'Investment', 'Output', 'Wag

set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0, 0.6, 0.8]);
```

B. Solving RBC with Dynare

```
% Econ 210C Homework 2
% Justin Abraham

%----------------------------------------------------------------
% 1. Defining variables
%----------------------------------------------------------------
% y = output (in log terms)
% c = consumption (in log terms)

var y I k a c w R r;
varexo e;

parameters alpha beta delta rho sigma sigmae;

%----------------------------------------------------------------
% 2. Calibration
%----------------------------------------------------------------

alpha  = 1/3;   // exponent on capital in the production function
beta   = 0.99;  // household's discount factor
delta  = 0.025;  // capital depreciation rate
rho    = 0.95;  // persistence of shock
sigma  = 1;     // degree of risk aversion
sigmae = 0.01;   // std of productivity shock

%----------------------------------------------------------------
% 3. Model
%----------------------------------------------------------------
% can also use the log-linearized system (Dynare will do this for you)
% index by (-1) has Dynare treat as state variable

model;

exp(c)^(-sigma) = beta*(exp(c(+1))^(-sigma))*(alpha*exp(a(+1))*exp(k)^(alpha-1) + (1-delta));
exp(y)   = exp(a)*exp(k(-1))^alpha;
exp(k)   = exp(k(-1))*(1-delta) + exp(I);
exp(y)   = exp(c) + exp(I);
exp(R)   = alpha*exp(y)/exp(k(-1)) - delta;
exp(w)   = (1-alpha)*exp(y);
exp(c)^(-sigma) = beta*(exp(c(+1))^(-sigma))*(1+r);
a = rho*a(-1) + e;

end;

%----------------------------------------------------------------
% 4. Computation
%----------------------------------------------------------------

initval;

k = log(30);
y = log(3);
I = log(0.5);
c = log(2.5);
a = 0;
r = (1/beta)-1;
```

```
R = log((1/beta)-(1-delta)));
w = log(1);

end;

shocks;

var e = sigmae^2;

end;

steady;

stoch_simul(order=1, irf=20) c y k I r a;
```