ECON 210C PROBLEM SET # 4

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1. Labor supply problem

2. Demand shock

(a). The consumption-leisure condition at time t is just equating marginal benefits of labor and leisure $\frac{W_t}{C_t} = v_t L_t^{\chi}$.

(b). The consumer consuming one less unit today means they are losing out on $\frac{1}{C_t}$ today. With that, they can buy $\frac{1}{P_t}$ units of capital today, and tomorrow they will get $P_{t+1} + d_{t+1}$ from that unit of capital. So their payoff tomorrow is

$$\frac{1}{P_t} \times (P_{t+1} + d_{t+1}) \times \frac{\beta}{C_{t+1}}$$

and optimality therefore implies we have

$$\frac{1}{C_t} = \frac{1}{P_t} \times (P_{t+1} + d_{t+1}) \times \frac{\beta}{C_{t+1}}$$

as the inter-temporal optimality condition.

(c).

3. Business cycle and external returns to scale

(a). Each firm sets wage equal to marginal product of labor, so we have

$$W_t = Y_t^{1-1/\gamma} \left(\frac{K_{it}}{L_{it}}\right)^{\alpha} Z_t^{1-\alpha}$$

and so we can find labor demand as a function of wages

$$L_{it} = (W_t Z_t^{\alpha-1} Y_t^{1/\gamma-1} K_{it}^{-\alpha})^{-\frac{1}{\alpha}}$$

which simplifies to

$$L_{it} = W_t^{-\frac{1}{\alpha}} Z_t^{\frac{1-\alpha}{\alpha}} Y_t^{\frac{1-1/\gamma}{\alpha}} K_{it}$$

(b). Integrating both sides over all firms, we have

$$L_t = W_t^{-\frac{1}{\alpha}} Z_t^{\frac{1-\alpha}{\alpha}} Y_t^{\frac{1-1/\gamma}{\alpha}} K_t$$

so we can start to solve for aggregate production, so we get

$$Y_t^{\frac{1-1/\gamma}{\alpha}} = \frac{L_t}{K_t} \times W_t^{\frac{1}{\alpha}} Z_t^{\frac{\alpha-1}{\alpha}}$$

and solving for Y gives

$$Y_t = \left(\frac{L_t}{K_t}\right)^{\frac{\alpha}{1-1/\gamma}} W_t^{\frac{1}{1-1/\gamma}} Z_t^{\frac{\alpha-1}{1-1/\gamma}}$$

4. Problems from Romer

4.1. **Problem 6.10.**

4.2. **Problem 6.11.**

4.3. **Problem 6.12.**

(a). First, we can substitute our wage expression $w = \theta p$ to get

$$p = \theta p + (1-\alpha)\ell - s \implies p = \frac{(1-\alpha)\ell - s}{1-\theta}$$

and we are given aggregate demand, so we have

$$y = m - p = m - \frac{(1-\alpha)\ell - s}{1-\theta}$$

and from our output equation we have

$$s + \alpha\ell = m - \frac{(1-\alpha)\ell - s}{1-\theta} \implies \ell = \frac{(1-\theta)m + \theta s}{1-\theta\alpha}$$

which we can substitute into our price equation to get

$$p = \frac{(1-\theta)m - s}{1-\theta\alpha}$$

and now we have our output

$$y = \frac{(1-\theta)\alpha m + s}{1-\theta\alpha}$$

and we can find wage using the original wage expression to get

$$w = \theta \times \frac{(1-\theta)m - s}{1-\theta\alpha}$$

and we can now find how employment responds to shocks.

We can take mixed second derivatives to obtain

$$\frac{\partial^2 \ell}{\partial m \partial \theta} = \frac{\alpha - 1}{(1-\theta\alpha)^2}$$

for how indexation moderates the effect of a monetary shock and

$$\frac{\partial^2 \ell}{\partial s \partial \theta} = \frac{1}{(1-\theta\alpha)^2}$$

for how indexation moderates the effect of a supply shock.

Since $\alpha - 1 < 0$, we have that greater indexation reduces the effect of a monetary shock, while $1 > 0$ tell us that greater indexation scales the effects of supply shocks up.

(b). With independence we can just use the formula for the variance of a linear combination of two random variables. So we have

$$\operatorname{Var}(\ell) = \left(\frac{1-\theta}{1-\theta\alpha}\right)^2 \operatorname{Var}(m) + \left(\frac{\theta}{1-\theta\alpha}\right)^2 \operatorname{Var}(s)$$

so minimizing this requires the first order condition

$$(1-\theta)(\alpha-1)\operatorname{Var}(m) + \theta \operatorname{Var}(s)$$

so solving for θ we get

$$\theta^* = \frac{(1-\alpha)\operatorname{Var}(m)}{(1-\alpha)\operatorname{Var}(m) + \operatorname{Var}(s)}$$

as the wage indexation that minimizes employment variance.

(c.i). We can easily see that we have

$$y_i - y = \alpha(\ell_i - \ell) \implies \ell_i = \ell + \frac{y_i - y}{\alpha} = \ell - \frac{\theta_i - \theta}{\alpha} \times \phi p$$

and we already have expressions for employment and the price levels that we can substitute to get

$$\ell_i = \frac{(1-\theta)\alpha - \phi(1-\alpha)(\theta_i - \theta))m + (\theta\alpha + \phi(\theta_i - \theta))s}{(1-\theta\alpha)\alpha}$$

for employment at firm i.

(c.ii). We now have

$$\operatorname{Var}(\ell_i) = \left(\frac{(1-\theta)\alpha - \phi(1-\alpha)(\theta_i - \theta)}{(1-\theta\alpha)\alpha}\right)^2 \operatorname{Var}(m) + \left(\frac{\theta\alpha + \phi(\theta_i - \theta)}{(1-\theta\alpha)\alpha}\right)^2 \operatorname{Var}(s)$$

so we must satisfy the first order condition

$$(\alpha-1)\phi[(1-\theta)\alpha - \theta_i(1-\alpha)\phi + \theta(1-\alpha)]\operatorname{Var}(m) + \phi[\theta\alpha + \phi\theta_i - \phi\theta]\operatorname{Var}(s) = 0$$

which allows us to solve for θ_i to get

$$\theta_i^* = \frac{(1-\alpha)\phi((1-\theta)\alpha + \theta\phi(1-\alpha))\operatorname{Var}(m) + \phi\theta(\alpha - \phi)\operatorname{Var}(s)}{(\phi(1-\alpha))^2 \operatorname{Var}(m) + \phi^2 \operatorname{Var}(s)}$$

(c.iii). The Nash equilibrium value implies that each firm's first order condition can have θ_i and θ identical. So we need

$$(\alpha - 1)\phi[(1-\theta)\alpha - \theta(1-\alpha)\phi + \theta(1-\alpha)] \operatorname{Var}(m) + \phi[\theta\alpha + \phi\theta - \phi\theta] \operatorname{Var}(s) = 0$$

which simplifies to

$$(1-\theta)\alpha(1-\alpha)\phi \operatorname{Var}(m) - \theta\alpha\phi \operatorname{Var}(s) = 0$$

allowing us to solve for the Nash value of θ as

$$\theta_{\text{Nash}} = \frac{(1-\alpha)\operatorname{Var}(m)}{(1-\alpha)\operatorname{Var}(m) + \operatorname{Var}(s)}$$

which is the same value as in part b.