ECON 210C PROBLEM SET # 2

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1. Investment and the Housing Market

(a).

(1) $I = \psi(P)$: Gross investment in housing is an increasing function of the price of houses. This specification implies that housing investment can be interpreted as the supply of new housing.

(2) $r + \delta = (R + \dot{P})/P$: This implies that the costs of investing into a house, namely forgone investment income and depreciation are equal to the benefits, namely rental payments and capital gains.

(3) $R = R(H)$: Rental cost is a decreasing function of the size of the housing stock.

(4) $\dot{H} = I - \delta H$: The housing stock can change in two ways, housing investment and depreciation.

(b). We merely substitute to obtain

$$\dot{H} = \psi(P) - \delta H$$
$$r + \delta = (R(H) + \dot{P})/P$$

(d). Rewriting the equation for $\dot{P}$ gives

$$\dot{P} + R(H) = P(r + \delta)$$

implying an increase in r must increase $R(H)$ correspondingly given $\dot{P} = 0$. Since R is a decreasing function of H, we have that H decreases and the $\dot{P} = 0$ locus shifts to the left.

(e). If we go from r to r^*, where $r < r^*$, then P immediately drops to the level $\frac{R(H)}{r^*+\delta}$, as the higher opportunity cost of investing in housing reduces the quantity of housing investment. As housing investment decreases, the quantity of housing stock decreases gradually until $R(H)$ goes up to the $\dot{P} = 0$ point. So we have that H moves gradually down to the new steady state, P drops immediately then rises to its new but lower level, I drops immediately then rises to its new lower level, and R rises gradually to its new steady state level.

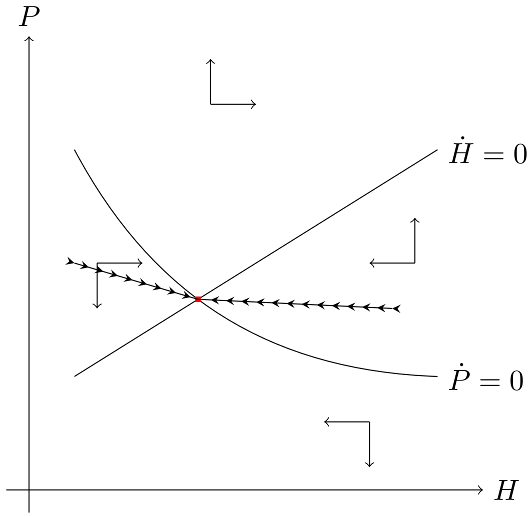

FIGURE 1. Phase diagram for 1c

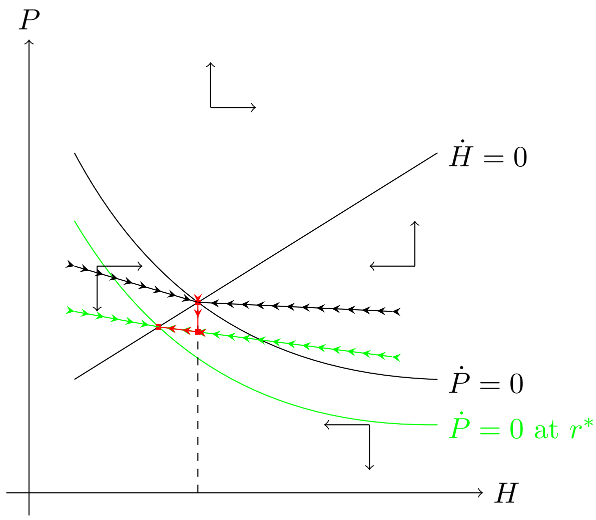

FIGURE 2. Change in the real interest rate

(f). P will still drop immediately, but to a level higher than $\frac{R(H)}{r^*+\delta}$, because we still have that the higher opportunity cost of investing in housing reduces the quantity of housing investment. As housing investment decreases, the quantity of housing stock decreases at first, leveling off during the period increased interest rates, after which it rises again forever back up to the original $\dot{P} = 0$ point. So we have that H moves gradually down until it starts rising again (before the increase is over) to the same steady state, P drops immediately then rises back to its original level (with a discrete jump when the increase is over), I drops immediately then rises back to its original level (with a discrete jump when the increase is over), and R rises gradually until it levels off at the same point H does, after which it gradually falls to the same steady state level.

(g). P will still drop immediately, and then gradually declines further until it hits $\frac{R(H)}{r^*+\delta}$ (with a discrete drop when the increase hits).

H declines gradually starting immediately toward the new lower steady state value.

I drops immediately, then rises to its new lower level.

R increases gradually starting immediately toward the new higher steady state value.

(h). In this case, we merely have

$$r + \delta = R(H)/P$$

so an increase in r implies an immediate fall in P

P will drop immediately, and then rises slowly to its new steady state value.

H declines gradually; (H, P) moves along the $\dot{P} = 0$ locus.

I drops immediately, then rises to its new lower level.

R increases gradually starting immediately toward the new higher steady state value.

(i). If people have static expectations and there is an interest rate shock, then the model matches everything in the housing crisis; prices and construction dropped rapidly, while there was little effect on rental costs.

2. Discount Factor Shocks

The only equation affected is the Euler equation, so we have our model:

$$\begin{aligned}
\lambda_t &= \frac{1}{C_t} \\
\frac{C_{t+1}}{C_t} &= E[\beta_{t+1}(1 + R_{t+1})] \\
W_t &= (1-\alpha)Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha} \\
R_t + \delta &= \alpha Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha-1} \\
L_t^{\frac{1}{\eta}} C_t &= W_t \\
C_t + I_t + G_t &= Z_t L_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha} \\
K_t &= (1-\delta)K_{t-1} + I_t
\end{aligned}$$

We have the endogenous variables $(\lambda_t, K_t, W_t, C_t, L_t, R_t, I_t\ )$ and exogenous variables (β_t, Z_t, G_t), and so we can follow the derivation from class, only replacing the Euler equation to get the $\check{\beta}_t$ term.

$$\Delta \check{K_t} = \frac{\bar{Y}}{\bar{K}}\left(1+\frac{1-\alpha}{\alpha+1/\eta}\right)\check{Z}_t + \left(\frac{\alpha(1-\alpha)}{\alpha+1/\eta}\frac{\bar{Y}_t}{\bar{K}_t}+\alpha\frac{\bar{Y}}{\bar{K}}-\delta\right)\check{K_{t-1}} + \left(\frac{\bar{C}}{\bar{K}}+\frac{\bar{Y}}{\bar{K}}\frac{(1-\alpha)}{\alpha+1/\eta}\right)\check{\lambda}_t - \frac{\bar{G}}{\bar{K}}\check{G}_t$$

$$\Delta\check{\lambda_{t+1}} = -\check{\beta_{t+1}} - \frac{\alpha\frac{\bar{Y}}{\bar{K}}}{\alpha\frac{\bar{Y}}{\bar{K}}+1-\delta}\left[\left(1+\frac{1-\alpha}{\alpha+1/\eta}\right)\check{Z_{t+1}} + (1-\alpha)\left(\frac{\alpha}{1/\eta+\alpha}-1\right)\check{K}_t + \frac{1-\alpha}{1/\eta+\alpha}\check{\lambda_{t+1}}\right]$$

By setting $\Delta\lambda_t = 0$, we have the locus of points where the marginal utility of consumption is constant.

Locus $\Delta\lambda_t = 0$.

$$\check{\lambda_{t+1}} = -\frac{1/\eta+\alpha}{1-\alpha}\left[\left(\frac{\alpha\frac{\bar{Y}}{\bar{K}}}{\alpha\frac{\bar{Y}}{\bar{K}}+1-\delta}\right)^{-1}\check{\beta_{t+1}} + \left(1+\frac{1-\alpha}{\alpha+1/\eta}\right)\check{Z_{t+1}} + (1-\alpha)\left(\frac{\alpha}{1/\eta+\alpha}-1\right)\check{K}_t\right]$$

Since $\check{\beta}_t$ does not change the slope of the $\Delta\lambda_t$ locus, the phase diagram for this economy is similar to the one we drew in class.

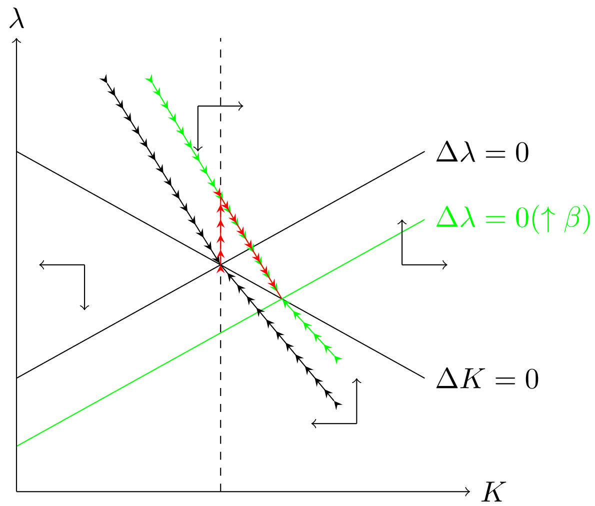

FIGURE 3. Change in β

Table 1. Response to permanent β shock

	Impact	Transition	Steady State
C	$\downarrow$	$\uparrow$	$\uparrow$
Y	$\uparrow$	?	$\uparrow$
I	$\uparrow$	$\downarrow$	$\uparrow$
L	$\uparrow$	?	?
W	$\downarrow$	$\uparrow$	$\uparrow$
R	$\uparrow$	$\downarrow$	$\downarrow$

Table 2. Response to temporary β shock

	Impact	Transition	Inflection	Transition	Steady State
C	$\downarrow$	$\uparrow$	0	$\downarrow$	0
Y	$\uparrow$	$\uparrow$	0	$\downarrow$	0
I	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	0
L	$\uparrow$	$\uparrow$	0	$\downarrow$	0
W	$\downarrow$	?	0	$\downarrow$	0
R	$\downarrow$	?	0	$\uparrow$	0

3. Labor Supply

(a). We have budget constraints

$$C + S_t = w_t N_t + S_{t-1}$$

$$\sum_{t=0}^{T} w_t N_t = C \times T$$

allowing us to get out first order conditions from

$$\mathcal{L} = \sum_{t=0}^{T} \left[\ln(1+N_t) + \lambda_t \left(C + S_t - w_t N_t - S_{t-1}\right)\right] + \psi \sum_{t=0}^{T} \left(w_t N_t - C \cdot T\right)$$

First order conditions for S_t and N_t are:

$$\lambda_t = \lambda_{t+1}$$

$$\frac{1}{1+N_t} + \psi w_t = \lambda_t w_t$$

Combining the above two equations gives us the Euler equation:

$$w_t(1+N_t) = w_{t+1}(1+N_{t+1})$$

(b). Rearranging, we have

$$\frac{w_{t+1}}{w_t} = \frac{1+N_t}{1+N_{t+1}}$$

implying that the ratio of wages between period are equal to the ratio of marginal utilities for leisure.