

ECON 210C PROBLEM SET # 3

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2. VARIABLE CAPITAL UTILIZATION IN AN RBC MODEL

(a). Firms choose capital utilization U , capital K , and labor demand N .

The production function that we can use directly (since the output is the numeraire) is

$$Y_t = (U_t K_{t-1})^\alpha (Z_t N_t)^{1-\alpha}$$

and since the firms own capital, they face the constraint

$$K_t = I_t + (1 - \delta(U_t))K_{t-1}$$

but they also have to pay wages $W_t N_t$ and invest I_t so we can set up the Lagrangian

$$\mathcal{L} = E \sum_s \left(\prod_{k=1}^s (1+r_{t+k})^{-1} \right) \left((U_{t+s} K_{t+s-1})^\alpha (Z_{t+s} N_{t+s})^{1-\alpha} - W_{t+s} N_{t+s} - I_{t+s} + q_{t+s} (-K_{t+s} + I_{t+s} + (1 - \delta(U_{t+s}))K_{t+s-1}) \right)$$

so we have first order conditions:

for labor we have

$$W_t = (1 - \alpha)(U_t K_{t-1})^\alpha Z_t^{1-\alpha} N_t^{-\alpha}$$

for investment we have

$$q_t = 1$$

for capital at time t we have

$$q_t = E \left[\frac{1}{1 + r_{t+1}} \left(\alpha U_{t+1}^\alpha K_t^{\alpha-1} (Z_{t+1} N_{t+1})^{1-\alpha} + q_{t+1} (1 - \delta(U_{t+1})) \right) \right]$$

and finally we have the condition for utilization

$$\alpha U_t^{\alpha-1} K_{t-1}^\alpha (Z_{t+1} N_{t+1})^{1-\alpha} = q_t K_{t-1} \delta'(U_t)$$