Econ 210C Homework 5: Suggested Solutions

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1.1 Romer 6.7

(1) Substitute the policy rule into the output equation to get:

$$\begin{aligned} y(t) &= -[i(t) - \pi(t)]/\theta \\ &= -\left[\max\{-\pi(t), r(y(t), \pi(t))\}\right]/\theta \\ &= \theta^{-1} \min\{\pi(t), -r(y(t), \pi(t))\} \end{aligned}$$

Consider when $\pi(t) \leqslant -r(y(t), \pi(t))$. This case corresponds to when the ZLB binds, and output is determined by inflation as $y(t) = \theta^{-1}\pi(t)$. In this regime, increases in inflation reduce the real interest rate (since the nominal rate is stuck at zero), which increases demand and therefore increases output. In (y, π)-space, outcomes during this regime are represented by a line with slope $-\theta^{-1}$ which intersects the origin.

Next, consider the regime where $\pi(t) > -r(y(t), \pi(t))$, which is when the ZLB does not bind. Then the nominal interest rate will vary via the policy function, and output must satisfy $y(t) = -\theta^{-1}r(y(t), \pi(t))$. Since r is strictly increasing in both output and inflation, there is a strictly decreasing implicit relationship between output and inflation. For example, suppose $r(y(t), \pi(t)) = r_0 + r_y y(t) + r_\pi \pi(t)$ with $r_y, r_\pi > 0$. Then $y(t) = -\frac{r_0}{1+r_y} - \frac{r_\pi}{1+r_y}\pi(t)$.

Given that the functions associated with each regime are strictly increasing and decreasing respectively, and the ZLB-regime function has range and domain of all of the real line, there is a unique crossing of the two lines and the crossing point is characterized by the unique point $(\tilde{y}, \tilde{\pi})$ such that $\tilde{\pi} + r(\tilde{y}, \tilde{\pi}) = 0$.

To construct the implied aggregate demand curve, we can consider two cases. In the first case, we have $\pi(t) > \tilde{\pi}$. Since there is a unique crossing, the term $\theta^{-1}\pi(t)$ is strictly increasing, and $-\theta^{-1}r(y(t), \pi(t)$ is strictly decreasing, it must be the case that while $\pi(t) > \tilde{\pi}$ we have $\theta^{-1}\pi(t) > -\theta^{-1}r(y(t), \pi(t))$. In the opposite case where $\pi(t) < \tilde{\pi}$ we must then have $\theta^{-1}\pi(t) < -\theta^{-1}r(y(t), \pi(t))$. Therefore, the function $-\theta^{-1}r(y(t), \pi(t))$ is active whenever $\pi(t) > \tilde{\pi}$ and the function $\theta^{-1}\pi(t)$ is active whenever $\pi(t) < \tilde{\pi}$. The implied aggregate demand curve (for linear r) is depicted in figure 1.

(2) (i) When $\tilde{y} > 0$, the aggregate demand curve intersects the vertical axis at a positive level of inflation as well as at zero inflation. Starting from a position where $\pi(0) > \tilde{\pi}$ and $y(0) < 0$ implies that outcomes lie on the portion of the aggregate demand curve where the ZLB does not bind.

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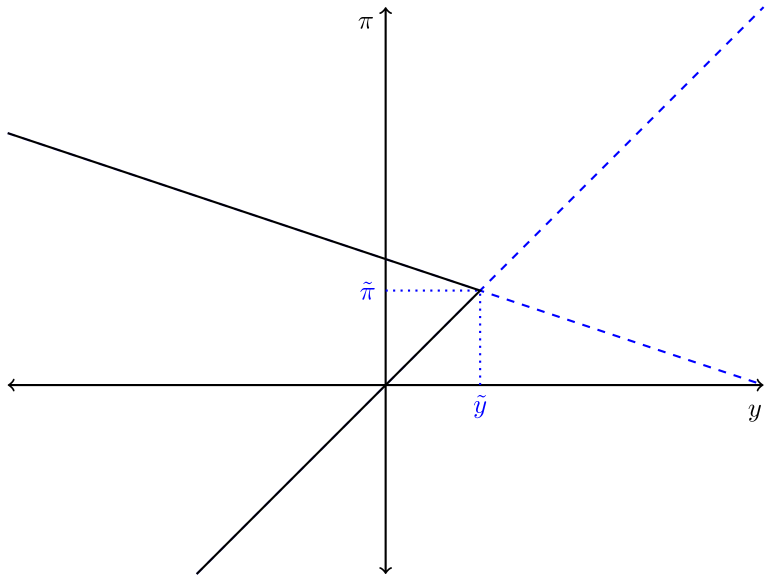
Figure 1: Aggregate Demand

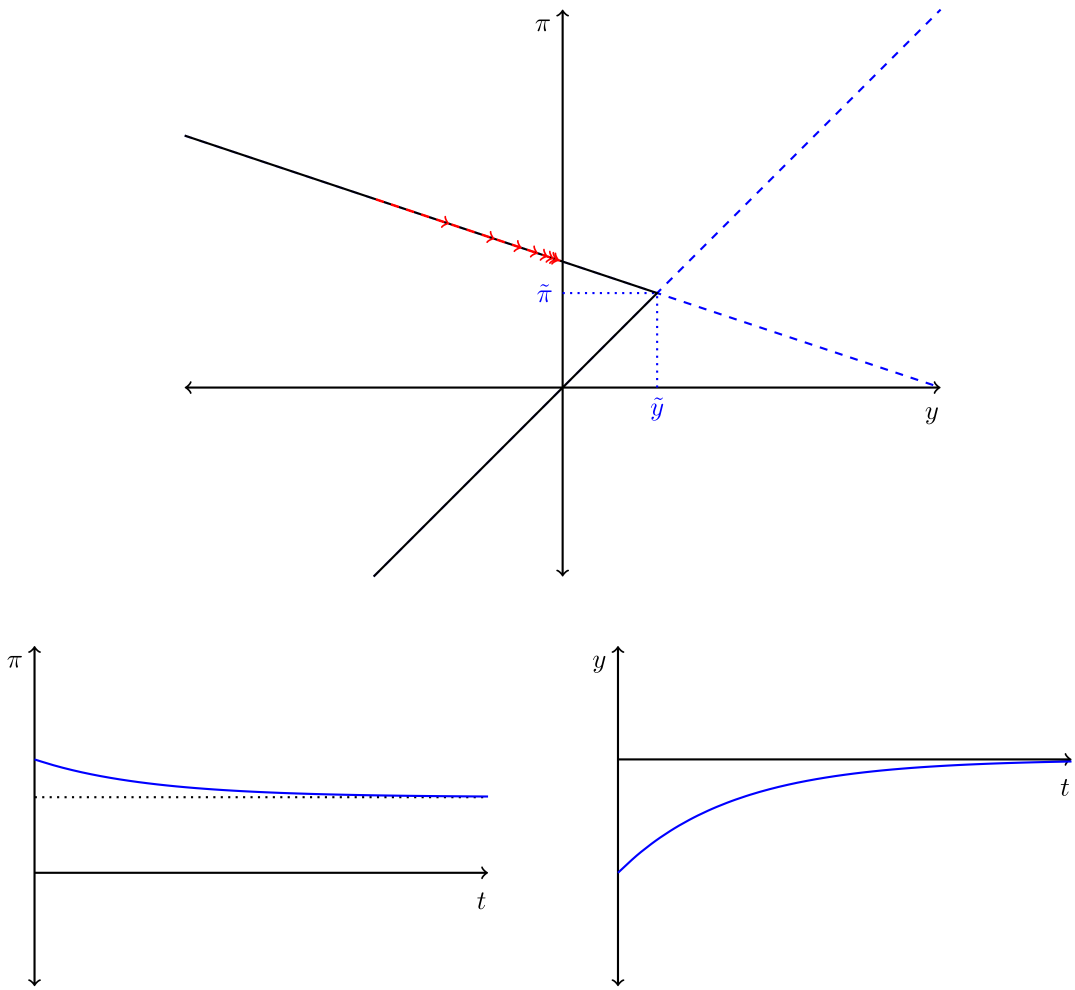


Given that output is below zero, inflation will be decreasing according to the dynamic equation: $\dot{\pi}(0) = \lambda y(0) < 0$. This decrease corresponds to a movement down along the aggregate demand curve. Inflation continues to decrease until the vertical axis is reached, at which point output is equal to zero and inflation becomes constant – the system reaches a steady state. This evolution is depicted in figure 2.

(ii) When $\tilde{y} < 0$ the intersection of the constrained and unconstrained curves is to the left of the vertical axis. This means that there is no point of the aggregate demand curve which intersects the vertical axis, and output must always be below zero. But if output is always below zero, inflation must always be decreasing. So, starting from any point along the aggregate demand curve, we will have falling inflation and as a result falling output. The dynamics undergo two phases. In the current case, inflation is initially above $\tilde{\pi}$. Initially, the ZLB does not bind and the economy moves along the segment of the aggregate demand curve corresponding to unconstrained policy. The dynamics during this period have falling inflation and rising output. However, because $\tilde{y} < 0$ the economy never quite reaches the vertical axis. Instead, it hits the point $(\tilde{y}, \tilde{\pi})$ where the nominal interest rate hits the ZLB. From here onwards, the ZLB binds and output begins to fall, driven by the continued falling inflation rate. From here, the economy is in free fall with continual disinflation and an increasingly negative output growth rate. Figure 3 depicts these dynamics.

(iii) Returning to the case where $\tilde{y} > 0$, if we assume that the initial inflation rate is below $\tilde{\pi}$ then the ZLB binds initially. If we also assume that output is negative, then we must also have falling inflation. But the falling inflation implies further movement along the ZLB-portion of the aggregate demand curve. The economy must stay the ZLB forever with disinflation and increasingly negative output growth. Figure 4 depicts the dynamics for this case.

Figure 2: Dynamics When The ZLB Does Not Bind



1.2. Romer 6.13

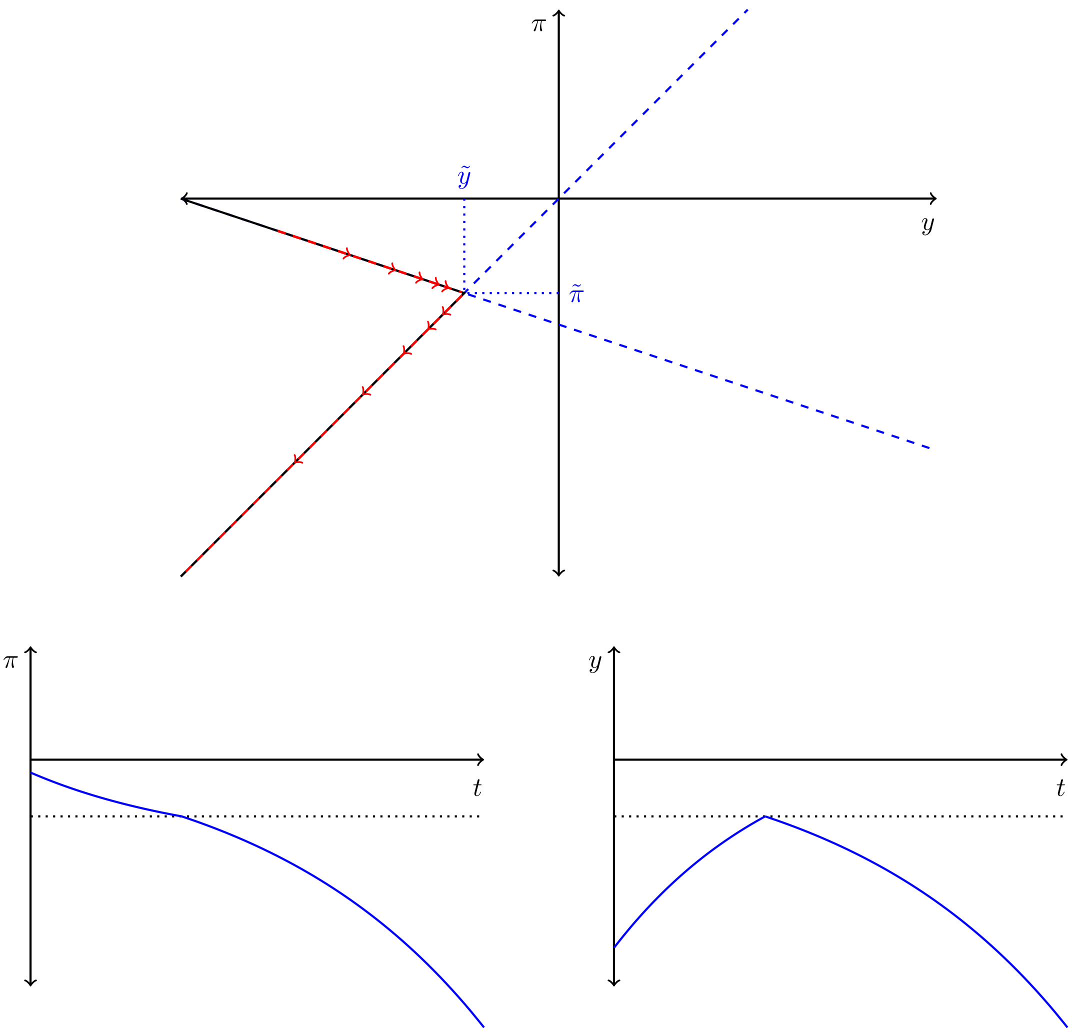
(a) The term rV_p represents the flow value of looking for palm trees, which must necessarily equal to the rate of finding palm trees times the gain in value from transitioning from state P to state C after picking a coconut, minus the cost of climbing the tree to get the coconut.

(b) The corresponding equation for the value of having a coconut is: $rV_c = aL(V_p - V_c + \tilde{u})$. This equation states that the flow value of having a coconut is the chance of finding another individual with a coconut times the value of transitioning from the coconut state to palm tree state plus the utility gain from consuming a coconut.

(c) Subtracting the two conditions gives

$$V_c - V_p = \frac{bc + aL\tilde{u}}{r + b + aL}$$

Figure 3: Unavoidable Liquidity Trap



Then, substitute this result into the original equations to get

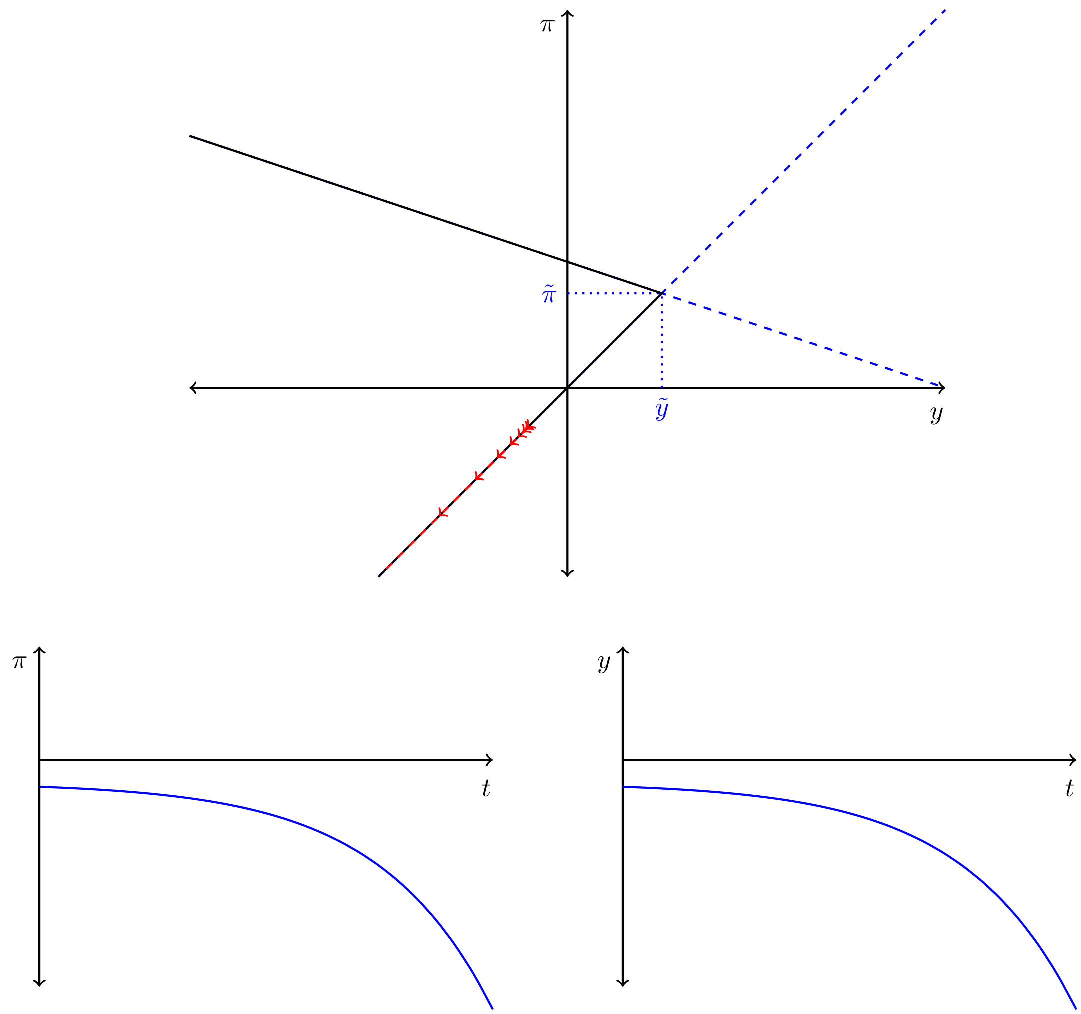
$$V_c = \frac{aL}{r}\frac{r\tilde{u} + b(\tilde{u} - c)}{r + b + aL}$$
$$V_p = \frac{b}{r}\frac{aL(\tilde{u} - c) - rc}{r + b + aL}$$

(d) If everyone climbs the palm trees that they find, then the dynamics of the number of individuals with coconuts is given by the differential equation:

$$\dot{L} = b(N - L) - aL^2$$

The first term is the chance that someone finds a palm tree times the number of individuals searching for palm trees, while the second term is the chance that two individuals with coconuts match times the number of individuals searching for coconuts.

Figure 4: Avoidable Liquidity Trap



In steady state, we must have that $0 = b(N - L) - aL^2$. This quadratic equation has the solutions:

$$L = \frac{-b \pm \sqrt{b^2 + 2abN}}{2a} = \frac{-b \pm 3b}{2a} = \frac{b}{a} \text{ and } -2\frac{b}{a}$$

under the assumption that $aN = 2b$. The only positive solution is then $L = b/a$.

(e) The value of choosing to climb a tree, pick a coconut, and switch from the P state to the C state is $V_c - V_p - c$. This is positive if and only if $bc + aL\tilde{u} > (r + b + aL)c$, so a tree-climbing steady state equilibrium with $L = b/a$ exists if and only if $c < \frac{aL\tilde{u}}{r+aL} = \frac{b\tilde{u}}{r+b}$.

(f) If no one else climbs trees, then the value of having a coconut is always zero. But if the value of having a coconut is zero, then there is no incentive to climb trees. There is only the cost of climbing followed by permanently entering the C state, which has no value. It is always an equilibrium for nobody to climb trees. This means that whenever a tree-climbing equilibrium exists, there also exists this no-climbing equilibrium, and there are multiple equilibria. However, everyone is strictly better off in the climbing equilibrium because they get to enjoy coconuts leading to positive net utility and in the no-climbing equilibrium utility is zero.

1.3. Romer 6.14

(a) The first order condition for Y_i is

$$0 = \mathbb{E}\left[\frac{P_i}{P} \mid P_i\right] - Y_i^{\gamma-1}$$

and so

$$Y_i = \mathbb{E}\left[\frac{P_i}{P} \mid P_i\right]^{\frac{1}{\gamma-1}}$$

Taking logs gives

$$y_i = \frac{1}{\gamma-1}\ln\mathbb{E}\left[\frac{P_i}{P} \mid P_i\right]$$

(b) Since ln is strictly concave, by Jensen's inequality we must have

$$y_i - y_i^{CE} = \frac{1}{\gamma-1}\left\{\ln\mathbb{E}\left[\frac{P_i}{P} \mid P_i\right] - \mathbb{E}\left[\ln\frac{P_i}{P} \mid P_i\right]\right\} > 0$$

(c) Given that $\ln(P_i/P) = \mathbb{E}[\ln(P_i/P) \mid P_i] + u_i$ with $u_i \sim \mathcal{N}(0, \sigma^2)$, we have

$$\begin{aligned}
\ln\mathbb{E}\left[\frac{P_i}{P} \mid P_i\right] &= \ln\mathbb{E}\left\{\exp\mathbb{E}\left[\ln(P_i/P) \mid P_i\right]\exp u_i \mid P_i\right\} \\
&= \mathbb{E}[\ln(P_i/P) \mid P_i] + \ln\mathbb{E}[\exp u_i \mid P_i] \\
&= \mathbb{E}[\ln(P_i/P) \mid P_i] + \frac{1}{2}\sigma^2
\end{aligned}$$

1.4. Romer 7.10

When a firm sets their price to P_t^* in period t, their future prices prior to the next re-optimization are given by $P_{t,t+\ell} = (P_{t+\ell-1}/P_{t-1})^\gamma P_t^*$. Letting $Q_{t,t+\ell}$ be the discount factor, the price setting problem of an individual firm is then

$$\max_{P_t^*}\mathbb{E}_t\sum_{\ell=0}^{\infty}Q_{t,t+\ell}\theta^\ell\left(\frac{(P_{t+\ell-1}/P_{t-1})^\gamma P_t^*}{P_{t+\ell}} - MC_{t+\ell}\right)\left(\frac{(P_{t+\ell-1}/P_{t-1})^\gamma P_t^*}{P_{t+\ell}}\right)^{-\epsilon}Y_{t+\ell}$$

The first order condition is

$$0 = \mathbb{E}_t\sum_{\ell=0}^{\infty}Q_{t,t+\ell}\theta^\ell\left((1-\epsilon)\frac{(P_{t+\ell-1}/P_{t-1})^\gamma}{P_{t+\ell}} + \epsilon\frac{1}{P_t^*}MC_{t+\ell}\right)Y_{t,t+\ell}$$

where $Y_{t,t+\ell} = \left(\frac{\Pi_{t,t+\ell}^\gamma P_t^*}{P_{t+\ell}}\right)^{-\epsilon}Y_{t+\ell}$. Re-arranging gives

$$0 = \mathbb{E}_t\sum_{\ell=0}^{\infty}Q_{t,t+\ell}\theta^\ell\left(\frac{P_t^*}{P_{t-1}^\gamma}\frac{P_{t+\ell-1}^\gamma}{P_{t+\ell}} - \frac{\epsilon}{\epsilon-1}MC_{t+\ell}\right)Y_{t,t+\ell}$$

Linearizing this condition relative to a zero inflation steady state gives

$$0 = \mathbb{E}_t \sum_{\ell=0}^{\infty} (\beta\theta)^{\ell} \left(\check{P}_t^* - \gamma \check{P}_{t-1} - (1-\gamma)\check{P}_{t+\ell} - \gamma \Pi_{t+\ell} - \check{MC}_{t+\ell}\right)$$

which implies that

$$\check{P}_t^* - \gamma \check{P}_{t-1} = (1-\beta\theta)\mathbb{E}_t \sum_{\ell=0}^{\infty} (\beta\theta)^{\ell} \left((1-\gamma)\check{P}_{t+\ell} + \gamma \Pi_{t+\ell} + \check{MC}_{t+\ell}\right)$$

Next, write this condition recursively as

$$\check{P}_t^* - \gamma \check{P}_{t-1} = (1-\beta\theta)\left((1-\gamma)\check{P}_t + \gamma \Pi_t + \check{MC}_t\right) + \beta\theta \underbrace{(1-\beta\theta)\mathbb{E}_t \sum_{\ell=1}^{\infty} (\beta\theta)^{\ell-1} \left((1-\gamma)\check{P}_{t+\ell} + \gamma \Pi_{t+\ell} + \check{MC}_{t+\ell}\right)}_{=\mathbb{E}_t[\check{P}_{t+1}^* - \gamma \check{P}_t]}$$

which can be re-written as

$$\check{P}_t^* - \check{P}_{t-1} = (1-\gamma\beta\theta)\check{\Pi}_t + (1-\beta\theta)\check{MC}_t + \beta\theta\mathbb{E}_t[\check{P}_{t+1}^* - \check{P}_t]$$

Using the definition of the price level, the inflation rate is given by

$$\begin{aligned}
\Pi_t = \frac{P_t}{P_{t-1}} &= \left[\int_0^1 \left(\frac{P_t(\nu)}{P_{t-1}}\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}\nu\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left[\theta \Pi_{t-1}^{\gamma\frac{\epsilon-1}{\epsilon}} + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}
\end{aligned}$$

Log-linearizing this expression gives an equation for inflation dynamics given the reset price relative to last period's price level:

$$\check{\Pi}_t = \theta\gamma\check{\Pi}_{t-1} + (1-\theta)(\check{P}_t^* - \check{P}_{t-1})$$

Combining to eliminate the reset price relative to last period's price level gives

$$\frac{\check{\Pi}_t - \theta\gamma\check{\Pi}_{t-1}}{1-\theta} = (1-\gamma\beta\theta)\check{\Pi}_t + (1-\beta\theta)\check{MC}_t + \beta\theta\mathbb{E}_t\frac{\check{\Pi}_{t+1} - \theta\gamma\check{\Pi}_t}{1-\theta}$$

which simplifies to

$$\check{\Pi}_t = \frac{\beta}{1+\beta\gamma}\mathbb{E}_t\check{\Pi}_{t+1} + \frac{\gamma}{1+\beta\gamma}\check{\Pi}_{t-1} + \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\beta\gamma)}\check{MC}_t$$

This equation is the New Keynesian Phillips Curve with partial indexation.

2. Rotemberg (1982)

(a) Quadratic costs (generally convex costs) can proxy for many possible smoothly varying costs to price adjustment. For instance, they might capture the idea that large price increases may lead to a loss in a firm's customer base and large price cuts could be viewed as signaling a change in product quality. The number of customers that find it worthwhile to pay some search cost and find a new supplier might be very small when prices adjust slightly, but then might increase rapidly as the firm's price change increases. This convexity could reflect heterogeneity in the intrinsic costs that customers face to search for a new supplier. Alternatively, one could imagine that customers might prefer smoother price paths due to convenience and cognitive costs of keeping tack of prices. In this case, the firm might provide a smoother price path (behaving as if they faced a convex adjustment cost) in exchange for higher average profits.

(b) The first order condition is

$$p_{i,t} - p^*_{i,t} + c(p_{i,t} - p_{i,t-1}) = \beta c(\mathbb{E}_t p_{i,t+1} - p_{i,t})$$

Under the assumption that there are some coefficients $\{v_\ell\}_{\ell=-1}^{\infty}$ such that

$$p_{i,t} = v_{-1} p_{i,t-1} + \sum_{\ell=0}^{\infty} v_\ell \mathbb{E}_t p^*_{i,t+\ell}$$

we must have

$$(1 + c + \beta c)p_{i,t} - p^*_{i,t} - cp_{i,t-1} = \beta c \mathbb{E}_t \left(v_{-1} p_{i,t} + \sum_{\ell=0}^{\infty} v_\ell \mathbb{E}_{t+1} p^*_{i,t+1+\ell} \right)$$

and so

$$[1 + c + (1 - v_{-1})\beta c] \left(v_{-1} p_{i,t-1} + \sum_{\ell=0}^{\infty} v_\ell \mathbb{E}_t p^*_{i,t+\ell} \right) - cp_{i,t-1} = p^*_{i,t} + c\beta \sum_{\ell=0}^{\infty} v_\ell \mathbb{E}_t p^*_{i,t+1+\ell}$$

Matching coefficients gives

$$\begin{aligned}
c &= [1 + c + (1 - v_{-1})\beta c] v_{-1} \\
1 &= [1 + c + (1 - v_{-1})\beta c] v_0 \\
\forall \ell \geqslant 0, \quad c\beta v_\ell &= [1 + c + (1 - v_{-1})\beta c] v_{\ell+1}
\end{aligned}$$

The roots of the first equation are

$$\begin{aligned}
v_{-1} &= \frac{(1 + c + \beta c) \pm \sqrt{(1 + c + \beta c)^2 - 4\beta c^2}}{2\beta c} \\
&= 1 + \frac{1 + (1 - \beta)c}{2\beta c} \pm \frac{\sqrt{(1 + c)^2 + 2\beta c(1 - c) + (\beta c)^2}}{2\beta c}
\end{aligned}$$

which implies that one root is above one and the other root is below one. Therefore the stable solution must be

$$\begin{aligned}
v_{-1} &= \frac{(1 + c + \beta c) - \sqrt{(1 + c + \beta c)^2 - 4\beta c^2}}{2\beta c} \\
\forall \ell \geqslant 0, \quad v_\ell &= \left(\frac{c\beta}{1 + c + (1 - v_{-1})\beta c} \right)^\ell \frac{1}{c} v_{-1}
\end{aligned}$$

(c) If $p^*_{i,t} = (1-\phi)p_t + \phi m_t$ then, given symmetry, the first order condition implies that

$$\begin{aligned} p_t + c(p_t - p_{t-1}) &= (1-\phi)p_t + \phi m_t + \beta c(\mathbb{E}_t p_{t+1} - p_t) \\ \implies \left(1 + \frac{\phi}{c} + \beta\right) p_t &= p_{t-1} + \frac{\phi}{c} m_t + \beta \mathbb{E}_t p_{t+1} \end{aligned}$$

Re-write as

$$p_t = \underbrace{\frac{c}{\phi + c + \beta c}}_{\equiv \rho} p_{t-1} + \underbrace{\frac{\phi}{\phi + c + \beta c}}_{= 1-\rho-\alpha} m_t + \underbrace{\frac{\beta c}{\phi + c + \beta c}}_{\equiv \alpha} \mathbb{E}_t p_{t+1}$$

and guess that $p_t = \gamma_{-1} p_{t-1} + \sum_{\ell=0}^{\infty} \gamma_\ell \mathbb{E}_t m_{t+\ell}$ to get

$$\begin{aligned} p_t &= (1-\rho-\alpha)m_t + \rho p_{t-1} + \alpha \mathbb{E}_t \left(\gamma_{-1} p_t + \sum_{\ell=0}^{\infty} \gamma_\ell \mathbb{E}_{t+1} m_{t+1+\ell}\right) \\ (1-\alpha\gamma_{-1})\left(\gamma_{-1} p_{t-1} + \sum_{\ell=0}^{\infty} \gamma_\ell \mathbb{E}_t m_{t+\ell}\right) &= (1-\rho-\alpha)m_t + \rho p_{t-1} + \alpha \mathbb{E}_t \sum_{\ell=0}^{\infty} \gamma_\ell m_{t+1+\ell} \end{aligned}$$

then match coefficients:

$$\begin{aligned} \rho &= (1-\alpha\gamma_{-1})\gamma_{-1} \\ (1-\rho-\alpha) &= (1-\alpha\gamma_{-1})\gamma_0 \\ \forall \ell \geqslant 0, \quad \alpha\gamma_\ell &= (1-\alpha\gamma_{-1})\gamma_{\ell+1} \end{aligned}$$

The stable solution is then

$$\begin{aligned} \gamma_{-1} &= \frac{1 - \sqrt{1 - 4\alpha\rho}}{2\alpha} \\ \forall \ell \geqslant 0, \quad \gamma_\ell &= \left(\frac{\alpha\gamma_{-1}}{\rho}\right)^\ell \frac{1-\rho-\alpha}{\rho} \gamma_{-1} \end{aligned}$$

(d) Given that money follows a random walk, we have

$$
\begin{aligned}
p_t &= \gamma_{-1} p_{t-1} + \sum_{\ell=0}^{\infty} \left(\frac{\alpha\gamma_{-1}}{\rho}\right)^{\ell} \frac{1-\rho-\alpha}{\rho} \gamma_{-1} \mathbb{E}_t m_{t+1} \\
&= \gamma_{-1} p_{t-1} + \frac{1-\rho-\alpha}{\rho} \gamma_{-1} m_t \left(\sum_{\ell=0}^{\infty} \left(\frac{\alpha\gamma_{-1}}{\rho}\right)^{\ell} \right) \\
&= \gamma_{-1} p_{t-1} + \frac{\frac{1-\rho-\alpha}{\rho}}{1 - \frac{\alpha\gamma_{-1}}{\rho}} \gamma_{-1} m_t \\
&= \gamma_{-1} p_{t-1} + \frac{1-\rho-\alpha}{\rho - \alpha\gamma_{-1}} \gamma_{-1} m_t \\
y_t &= m_t - p_t = \left[1 - \frac{1-\rho-\alpha}{\rho-\alpha\gamma_{-1}}\gamma_{-1}\right] m_t - \gamma_{-1} p_{t-1} = \left[\frac{\rho - (1-\rho)\gamma_{-1}}{\rho - \alpha\gamma_{-1}}\right] m_t - \gamma_{-1} p_{t-1} \\
y_t &= \left[\frac{\rho - (1-\rho)\gamma_{-1}}{\rho - \alpha\gamma_{-1}}\right] m_t + \gamma_{-1}(y_{t-1} - m_{t-1}) \\
y_t &= \left[\frac{\rho - (1-\rho)\gamma_{-1}}{\rho - \alpha\gamma_{-1}} - \gamma_{-1}\right] m_{t-1} + \gamma_{-1} y_{t-1} + \frac{\rho - (1-\rho)\gamma_{-1}}{\rho - \alpha\gamma_{-1}} \varepsilon_t \\
&= \frac{\rho - \gamma_{-1} + \alpha\gamma_{-1}^2}{\rho - \alpha\gamma_{-1}} m_{t-1} + \gamma_{-1} y_{t-1} + \frac{\rho - (1-\rho)\gamma_{-1}}{\rho - \alpha\gamma_{-1}} \varepsilon_t \\
&= \gamma_{-1} y_{t-1} + \frac{\rho - (1-\rho)\gamma_{-1}}{\rho - \alpha\gamma_{-1}} \varepsilon_t
\end{aligned}
$$

where the last line follows since $\rho = (1-\alpha\gamma_{-1})\gamma_{-1}$. We can see that a permanent money shock will create a temporary increase in output, which follows an AR(1) process.

(e) Up to log-linearization the two models make identical predictions.