# ECON 210C PROBLEM SET # 4

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## 1. Labor supply problem

## 2. Demand shock

**(a).** The consumption-leisure condition at time $t$ is just equating marginal benefits of labor and leisure $\frac{W_t}{C_t} = v_t L_t^{\chi}$.

**(b).** The consumer consuming one less unit today means they are losing out on $\frac{1}{C_t}$ today. With that, they can buy $\frac{1}{P_t}$ units of capital today, and tomorrow they will get $P_{t+1} + d_{t+1}$ from that unit of capital. So their payoff tomorrow is

$$\frac{1}{P_t} \times (P_{t+1} + d_{t+1}) \times \frac{\beta}{C_{t+1}}$$

and optimality therefore implies we have

$$\frac{1}{C_t} = \frac{1}{P_t} \times (P_{t+1} + d_{t+1}) \times \frac{\beta}{C_{t+1}}$$

as the inter-temporal optimality condition.

## 3. Business cycle and external returns to scale

**(a).** Each firm sets wage equal to marginal product of labor, so we have

$$W_t = Y_t^{1-1/\gamma} \left(\frac{K_{it}}{L_{it}}\right)^{\alpha} Z_t^{1-\alpha}$$

and so we can find labor demand as a function of wages

$$L_{it} = (W_t Z_t^{\alpha-1} Y_t^{1/\gamma-1} K_{it}^{-\alpha})^{-\frac{1}{\alpha}}$$

which simplifies to

$$L_{it} = W_t^{-\frac{1}{\alpha}} Z_t^{\frac{1-\alpha}{\alpha}} Y_t^{\frac{1-1/\gamma}{\alpha}} K_{it}$$

**(b).** Integrating both sides over all firms, we have

$$L_t = W_t^{-\frac{1}{\alpha}} Z_t^{\frac{1-\alpha}{\alpha}} Y_t^{\frac{1-1/\gamma}{\alpha}} K_t$$

so we can start to solve for aggregate production, so we get

$$Y_t^{\frac{1-1/\gamma}{\alpha}} = \frac{L_t}{K_t} \times W_t^{\frac{1}{\alpha}} Z_t^{\frac{\alpha-1}{\alpha}}$$

and solving for $Y$ gives

$$Y_t = \left(\frac{L_t}{K_t}\right)^{\frac{\alpha}{1-1/\gamma}} W_t^{\frac{1}{1-1/\gamma}} Z_t^{\frac{\alpha-1}{1-1/\gamma}}$$

## 4. Problems from Romer

### 4.1. **Problem 6.10.**

### 4.2. **Problem 6.11.**

### 4.3. **Problem 6.12.**