7: Indices and logarithms

EN1106 - Introductory Mathematics

Level I - Semester 1

7.3 Multiplication and division by powers of 10

Definitions

- To multiply and divide decimal fractions by powers of 10 is particularly simple.
- In general, to multiply a number by 10^n, the decimal point is moved n places to the right if n is a positive integer, and n places to the left if n is a negative integer.
- For Example:
  - To multiply 256.875 by 10 the decimal point is moved one place to the right, that is 256.875 x 10 (or 10^1)= 2568.75
  - To multiply by 100 the decimal point is moved two places to the right. So 256.875 x 100 (or 10^2) = 25687.5

Definitions

- To divide a number by 10, the decimal point is moved one place to the left. This is equivalent to multiplying by 10^{-1}.
- To divide by 100, the decimal point is moved two places to the left. This is equivalent to multiplying by 10^{-2}.
- Example:
  - $256.875 \times 10^{-1} = 25.6875$
  - $256.875 \times 10^{-2} = 2.56875$
  - $256.875 \times 10^{-3} = 0.256875$

7.4 Scientific Notation

Scientific Notation

- It is often necessary to use very large numbers such as 65000000000 or very small numbers such as 0.000000001. Scientific notation can be used to express such numbers in a more concise form, which avoids writing very lengthy strings of numbers.
- Each number is written in the form **a** x 10^n where **a** is usually a number between 1 and 10.
- We also make use of the fact that
  - $10 = 10^1$, $100 = 10^2$, $1000 = 10^3$ and so on
  - $10^{-1} = 1/10 = 0.1$, $10^{-2} = 1/100 = 0.01$ and so on
  - 68000 can be written $6.8 \times 10000 = 6.8 \times 10^4$
  - 0.09 can be written $9 \times 0.01 = 9 \times 10^{-2}$

7.7 Solving Equations with Logarithms

- This section illustrates the use of logarithms in solving certain types of equations.
- Earlier we have learned that if a and y be positive real numbers and let $a^x = y$: Then x is called the logarithm of y to the base a.
- We write this as $\log_a y = x$
- Also, from the laws of Algorithms,
  - $\log A + \log B = \log AB$
  - $\log A - \log B = \log(A/B)$
  - $\mathrm{n} \log \mathrm{A} = \log \mathrm{A}^{\mathrm{n}}$

- Logarithms to base 10 are often denoted by 'log' or '$\log_{10}$'; logarithms to base **e** are denoted by 'ln' or '$\log_e$'

- Examples

1. $10^x = 59$

Taking logs to base 10 gives

$\log_{10}10^x = \log_{10}59$

$x\log_{10}10 = \log_{10}59$

$x = \log 59 = 1.7709$

2. $e^x = 0.5$

Taking logs to base e gives

$\log_e e^x = \log_e 0.5$

$\ln e^x = \ln 0.5$

$x \ln e = \ln 0.5$

$x = \ln 0.5 = 0.6931$

- log(3x) = 0.76

$$\log 3x = 0.76$$

$$3x = 10^{0.76}$$

$$x = \frac{10^{0.76}}{3} = 1.9181$$

- ln (x/2) = 2.6

$$\ln\left(\frac{x}{2}\right) = 2.6$$

$$\frac{x}{2} = e^{2.6}$$

$$x = 2e^{2.6} = 26.9275$$