



UNIVERSITY OF COLOMBO, SRI LANKA



UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING



DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2019 – 2nd Year Examination – Semester 3

IT3305: Mathematics for Computing-II

PART 2 - Structured Question Paper

12th May 2019

(ONE HOUR)

To be completed by the candidate

BIT Examination Index No:

Important Instructions:

- The duration of the paper is **1 (One) hour**.
- The medium of instruction and questions is English.
- This paper has **3 questions** and **14 pages**.
- **Answer all questions.**
- **Question 1 carries 40% marks and the other questions carry 30% marks each.**
- **Write your answers** in English using the space provided **in this question paper**.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
If a page is not printed, please inform the supervisor immediately.
- Calculators are **NOT ALLOWED**

Questions Answered

Indicate by a cross (x), (e.g. ☐) the numbers of the questions answered.

To be completed by the candidate by marking a cross (x).	1	2	3	
To be completed by the examiners:				

1)

(a) Consider the following three matrices:

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 3 & -1 \end{bmatrix}$$

Find D such that $CD - AB = 0$.

(10 marks)

(b) Let $L = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & -1 & -1 \end{bmatrix}$. Find $|L|$ and L^{-1}

Hence solve the following system of three linear equations as a function of t.

$$\begin{aligned} x + 2y + 3z &= 0 \\ 3x + 2y + z &= t \\ 2x - y - z &= 3 \end{aligned}$$

(30 marks)

ANSWER IN THIS BOX

(1) (a)

$$\text{Let } D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

$$\begin{bmatrix} 0 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} c & d \\ 3a - c & 3b - d \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} c - 2 & d + 2 \\ 3a - c - 1 & 3b - d - 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$c=2, d=-2, a=1, b=1$$

(b)

$$|L| = -12$$

$$L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = L^{-1} \begin{bmatrix} 0 \\ t \\ 3 \end{bmatrix}$$

$$L^{-1} = \frac{1}{|L|} \text{Adj } L = -\frac{1}{12} \begin{bmatrix} -1 & -1 & -4 \\ 5 & -7 & 8 \\ -7 & 5 & -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{12} \begin{bmatrix} -1 & -1 & -4 \\ 5 & -7 & 8 \\ -7 & 5 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ t \\ 3 \end{bmatrix}$$

$$x = \frac{t}{12} + 1 \quad y = \frac{7t}{12} - 2 \quad z = \frac{-5t}{12} - 1$$

- 2) (a) Find the derivative of $f(x) = x^n \ln x$, $x > 0$ and n is a non-zero integer.

(15 marks)

- (b) Hence or otherwise find $\int x^{n-1} \ln x^n dx$ and $\int_1^e x^5 \ln x dx$.

(15 marks)

ANSWER IN THIS BOX

(a)

$$f'(x) = n x^{n-1} \ln x + x^{n-1}$$

$$\text{Therefore } x^n \ln x = f(x) = n \int x^{n-1} \ln x dx + \frac{x^n}{n} + C$$

(b)

$$\text{Hence } \int x^{n-1} \ln x dx = \frac{1}{n} \left[x^n \ln x - \frac{x^n}{n} - C \right] \text{ and}$$

$$\begin{aligned} \int_1^e x^5 \ln x dx &= \frac{1}{6} \int_1^e x^{6-1} \ln x^6 dx = \frac{1}{6} \times \frac{1}{5} \left[x^6 \ln x - \frac{x^6}{6} \right]_1^e \\ &= \frac{1}{30} \left[\left(e^6 - \frac{e^6}{6} \right) - \left(0 - \frac{1}{6} \right) \right] \\ &= \frac{5e^6 + 1}{180}. \end{aligned}$$

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- 3) The probability density function of a continuous random variable X is given below. Here “a” is a constant.

$$f(x) = \begin{cases} ax(x+3) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Calculate the value of constant a .
- Calculate the expected value of X
- Calculate the variance of X.
- Calculate the probability that X is greater than 0.20.
- Calculate the variance of Z, where $Z = 2X + 1$.

(30 marks)

ANSWER IN THIS BOX

- (a) Calculate the value of constant a .

It is known that $\int_{-\infty}^{+\infty} f(x)dx = 1$

So,

$$\int_0^1 ax(x+3)dx = \int_0^1 (ax^2 + 3ax)dx = \int_0^1 ax^2dx + \int_0^1 3axdx$$

$$= \left(\frac{ax^3}{3} + \frac{3ax^2}{2} \right)_0^1$$

$$= \left(\frac{a}{3} + \frac{3a}{2} \right)$$

$$= \frac{2a+9a}{6}$$

$$= \frac{11a}{6}$$

$$\text{Then, } \frac{11a}{6} = 1$$

$$\text{Therefore, } a = \frac{6}{11}$$

(b) Calculate the expected value of X

$$E(X) = \int_0^1 xf(x)dx$$

$$= \int_0^1 x \frac{6}{11} x(x+3)dx = \frac{6}{11} \int_0^1 (x^3 + 3x^2) dx$$

$$= \frac{6}{11} \left(\frac{x^4}{4} + \frac{3x^3}{3} \right)_0^1$$

$$= \frac{6}{11} \left(\frac{1}{4} + \frac{3}{3} \right)$$

$$= \frac{6}{11} \left(\frac{1}{4} + 1 \right)$$

$$= \frac{6}{11} * \frac{5}{4}$$

$$= \frac{30}{44} = \frac{15}{22} = 0.6818$$

(c) Calculate the variance of X.

$$V(X) = E(X - E(X))^2 = E(X^2) - (E(X))^2$$

$$E(X^2) = \int_0^1 x^2 f(x) dx$$

$$= \int_0^1 x^2 \frac{6}{11} x(x+3) dx$$

$$= \frac{6}{11} \int_0^1 (x^4 + 3x^3) dx$$

$$= \frac{6}{11} \left(\frac{x^5}{5} + \frac{3x^4}{4} \right)_0^1$$

$$= \frac{6}{11} \left(\frac{1}{5} + \frac{3}{4} \right)$$

$$= \frac{6}{11} * \frac{19}{20}$$

$$= \frac{114}{220} = \frac{57}{110} = 0.5182$$

Therefore

$$V(X) = E(X^2) - (E(X))^2$$

$$= 0.5182 - 0.6818^2$$

$$= 0.5182 - 0.4649$$

$$= 0.0533$$

(d) Calculate the probability that X is greater than 0.20.

$$\begin{aligned}
 P(X > 0.20) &= \int_{0.20}^1 \frac{6}{11} x(x+3) dx = \int_{0.20}^1 \frac{6}{11} (x^2 + 3x) dx = \frac{6}{11} \left(\frac{x^3}{3} + \frac{3x^2}{2} \right)_{0.2}^1 \\
 &= \frac{6}{11} \left[\left(\frac{1}{3} + \frac{3}{2} \right) - \left(\frac{0.2^3}{3} + \frac{3(0.2)^2}{2} \right) \right] \\
 &= \frac{6}{11} [(0.33 + 1.5) - (0.0027 + 0.06)] \\
 &= \frac{6}{11} [(1.83) - (0.0627)] \\
 &= \frac{6}{11} * 1.7673 \\
 &= 0.9639
 \end{aligned}$$

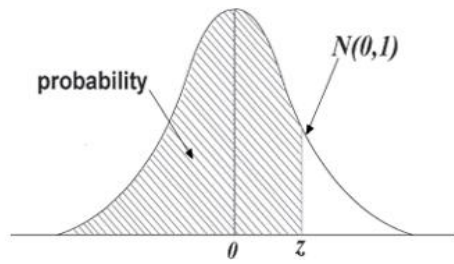
(e) Calculate the variance of Z, where $Z = 2X + 1$

$$V(Z) = V(2X + 1) = 4V(X) = 4 * 0.0533 = 0.2132$$

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The Standard Normal Distribution Table



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of Z , the standardized normal deviate, (the proportion P , of the distribution less than Z) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value X is obtained by calculating $Z = (X - \mu) / \sigma$ and reading the proportion corresponding to this value of Z .

Z	P	Z	P	Z	P
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997