

5: Techniques of Counting

IT2106 - Mathematics for Computing I

Level I - Semester 2





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Learning Outcomes

After completing this module students will be able to

- •Calculate the number of elements in certain mathematically defined sets where ordinary methods of counting are tedious
- •Calculate number of possible outcomes of elementary combinatorial processes such as permutations and combinations

A permutation is an arrangement of objects in different orders.

The order of the arrangement is important!! For example, the number of different ways 3 students can enter school can be shown as 3!, or $3 \cdot 2 \cdot 1$, or 6. There are six different arrangements, or permutations, of the three students in which all three of them enter school.

The notation for a permutation:
n is the **total** number of objects
r is the number of objects chosen (want)

(Note if
$$n = r$$
 then $_{n}P_{r} = n!$)
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- To find the permutations of {A, B, C, D, E}, there are:
 - Five possible choices for the first item
 - Four possible choices for the second item
 - Three remaining possible choices for the third item
 - Two remaining possible choices for the fourth item, and
 - Only one possible "choice" for the final item
- For any positive integer N, we define N! ("N factorial") as the product of all the positive integers up to and including N
 - Example: 5! = 1 * 2 * 3 * 4 * 5 = 120
- Given any N distinct items, there are N! possible permutations of those items

By the rule of product,

The number of permutations of n things taken r at a Time

$$P(n,r) = n(n-1)(n-2)...(n-r+1)$$

Note:

$$P(n,r) = \frac{n!}{(n-r)!}$$

with Specific Arrangements

EXAMPLE:

Use the letters in the word "square" and tell how many 6-letter arrangements, with no repetitions, are possible if the :

- a) first letter is a vowel.
- b) vowels and consonants alternate, beginning with a consonant.

Solution:

Part a:

When working with arrangements, I put lines down to represent chairs. Before starting a problem I decide how many chairs I need to fill and then work from there

I need six "chairs" (6-letter arrangements)

The first of the six chairs must be a vowel (u,a,e). There are three ways to fill the first chair.

After the vowel has been placed in the first chair, there are 5 letters left to be arranged in the remaining five chairs.

 $\underline{3} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$ or

 $3 \cdot {}_{5}P_{5} = 3 \cdot 120 = 360$

Solution:

Part b:

I need six "chairs" (6-letter arrangements)

Beginning with a consonant, every other chair must be filled with a consonant. (s,q,r)

<u>3</u> · <u>2</u> · <u>1</u> · ____

The remaining chairs have the three vowels to be arranged in them:

<u>3</u> · <u>3</u> · <u>2</u> · <u>1</u> · <u>1</u>

= 36

with Repetition

In general, repetitions are taken care of by dividing the permutation by the number of objects that are identical!

Example1:

1. How many different 5-letter words can be formed from the word APPLE?

$$\frac{{}_{5}P_{5}}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{120}{2} = 60 words$$

You divide by 2! Because the letter P repeats twice

with Repetition

Example:

2. How many different six-digit numerals can be written using all of the following six digits:

$$\frac{{}_{6}P_{6}}{2!3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{720}{12} = 60$$

Two 4s repeat and three 5s repeat

with Repetition

The number of different permutations of n objects, where there are n1 indistinguishable objects of type 1, n2 indistinguishable objects of type 2, ..., and nk indistinguishable objects of type k, is

$$\frac{n!}{n_1!n_2!..n_k!}$$

Combinations

A **combination** is a set of objects in which order is *not* important.

The number of combinations of n things taken r at a time

$$C(n,r) = {n \choose r} = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!r!}$$

or

$$_{n}C_{r}=\frac{_{n}P_{r}}{r!}$$

Combinations

Example1: Evaluate
$$7C2 = \frac{7 \cdot 6}{2 \cdot 1} = \frac{42}{2} = 21$$

Example2: There are 12 boys and 14 girls in Mrs.Schultzkie's math class. Find the number of ways Mrs. Schultzkie can select a team of 3 students from the class to work on a group project. The team consists of 1 girl and 2 boys.

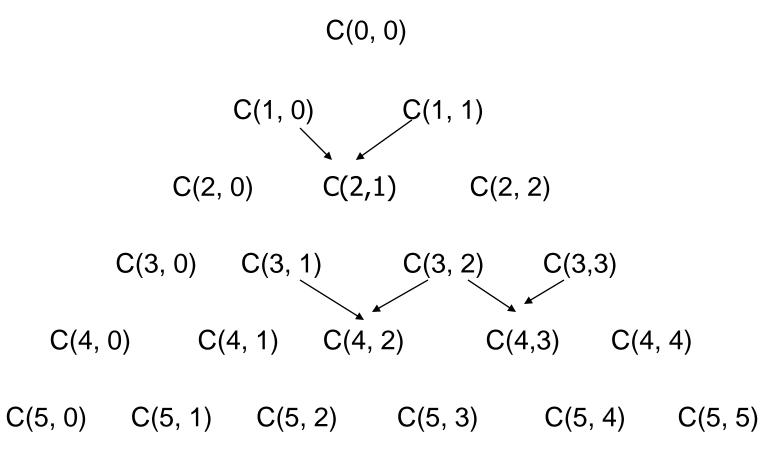
Binomial Theorem

Let x and y be variables, and let n be a positive integer. Then

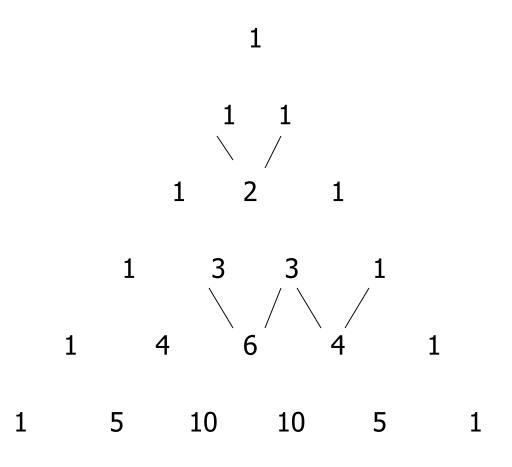
$$(x+y)^n = \sum_{j=0}^n C(n,j)x^{n-j}y^j$$

$$= \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \dots + \binom{n}{n-1}x^1y^{n-1} + \binom{n}{n}y^n$$

Pascal's Triangle



Pascal's Triangle



The Pigeonhole Principle

Pigeonhole principle

If n pigeon holes are occupied by n+1 or more pigeons, then at least one pigeon hole is occupied more than one pigeon.

Example:

Suppose a department contains 13 professors. Then two of the professors (pigeons) were born in the same month (pigeon holes).

The Pigeonhole Principle

Generalized pigeonhole principle

If n pigeonholes are occupied by kn+1 or more pigeons, where k is a positive integer, the at least one pigeon hole is occupied by k+1 or more pigeons.

Example:

Find the minimum number of students in a class to be sure that three of them are born in the same month.

Here the n=12 months are the pigeonholes and k+1=3, or k=2.Hence among any kn+1=25students (pigeons), three of them are born in the same month.