

UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING



DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2011/2012 – 2nd Year Examination – Semester 3

IT3304: Mathematics for Computing-II PART 2 - Structured Question Paper 24th February 2012 (ONE HOUR)

To be completed by the candidate	
BIT Examination Index No:	

Important Instructions:

- The duration of the paper is 1 (One) hour.
- The medium of instruction and questions is English.
- This paper has 3 questions and 09 pages.
- Answer all questions.
- Question 1 (40% marks) and other questions (30% marks each).
- Write your answers in English using the space provided in this question paper.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
 If a page is not printed, please inform the supervisor immediately.

Questions Answered _							
Indicate by a cross (×), (e.g.	Ж) the nu	mbers o	of the	ques	tions ans	wered
be completed by the cand	dida	te by					

To be completed by the candidate by marking a cross (x).	1	2	3	
To be completed by the examiners:				

1)	Suppose A and B are two square	e matrices each of order n	. When is B said to be the inverse of A	Α?
1,	Duppose II and D are two square	induited each of order n.	. When is b said to be the inverse of a	

(5marks)

Let
$$A = \begin{pmatrix} 3 & -3 & 1 & -1 \\ 1 & 1 & 3 & 3 \\ -1 & 1 & 3 & -3 \\ 3 & 3 & -1 & -1 \end{pmatrix}$$
.

Find

(i) $A(A)^T$

(10marks)

(ii) $(A)^{T}A$

(10marks)

(iii) Does A⁻¹ exist? If your answer is yes, find it.

(15marks)

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B is said to be the inverse of A when AB = BA = I where I is the identity matrix of order n

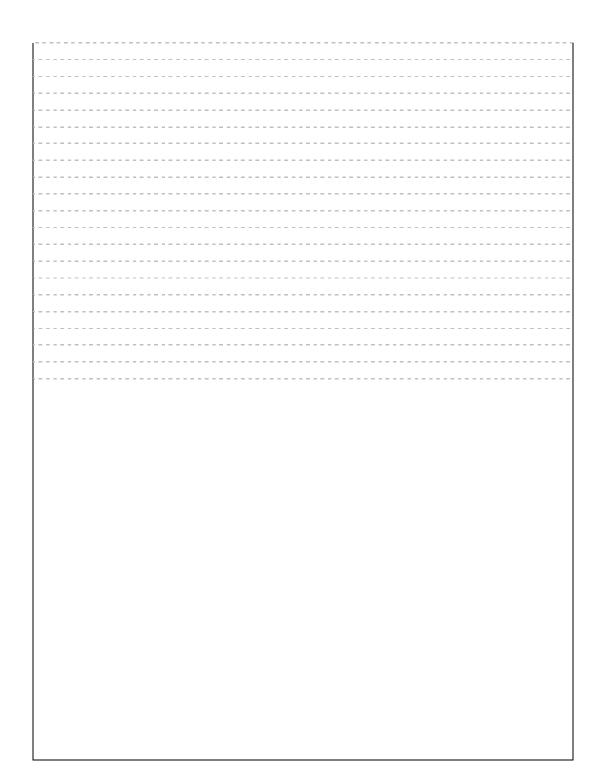
_ (i)

$$\mathbf{A}\mathbf{A}^{\mathrm{T}} = \begin{pmatrix} 3 & -3 & 1 & -1 \\ 1 & 1 & 3 & 3 \\ -1 & 1 & -3 & -3 \\ 3 & 3 & -1 & -1 \end{pmatrix} \begin{pmatrix} 3 & 1 & -1 & 3 \\ -3 & 1 & 1 & 3 \\ -1 & -3 & -3 & -1 \\ -1 & 3 & -3 & -1 \end{pmatrix} = \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 20 \end{pmatrix}$$

(ii)

$$\mathbf{A}^{\mathsf{T}}\mathbf{A} = \begin{pmatrix} 3 & 1 & -1 & 3 \\ -3 & 1 & 1 & 3 \\ 1 & 3 & 3 & -1 \\ -1 & 3 & -3 & -1 \end{pmatrix} \begin{pmatrix} 3 & -3 & 1 & -1 \\ 1 & 1 & 3 & 3 \\ -1 & 1 & 3 & -3 \\ -3 & -3 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & -0 & -0 & -20 \end{pmatrix}$$

(iii)
$$A^{-1} = \frac{1}{20} \begin{pmatrix} 3 & 1 & -1 & 3 \\ -3 & 1 & 1 & 3 \\ 1 & 3 & 3 & -1 \\ -1 & 3 & -3 & -1 \end{pmatrix}$$



(a) Let S_n denote the n^{th} partial sum of the series $\sum_{n=1}^{\infty} 3n \left(\frac{1}{2}\right)^{n-1}$.

By considering $S_n - \frac{1}{2} S_n$ prove that $S_n = 12 - (12 + 6n) \left(\frac{1}{2}\right)^n$.

Hence prove that $\sum_{n=1}^{\infty} 3n \left(\frac{1}{2}\right)^{n-1} = 12.$

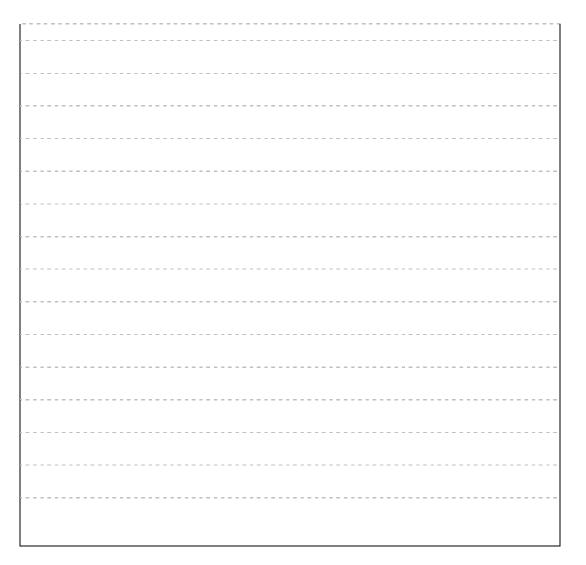
(20 marks)

(b) Evaluate
$$\int_{1}^{\sqrt{3}} \frac{1}{x^2(x^2+1)} dx$$
.

(10 marks)

ANSWER IN THIS BOX
(a) $S_n = 3 + 3(2)\left(\frac{1}{2}\right) + 3(3)\left(\frac{1}{2}\right)^2 + 3(4)\left(\frac{1}{2}\right)^3 + \dots + 3(n)\left(\frac{1}{2}\right)^{n-1}$
$\therefore \frac{1}{2}S_n = 3\left(\frac{1}{2}\right) + 3(2)\left(\frac{1}{2}\right)^2 + 3(3)\left(\frac{1}{2}\right)^3 + 3(4)\left(\frac{1}{2}\right)^4 + \dots + 3(n)\left(\frac{1}{2}\right)^n$
Therefore
$S_n - \frac{1}{2} S_n = 3 + 3\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + \dots + 3\left(\frac{1}{2}\right)^{n-1} - 3n\left(\frac{1}{2}\right)^n$
$\therefore \frac{1}{2}S_n = 3\left[\frac{1-(1/2)^n}{1-1/2}\right] - 3n\left(\frac{1}{2}\right)^n$
$\therefore S_n = -12 - (12 + 6n) \left(\frac{1}{2}\right)^n.$
$\therefore \sum_{n=1}^{\infty} 3n \left(\frac{1}{2}\right)^{n-1} = \lim_{n \to \infty} S_n = \lim_{n \to \infty} \left[12 - (12 + 6n) \left(\frac{1}{2}\right)^n\right] = 12.$
$n=1-\binom{2}{j}$
·

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3) The number of claims for missing baggage in a day at a particular airport in a small city is a discrete random variable with the following probability distribution function.

X	0	1	2	3	4	5	6	7	8	9	≥10
Probability	0.05	2 <i>a</i>	0.15	0.1	A	0.15	0.05	b	0.05	0.05	0

- (a) It is given that the probability of X being greater than or equal to 5 is 0.4. Calculate the values of a and b.
- (b) Calculate the probability of getting at most one claim.
- (c) Calculate the probability of getting at least 6 claims.
- (d) Calculate the probability of getting less than 4 claims.
- (e) Calculate the probability of the number of claims being between 3 and 6 inclusive.
- (f) Calculate the expected number of claims.

(30 marks)

ANSWER IN THIS BOX (a) $P(X \ge 5) = P(5) + P(6) + P(7) + P(8) + P(9)$ = 0.15 + 0.05 + b + 0.05 + 0.05= 0.3 + b = 0.4 \tilde{O} b = 0.1 $P(X \le 4) = P(0) + P(1) + P(2) + P(3) + P(4)$ = 0.05 + 2a + 0.15 + 0.1 + a= 3a + 0.3 = 0.6ð 3a = 0.3ð a = 0.1**(b)** $P(X \le 1) = P(0) + P(1)$ =0.05+0.2=0.25(c) $P(X \ge 6) = P(6) + P(7) + P(8) + P(9)$ = 0.05 + 0.1 + 0.05 + 0.05= 0.25(d) P(X < 4) = P(0) + P(1) + P(2) + P(3)= 0.05 + 0.2 + 0.15 + 0.1(e) $P(3 \le X \le 6) = P(3) + P(4) + P(5) + P(6)$ =0.1+0.1+0.15+0.05= 0.4(f) E(X) = 0P(0) + 1P(1) + 2P(2) + 3P(3) + 4P(4) + 5P(5) + 6P(6) + 7P(7) +8P(8) + 9P(9)= (0*0.05) + (1*0.2) + (2*0.15) + (3*0.1) + (4*0.1) + (5*0.15) +(6*0.05) + (7*0.1) + (8*0.05) + (9*0.05)= 0 + 0.2 + 0.3 + 0.3 + 0.4 + 0.75 + 0.3 + 0.7 + 0.4 + 0.45= 3.8

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