UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2016 – 2nd Year Examination – Semester 3

IT3305: Mathematics for Computing-II

PART 2 - Structured Question Paper

6th May 2016

(ONE HOUR)

To be completed by the candidate

BIT Examination Index No: .....

Important Instructions:

- The duration of the paper is **1 (One) hour**.
- The medium of instruction and questions is English.
- This paper has **3 questions** and **13 pages**.
- **Answer all questions**.
- **Question 1 carries** 40% marks **and the other questions carry** 30% marks each.
- **Write your answers** in English using the space provided **in this question paper**.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
  If a page is not printed, please inform the supervisor immediately.
- Note that **The Standard Normal Distribution Table** is attached with the paper.

Questions Answered

Indicate by a cross (×), (e.g. [X]) the numbers of the questions answered.

To be completed by the candidate by marking a cross (×).	1	2	3	
To be completed by the examiners:				

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1) Suppose X and Y are two square matrices of order *n*. When is Y said to be the inverse of X?

(5 marks)

(a) Consider the following square matrix of order 4.

$$X = \begin{pmatrix} 2 & -2 & 1 & -1 \\ 1 & 1 & 2 & 2 \\ -1 & 1 & 2 & -2 \\ 2 & 2 & -1 & -1 \end{pmatrix}$$

(15 marks)

Show that $X(X)^T=(X)^TX$. Hence or otherwise find X^{-1}.

(b) Let $Y=\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$.

Find the determinant of Y and the adjoint of Y. Hence find Y^{-1}.

(20 marks)

ANSWER IN THIS BOX

(1) (a)

Y is said to be the inverse of X if XY = YX = I where I is the identity matrix of order n.

$$\mathbf{XX^T} = \begin{pmatrix} 2 & -2 & 1 & -1 \\ 1 & 1 & 2 & 2 \\ -1 & 1 & 2 & -2 \\ 2 & 2 & -1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 & -1 & 2 \\ -2 & 1 & 1 & 2 \\ 1 & 2 & 2 & -1 \\ -1 & 2 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

$$\mathbf{X^TX} = \begin{pmatrix} 2 & 1 & -1 & 2 \\ -2 & 1 & 1 & 2 \\ 1 & 2 & 2 & -1 \\ -1 & 2 & -2 & -1 \end{pmatrix}\begin{pmatrix} 2 & -2 & 1 & -1 \\ 1 & 1 & 2 & 2 \\ -1 & 1 & 2 & -2 \\ 2 & 2 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

$\mathbf{XX^T = X^TX}$

$$\mathbf{X^{-1}} = \frac{1}{10}\begin{pmatrix} 2 & 1 & -1 & 2 \\ -2 & 1 & 1 & 2 \\ 1 & 2 & 2 & -1 \\ -1 & 2 & -2 & -1 \end{pmatrix}$$

Continued…

(b)

Determinant of Y =|Y| = 3.

Cofactor of A = $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & 0 \\ 1 & -1 & 1 \end{bmatrix}$. Therefore adjoint of A = $\begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

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Index No: ……..

2) (a) The position vectors of the points A, B and C are given by a**i** + **j** + **k**, **i** + b**j** + **k** and **i** – 3**k** respectively. Find values for a and b such that

(Detected a misprint of the position vector of the point B. It should be corrected as **i** + b**j** – **k. The following answers is given for the corrected question. However, the marking scheme will be adjusted to give maximum credit to students who attempted reasonably.)**

(i). A, B and C are collinear

(6 marks)

(ii). AB is perpendicular to BC

(8 marks)

(iii). AB is parallel to OC, where O is the origin

(6 marks)

(b) Differentiate the following functions with respect to *x* where *x* > 0.

(i). $f(x) = x^x$

(5 marks)

(ii). $g(x) = x^{(x^x)}$

(5 marks)

ANSWER IN THIS BOX

(2) (a)

(i) If A, B, C are collinear then $\overrightarrow{AB} = \lambda \overrightarrow{BC}$ for some $\lambda \in R$.

Therefore

$$\begin{pmatrix} 1-a \\ b-1 \\ -2 \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ -b \\ -2 \end{pmatrix}$$

And hence a = 1, $\lambda = 1$, b = 0.5.

(ii) If AB is perpendicular to BC, then, $\overrightarrow{AB}.\overrightarrow{BC} = 0$

$$\begin{pmatrix} 1-a \\ b-1 \\ -2 \end{pmatrix}.\begin{pmatrix} 0 \\ -b \\ -2 \end{pmatrix} = 0$$

Therefore $0 + (-b)(b-1) + 4 = 0$

$-b^2 + b + 4 = 0$

$b^2 - b - 4 = 0$

$(b - 0.5)^2 = \frac{17}{4}$

Therefore $b = \frac{1 \pm \sqrt{17}}{2}$ and $a \in R$.

Continued…

(iii) If AB is parallel to OC, then
$\overrightarrow{AB} = \mu\,\overrightarrow{OC}$ for some $\mu \in R$.

$$\begin{pmatrix} 1-a \\ b-1 \\ -2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$$

Therefore b = 1, $\mu = \frac{2}{3}$ and a = $\frac{1}{3}$.

(b) (i) $f(x) = x^x = e^{x\ln x}$

$$f'(x) = e^{x\ln x} \cdot \frac{d}{dx}\,[x\ln x]$$
$$= x^x(1 + \ln x)$$

(ii) $g(x) = x^{(x^x)}$

$$g(x) = x^{f(x)} = e^{f(x)\ln x}$$

$$\therefore g'(x) = e^{f(x)\ln x} \cdot \frac{d}{dx}[\,f(x)\ln x]$$
$$= x^{x^x} \cdot \left[\frac{f(x)}{x} + f'(x)\ln x\right]$$
$$= x^{x^x}[x^{x-1} + x^x\,(1 + \ln x)\,\ln x]$$

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Index No: ……..

3) The call center of a certain web-based shopping company has five hot-line numbers. Let *X* denote the number of hot-lines in use (busy) at a specific time. The probability function of *X* is given in the table below

X	0	1	2	3	4	5	More than 5
Probability	b	2a	0.15	2a	2b	0.20	0

From experience the company knows that the probability that less than three hotline numbers being busy in a specific time is 0.4.

(a) Calculate the values of *a* and *b*.

(5 marks)

In a specific time;

(b) what is the mode of the number of busy hot-lines?

(5 marks)

(c) calculate the probability that at least one hot-line is busy.

(5 marks)

(d) calculate the probability that less than 4 hot-lines are busy.

(5 marks)

(e) calculate the probability that less than 2 hot-lines or more than 4 hot-lines will be busy.

(5 marks)

(f) calculate the expected value of X.

(5 marks)

ANSWER IN THIS BOX

(a)

$P(X<3) = 0.4 \Rightarrow b+2a+0.15 = 0.4$

$b+2a = 0.25$ --------(1)

Total probability =1

$b+2a+0.15+2a+2b+0.2 = 1$

$3b+4a = 0.65$ --------(2)

$(2) - 2x(1) \Rightarrow (3b+4a) - (2b+4a) = 0.65-0.5$

$b = 0.15$

by (1) $0.15+2a = 0.25$

$2a = 0.1$

$a = 0.05$

Continued…

(b)

Value with the highest probability is the mode. Here it is 4.

(c)

$$\mathbf{P(X\geq 1) = 1 - P(X < 1) = 1 - [P(X=0)]}$$

$$\mathbf{= 1 - [0.15]}$$

$$\mathbf{= 1 - 0.15 = 0.85}$$

(d)

$$P(X<4) = 1 - P(X\geq 4) = 1 - [P(X=4)+P(X=5)]$$

$$= 1 - [0.3+0.2] = 0.5$$

(e)

$$P(X<2) + P(X>4) = P(X=0)+ P(X=1)+P(X=5)$$

$$= 0.15 + 0.1 + 0.2$$

$$= 0.45$$

(f)

X	P(x)	Xp(x)
0	0.15	0
1	0.1	0.1
2	0.15	0.3
3	0.1	0.3
4	0.3	1.2
5	0.2	1
Total	1	2.9

The expected value $= E(X) = \sum xp(x) = 2.9$

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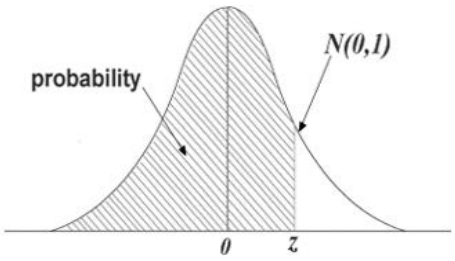
The Standard Normal Distribution Table



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of ***Z***, the standardized normal deviate, (the proportion ***P***, of the distribution less than ***Z***) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value ***X*** is obtained by calculating $Z = (X - \mu)/\sigma$ and reading the proportion corresponding to this value of ***Z***.

Z	*P*	*Z*	*P*	*Z*	*P*
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997