UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2017 – 2nd Year Examination – Semester 3

IT3305: Mathematics for Computing-II

PART 2 - Structured Question Paper

2nd June 2017

(ONE HOUR)

To be completed by the candidate

BIT Examination Index No: .....

Important Instructions:

- The duration of the paper is **1 (One) hour**.
- The medium of instruction and questions is English.
- This paper has **3 questions** and **10 pages**.
- **Answer all questions**.
- **Question 1 carries** 40% marks **and the other questions carry** 30% marks each.
- **Write your answers** in English using the space provided **in this question paper**.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
  If a page is not printed, please inform the supervisor immediately.

Questions Answered

Indicate by a cross (×), (e.g. [X]) the numbers of the questions answered.

To be completed by the candidate by marking a cross (×).	1	2	3	
To be completed by the examiners:				

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1)

(a) Write down the **three elementary row operations** used in matrix algebra.

(05 marks)

(b) Consider the following system of three linear equations

$$2x + 4y + 6z = 22$$
$$3x - 2y + 2z = 5$$
$$2x + y + 3z = 5$$

(i) Transform this system of linear equations into matrix form.

(10 marks)

(ii) Using elementary row operations solve the given system of linear equations.

(15 marks)

(iii) Does the system have no solution, a unique solution or infinitely many solutions?

(10 marks)

ANSWER IN THIS BOX

(1) (a)

(i)

$$\begin{bmatrix} 2 & 4 & 6 \\ 3 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 22 \\ 5 \\ 5 \end{bmatrix}$$

(ii)

Multiplying the 1st row by 1/2

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \\ 5 \end{bmatrix}$$

Multiplying 1st row by -3 and adding to 2nd row/ multiplying1st row by -2 and adding to 3rd row

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -4 \\ 0 & 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -6 \\ 13 \end{bmatrix}$$

Multiplying 2nd row by 1/2

$$\begin{bmatrix}1 & 2 & 3\\0 & 1 & -2\\0 & 3 & 5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}11\\-3\\-13\end{bmatrix}$$

Multiplying 2nd row by -2 and adding to 1st row/ multiplying 2nd row by -3 and adding to 3rd row

$$\begin{bmatrix}1 & 0 & 7\\0 & 1 & -2\\0 & 0 & 11\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}17\\-3\\22\end{bmatrix}$$

Multiplying 3rd row by 1/22

$$\begin{bmatrix}1 & 0 & 7\\0 & 1 & -2\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}17\\-3\\2\end{bmatrix}$$

Multiplying 3rd row by 2 and adding to 2nd row/ multiplying 3rd row by -7 and adding to 1st row

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\1\\2\end{bmatrix}$$

x=3, y=1, z= -2

(iii) The system has a unique solution

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2) (a) A particle P at position $\underline{i} + \underline{j}$ at time t=0 moves with velocity $2\underline{i} + 3\underline{j}$. Another particle Q at position $10\underline{i} + \underline{j}$ at time t=0 moves with velocity $-2\underline{i} + 3\underline{j}$. If they meet at position R, find the time taken to meet and the distances *PR* and *QR*.

(b) Find the area enclosed by the *x*-axis and the curves $x = 5$, $y = 2 + x$ and $y = 3/x$ in the first quadrant.

(30 marks)

ANSWER IN THIS BOX

(2)

(a)

Suppose that at time t the two particles meet, then

$$\begin{pmatrix}1\\1\end{pmatrix} + t\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}10\\1\end{pmatrix} + t\begin{pmatrix}-2\\3\end{pmatrix}$$

$$1 + 2t = 10 - 2t$$

$$4t = 9$$

$$T = 2.25$$

$$R \equiv \begin{pmatrix}1\\1\end{pmatrix} + \frac{9}{4}\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}5 \cdot 5\\7.75\end{pmatrix}$$

$PR = \sqrt{4.5^2 + 6.75^2}$=8.11

$QR = \sqrt{4.5^2 + (-6.75)^2}$=8.11

(b)

$2+x = 3/x$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=1$ in the first quadrant.

So the curves $y = 2 + x$ and $y = 3/x$ meet at (1, 3). The required area is

$$A = \int_0^1 (x+2)dx + \int_1^5 \frac{3}{x}\, dx$$

$$= [x^2/2 + 2x]^1_0 + [3lnx]^5_1$$

$$= 5/2+3ln5.$$

Continued…

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3) The time taken to complete a transaction from an automatic teller machine (ATM) of a certain bank has a normal distribution with mean 65 seconds and variance 25 seconds.

(a) Calculate the probability that the time taken to complete a transaction is less than one minute.

(b) Calculate the probability that the time taken to complete the transaction is more than 75 seconds.

(c) Calculate the probability that the time taken to complete the transaction is between 57.5 seconds and 70 seconds.

(d) Find t where about 4% of the transactions are less than t seconds?

(30 marks)

ANSWER IN THIS BOX

(a) $P(X < 60) = P\left(\frac{X-\mu}{\sigma} < \frac{60-65}{5}\right) = P(Z < -1) = 0.1587$

(b) $P(X > 75) = P\left(\frac{X-\mu}{\sigma} > \frac{75-65}{5}\right) = P(Z > 2) = 1 - P(Z < 2) = 1 - 0.9772 = 0.0228$

(c) $P(57.5 < X < 70) = P\left(\frac{57.5-65}{5} < \frac{X-\mu}{\sigma} < \frac{70-65}{5}\right) = P(-1.5 < Z < 1)$

$= P(Z < 1) - P(Z < -1.5) = 0.8413 - 0.0668 = 0.7745$

(d)

For 0.04 area (that is P(Z<z)=0.04) the z = - 1.75

That is $z = -1.75$

$$\frac{x-\mu}{\sigma} = -1.75$$

$$\frac{x-65}{5} = -1.75$$

$$x = 65 - 1.75 * 5$$

$$x = 65 - 8.75$$

$$x = 56.25$$

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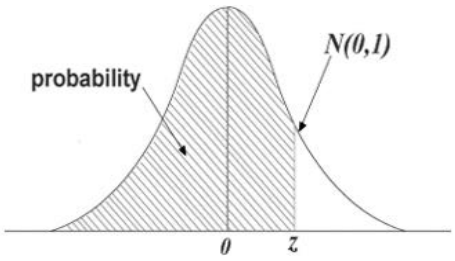
The Standard Normal Distribution Table



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of ***Z***, the standardized normal deviate, (the proportion ***P***, of the distribution less than ***Z***) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value ***X*** is obtained by calculating $Z = (X - \mu)/\sigma$ and reading the proportion corresponding to this value of ***Z***.

Z	*P*	*Z*	*P*	*Z*	*P*
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997