Problem 4

a)

We need to show that for all constants c > 0, there exists a constants $n_0 > 0$ such that $15 + 7n \leq c * n \log n$ for all $n \geq n_0$

Let $c > 0$ be given,

$$15 + 7n \leq 15n \leq \frac{15}{\log n} n \log n \quad \forall n \geq 2$$
$$\leq c * n \log n$$

Thus, $c \geq \frac{15}{\log n}$

For any c, we choose $n_0 = 2^{\frac{15}{c}}$

b)

$f(n) \in \Theta(n^{\frac{3}{2}})$

$g(b) \in \Theta(n(\log n)^2)$

Let $L = \lim_{n->\infty} \frac{f(n)}{g(n)} = \lim_{n->\infty} \frac{n^{\frac{3}{2}}}{n(\log n)^2} = \lim_{n->\infty} \frac{\sqrt{n}}{(\log n)^2} = 0$

Therefore, $f(n) \in o(g(n))$

c)

$\sum_{i=1}^{n-1} \sum_{j=1}^{n} \sum_{k=i}^{j} 1 + \Theta(\log n) + \Theta(\frac{n}{4}) = \sum_{i=1}^{n-1} \sum_{j=i}^{n} (j - i + 1) + \Theta(\log n) + \Theta(\frac{n}{4})$
$= \sum_{i=1}^{n-1} \frac{(n+1-i)(n+2-i)}{2} + \Theta(\log n) + \Theta(\frac{n}{4}) \in \Theta(n^3)$

d)

For $i = 6, 7$, we have 7 starts

Otherwise,

If i = 1, then print n starts

If i = 2, then print 2n starts

If i = 3, then print 3n-1 starts

If i = 4, then print 4n-1 starts

If i = 5, then print 5n-2 starts

Generally, the expected number of starts should be $\frac{2}{7} * 7 + \frac{1}{7} * n + \frac{1}{7} * 2n + \frac{1}{7} * (3n - 1) + \frac{1}{7} * (4n - 1) + \frac{1}{7} * (5n - 2) = \frac{15n+9}{7}$ starts