

Problem 2

- a) The worst case is that in each recursion depth, we have n elements to be sorted (i.e. will not end because only one element in the group). And the maximum number of digits is the same number of digits of n^2 , which should be $\Theta(\log_{10} n^2) = \Theta(\log n)$. And we assume the radix is a constant, then the total runtime for worst-case should be $\Theta(n \log n) \in o(n^2)$
- b) The best case is that all \sqrt{k} patterns are found at the beginning of the text.

For the first guess, it costs $\Theta(\sqrt{k} \cdot m + k - \sqrt{k}) = \Theta(k + \sqrt{k} \cdot m)$

For the rest guesses, each costs $\Theta(k - \sqrt{k}) = \Theta(k)$

Therefore, totally, we have $n - m + 1$ guesses, which is $\Theta(n)$

So the best-case expected runtime is $\Theta(nk + \sqrt{k} \cdot m) = \Theta(nk)$

The worst case is that for each guess, the hash value matches, and we need to do a `strcmp()` for each pattern. And finally, we will find the matched pattern in the last guess.

For every guess, each costs $\Theta(k \cdot m)$, similar to the best case, we have $\Theta(n)$ guesses in total. Therefore, the worst-case runtime is $\Theta(nkm)$