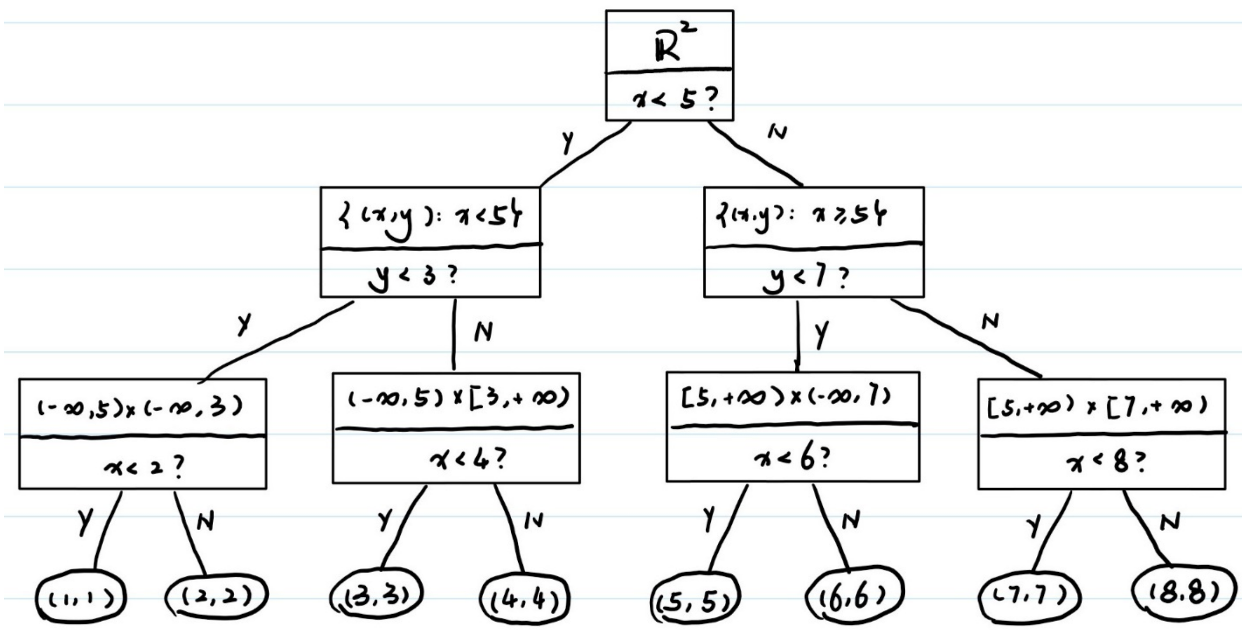
Assignment 4 Problem 5

a) Draw the kd-tree representing the set of 2D points
$S = \{p_1, \ldots, p_8\} = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8)\}$.

b) Give an algorithm using pseudocode for finding a point with the smallest x-coordinate in a kd-tree storing 2D points. Your algorithm should be as efficient as possible. State and explain the worst case running time of your algorithm.

solution:

a)



b) The algorithm is given as follow:

```
Recurse_find(n)
n: a node in the kd-tree
   if n is leaf, return n.x
   if n is separating the area horizontally
           return min{Recurse_find(n.left), Recurse_find(n.right)}
   if n is separating the area vertically
           return Recurse_find(n.left)


Find_min_x(T)
T: the kd-tree stroing 2D points
   return Recurse_find(T->root)
```

As the code shows, what we do is recursively visit the nodes of the kd-tree. Therefore, consider a kd-tree of n points, since a kd-tree is balanced, therefore, the height h of the kd-tree should be $O(\log n)$

The worst-case should be the second last level of the kd-tree separate the area horizontally, which means that we need to compare the x-value of left and right child. In this case, assume the height of kd-tree is h. Then there are h levels of kd-tree that separating occurs, since the last one is vertical separate, therefore, there are $\frac{k}{2}$ horizontal separate (because the first separate is always vertical), since each horizontal separate let us visit two more nodes, and each vertical separate let us just visit another node, the total runtime should be $\Theta\left(1 + 1 + 2 + 2 + 3 + 3 + \cdots + \frac{k}{2} + \frac{k}{2}\right) = \Theta\left(2 \cdot 1 + 2 + \ldots + \frac{k}{2}\right) = \Theta\left(\frac{k}{2} \cdot \frac{k+2}{2}\right) = \Theta(k^2)$

Since $k \in O(\log n)$, we have the runtime should be $O(\log^2 n)$