University of Waterloo
CS240 Fall 2021
Assignment 1

Due Date: Wednesday, Sept 22 at 5:00pm

The integrity of the grade you receive in this course is very important to you and the University of Waterloo. As part of every asessment in this course you must read and sign an Academic Integrity Declaration before you start working on the assessment and submit it **before the deadline of Sept 22** along with your answers to the assignment; i.e. **read, sign and submit A01-AcInDe.txt now or as soon as possible**. The agreement will indicate what you must do to ensure the integrity of your grade. If you are having difficulties with the assignment, course staff are there to help (provided it isn't last minute).

The Academic Integrity Declaration must be signed and submitted on time or the assessment will not be marked.

Please read `http://www.student.cs.uwaterloo.ca/~cs240/f21/guidelines.pdf` for guidelines on submission. **Each question must be submitted individually to MarkUs as a PDF** with the corresponding file names: a1q1.pdf, a1q2.pdf, ... , a1q5.pdf . It is a good idea to submit questions as you go so you aren't trying to create several PDF files at the last minute.

Late Policy: Assignments are due at 5:00pm. To accommodate any small time differences with our submission server, there is a grace period of 5 minutes; i.e. we will accept assignments until 5:05pm without penalty. Assignments submitted after 5:05pm will incur a penalty of 1 mark per minute to a maximum of 10 marks.
Assignments submitted after 5:15pm will not be accepted but may be reviewed (by request) for feedback purposes only.

Note: you may assume all logarithms are base 2 logarithms: $\log = \log_2$.

Problem 1 [4+4+4+4=16 marks]

Provide a complete proof of the following statements from first principles (i.e., using the original definitions of order notation).

a) $1665n^2 + 314n + 42 \in \Theta(n^2)$

b) $n^2(\log n)^{1.0001} \in \Omega(n^2)$

c) $n\log(\log(n)) \in o(n(\log(\log(n)))^2)$

d) n^n is $\omega(n^{17})$

Problem 2 [4+4+4+4=16 marks]

For each pair of the following functions, fill in the correct asymptotic notation among Θ, o, and ω in the statement $f(n) \in \sqcup(g(n))$. Prove the relationship using any relationship or technique described in class.

a) $f(n) = n^4(7 + 3\cos 2n)$ versus $g(n) = 7n^4 + 5n^3 + 3n$

b) $f(n) = 10^n + 99n^{10}$ versus $g(n) = 75^n + 25n^{27}$

c) $f(n) = n^{\frac{3}{2}}$ versus $g(n) = n \log n$

d) $f(n) = \log\log n$ versus $g(n) = (\log\log\log n)^8$

Problem 3 [4+4+4+4=16 marks]

Prove or disprove each of the following statements. To prove a statement, you should provide a formal proof that is based on the definitions of the order notations. To disprove a statement, you can either provide a formal proof that the statement is false or give a counter example and explain it. All functions are positive functions.

a) $f(n) \notin o(g(n))$ and $f(n) \notin \omega(g(n)) \Rightarrow f(n) \in \Theta(g(n))$

b) $n\log(4^n) \in \Omega(n^2)$

c) $f(n) \in \Theta(g(n)) \Rightarrow 2^{f(n)} \in \Theta(2^{g(n)})$

d) $\min(f(n), g(n)) \in \Theta\left(\frac{f(n)g(n)}{f(n)+g(n)}\right)$

Problem 4 [4+4+4+4=16 marks]

Analyze the following piece of pseudocode and give a tight (Θ) bound on the running time as a function of n. Show your work. A formal proof is not required, but you should justify your answer (in all cases, n is assumed to be a positive integer).

a)

```
x = 0
for i = 1 to n + 42 do
   x = x * 4
   for j = 762 to 20210 do
      x = x * 20
```

b)

```
x = 1
y = 0
for i = 1 to n do
   x = x * 5
   for j = 1 to x do
      y = y + 1
```

c)
```
x = 1
i = 2
while (i < n) do
   for j = 1 to n do
      x = x + 1
   i = i * i
```

d)
```
x = 0
for i = 1 to sqr(n)     // i.e. n^2
   for j = 1 to ceiling( log(i) )
      x = x + 1
```

Problem 5 [2+4=6 marks]

Consider two algorithms, Algo1 and Algo2, that solve the same problem. For any input of size n, Algo1 takes time $T_1(n)$ and Algo2 takes time $T_2(n)$. Prove or disprove each of the following statements. To prove a statement, you should provide a formal proof that is based on the definitions of the order notations. To disprove a statement, provide a counter example and explain it.

a) Suppose that $T_1(n) \in O(n^2)$ and $T_2(n) \in O(n^3)$. There exists a value n_0 such that for all $n > n_0$, $T_1(n)$ runs as fast as or faster than $T_2(n)$; i.e. $T_1(n) \leq T_2(n)$.

b) Suppose that $T_1(n) \in \Theta(n^2)$ and $T_2(n) \in \Theta(n^3)$. There exists a value n_0 such that for all $n > n_0$, $T_1(n)$ runs as fast as or faster than $T_2(n)$; i.e. $T_1(n) \leq T_2(n)$.