

Problem 3

a) Let $c > 0$ be given.

Note that $7n + 3 < 8n$ for all $n > 3$

And since $8n = \frac{8}{\log n} * n \log n$

When $\frac{8}{\log n} \leq c$, which implies $n_0 \geq \frac{8}{c}$

This means for all $n_0 \geq \frac{8}{c}$, we have that for all constant c , there exists

constant $n_0 \geq 0$ such that $|f(n)| \leq c|g(n)| \forall n \geq n_0$

Where $f(n) = 7n+3$, and $g(n) = n \log n$

b) For the inner loop, each iteration cost $\Theta(1)$

So the total runtime of the inner loop is $\Theta(10000) = \Theta(1)$

For the outer loop, each iteration cost $\Theta(1 + 1 + 1 + 1) = \Theta(1)$

So the runtime of the code is only depends on the number we run the outer loop.

Note that initially, $i = 0$, after one iteration, $i = 1$, after the second iteration, $i = 1 + 3$, etc.

Therefore, after k iteration, $i = \sum_{j=0}^{k-1} (2j + 1) = k^2$

When $i \geq n$, the program stops. Therefore, the runtime should be $\Theta(\sqrt{n})$