

Problem 2

a)

The expected running time to search a key k should be $\theta(n \log n)$

To simplify the question, we can consider the keys for such BST are in $\{1, 2, \dots, n\}$

For a BST, the average height is in $\Theta(\log n)$

Since k can be arbitrary, if k is not in $\{1, 2, \dots, n\}$, then we probably reach the bottom of the tree.

In this case, the running time for a search is $\Theta(\log n)$

And since this kind of k 's are infinitely many.

Therefore, The expected running time to search for a key k should be $\theta(n \log n)$

b)

Considering a decision tree, there need to have at least $5! = 120$ leaves.

So the tree need to have height at least 7, which means there is at least 7 comparisons.