Assignment 3 Problem 1: Greedy Algorithms: Skyline Antenna Optimization

The algorithm should firstly, sort the "buildings" with their x-coordinates of edge which has a larger x-coordinate (that is, the right edge in the example provided) in non-decreasing order. Then, traverse the sorted "buildings" in order, and do the judgment on each of them: If the building does not contain any antenna, then we put an antenna at the right edge of this building.

The pseudo code is as follows:

$myAlgorithm(n, left[], right[])$
    $structure\ building[i] = \{left[i], right[i]\}\ for\ all\ i\ from\ 1\ to\ n$
    $sort\ the\ building[]\ in\ nondecreasing\ order\ based\ on\ building[i].right$
    $pre\_antenna = MIN\_INT$
    $for\ i = 1\ to\ n\ do$
        $if\ building[i].left > pre\_antenna$
            $add\ building[i].right\ to\ ans[]$
            $pre\_antenna = building[i].right$
    $output\ ans[]$

To show the correctness, suppose that there exists a optimized answer that contains less number of antennas than the greedy algorithm produces.

Then we only consider the buildings that we add an antenna to its right edge during the algorithm. Since the pre_antenna variable in the algorithm always increasing, if the judge statement is true, then this means that the right edge of the previous building is less than the left edge of the current building, which means that these two buildings are "disjoint". If the number of antennas is less than the algorithm produces, there is no way to contain all selected buildings. Also, since we are traverse all buildings and guarantee that there is an antenna in their range. Therefore, the output is optimal. And the runtime of the algorithm is $O(n \log n + n) = O(n)$