Assignment 6 Problem 1

Since the graph is finite, and $C_i, R_i \in R_{>0}$. The only possible that can let Runner gets infinitely larger than the Coyote is there exists a cycle such that there is a path from s to that cycle, and each time passing the cycle, the value of $\frac{size\ of\ Runner}{size\ of\ Coyote}$ getting greater, which means that the product of all C_i in that cycle is less than the product of all R_i in that cycle. By go through that cycle again and again, $\frac{size\ of\ Runner}{size\ of\ Coyote}$ will getting greater and greater which will go to infinity. Naming this kind of cycle "special cycle".

To find the cycle, applying DFS on vertex s. When we meet some vertex that is discovered already (which means we have found a cycle!!), then we back track it, by track its parents, and track parents' parents, and so on. Until we reach the last occurrence of such vertex. During the back track process, also calculate the value of $\frac{\prod R_i}{\prod C_i}$ for all edges that we tracking through (these edges are in the cycle). After going through such cycle, if we find the value of $\frac{\prod R_i}{\prod C_i}$ is greater than 1, then such cycle is a 'special cycle', and thus by going through this cycle again and again, then Runner gets infinitely larger than the Coyote.

Pseudo code:

```
DFS(v)
    mark(v) = discovered
    for u ∈ AdjacencyList(v) do
        if u is undiscovered then
            DFS(u); parent(u) = v;
```

else

back track the cycle through parent array

in the same time calculate $\frac{\prod R_i}{\prod C_i}$ for all edges in the cycle

if $\frac{\prod R_i}{\prod C_i} > 1$ return answer exist 'special cycle'

fi

Time analysis:

For traditional DFS, O(n+m). For back track, the largest cycle is of length n, the number of cycles for each discovered u is O(n). Therefore, for back tracking and calculating $\frac{\prod R_i}{\prod C_i}$, the time it cost is O(n), for each vertex discovered u, the running time should be $O(n^2)$, therefore, the total running time is $O\left(n^2(n+m)\right) \in O((n+m)^{10})$