Problem 2: Reductions

a) Extend the array A[ ] into three arrays named X[ ], Y[ ], and Z[ ].
And the contents of X, Y and Z are following:
- X[ ] has the same length, same entries as A[ ].
- Y[ ] has the same length, but for all m in the range, Y[m] = $-\frac{1}{2}\cdot$ A[m]
- Z[ ] has the same length, same entries as A[ ].

By doing this, we have already reduced the 3SUM0 to ARITHPROG.
Since the ARITHPROG is to find i, j, k such that $Y[j] - X[i] = Z[k] - Y[j]$,
Rearrange we get $X[i] - 2Y[j] + Z[k] = 0$, now change the values in terms of A[ ], then we have $A[i] + A[j] + A[k] = 0$, and in this case i, j, k has no restriction to be distinct.

b) Firstly, modify the arrays X[ ],Y[ ], and Z[ ] as follow:
For m in range 1 to n,
- X[m] = 100*X[m] + 3
- Y[m] = 100*Y[m] + 4
- Z[m] = 100*Z[m] – 7

Then, we combine three arrays into one array namely A[ ] with length 3n.
This time, we just need to run the 3SUM0 on A[ ], and the result of the 3SUM0 is exactly the same as ARITHPROG.
To prove the correctness, firstly, we considering the last digit of X[i] + Y[j] +Z[k] we obtained. Since the 3SUM0 returns true if there are three entries adds to 0, so the only possible combination is one entry from X[ ], one entry from Y[ ], and one entry from Z[ ]. Any other combinations will not adds to 0.
Secondly, we considering the sum. Consider the three entries that 3SUM0 found in process that adds to 0 corresponds to X[i], Y[j] and Z[k]. Then during 3SUM0, it founds
$100X[i] + 3 + 100Y[j] + 100Z[k] - 7 = 0$
Rearranging, we get $100(X[i] + Y[j] + Z[k]) = 0$ which implies $X[i] + Y[j] + Z[k] = 0$. Which is exactly what we want.