Problem 2: Reductions

1. Extend the array A[ ] into three arrays named X[ ], Y[ ], and Z[ ].

And the contents of X, Y and Z are following:

* X[ ] has the same length, same entries as A[ ].
* Y[ ] has the same length, but for all m in the range, Y[m] = A[m]
* Z[ ] has the same length, same entries as A[ ].

By doing this, we have already reduced the 3SUM0 to ARITHPROG.

Since the ARITHPROG is to find such that ,

Rearrange we get , now change the values in terms of A[ ], then we have , and in this case has no restriction to be distinct.

1. Firstly, modify the arrays X[ ],Y[ ], and Z[ ] as follow:

For m in range 1 to n,

* X[m] = 100\*X[m] + 3
* Y[m] = 100\*Y[m] + 4
* Z[m] = 100\*Z[m] – 7

Then, we combine three arrays into one array namely A[ ] with length 3n.

This time, we just need to run the 3SUM0 on A[ ], and the result of the 3SUM0 is exactly the same as ARITHPROG.

To prove the correctness, firstly, we considering the last digit of X[i] + Y[j] +Z[k] we obtained. Since the 3SUM0 returns true if there are three entries adds to 0, so the only possible combination is one entry from X[ ], one entry from Y[ ], and one entry from Z[ ]. Any other combinations will not adds to 0.

Secondly, we considering the sum. Consider the three entries that 3SUM0 found in process that adds to 0 corresponds to X[i], Y[j] and Z[k]. Then during 3SUM0, it founds

Rearranging, we get which implies . Which is exactly what we want.