Assignment 6 Problem 2

1. Let = weight of shortest path from to using exactly edges

Then

* 0 if
* if

And we want .

Compute from : if x exists

Where x are vertices that exists an edge that points from x to v.

Otherwise,

To prove the correctness, we consider all possibilities of , then correct by induction on .

Base case, for , only there exists a path with length 0, otherwise no path exists. Thus, the base case holds.

Induction hypothesis: if correct.

Induction steps, since is correct, then we just need to find from , consider . To get this, we should consider all adjacent edges that points to v. Say the starting point of one of such edge is x, and the end point is v. Then the lightest path from u which contains x to v should be . By comparing all such edges and find the minimum one, then we get the lightest path from u to v with i edges. Also, if such edges do not exist, then this means that point v is isolated, and no shortest path from u to v exist, then .

Therefore, is correct. By induction, the algorithm is correct.

Pseudo code:

Initialize for all as above.

for from 1 to h do

for do

for do

for each edge do

od

od

od

od

The runtime is obviously

1. for all

This is true, because consider a length path, after first edges, we must at some vertex. So we just need to minimize the sum of weights of the first edges and the last edges. By considering all vertices after edges, we have considered all possibilities.

The runtime is obviously , for all , another for trying all vertices.