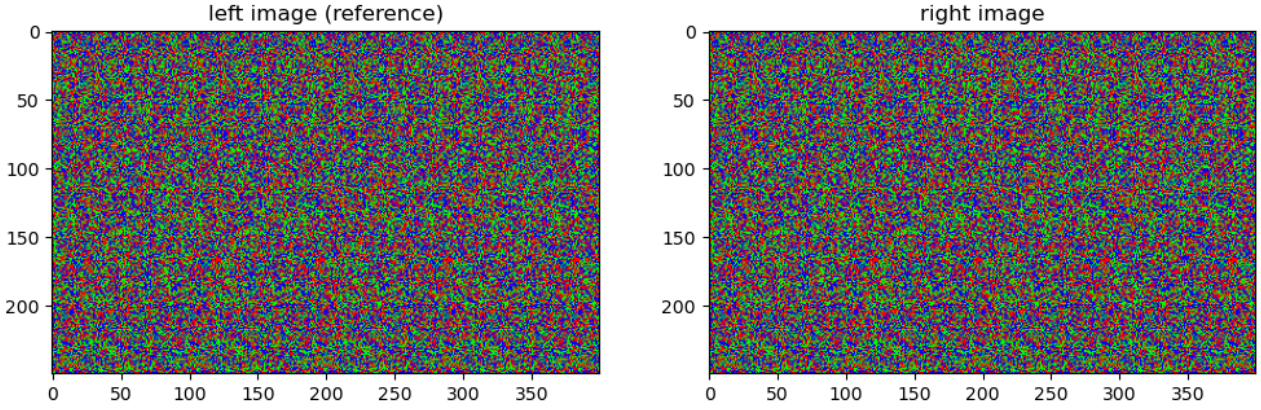
Assignment 3

Part I: Windows-based stereo
$\longleftarrow 70\ pts$

In [1]:
```
%matplotlib inline
```

In [2]:
```
import numpy as np
import math
import matplotlib.pyplot as plt
import matplotlib.image as image
from matplotlib.colors import LogNorm
from skimage import img_as_ubyte
from skimage.color import rgb2gray
from numpy import linalg as la
```

In [3]:
```
# you should use this random dot stereo pair for code developing/testing in
im_left = image.imread("images/stereo_pairs/rds_left.gif")
im_right = image.imread("images/stereo_pairs/rds_right.gif")

fig = plt.figure(figsize = (12, 5))
plt.subplot(121)
plt.title("left image (reference)")
plt.imshow(im_left)
plt.subplot(122)
plt.title("right image")
plt.imshow(im_right)

# the range of disparities for this random dot stereo pair
d_min = 0
d_max = 2
```

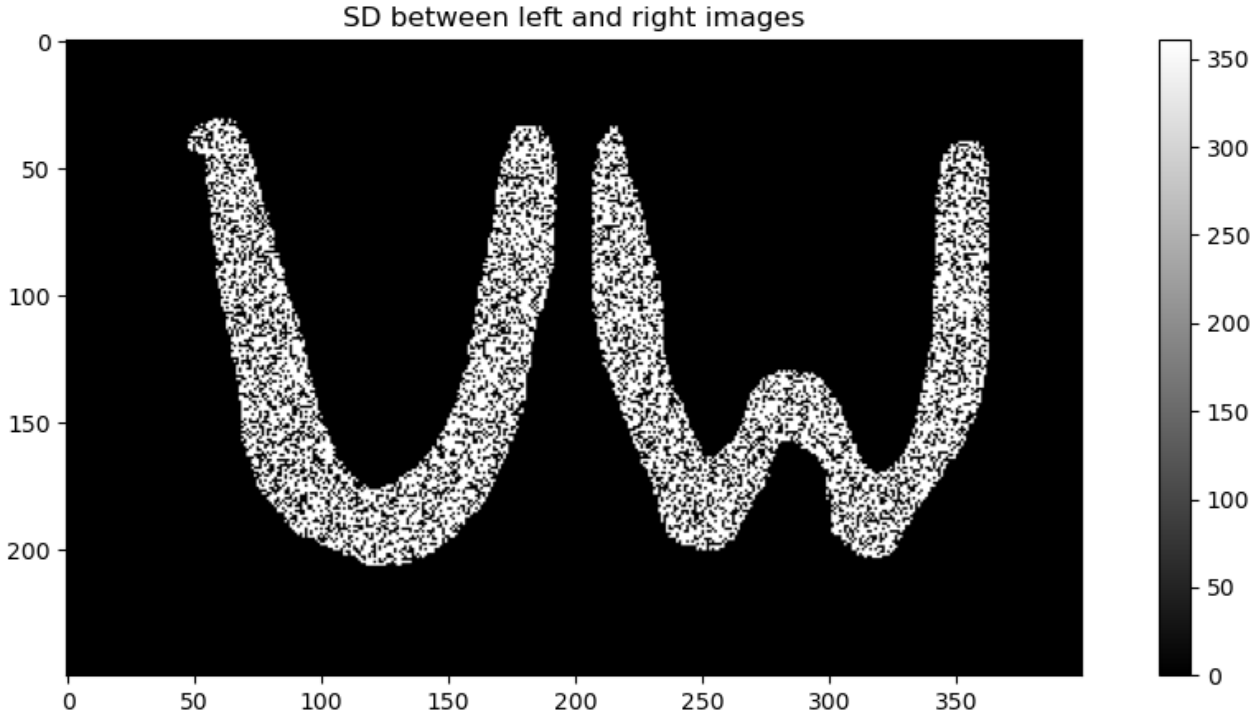


Problem 1: compute and visualize (as an image) an array of "squared differences" between RGB pixel values in the left and the right images. Treat each RGB pixel value as 3-vector and interprete "squared difference" as squared L2 norm of the difference between the corresponding vectors.

HINT (important here and later): convert R, G, B values to floats to avoid severe "overflow" bugs while adding small-range types (one-byte for $char$) or subtracting unsigned types. Note that $imshow$ function can display (as an image) any 2D array of floats.

In [4]:
```
SD = np.zeros(np.shape(im_left))

left_copy = im_left.copy()[:,:,:].astype(float)
right_copy = im_right.copy()[:,:,:].astype(float)

SD = left_copy - right_copy

SD = la.norm(SD, axis=2)


fig = plt.figure(figsize = (12, 5))
plt.title("SD between left and right images")
plt.imshow(SD, cmap = "gray")
plt.colorbar()
```

Out[4]:
```
<matplotlib.colorbar.Colorbar at 0x7f89b9cdf2b0>
```



Problem 2: write function for computing squared differences between RGB pixel values in the reference (left) image and the "shifted" right image for ALL shifts/disparities Δ in the range $\Delta \in [d_{min}, d_{max}]$. You should think about the correct direction of the shift. The output should be array SD such that $SD[i]$ is an image of Squared Differences for shift $\Delta = d_{min} + i$ for any $i \in [0, d_{max} - d_{min}]$.

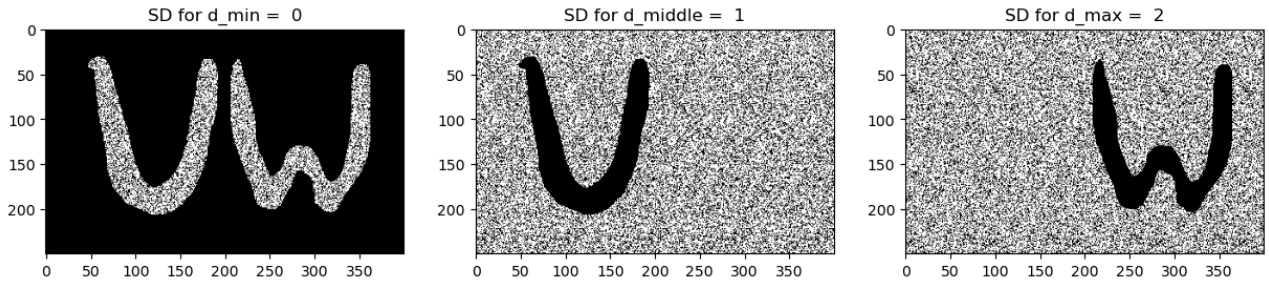
In [5]:
```
def SD_array(imageL, imageR, d_minimum, d_maximum):
    # initialization of the array of "squared differences" for different shi
    SD = np.zeros((1+d_maximum-d_minimum,np.shape(imageL)[0],np.shape(imageL
    i = d_minimum
    index = 0
    while i <= d_maximum:
        temp = imageL - np.roll(imageR, i, axis = 1)
        SD[index] = la.norm(temp, axis=2)
        i = i + 1
        index = index + 1

    return SD
```

Use $\mathrm{SD\_array}$ function to compute SD images for the random dot stereo pair. Visualize such squared difference images for $\Delta = d_{min}$, $\Delta = d_{mid} \approx \frac{d_{min}+d_{max}}{2}$, and $\Delta = d_{max}$. Note that the first image should be identical to the one in Problem 1. (fully implemented)

In [6]:
```
SD = SD_array(im_left, im_right, d_min, d_max)
print(np.shape(im_left))
print(np.shape(im_right))
print(np.shape(SD))

fig = plt.figure(figsize = (15, 4))
plt.subplot(131)
plt.title('SD for d_min = {:>2d}'.format(d_min))
plt.imshow(SD[0], cmap = "gray")
plt.subplot(132)
d_middle = round((d_min+d_max)/2)
plt.title('SD for d_middle = {:>2d}'.format(d_middle))
plt.imshow(SD[d_middle-d_min], cmap = "gray")
plt.subplot(133)
plt.title('SD for d_max = {:>2d}'.format(d_max))
plt.imshow(SD[d_max-d_min], cmap = "gray")
#plt.colorbar(cax=plt.axes([0.91, 0.25, 0.01, 0.5]))
```

```
(250, 400, 4)
(250, 400, 4)
(3, 250, 400)
```

Out[6]:
```
<matplotlib.image.AxesImage at 0x7f89aaa135e0>
```



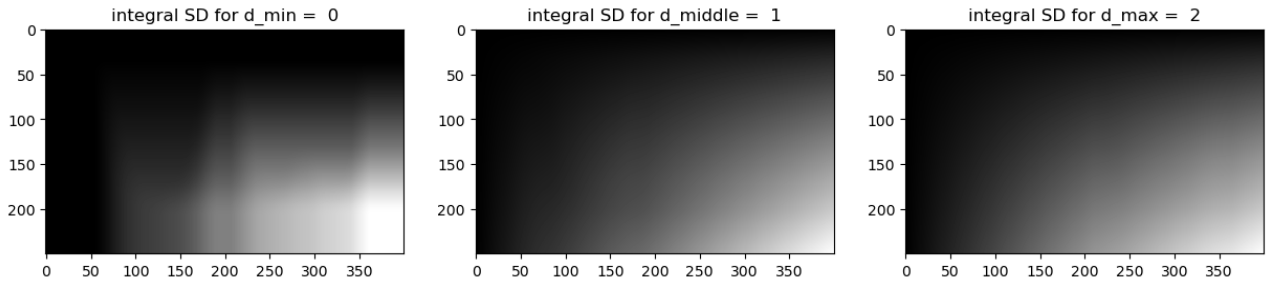
Problem 3: write function to compute an "integral image" for any given "scalar" image

In [7]:
```
# Function integral_image can be applied to any scalar 2D array/image.
# This function should return a double/float64 (precision) array/image of th
# NOTE: it is safer to explicitly specify double/float64 precision for integ
# later we will be adding/subtracting ("differenting") their values in nearb

def integral_image(img):
    img = img.astype(np.double)
    for i in range(img.shape[0]):
        for j in range(img.shape[1]):
            if i==0 and j==0:
                continue
            elif i==0:
                img[i][j] = img[i][j-1] + img[i][j]
            elif j==0:
                img[i][j] = img[i-1][j] + img[i][j]
            else:
                img[i][j] = img[i-1][j] + img[i][j-1] - img[i-1][j-1] + img[
    return img
```

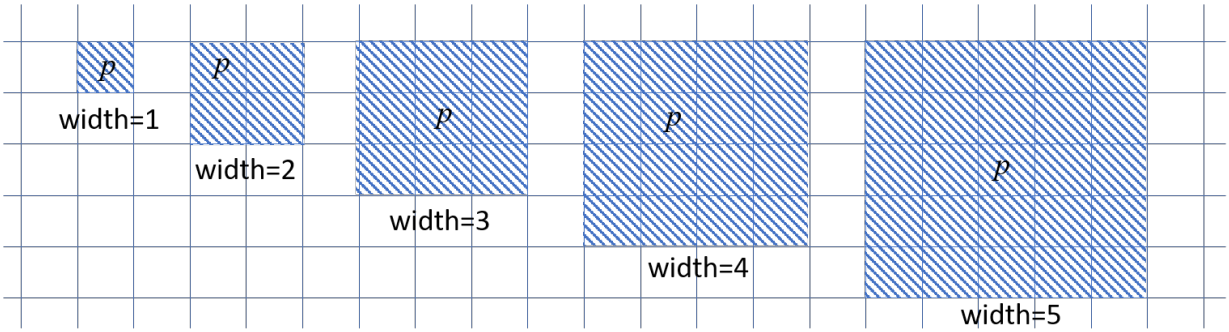
apply intergal_image function to the "squared differences" (SD) for each disparity (fully implemented)

In [8]:
```
integral_SD = np.zeros(np.shape(SD))
print(np.shape(integral_SD),np.shape(SD))

for Delta in range(1+d_max-d_min):
    integral_SD[Delta] = integral_image(SD[Delta])

fig = plt.figure(figsize = (15, 4))
plt.subplot(131)
plt.title('integral SD for d_min = {:>2d}'.format(d_min))
plt.imshow(integral_SD[0], cmap = "gray")
plt.subplot(132)
d_middle = round((d_min+d_max)/2)
plt.title('integral SD for d_middle = {:>2d}'.format(d_middle))
plt.imshow(integral_SD[d_middle-d_min], cmap = "gray")
plt.subplot(133)
plt.title('integral SD for d_max = {:>2d}'.format(d_max))
plt.imshow(integral_SD[d_max-d_min], cmap = "gray")
#plt.colorbar(cax=plt.axes([0.91, 0.2, 0.01, 0.6]))
```

Out[8]:
```
(3, 250, 400) (3, 250, 400)
<matplotlib.image.AxesImage at 0x7f8988acdb20>
```



Problem 4: write function that sums the elements of the input image within fixed-size windows around image pixels. Note that this function should work for any (odd or even) values of parameter $\mathrm{window\_width} \in \{1, 2, 3, 4, 5, \ldots\}$ according to the windows illustration below:

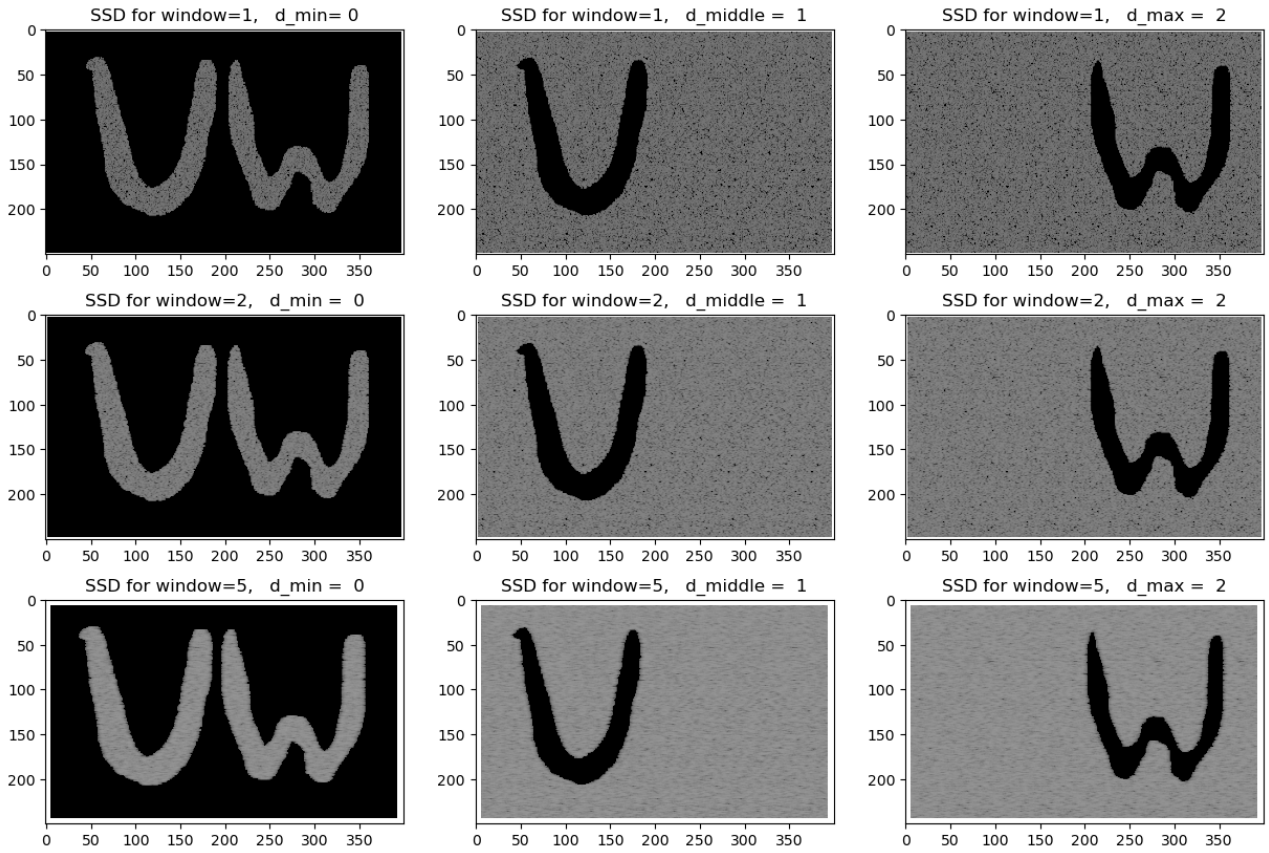


In [9]:
```
# function windSum can be applied to any scalar 2D array/image. It should re
# each element (pixel p) is the "sum" of the values in the input array/image
# The return image should be of the same size/type and have its margins (arc
# NOTE: you should use function integral_image implemented earlier.
# HINT: you should use standard np.roll function to avoid double or triple f
INFTY = np.inf

def windSum(img, window_width):
    rb = math.floor(window_width)
    lt = -1
    if window_width%2==0:
        lt = rb - 1
    else:
        lt = rb
    I = np.ones(window_width)
    temp = img.copy()
    for i in range(img.shape[1]):
        if i<=lt:
            continue
        elif i>img.shape[1]-1-rb:
            continue
        temp[:,i] = np.matmul(np.roll(img, i-lt, axis=1)[:,:window_width],I)
    ans = temp.copy()
    for j in range(img.shape[0]):
        if j<=lt:
            continue
        elif i>img.shape[0]-1-rb:
            continue
        ans[j] = np.matmul(np.roll(temp, j-lt, axis=0)[:window_width,:],I)
    img = ans.copy()
    for i in range(img.shape[0]):
        if i<=lt:
            img[i] = np.full(img.shape[1],INFTY)
        elif i>img.shape[0]-1-rb:
            img[i] = np.full(img.shape[1],INFTY)
    for j in range(img.shape[1]):
        if j<=lt:
            img[:,j] = np.full(img.shape[0],INFTY)
        elif j>img.shape[1]-1-rb:
            img[:,j] = np.full(img.shape[0],INFTY)
    img = np.flip(img, axis=1)
    return img
```

Compute SSD images for windows of different widths and for different disparities by applying $\mathrm{windSum}$ function to the "squared differences" SD. Note that the results for windows of width 1 (the first row of the figure below) should look identical (except for the narrow "margin") to the results in Problem 2. (fully implemented)

In [10]:
```
SSD1 = np.zeros(np.shape(SD))
SSD2 = np.zeros(np.shape(SD))
SSD5 = np.zeros(np.shape(SD))

for Delta in range(1+d_max-d_min):
    SSD1[Delta] = windSum(SD[Delta],1)
    SSD2[Delta] = windSum(SD[Delta],2)
    SSD5[Delta] = windSum(SD[Delta],5)

d_middle = round((d_min+d_max)/2)

fig = plt.figure(figsize = (15, 10))
plt.subplot(331)
plt.title('SSD for window=1,   d_min={:>2d}'.format(d_min))
plt.imshow(1+SSD1[0], cmap = "gray", norm=LogNorm(vmin=1, vmax=200000))
plt.subplot(332)
plt.title('SSD for window=1,   d_middle = {:>2d}'.format(d_middle))
plt.imshow(1+SSD1[d_middle-d_min], cmap = "gray", norm=LogNorm(vmin=1, vmax=
plt.subplot(333)
plt.title('SSD for window=1,   d_max = {:>2d}'.format(d_max))
plt.imshow(1+SSD1[d_max-d_min], cmap = "gray", norm=LogNorm(vmin=1, vmax=200
plt.subplot(334)
plt.title('SSD for window=2,   d_min = {:>2d}'.format(d_min))
plt.imshow(1+SSD2[0], cmap = "gray", norm=LogNorm(vmin=1, vmax=200000))
plt.subplot(335)
plt.title('SSD for window=2,   d_middle = {:>2d}'.format(d_middle))
plt.imshow(1+SSD2[d_middle-d_min], cmap = "gray", norm=LogNorm(vmin=1, vmax=
plt.subplot(336)
plt.title('SSD for window=2,   d_max = {:>2d}'.format(d_max))
plt.imshow(1+SSD2[d_max-d_min], cmap = "gray", norm=LogNorm(vmin=1, vmax=200
plt.subplot(337)
plt.title('SSD for window=5,   d_min = {:>2d}'.format(d_min))
plt.imshow(1+SSD5[0], cmap = "gray", norm=LogNorm(vmin=1, vmax=200000))
plt.subplot(338)
plt.title('SSD for window=5,   d_middle = {:>2d}'.format(d_middle))
plt.imshow(1+SSD5[d_middle-d_min], cmap = "gray", norm=LogNorm(vmin=1, vmax=
plt.subplot(339)
plt.title('SSD for window=5,   d_max = {:>2d}'.format(d_max))
plt.imshow(1+SSD5[d_max-d_min], cmap = "gray", norm=LogNorm(vmin=1, vmax=200
```

Out[10]:
```
<matplotlib.image.AxesImage at 0x7f89ab82b280>
```



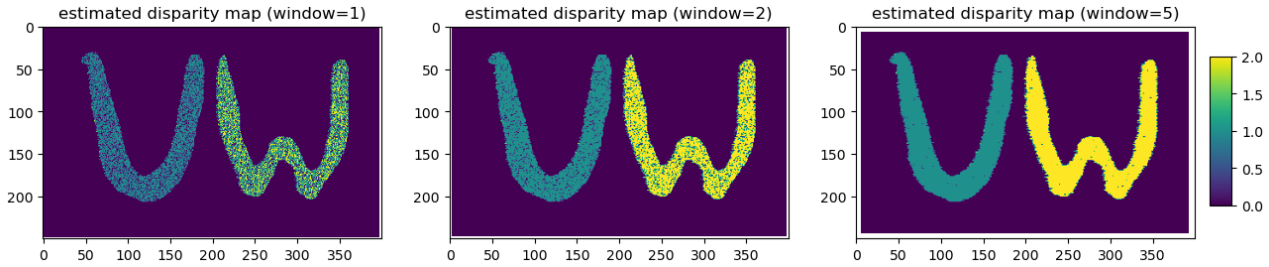
Problem 5: write code for function computing "disparity map" from SSD arrays (as above) for each disparity in the range specified by integers d_{min}, d_{max}. It should return a disparity map (image). At each pixel, disparity map image should have disparity value corresponding to the minimum SSD at this pixel. For pixels at the margins, the disparity map should be set to zero. HINT: margin pixels are characterized by $INFTY$ values of SSD.

In [11]:
```
# You should use functions np.where (pointwise "if, then, else" operation) a
# These functions will help to avoid double loops for traversing the pixels.
# WARNING: there will be a deducton for double-loops traversing pixels, but

def SSDtoDmap(SSD_array, d_minimum, d_maximum):
    records = np.full(np.shape(SD[0]),INFTY)
    dMap = np.full(np.shape(SD[0]),INFTY)
    for i in range(SSD_array.shape[0]):
        dMap = np.where(records<=SSD_array[i], dMap, i+d_minimum).copy()
        records = np.where(records<=SSD_array[i], records, SSD_array[i]).cop

    return dMap
```

Compute and show disparity map (fully implemented)

In [12]:
```
dMap1 = SSDtoDmap(SSD1,d_min,d_max)
dMap2 = SSDtoDmap(SSD2,d_min,d_max)
dMap5 = SSDtoDmap(SSD5,d_min,d_max)

fig = plt.figure(figsize = (15, 3))
plt.subplot(131)
plt.title("estimated disparity map (window=1)")
plt.imshow(dMap1, vmin = 0, vmax = d_max)
plt.subplot(132)
plt.title("estimated disparity map (window=2)")
plt.imshow(dMap2, vmin = 0, vmax = d_max)
plt.subplot(133)
plt.title("estimated disparity map (window=5)")
plt.imshow(dMap5, vmin = 0, vmax = d_max)
plt.colorbar(cax=plt.axes([0.91, 0.25, 0.015, 0.5]))
```

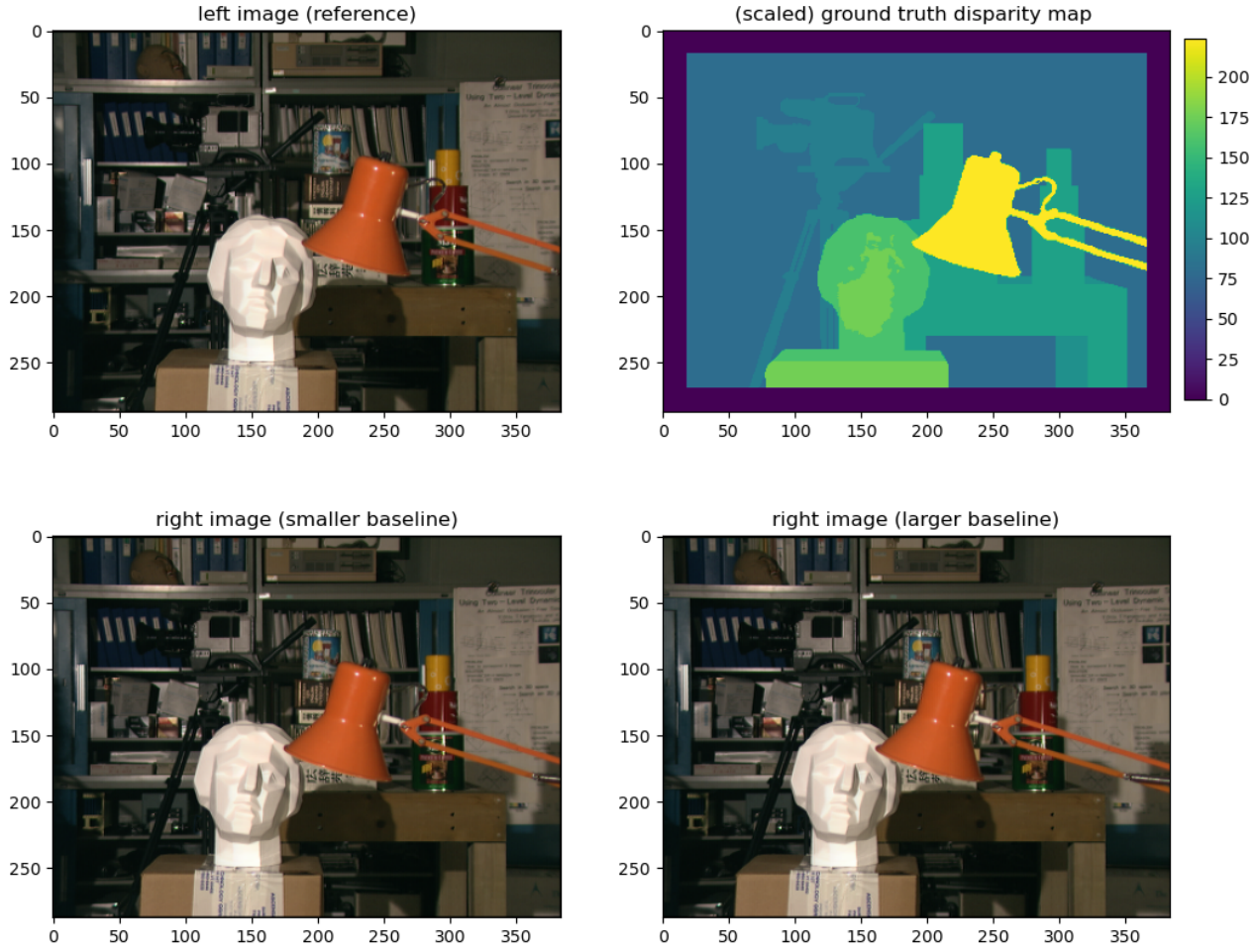
Out[12]:
```
<matplotlib.colorbar.Colorbar at 0x7f895811b3d0>
```



Problem 6: test your code on a real stereo pair with ground truth (Tsukuba)

In [13]:
```
# images/tsukuba subdirectory contains (a subset of) "Tsukuba" stereo images
# the oldest stereo data with dense ground-truth produced at the University
# The full Tsukuba dataset and many other stereo images with ground-truth di
# downloaded from well-known Middlebury repository  http://vision.middlebury
im_left = image.imread("images/stereo_pairs/tsukuba/scene1.row3.col3.ppm")
im_gt = image.imread("images/stereo_pairs/tsukuba/truedisp.row3.col3.pgm")
im_right = image.imread("images/stereo_pairs/tsukuba/scene1.row3.col4.ppm")
im_right2 = image.imread("images/stereo_pairs/tsukuba/scene1.row3.col5.ppm")

fig = plt.figure(figsize = (12, 10))
plt.subplot(221)
plt.title("left image (reference)")
plt.imshow(im_left)
plt.subplot(222)
plt.title("(scaled) ground truth disparity map ")
plt.imshow(im_gt)
plt.colorbar(cax=plt.axes([0.91, 0.557, 0.015, 0.3]))
plt.subplot(223)
plt.title("right image (smaller baseline)")
plt.imshow(im_right)
plt.subplot(224)
plt.title("right image (larger baseline)")
plt.imshow(im_right2)
```

Out[13]:
```
<matplotlib.image.AxesImage at 0x7f89994c8550>
```



Note that the integer-valued ground truth image above represents scaled disparity values for the pixels in the reference (left) mage. The scale w.r.t. the smaller baseline right image (im_right) is 16 and for the larger baseline image (im_right2) is 8. Below, you should use the smaller-baseline right image (im_right).

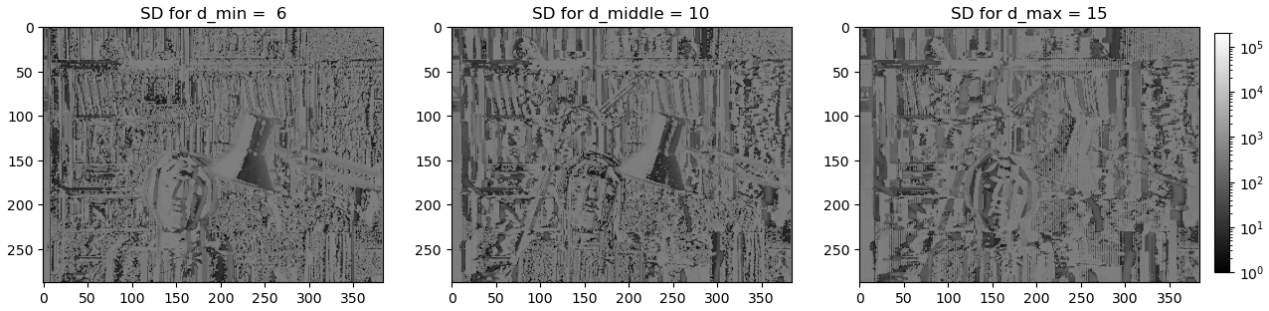
Problem 6a: Using ground truth disparity map, estimate the range of disparity values between pixels in the left image (im_left) and the right image (im_right) .

In [14]:
```
# Solution: use standard functions to find min and max values in the ground
# You should ignore 0-valued margin!

d_min = 6 # change me
d_max = 15 # change me
```

Compute squared differences using $\mathrm{SD\_array}$ function and visualize the results using logarithmic scale. Note that linear scale would make it hard to see smaller squared differences since there are many very large ones. (fully implemented)

In [15]:
```
SD = SD_array(im_left, im_right, d_min, d_max)

fig = plt.figure(figsize = (15, 4))
plt.subplot(131)
plt.title('SD for d_min = {:>2d}'.format(d_min))
plt.imshow(im_left)
plt.imshow(1+SD[0], cmap = "gray", norm=LogNorm(vmin=1, vmax=200000))
plt.subplot(132)
d_middle = round((d_min+d_max)/2)
plt.title('SD for d_middle = {:>2d}'.format(d_middle))
plt.imshow(1+SD[d_middle-d_min], cmap = "gray", norm=LogNorm(vmin=1, vmax=20
plt.subplot(133)
plt.title('SD for d_max = {:>2d}'.format(d_max))
plt.imshow(1+SD[d_max-d_min], cmap = "gray", norm=LogNorm(vmin=1, vmax=20000
plt.colorbar(cax=plt.axes([0.91, 0.2, 0.01, 0.6]))
```

Out[15]:
```
<matplotlib.colorbar.Colorbar at 0x7f89aa9144c0>
```



Problem 6b: Explain the differences you observe above:

answer: For d_min = 6, the background is the most clearly idenfiable one, and its the only one for me that the video camera can be recognized. As d_middle = 10, the background has more noise than the d_min one, but the sculpture seems more clearly compared to the d_min and d_max. Ford_max = 15, there are much more noise compared to the d_middle one, the only thing that can be clearly identified is the lamp.

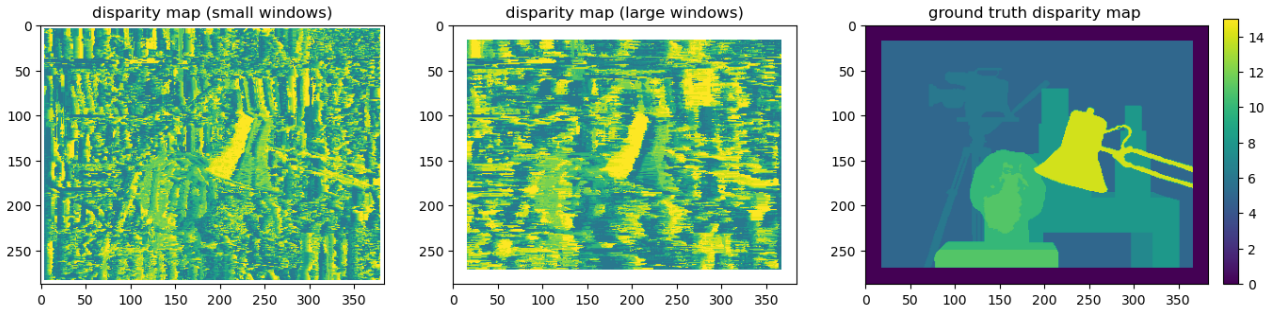
Problem 6c: Write function Dmap_Windows that returns disparith map from a given stereo pair (left and right image), specified disparity range, and window size. Your implementation should combine functions implemented and debugged earlier (SD_array, windSum, and SSDtoDmap).

In [16]:
```
def Dmap_Windows(imageL, imageR, d_minimum, d_maximum, window_width):
    SD = SD_array(imageL, imageR, d_minimum, d_maximum)
    SSD = np.zeros(np.shape(SD))
    for Delta in range(1+d_max-d_min):
        SSD[Delta] = windSum(SD[Delta],window_width)
    dMap = SSDtoDmap(SSD,d_minimum,d_maximum)
    return dMap
```

Compute and show disparity maps for Tsukuba using small and large windows. (fully implemented)

In [17]:
```
dispMap_small = Dmap_Windows(im_left, im_right, d_min, d_max, 4)
dispMap_large = Dmap_Windows(im_left, im_right, d_min, d_max, 15)

fig = plt.figure(figsize = (16, 7))
plt.subplot(131)
plt.title("disparity map (small windows)")
plt.imshow(dispMap_small, vmin = 0, vmax = d_max)
plt.subplot(132)
plt.title("disparity map (large windows)")
plt.imshow(dispMap_large, vmin = 0, vmax = d_max)
plt.subplot(133)
plt.title("ground truth disparity map ")
plt.imshow(im_gt/16, vmin = 0, vmax = d_max)
plt.colorbar(cax=plt.axes([0.91, 0.3, 0.01, 0.4]))
```

Out[17]:
```
<matplotlib.colorbar.Colorbar at 0x7f897850a700>
```



Part II: Scan-line stereo $\longleftarrow 30\ pts$

Problem 7(a): Program $Viterbi$ approach discussed in class and apply it to Tsukuba example. For the photo-consistency term of the loss function (objective function) you can use previously implemented $\mathrm{SD\_array}$

$$D_p(d) = |I_p - I_{p+d}|^2 \qquad \longleftarrow \qquad \mathrm{SD\_array}[d][p]$$

that for every pixel p defines the cost for every possible disparity value $d \in \{d_{min}, \ldots, d_{max}\}$. The regularization term should be

$$V_{pq}(d_p, d_q) = w|d_p - d_q|$$

where you can select some value for parameter w empirically (start from $w \approx 0$). Discuss the differences with the results of the window-based stereo above.

NOTE: You should implement $Viterbi$ optimization yourself - it was fully covered in class. Organize your code (e.g. add cells, introduce functions, write comments, ect) as part of your mark will depend on clarity. The main iteration of the forward pass in Viterbi (m^2-complexity operation for each pair of neighboring pixels on a scan-line) can be implemented as a separate function. You can avoid double for-loops using functions like $np.where$, $np.minimum$, $np.square$, $np.ogrid$ or others similar general "vectorized" functions in numpy that alow to avoid multi-loops over matrix (image) elements (pixels).

In [ ]:

Problem 7(b): Test the case where the photoconsistency term $D_p(d)$ is computed by averaging SD in some small window of size h. That is, for each disparity d you should replace 2D array $\mathrm{SD\_array}(d)$ in Problem 7(a) by

$$\mathrm{SD\_array}(d) \quad \longleftarrow \quad windSum(SD(d), h).$$

Compare the results for different window sizes $h \in \{1, 3, 5\}$. Feel free to fine tune regularization parameter w for each case trying to obtain the best results possible.

NOTE 1: $h = 1$ should be equivalent to Problem 7(a) above.

NOTE 2: this version combines window-based stereo with regularization along scan-lines. The case when $w = 0$ should give the same results as in Problem 6(c).

In [ ]:

Problem 8 (aligning disparity boundaries with intensity edges): test the following case where regularization weights w_{pq} depend on a specific pair of neighboring points

$$V_{pq}(d_p, d_q) = w_{pq}|d_p - d_q|$$

rather than all being equal to one constant $w_{pq} = w$, as in Problem 7. For such locally adaptive regularization weights w_{pq} it is common to use local intensity contrast (in the reference image)

$$w_{pq} = w \exp \frac{-\|I_p - I_q\|^2}{2\sigma^2}$$

which weighs the overall regularization constant w by a Gaussian kernel (in RGB or grey-scale space). The latter makes it cheaper to draw large disparity jumps at "contrast edges" or "contrast boundaries" that are likely to happen at object boundaries. Note that bandwith parameter σ is important - it controls sensitivity to contrast edges. Discuss the results.

In [ ]:

Problem 9 [optional, small bonus]: experiment with quadratic and (robust) truncated-quadratic regularization terms

$$V_{pq}(d_p, d_q) = w_{pq}|d_p - d_q|^2 \quad \text{and} \quad V_{pq}(d_p, d_q) = w_{pq}\, min\{|$$

Discuss the observed differences in the results, if any.

In [ ]: