CS489/698 - Assignment 2 - Spring 2023

Date Due: *Tuesday, July 11th, 2023, @8AM.*

Topics Covered and General Instructions: Probability density functions, radiometric concepts and stochastic computation of form factors.

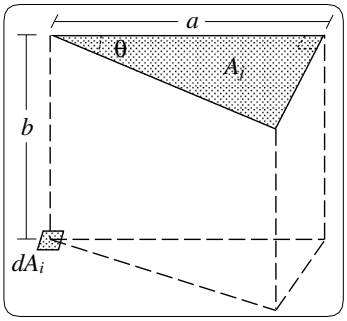
General Instructions:

- For the generation of random numbers, use the Matlab function *rand* with the specified *seed*.
- Your computation of form factors should employ a Monte Carlo approach based on ray casting. You should assume that all surfaces are diffuse, *i.e.*, the rays should follow a cosine distribution.

Questions

1. **(25 points)** Consider a simple two dimensional model of neutrons traversing a lead wall. It is assumed that each neutron enters the wall at a right angle (normal direction), and travels a unit distance. It then collides with a lead atom, and it is scattered in a random direction. After this scattering event, it travels a unit distance before colliding with another lead atom and being scattered in a random direction again. This process continues until all the neutron energy is spent, which you can assume that occurs after 6 collisions. You can also assume that the lead wall is 4 units thick in the x direction and infinitely thick in the y direction. Write a Matlab program ($seed = 2$) to determine the percentage of "surviving" neutrons (s), *i.e.*, those that can be expected to emerge from the other side of the lead wall, and plot the graph of $s(\%)$ as a function of the number of samples (random points). For the axes limits consider $n \in [1000, 10000]$ and $s(\%) \in [0, 10]$. Briefly explain the reasoning behind your solution.

2. **(35 points)** Write a Matlab program to compute the form factor F linking a differential element dA_i to a right triangle A_j in a plane parallel to the plane of dA_i, with the normal of dA_i passing through a vertex of A_j as indicated in the figure below. For this question, consider $a = 2m$, $b = 3m$, $\theta = 30°$ and dA_i located in the point $(0, 0, 0)$. Plot the graph of F as a function of the number n of rays. For the axes limits consider $n \in [1000, 100000]$ and $F \in [0, 0.04]$, and use $seed = 1$. Provide a detailed explanation of your solution, your documented Matlab program and the resulting plot.



3. **(10 points)** What is the relative error of your result (for $n = 100000$) obtained in question 2 with respect to the analytical value obtained for the same geometry? Your values can be rounded to the fourth decimal digit. Provide the analytical **closed** formula used in your calculation and its source/reference (from a book or research paper).

4. **(10 points)** Assuming that the triangle in question 2 has a reflectance equal to 0.25, and the power emitted by dA_i is equal $100W$, what is the value of its radiance? Mathematically justify your answer. Your result can be rounded to the second decimal digit.

5. (20 points) Consider a disk of radius R in which random points are distributed according to probability density function given by $p(x, y) = k(1 - r/R)$, where r represents the radial distance of the point from the disk center. What is the probability of having points with $r < R/2$? Provide a detailed derivation of your solution, which should be a closed formula (it should not allow any further simplification).

Bonus Question (20 points) Consider a probability density function given by

$$p(\theta, \phi) = \frac{1}{4\pi} \left(\frac{1 - g^2}{(1 + g^2 - 2g\cos\theta)^{\frac{3}{2}}} \right).$$

Derive warping functions for the polar angle $\theta \in [0, \pi]$ and the azimuthal angle $\phi \in [0, 2\pi]$ given in terms of random variables ξ_1 and ξ_2 uniformly distributed in $[0, 1]$. You can assume that p is separable and azimuthally symmetric. Provide a detailed derivation of your solution.

Hand In

- Students should prepare an assignment report (to be prepared using Latex) and submit it in a PDF format using Learn. The report should include the answers to the questions, plots (when required in the specification of a given question) and the Matlab programs (use the latex verbatim environment).
- The report should be identified using the terms "A2" followed by the student's last name and the last three digits of her/his ID (*e.g.,* A2-Bond-007.PDF).
- The student's name and ID should be also provided at the top of the PDF document as well.
- The files should be submitted via Learn following the steps indicated below:
  1. Log into Learn using their WatIAm credentials.
  2. Select "`CS 489 - Spring 2023 CS489_gvgbaran_1235`" from the course selector.
  3. Choose "Dropbox" from the "Submit" menu.
  4. Select the "`Assignment 2`" folder.
  5. Click "Add a File".
  6. Click "Upload" to use a file picker to choose the file, or drag-and-drop the file into the upload window.
  7. Click "Add".
  8. Click "Submit".
- Assignment submissions that do not follow these instructions will loose marks.

Collaboration

This assignment must be done individually. You can modify existing algorithms and/or use algorithms that you find from reputable sources (which include books, but not your peers). When someone else's code is used like this, you must acknowledge the source explicitly in your written report.