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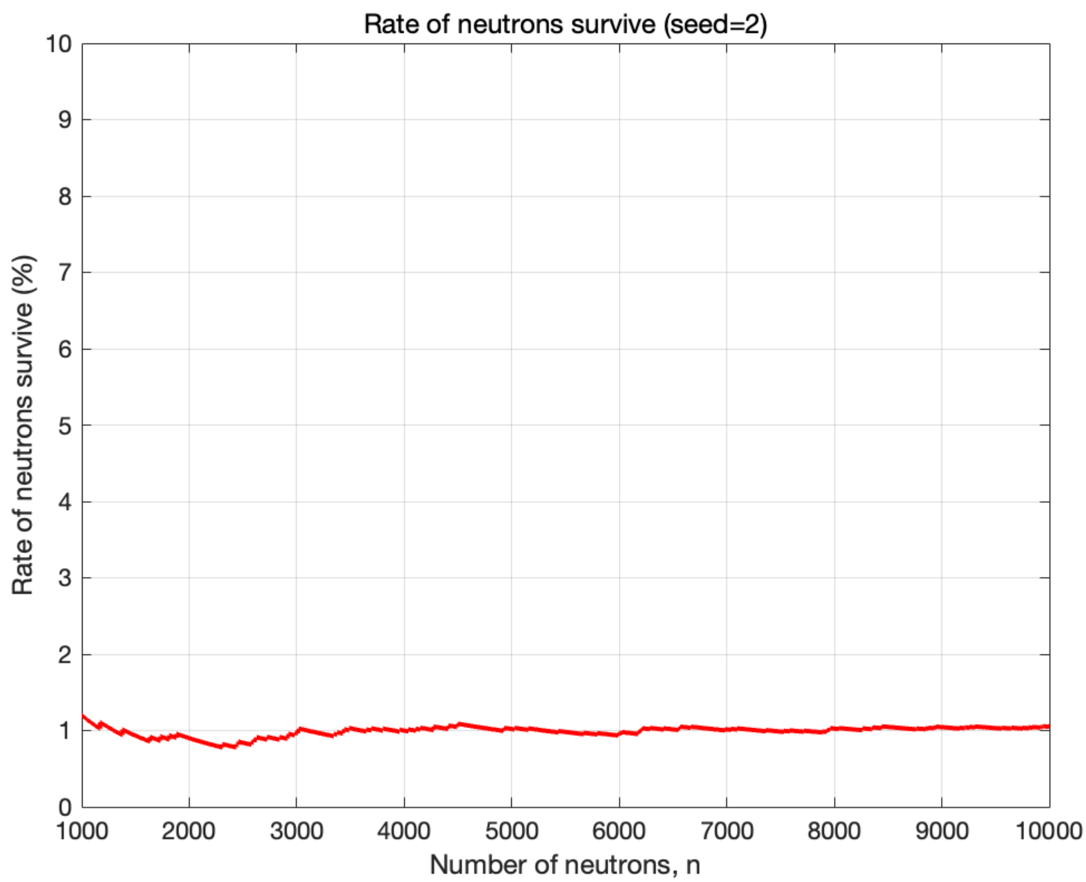
Question 1

```
close all;
nSamples=10000;
seed=2;
rand('seed',seed);
NSurvive=0;
rateSurvive=zeros(nSamples,1);

for n=1:nSamples
    xDist=0;
    for i=1:6
        r=rand(1,1);
        direction=2*pi*r;
        xDist=xDist+cos(direction);
        if(xDist>=4)
            NSurvive=NSurvive+1;
            break;
        end
    end
    rateSurvive(n)=100*NSurvive/n;
end

plot(rateSurvive, 'r-', 'linewidth', 2);
title(sprintf('Rate of neutrons survive (seed=%d)',seed));
xlabel('Number of neutrons, n');
ylabel('Rate of neutrons survive (%)');
axis([1000,10000 0 10]);

grid on

pause
print -dpsc2 volumn.eps
close
```

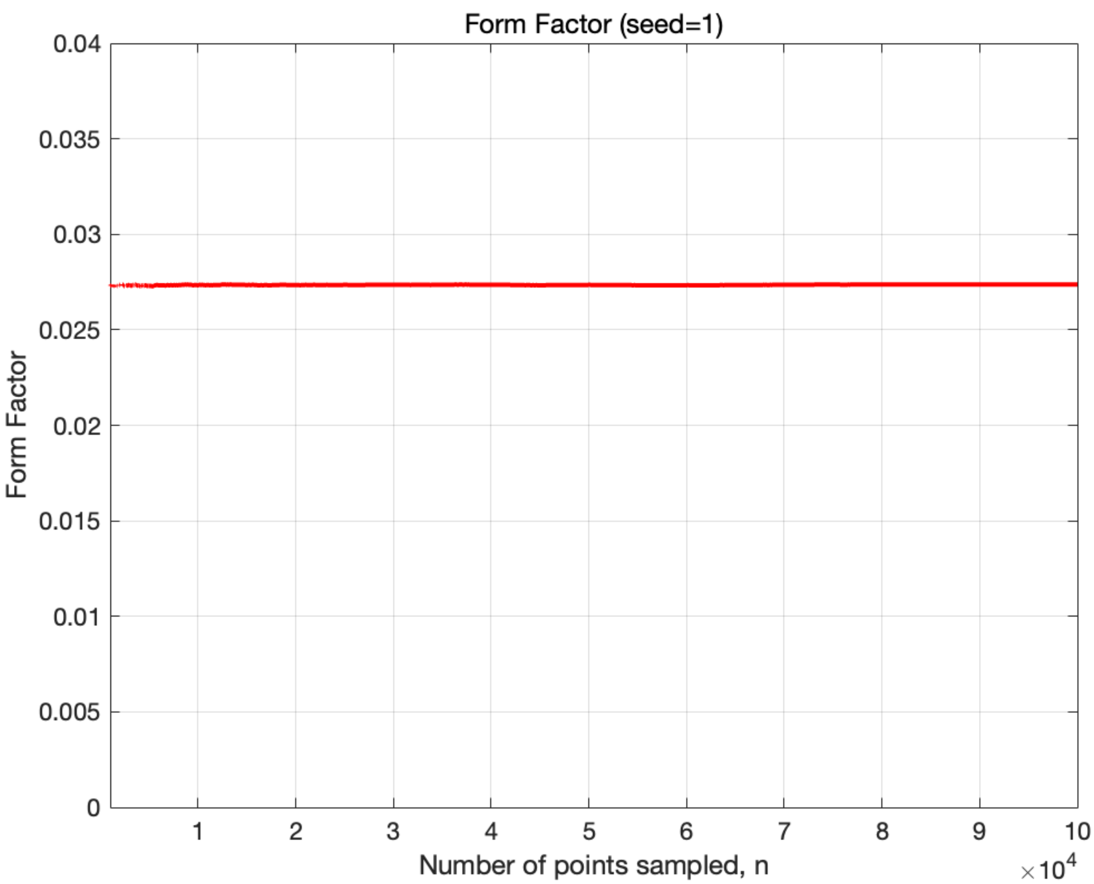
- For each neutrons, process a loop for 6 time. Each time randomly select a direction, then move 1 unit length long, and recorded the x-change in to xDist, and do a check whether $xDist \geq 4$ (if yes, then the neutron survives , otherwise, continue the loop until all energy is spent).


Rate of neutrons survive (seed=2)
Rate of neutrons survive (%)
Number of neutrons, n
0
1
2
3
4
5
6
7
8
9
10
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

Question 2

```
close all;
nSamples=100000;
seed = 1;
rand('seed',seed);
FormFac=zeros(nSamples,1);
sum=0;

for n=1:nSamples
    % when sampling points in the right triangle, we can simply sample in
    % the rectangle, for those points out of triangle, we can map it in to
    % the corresponding points in the triangle due to the centrosymmetric
    % to guarantee the sampling is uniform.
    r1=rand(1,1);
    x=2*r1;
    r2=rand(1,1);
    y=r2*(-2)/sqrt(3);
    if(y<(x*(-1)/sqrt(3)))
        % this case the point is out of triangle
       x=2-x;
       y=((-2)/sqrt(3))-y;
    end
    % due to parallel, the two angles in the form factor formula should be
    % the same, for simplicity, we will calculate the cos of it by
    % definition, which is b divided by the distance between random sampled point to
    % origin. Also, visibility term is 1 due to no obstacle.
    dist=sqrt(x^2+y^2+3^2);
    cosepsilon=3/dist;
    sum=sum+(cosepsilon^2/(pi*(dist^2)));
    FormFac(n)=(sum/n)*(2/sqrt(3));
end

plot(FormFac, 'r-', 'linewidth', 2);
title(sprintf('Form Factor (seed=%d)',seed));
xlabel('Number of points sampled, n');
ylabel('Form Factor');
axis([1000,100000 0 0.04]);

grid on
pause
print -dpsc2 FormFac.eps
close
```

- The method of sampling and getting cosine is mentioned in the comment.
- For mapping the out-of-range point to in-range point, the method is do a symmetric based on the center of the rectangle (i.e. for old point (x_{old}, y_{old}), do a symmetric based on (x_c, y_c), the result should be $(2x_c - x_{old}, 2y_c - y_{old})$). Since we are doing all things in a same plane (z=3), so z-coordinate is not a problem, it always remains 3
- Also, the result seems converges really fast (about n=300).


Form Factor (seed=1)
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Form Factor
1
2
3
4
5
6
7
8
9
10
Number of points sampled, n
×10^4

Question 3

- The result obtained in question 2 when n=100000 is 0.0274
- For closed form, $F_{dA_i->A_j} = \frac{1}{2\pi}\sum_{i=1}^{n} \beta_i \cos\alpha_i$, where α_i is the angle subtended by v_i, v_{i+1} from origin, and β_i is the angle between the plane containing $v_i, vi+1$ and origin, and the normal to the differential element.
- in the specific case of Q2, we have $\cos\alpha_1 = 0, \cos\alpha_3 = 0$
- Also, since $\cos\alpha_1 = \cos\alpha_3 = 0, \beta_1$ and β_2 do not need to be calculated for the final answer.
- Note that $\beta_2 = arctan(\frac{\frac{2}{\sqrt{3}}}{\sqrt{13}})$
- To get $\cos\alpha_2$, first we want the normal vector for plane which contain v_2, v_3, and origin
- Note that vector (2,0,3) and vector (0,1,0) is in the plane, thus normal vector can be easily get from observing, thus the normal vector is (-3,0,2)
- Next, we have $\cos\alpha_2 = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|} = \frac{2}{\sqrt{13}}$
- Finally, substitue all values in to the formula we get $F = 0.0273621155$
- Relative error of my result is $\frac{0.0274-0.0273621155}{0.0273621155} \approx 0.1385\%$
- Reference for this question:

- Stark, Michael M. and Richard F. Riesenfeld. "Exact Radiosity Reconstruction and Shadow Computation Using Vertex Tracing." (1999).
- Cohen, Michael F. and John R. Wallace. "Radiosity and realistic image synthesis." (2016) pp.70.

Question 4

- According to $L_i = \frac{\rho_i}{\pi} \frac{\Phi_i}{A_i}$
- the value of radiance of triangle should be $\frac{0.25}{\pi} \frac{0.0274 \times 100W}{0.5 \times 2 \times \frac{2}{\sqrt{3}}} \approx 0.1888 \frac{W}{m^2 sr} \approx 0.19 \frac{W}{m^2 sr}$

Question 5

$$\int_0^{2\pi}\int_0^{R} k(1-\frac{r}{R})drd\theta = k\cdot\int_0^{2\pi}\int_0^{R}(1-\frac{r}{R})drd\theta \tag{1}$$

$$= k\cdot\int_0^{2\pi}[r-\frac{r^2}{2R}]_0^R d\theta \tag{2}$$

$$= k\cdot\int_0^{2\pi}\frac{R}{2}d\theta \tag{3}$$

$$= \pi kR = 1 \implies k = \frac{1}{\pi R} \tag{4}$$

$$\int_0^{2\pi}\int_0^{\frac{R}{2}} k(1-\frac{r}{R})drd\theta = k\cdot\int_0^{2\pi}\int_0^{\frac{R}{2}}(1-\frac{r}{R})drd\theta \tag{5}$$

$$= k\cdot\int_0^{2\pi}[r-\frac{r^2}{2R}]_0^{\frac{R}{2}} d\theta \tag{6}$$

$$= k\cdot\int_0^{2\pi}\frac{3R}{8}d\theta \tag{7}$$

$$= k\cdot\frac{3}{4}R\pi = \frac{3}{4} = 0.75 \tag{8}$$

Bonus Question

$$\int_0^\phi \int_0^\theta \frac{1}{4\pi}(\frac{1-g^2}{(1+g^2-2gcos\theta)^{\frac{3}{2}}})d\theta d\phi \tag{9}$$

$$\int_0^\phi \frac{1}{2\pi}d\phi = \frac{\phi}{2\pi} \tag{10}$$

$$= \xi_2 \implies \phi = 2\pi\xi_2 \tag{11}$$

$$\int_0^\theta \frac{1}{2}\frac{1-g^2}{(1+g^2-2gcos\theta)^{\frac{3}{2}}}d\theta \tag{12}$$