CS489/698 - Assignment 1 - Spring 2023

Date Due: *Tuesday, June 6th, 2023, @8AM.*

Topics Covered and General Instructions

   Introduction to Monte Carlo concepts and techniques.

General Instructions

   The questions should be solved using Monte Carlo techniques. Analytical solutions can be used to check the correctness of your answers. For the generation of random numbers use the Matlab function *rand*. All plots should be properly labeled.

Questions

1. **(20 points)** Write a Matlab program to generate and plot 2000 random points uniformly distributed inside the ellipse defined by $x^2 + 4y^2 = 4$. For the axes limits consider $x \in [-2, 2]$ and $y \in [-1, 1]$. Provide the resulting plot.

2. **(20 points)** Consider a sequence of numbers $x_1, x_2, x_3, \cdots$ uniformly distributed in the interval (0,1). Write a Matlab program to determine what proportion of these numbers would satisfy the inequality $40x^2 + 7 > 43x$ considering:

   - 1000 random points.
   - 10000 random points.

   Provide the estimate values. Plots do not need to be provided.

3. **(30 points)** Consider the region whose points satisfy the following inequalities:

$$\begin{cases} 0 \leq x \leq 1 \quad 0 \leq y \leq 1 \quad 0 \leq z \leq 1 \\ x^2 + \sin y \leq z \\ x - z + e^y \leq 1 \end{cases}$$

   Write a Matlab program to compute the volume (v) of the region described above, and plot the graph of v as a function of the number of samples (random points). For the axes limits, consider $n \in [1000, 10000]$ and $v \in [0.1, 0.2]$.

4. **(40 points)** Consider a stick of unit length dropped onto a board ruled by parallel lines one unit length apart. Assume that the board is infinitely long in the x and y directions. You can also assume that the center of the stick lands between lines at a random point, the angular orientation of the stick is another random variable, and all random variables are drawn from a uniform distribution. Write a Matlab program to determine the probability (p) that the stick intersects one of the lines, and plot the graph of the p values as a function of the number of samples (random points). For the axes limits consider $n \in [1000, 100000]$ and $p(\%) \in [60, 70]$.

Briefly explain the reasoning behind your solution (make sure that your program is well documented).

Hand In

- Students should prepare an assignment report (using Latex) and submit it in a PDF format using Learn. The report should include the answers to the questions, plots (when required in the specification of a given question) and the Matlab programs (use the latex verbatim environment).

- The report should be identified using the terms "A1" followed by the student's last name and the last three digits of her/his ID (*e.g.,* A1-Bond-007.txt, A1-Bond-007.png and A1-Bond-007.PDF).

- The student's name and ID should be also provided at the top of the PDF document as well.

- The files should be submitted via Learn following the steps indicated below:

  1. Log into Learn using their WatIAm credentials.
  2. Select "`CS 489/698 - Spring 2023 CS489_gvgbaran_1235`" from the course selector.
  3. Choose "Dropbox" from the "Submit" menu.
  4. Select the "`Assignment 1`" folder.
  5. Click "Add a File".
  6. Click "Upload" to use a file picker to choose the file, or drag-and-drop the file into the upload window.
  7. Click "Add".
  8. Click "Submit".

- Assignment submissions that do not follow these instructions will loose marks.

Collaboration

This assignment must be done individually. You can modify existing algorithms and/or use algorithms that you find from reputable sources (which include books, but not your peers). When someone else's code is used like this, you must acknowledge the source explicitly in your written report.