CHAPTER 5

TREES

5.1 Introduction

5.1.1 Terminology

- A **tree** is a finite set of one or more nodes such that
  1) There is a specially designated node called **root**
  2) The remaining nodes are partitioned into n (≥ 0) disjoint set $T_1, \ldots, T_n$, where T_i: the **subtrees** of the root

This is a recursive definition; Every item (node) in a tree is the root of some subtree

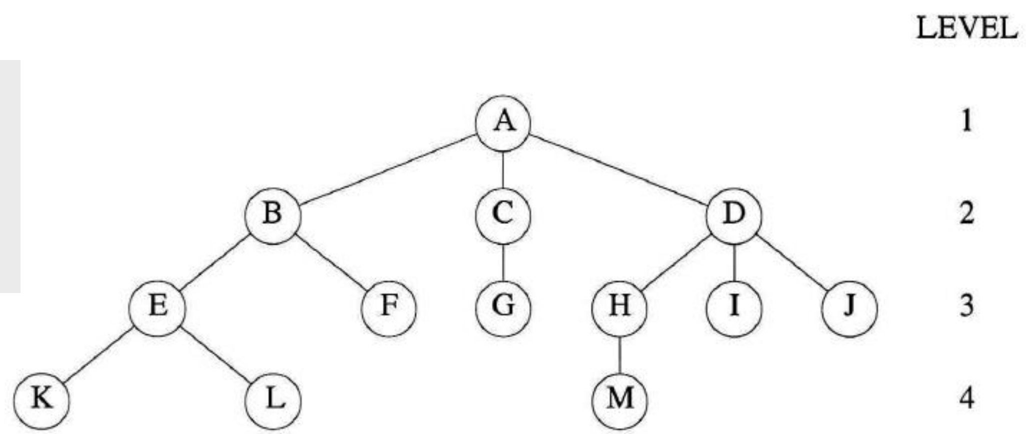


Figure 5.2: A sample tree

- Some Terminology
  - node: …
  - degree of a node: ….
  - terminal (or leaf) nodes :
  - nonterminal nodes:
  - children, parent, siblings :
  - degree of a tree
    - max{degree of the nodes}
  - ancestors : …
  - level of a node : …
  - height (depth) of a tree
    - max{level of the nodes}

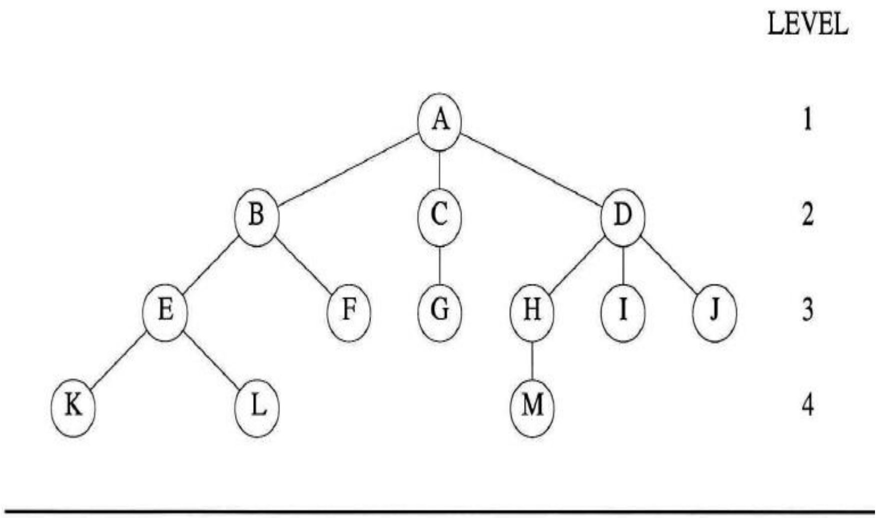
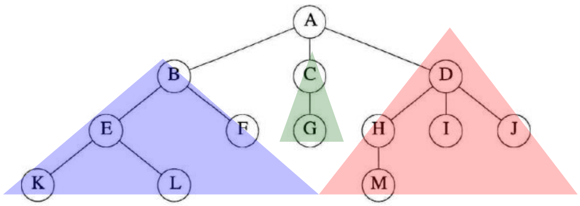


Figure 5.2: A sample tree

5.1.2 Representation of Trees



1) List Representation

- The root comes first,
  followed by a list of sub-trees
- **(A (** B (E (K, L), F), C (G), D(H (M), I, J) **))**

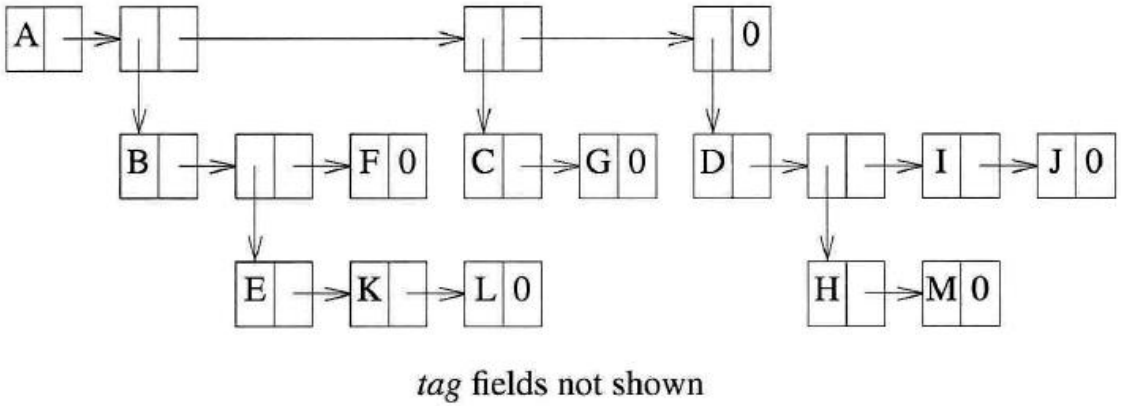


Figure 5.3: List representation of the tree of Figure 5.2

- Node representation

DATA	CHILD 1	CHILD 2	···	CHILD *k*

Figure 5.4: Possible node structure for a tree of degree *k*

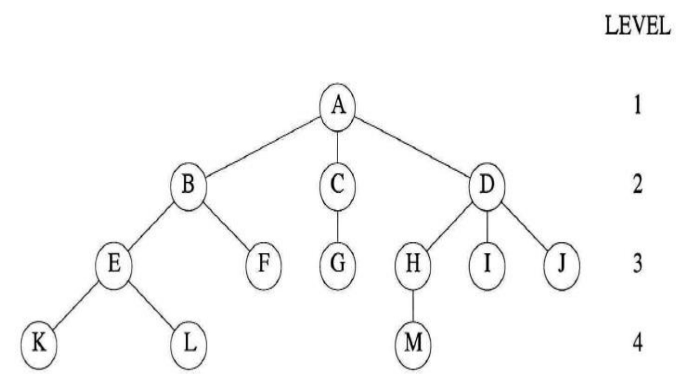


Figure 5.2: A sample tree

  - # of zero fields: $nk-(n-1)$
  - $n(k-1)+1$ of the nk child fields are 0
  - Wasteful of space

이만큼이 0이다 → 메모리 낭비

2) Left Child-Right Sibling(LCRS) Representation

이진 트리 트리

data	
left child	right sibling

Figure 5.5: Left child-right sibling node structure

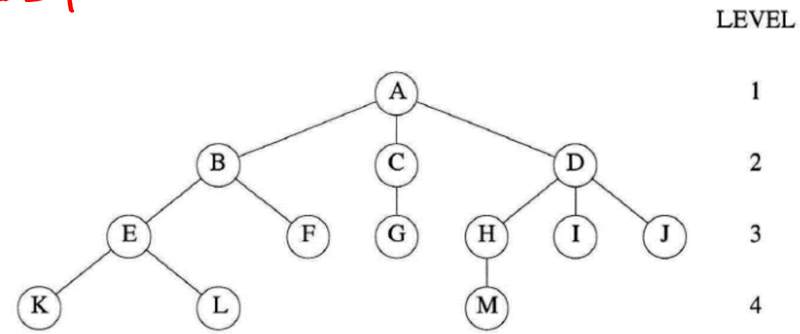


Figure 5.2: A sample tree

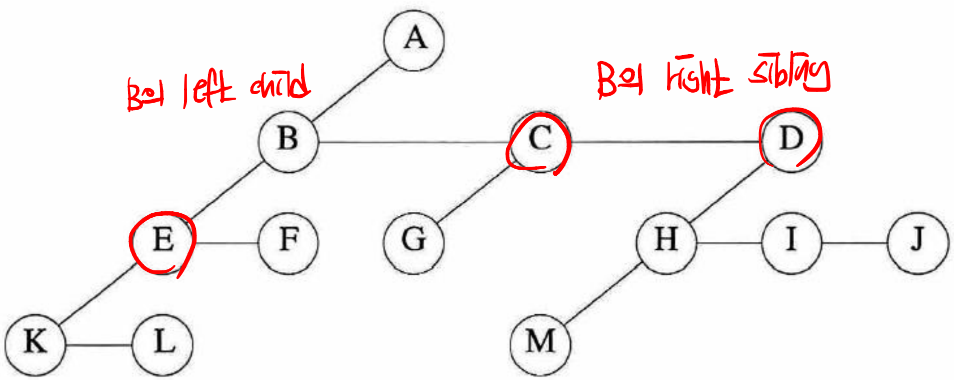


Figure 5.6: Left child-right sibling representation of tree of Figure 5.2

3) Representation as a Degree Two Trees

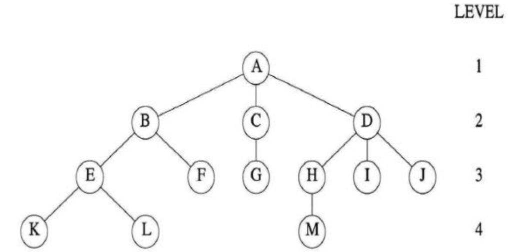
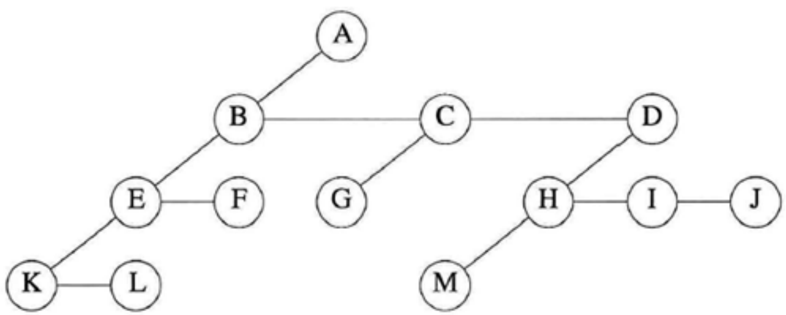
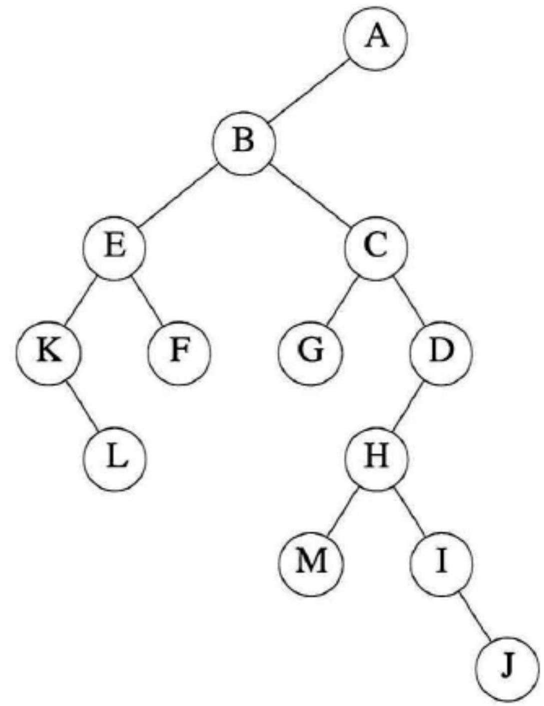


Figure 5.2: A sample tree

Data	
Left Child	Right Child





re 5.7: Left child-right child tree representation of tree of Figure 5.2

Rotate the right-sibling pointers in a LCRS tree clockwise by 45 degrees

Every tree can be transformed into a binary tree

항상 이진 트리로 바꿀 수 있음

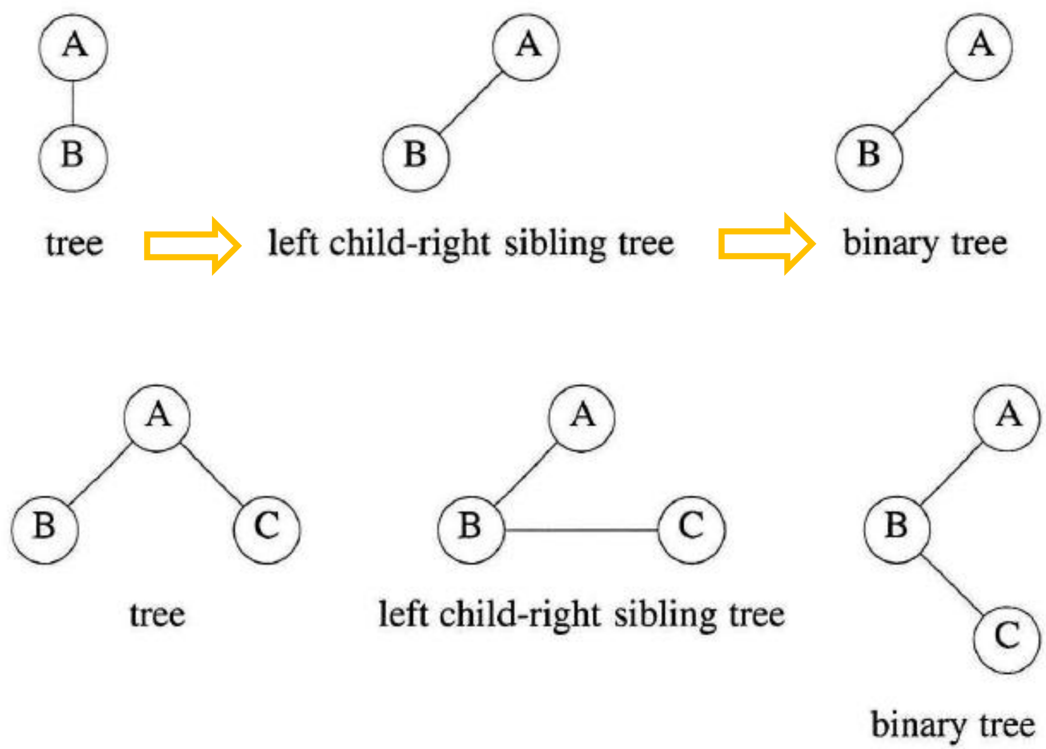
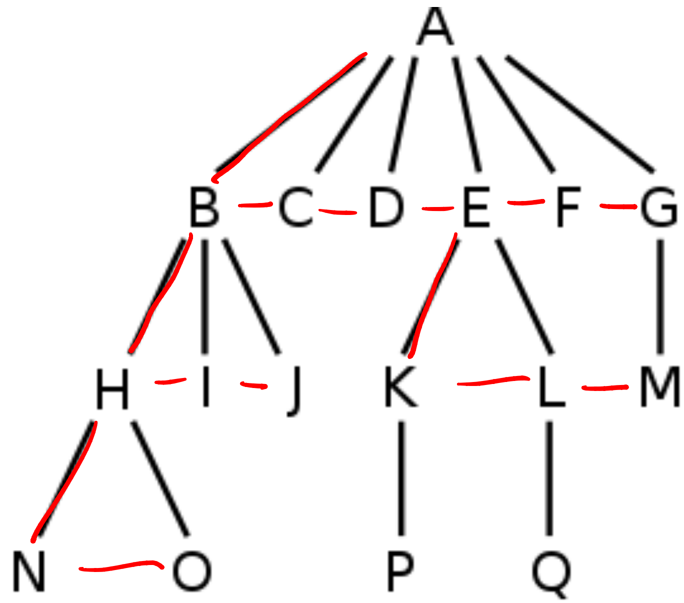


Figure 5.8: Tree representations


A
B
C
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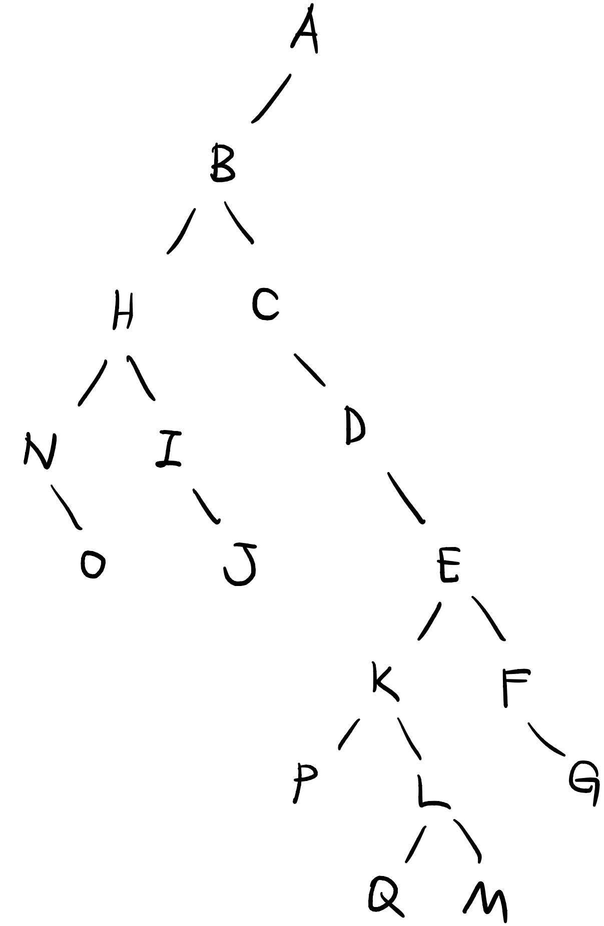
E
F
G
H
I
J
K
L
M
N
O
P
Q


A
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G
Q
M

5.1 Introduction

5.2 Binary Trees

5.3 Binary Trees Traversals

5.4 Additional Binary Tree Operations

5.5 Threaded Binary Trees

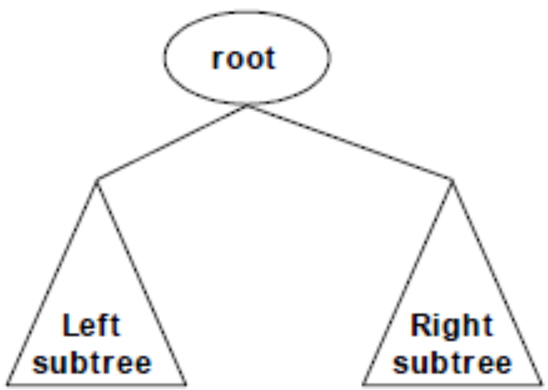
5.6 Heaps

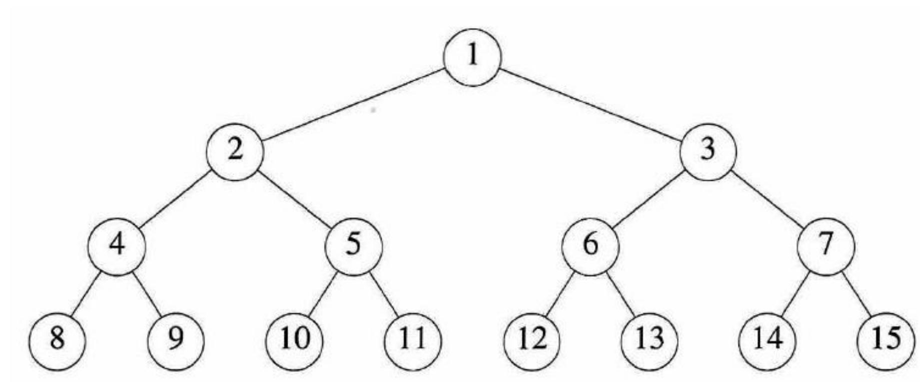
5.7 Binary Search Trees

5.2.1 The Abstract Data Type

→ 얘도 binary tree

- A **binary tree** is a finite set of nodes that is either **empty** **or** consists of a **root** and **two disjoint binary trees** (called the left subtree and the right subtree)





The degree of any given node must not exceed two

모든 node의 최대 degree가 2를 넘지않는 것

ADT *Binary_Tree* (abbreviated *BinTree*) is

objects: a finite set of nodes either empty or consisting of a root node, left *Binary_Tree*, and right *Binary_Tree*.

functions:

for all $bt, bt1, bt2 \in BinTree$, $item \in element$

BinTree Create()	::=	creates an empty binary tree
Boolean IsEmpty(*bt*)	::=	**if** (*bt* == empty binary tree) **return** *TRUE* **else return** *FALSE*
BinTree MakeBT(*bt*1, *item*, *bt*2)	::=	**return** a binary tree whose left subtree is *bt*1, whose right subtree is *bt*2, and whose root node contains the data *item*.
BinTree Lchild(*bt*)	::=	**if** (IsEmpty(*bt*)) **return** error **else return** the left subtree of *bt*.
element Data(*bt*)	::=	**if** (IsEmpty(*bt*)) **return** error **else return** the data in the root node of *bt*.
BinTree Rchild(*bt*)	::=	**if** (IsEmpty(*bt*)) **return** error **else return** the right subtree of *bt*.

ADT 5.1: Abstract data type *Binary_Tree*

- Differences between a **Tree** & a **Binary Tree**
  - There is no tree having zero nodes, but there is an empty binary tree
  - In a binary tree, we distinguish between the order of the children while in a tree we do not
  - e.g)

binary tree는 left child와 right child의 order를 구분한다

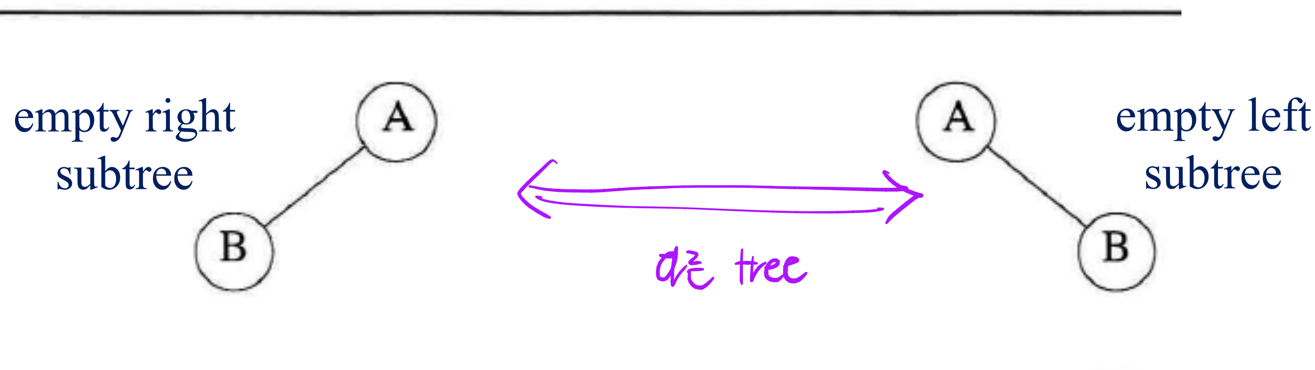


Figure 5.9: Two different binary trees

- Two special kinds of binary trees

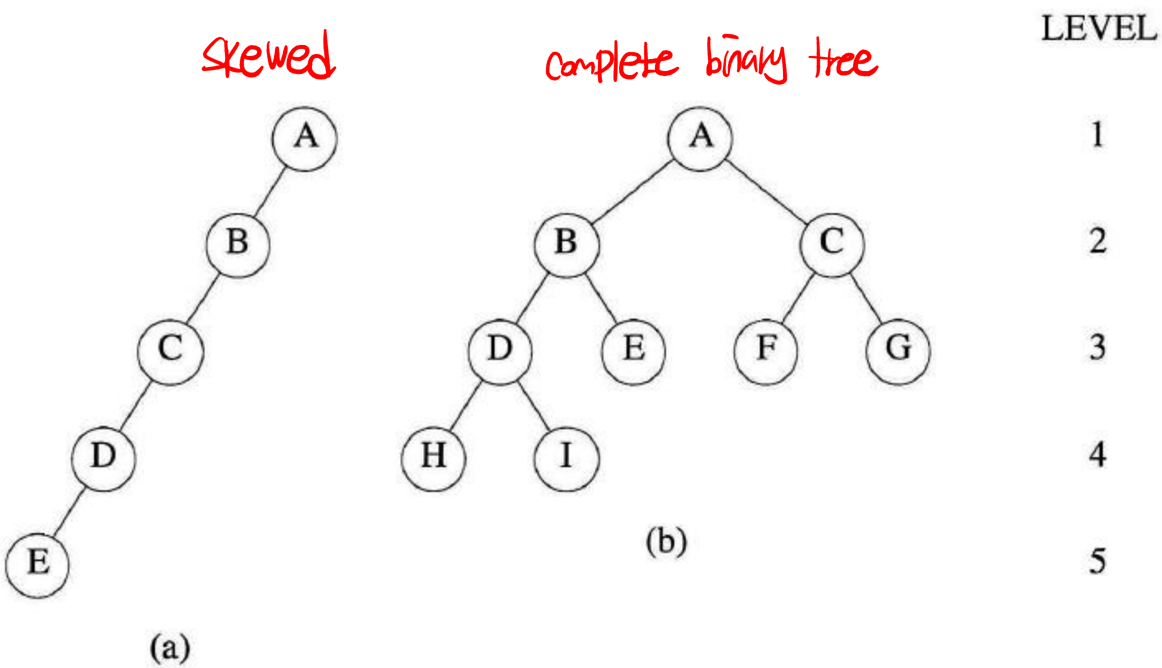
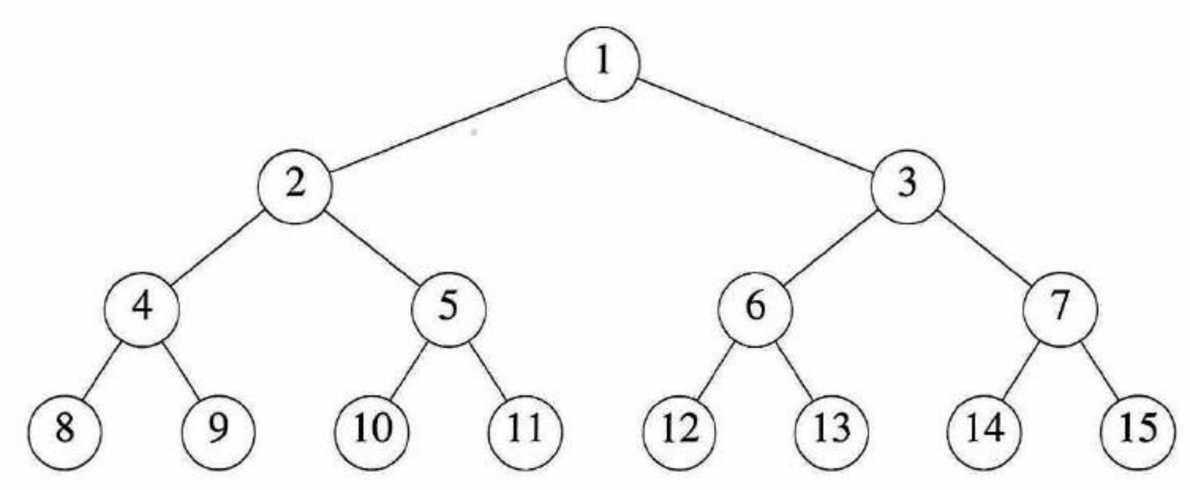


Figure 5.10: Skewed and complete binary trees

5.2.2 Properties of Binary Trees

- Max number of nodes
  - The max # of nodes on level i is 2^{i-1} $(i \geq 1)$
  - The max # of nodes in a BT of depth k is $2^k - 1$ $(k \geq 1)$

→ $$\sum_{i=1}^{k} (\text{maximum number of nodes on level } i) = \sum_{i=1}^{k} 2^{i-1} = 2^k - 1$$



1
2
3
4

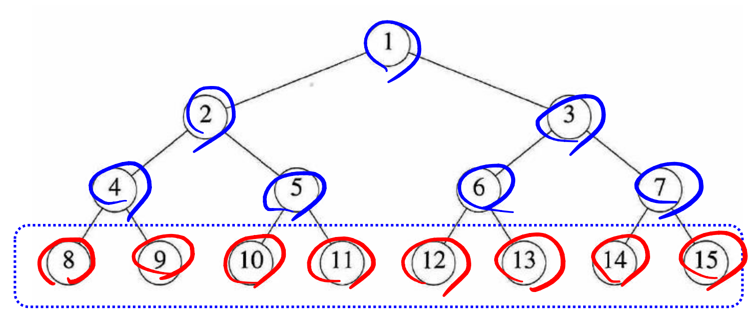
- Relation between # of leaf nodes and # of degree-2 nodes:

  $n_0 = n_2 + 1$

  – n_0 : # of leaf nodes 단말 노드=8

  – n_2 : # of nodes of degree 2

  degree가 2인 노드=7



[Proof]

  – n: total # of nodes

  – B: # of branches

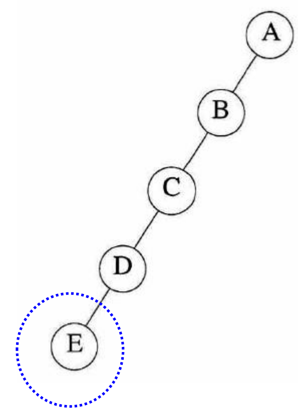
  – $n = n_0 + n_1 + n_2$

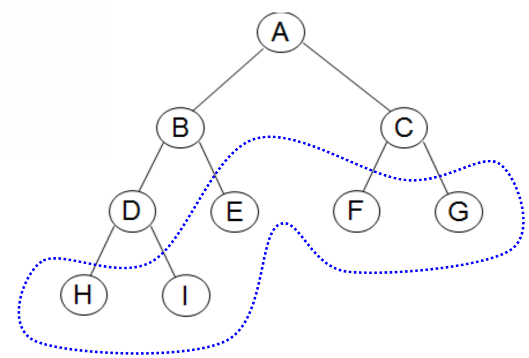
  – $n = B + 1$ B= n_1+2n_2

  $= n_1 + 2n_2 + 1$

  → $n_0 + n_1 + n_2 = n_1 + 2n_2 + 1$

  → $n_0 = n_2 + 1$





포화

- **Full** binary tree
  - Every node other than the leaves has two children.
  - BT of depth k having 2^k-1 nodes, $k \geq 0$
  - Nodes: numbered from 1 to 2^k-1

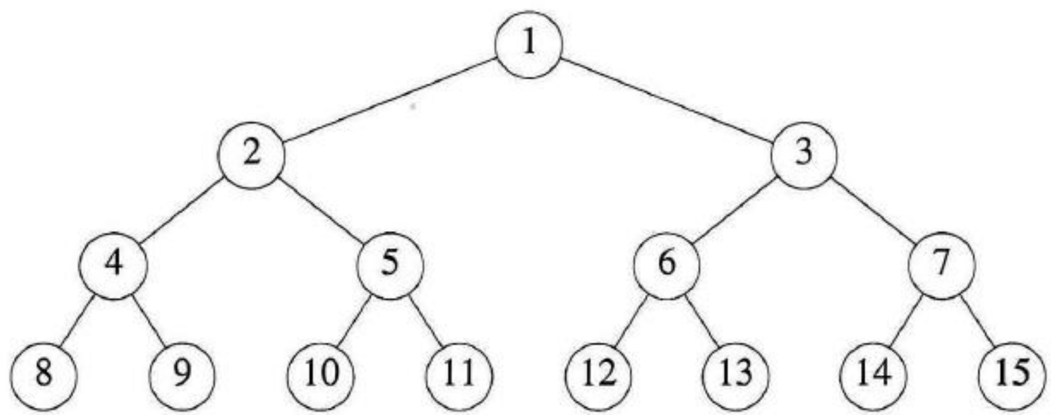


Figure 5.11: Full binary tree of depth 4 with sequential node numbers

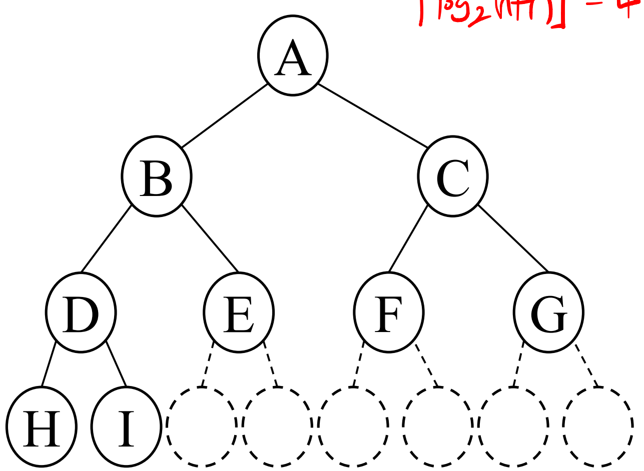
Full과 차이?

sol) Full은 각 level의 노드가 꽉 차있어야함

- **Complete** binary tree → 마지막 레벨을 제외하고 다른 레벨은 꽉차있음
  - Its nodes correspond to the nodes numbered from 1 to n in the full BT of depth k.
  - Height of a complete binary tree with n nodes: $\lceil \log_2(n+1) \rceil$ 올림

ex) $n=9$

$\lceil \log_2(n+1) \rceil = 4$



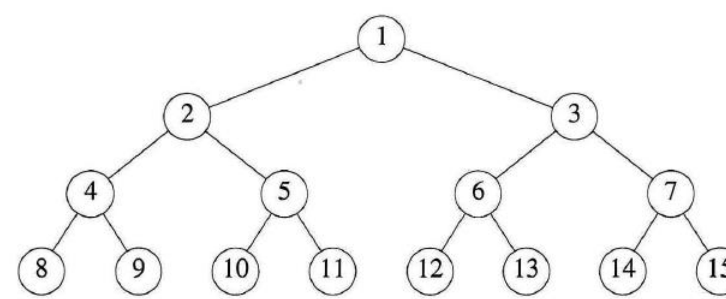
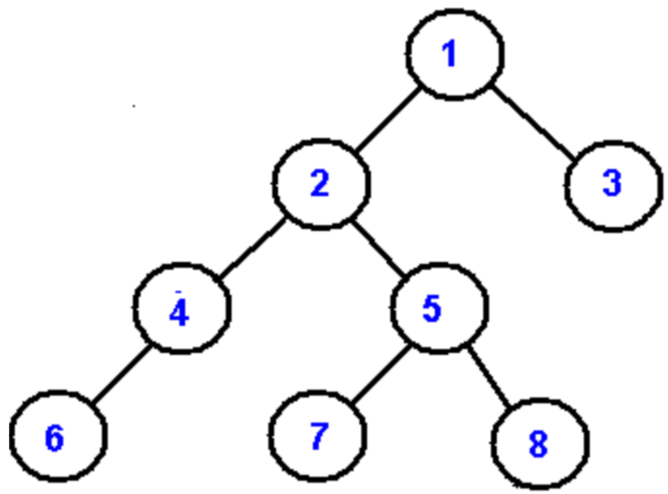


Figure 5.11: Full binary tree of depth 4 with sequential node numbers

binary tree (o)

Complete 〃 (x)

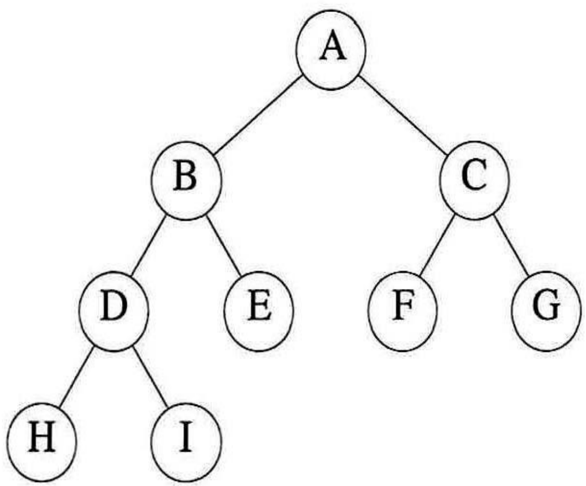
Full (x)



5.2.3 Binary Tree Representations

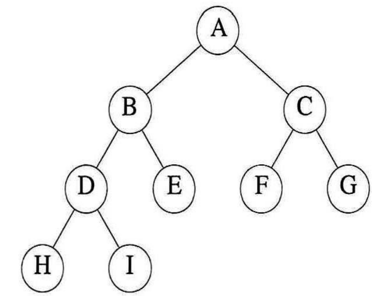
1) Array Representation

완전 이진 트리일때는 Array가 좋음



	tree
[0]	–
[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I

(b) Tree of Figure 5.10(b)

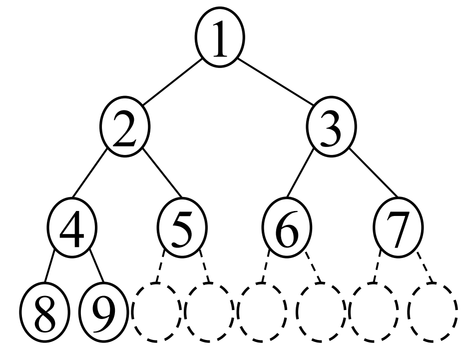


- Case : Complete BT with $\overset{9}{n}$ nodes

(1) parent(i): $\lfloor i/2 \rfloor$ 내림 if $i \neq 1$

(2) LeftChild(i): $2i$ if $2i \leq n$
no left child if $2i > n$

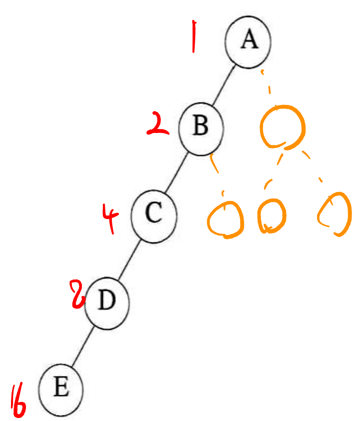
(3) RightChild(i) $2i+1$ if $2i+1 \leq n$
no left child if $2i+1 > n$



	tree
[0]	–
[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I

(b) Tree of Figure 5.10(b)

- Disadvantages skewed tree는 array로 표현하면 memory wasted
  - Waste space;
  - A skewed tree of depth k will require 2^k-1 spaces (only k will be used)
  - Insertion or deletion of nodes : …



	tree
[0]	–
[1]	A
[2]	B
[3]	–
[4]	C
[5]	–
[6]	–
[7]	–
[8]	D
[9]	–
.	.
.	.
.	.
[16]	E

2) Linked Representation

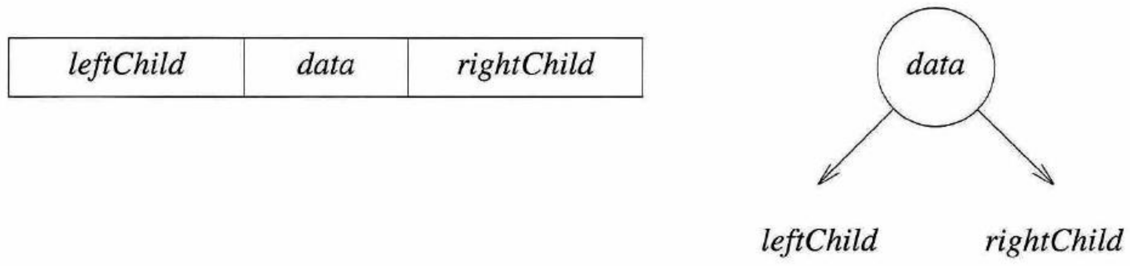


Figure 5.13: Node representations

```
typedef struct node *treePointer;
typedef struct node{
        int data;
        treePointer leftChild, rightChild;
        };
```

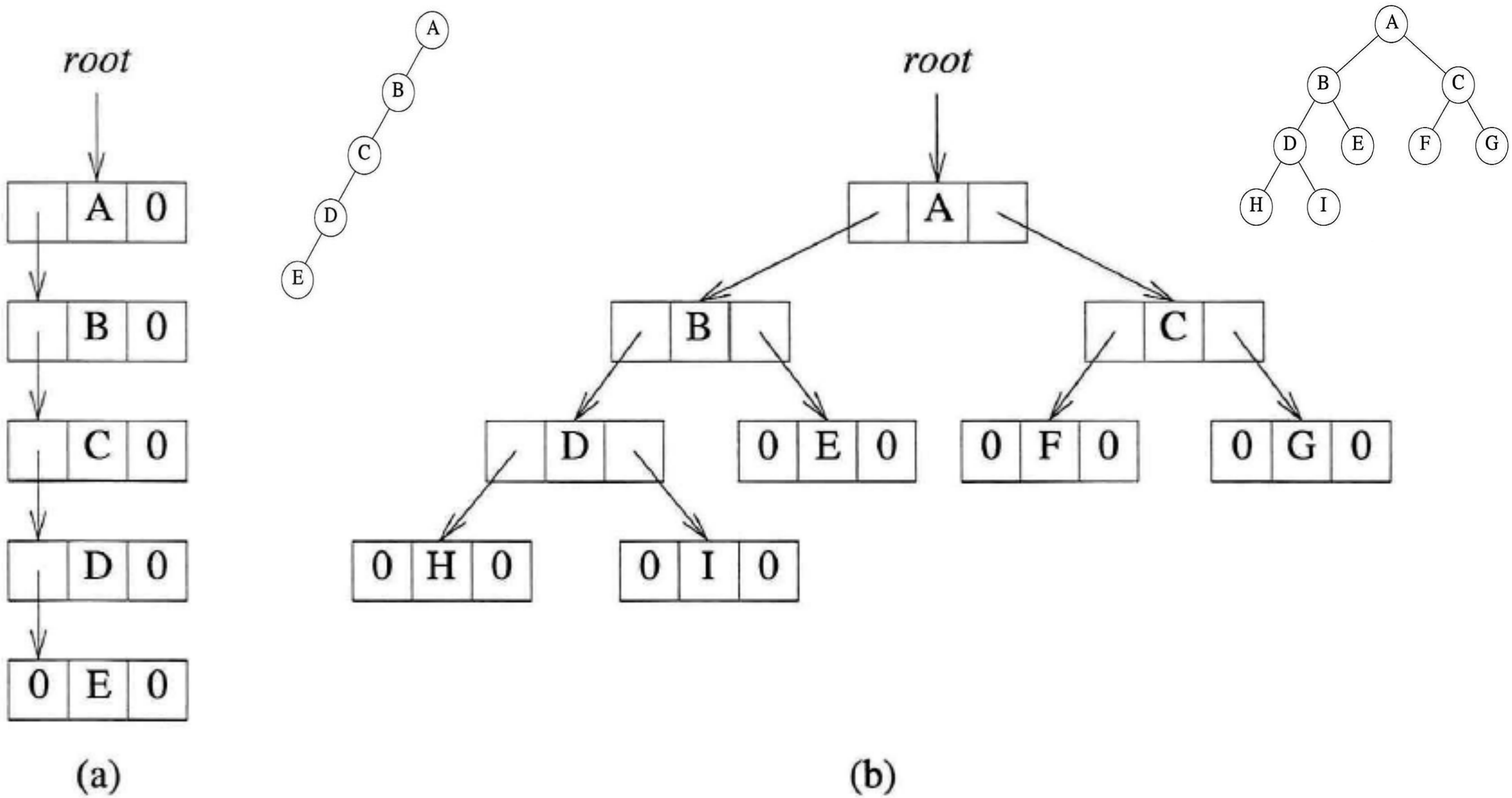


Figure 5.14: Linked representation for the binary trees of Figure 5.10

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