

习题 3.

习题 4.

习题 5.

习题 6.

习题 7.

习题 8.

习题 3.

$$\begin{aligned} & \text{已知 } \overrightarrow{m} = \overrightarrow{a} - \overrightarrow{b}, \quad \overrightarrow{n} = \overrightarrow{b} - \overrightarrow{a}, \quad \overrightarrow{p} = \overrightarrow{a} + \overrightarrow{b}. \\ & \text{求 } \overrightarrow{m}^2 + \overrightarrow{n}^2 + \overrightarrow{p}^2. \end{aligned}$$

解: $\overrightarrow{m}^2 = (\overrightarrow{a} - \overrightarrow{b})^2 = \overrightarrow{a}^2 - 2\overrightarrow{a}\cdot\overrightarrow{b} + \overrightarrow{b}^2$,
 $\overrightarrow{n}^2 = (\overrightarrow{b} - \overrightarrow{a})^2 = \overrightarrow{b}^2 - 2\overrightarrow{b}\cdot\overrightarrow{a} + \overrightarrow{a}^2$,
 $\overrightarrow{p}^2 = (\overrightarrow{a} + \overrightarrow{b})^2 = \overrightarrow{a}^2 + 2\overrightarrow{a}\cdot\overrightarrow{b} + \overrightarrow{b}^2$.
 $\therefore \overrightarrow{m}^2 + \overrightarrow{n}^2 + \overrightarrow{p}^2 = (\overrightarrow{a}^2 - 2\overrightarrow{a}\cdot\overrightarrow{b} + \overrightarrow{b}^2) + (\overrightarrow{b}^2 - 2\overrightarrow{b}\cdot\overrightarrow{a} + \overrightarrow{a}^2) + (\overrightarrow{a}^2 + 2\overrightarrow{a}\cdot\overrightarrow{b} + \overrightarrow{b}^2) = 3\overrightarrow{a}^2 + 3\overrightarrow{b}^2.$

习题 4.

$$\begin{aligned} & \text{已知 } A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \\ & \text{求 } A^2 - B^2. \end{aligned}$$

解: $A^2 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 30 & 36 & 42 \\ 64 & 80 & 96 \\ 98 & 114 & 130 \end{pmatrix}$,
 $B^2 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 30 & 36 & 42 \\ 64 & 80 & 96 \\ 98 & 114 & 130 \end{pmatrix}$,
 $\therefore A^2 - B^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$

习题 5.

$$\begin{aligned} & \text{已知 } A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \\ & \text{求 } (A^2 - B^2)^{-1}. \end{aligned}$$

解: 由上题得 $A^2 - B^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,
 $\therefore (A^2 - B^2)^{-1} = \text{不存在}.$

习题 6.

$$\begin{aligned} & \text{已知 } A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \\ & \text{求 } A^2 - B^2. \end{aligned}$$

解: 由上题得 $A^2 - B^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

习题 7.

$$\begin{aligned} & \text{已知 } A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \\ & \text{求 } (A^2 - B^2)^{-1}. \end{aligned}$$

解: 由上题得 $A^2 - B^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,
 $\therefore (A^2 - B^2)^{-1} = \text{不存在}.$

习题 8.

$$\begin{aligned} & \text{已知 } A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \\ & \text{求 } (A^2 - B^2)^{-1}. \end{aligned}$$

解: 由上题得 $A^2 - B^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,
 $\therefore (A^2 - B^2)^{-1} = \text{不存在}.$