What would the enclosed mass $M_r(r)$ and the gravitational acceleration $g(r)$ look like if we made a guess for how the density varies inside of the star.

1. If the density is $\rho_r = \rho_o$ (if $r \leq R_\star$), what is
   a. The density at the center of the star:

   b. The density at the surface of the star:

2. Our differential equation can be integrated on both sides:

$$\int_{M_r=\boxed{}}^{M_r} dM_r = \int_{r'=\boxed{}}^{r=r'} 4\pi\rho(r')r'^2 dr'$$

We need a boundary condition (a location r in the star at which we know the value of M_r).

Which of these two possibility is the most useful?

[ ] At the center: $M_r(r = 0) = 0$
[ ] At the surface: $M_r(r = R_\star) = M_\star$

3. Fill in the left side, and on the right-side (1) replace $\rho(r)$ with its equation and (2) make a change of variable for $x = r/R_\star$.

$\boxed{} - \boxed{} = \boxed{}$

4. Now pull all of the constants in front of the integral

$\boxed{} = \boxed{}\ \boxed{}$

4b. Parenthesis: What are the units of the combined bunch of constant in front? Does it make sense?

5. Now integrate (and evaluate at the bounds!)

$$M_r(r) = \boxed{}\left[\boxed{} - \boxed{}\right]$$

6. According to the equation above, what is the expression for M_r when $r = R_\star$ (or in other words, what is the value of $M_\star$?)

$M_r(r = R_\star)$ aka $M_\star =$

7. Parenthesis: in the notebook, we will calculate the value for ρ_o that would be implies by our choice of density variation for a star that has a mass of $1M_\odot$ and a radius of $1R_\odot$. So take a minute here to isolate ρ_o in the equation above, for safekeeping

$\rho_o =$

8. As we will see later on, it is actually useful to use the change of variable $x = r/R_\star$ when we compare e.g. the density/temperature/pressure/etc variation in stars of different size. This means that all of our graph are contained between $x = 0$ and $x = 1$. It is also very useful for computational work to have quantities that are unit-less.

So now, let's make the left side of our equation (which will be the y-axis in our graphs) also unit less, by dividing the equation for M_r in #5 with the equation for $M_\star$ in #6.

$\frac{M_r}{M_\star} =$

9. Finally, we can use this result to compute $g(r)$. We know that $g(r) = \frac{GM_r(r)}{r^2}$. We can scale this equation to make computation easy.
What is the expression for the gravitational acceleration at the *surface* of the star?

$g_\star = g(r = R_\star) =$

10. Now write down an expression for:

$\frac{g(r)}{g_\star} =$