Week 1 Thursday

L-2

Gravity field

Agenda for L2 - Gravity field

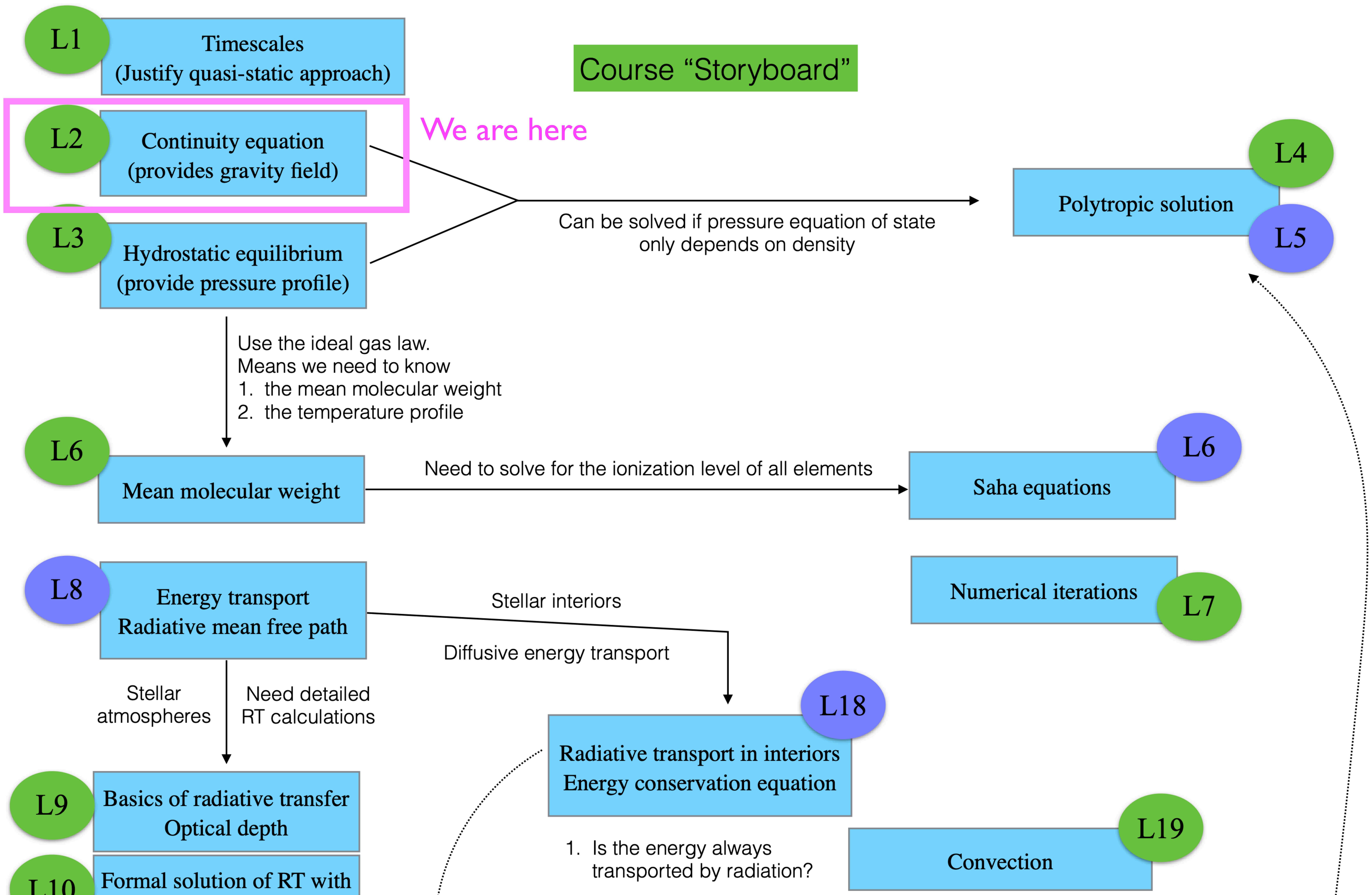
- Some notes about math

- Definition of enclosed mass and gravitational acceleration
  - I can explain (with words and sketches) the relation between the gravitational acceleration and the enclosed mass.
- How can we calculate the enclosed mass? Introducing the density ρ and the ‘continuity equation’
  - I can write the integral form of the equation for the enclosed mass ($M_r(r)$) in term of the density $\rho(r)$.

We cannot yet solve the both $M_r(r)$ and $\rho(r)$, because we only have 1 equation yet.

But before we move on, let’s do a few exercises to give us an idea of how $M_r(r)$ and $g(r)$ would look like for some guessed $\rho(r)$

- Worksheet on solving for the enclosed mass M_r if given a functional form for the density $\rho(r) = \ldots$
  - [Math]: I can make a change of variable to an integral to make the integrant unit-less.



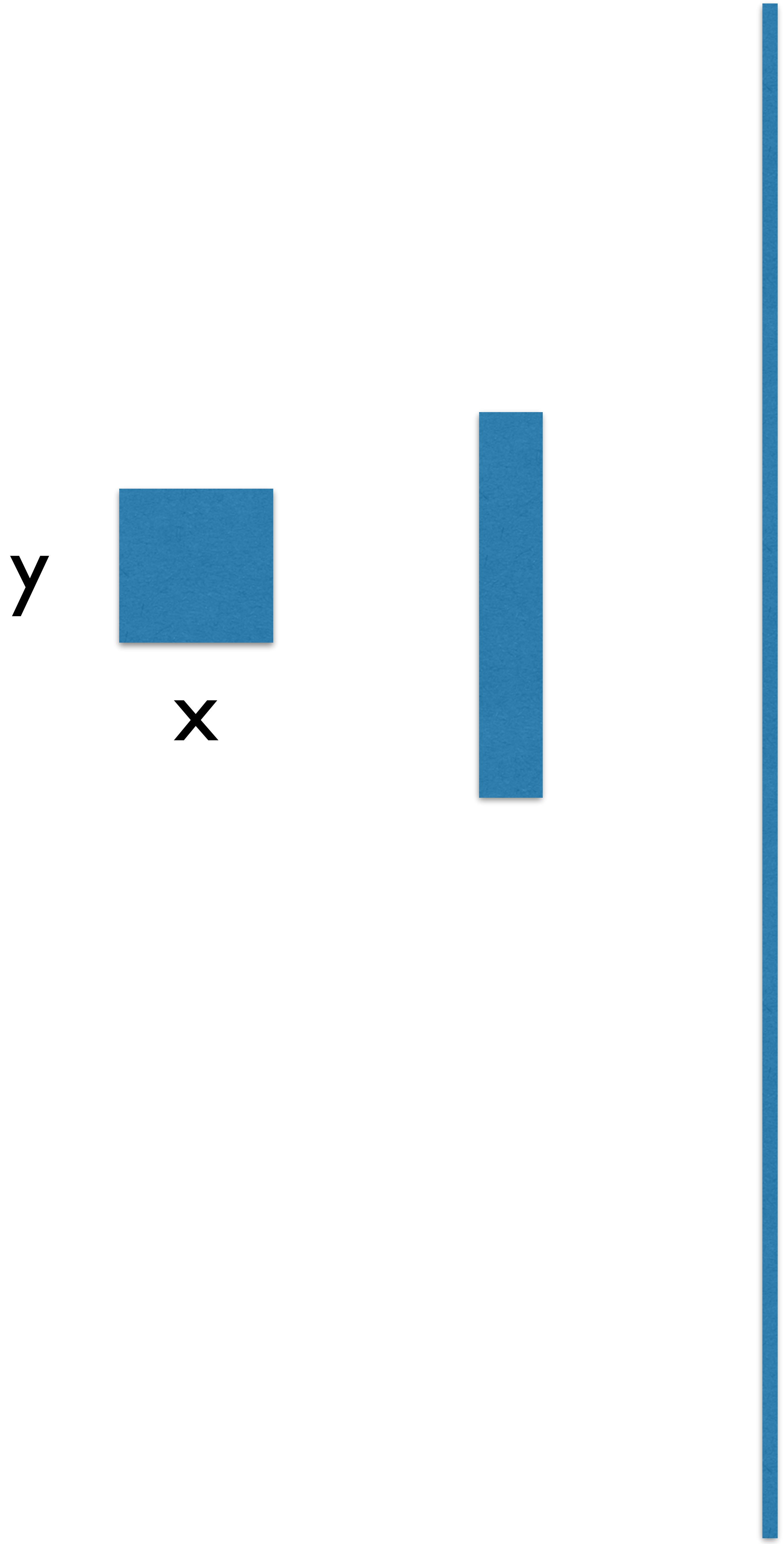
Course "Storyboard"
L1
Timescales
(Justify quasi-static approach)
L2
Continuity equation
(provides gravity field)
We are here
L3
Hydrostatic equilibrium
(provide pressure profile)
Can be solved if pressure equation of state only depends on density
L4
Polytropic solution
L5
Use the ideal gas law.
Means we need to know
1. the mean molecular weight
2. the temperature profile
L6
Mean molecular weight
Need to solve for the ionization level of all elements
L6
Saha equations
Numerical iterations
L7
L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Stellar atmospheres
Need detailed RT calculations
L18
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with
1. Is the energy always transported by radiation?
L19
Convection

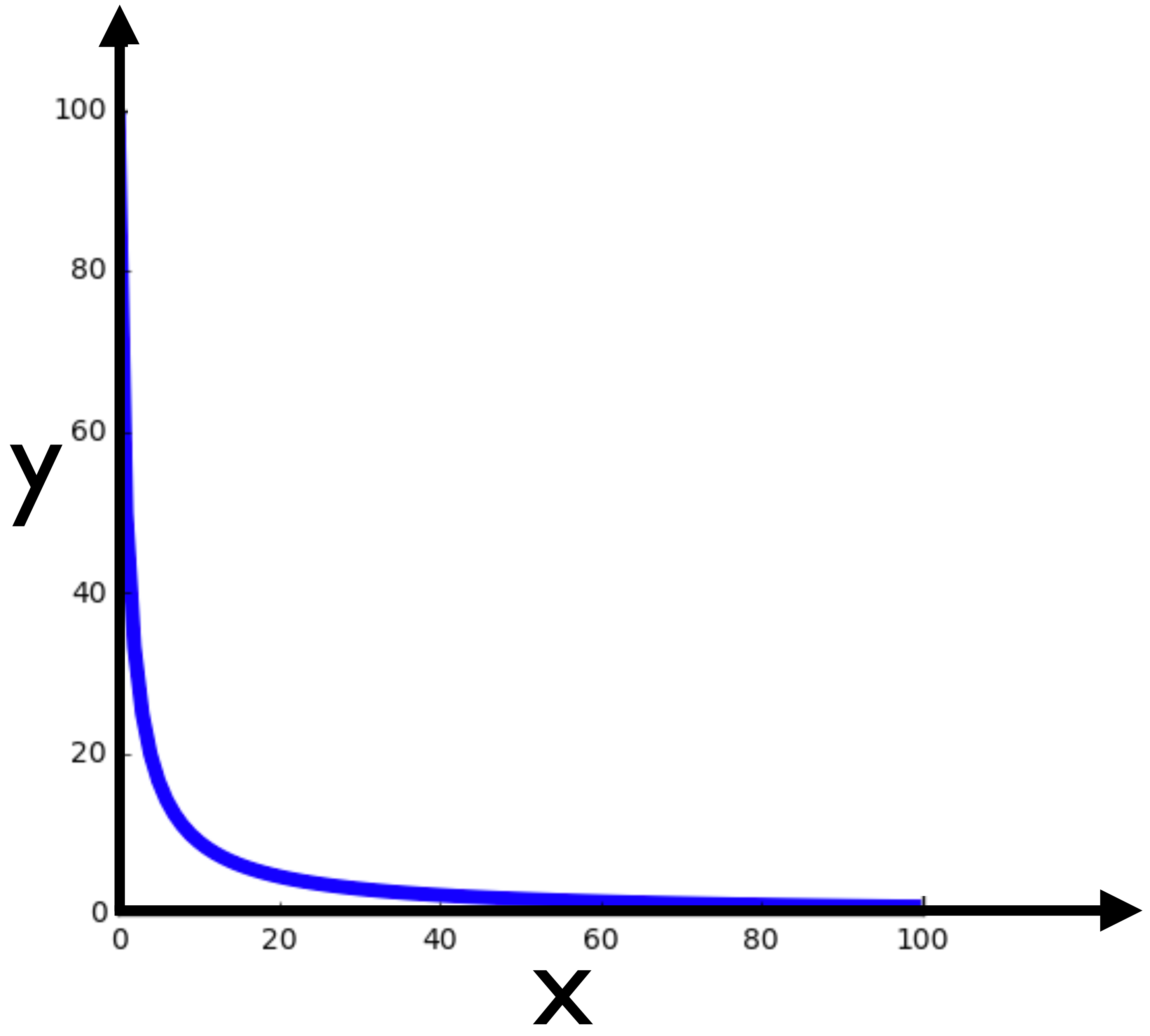
A short reminder of two important math concepts

I want an area of 100m².

$$x * y = 100$$

One equation, two unknowns





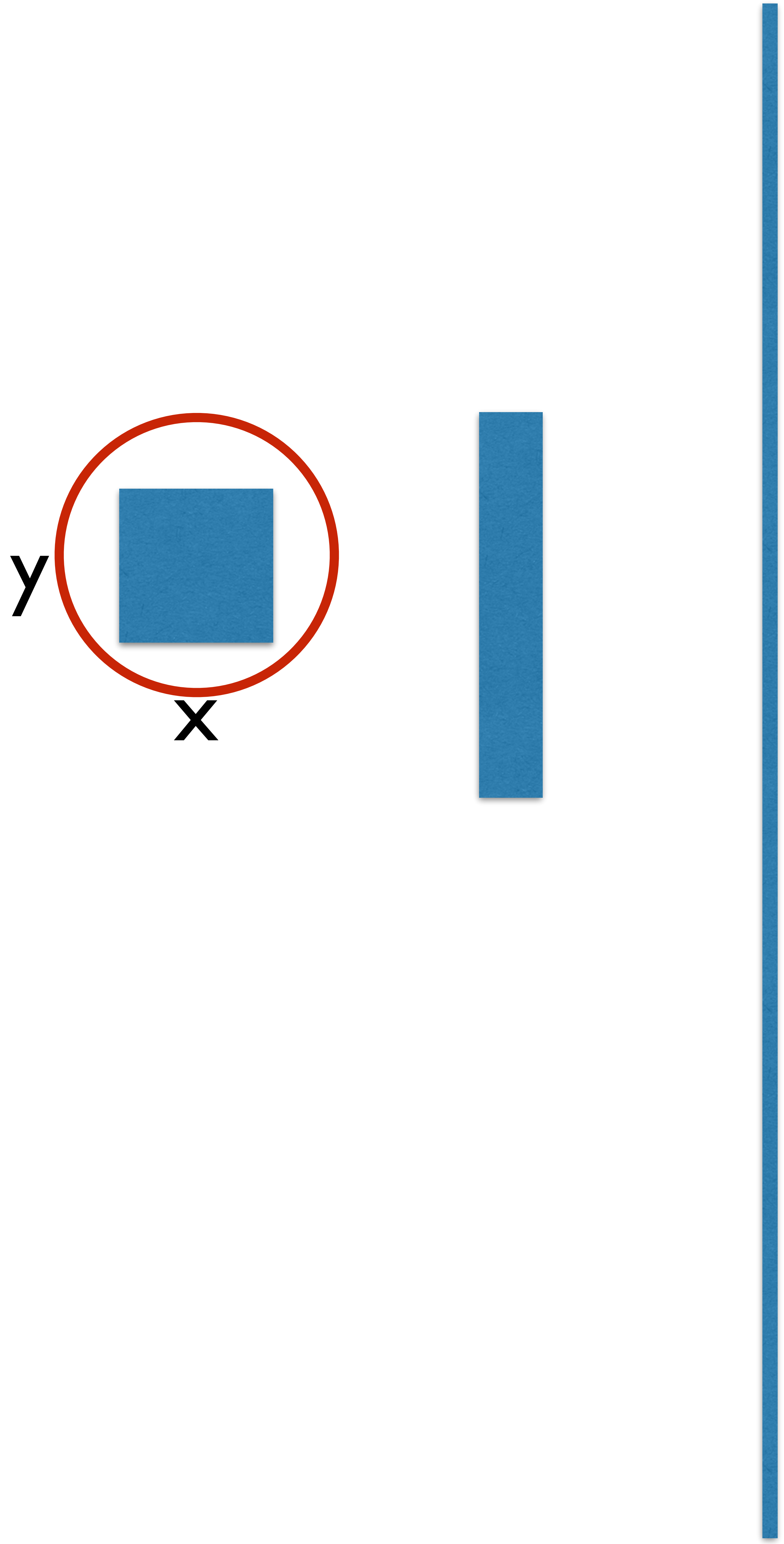
I want an area of $100m^2$.

$$x * y = 100$$

I want a square

$$x = y$$

Two equations, two unknowns



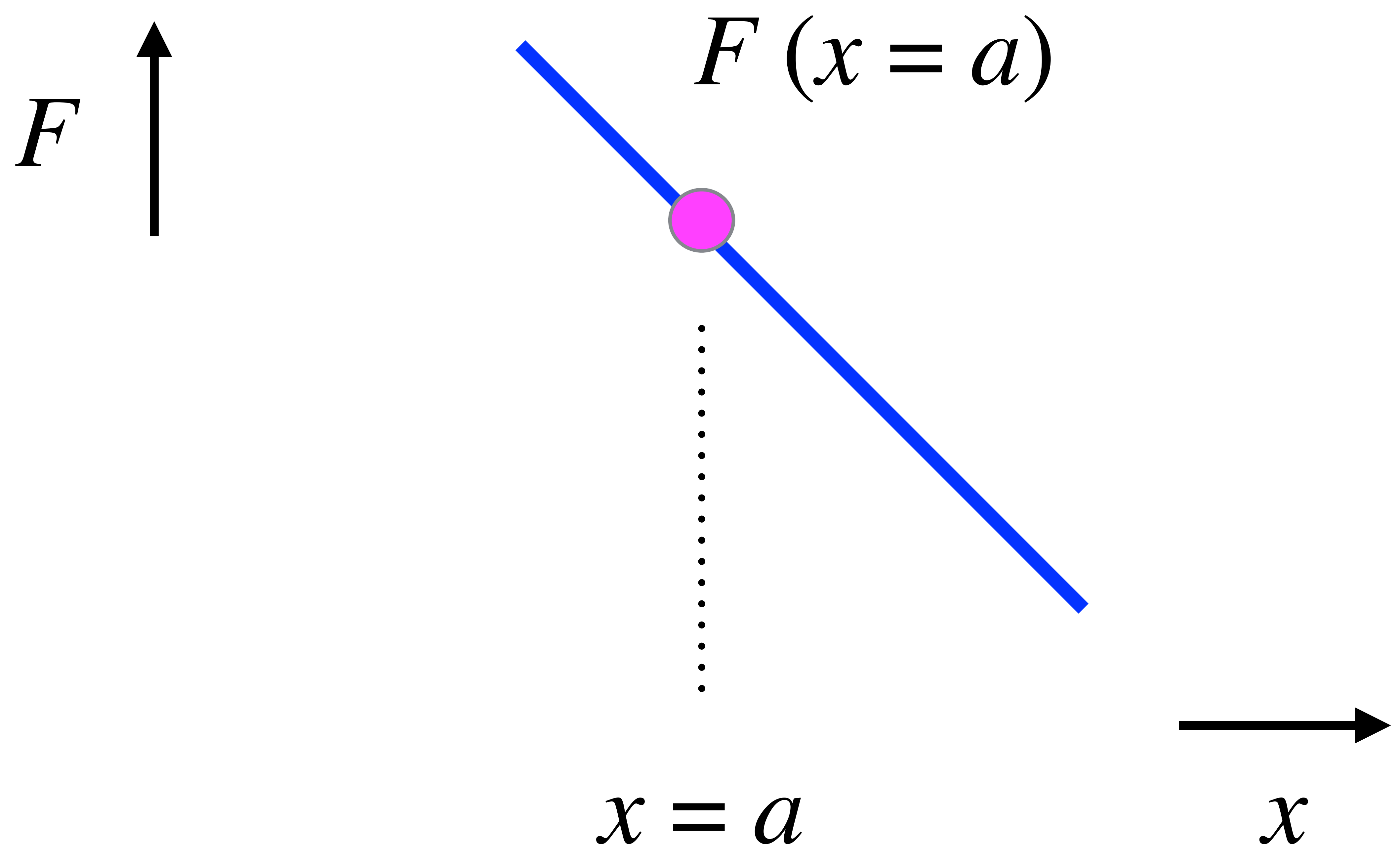
$$\int dF \neq F$$

$$\int_{1}^{2} dF = F(2) - F(1)$$

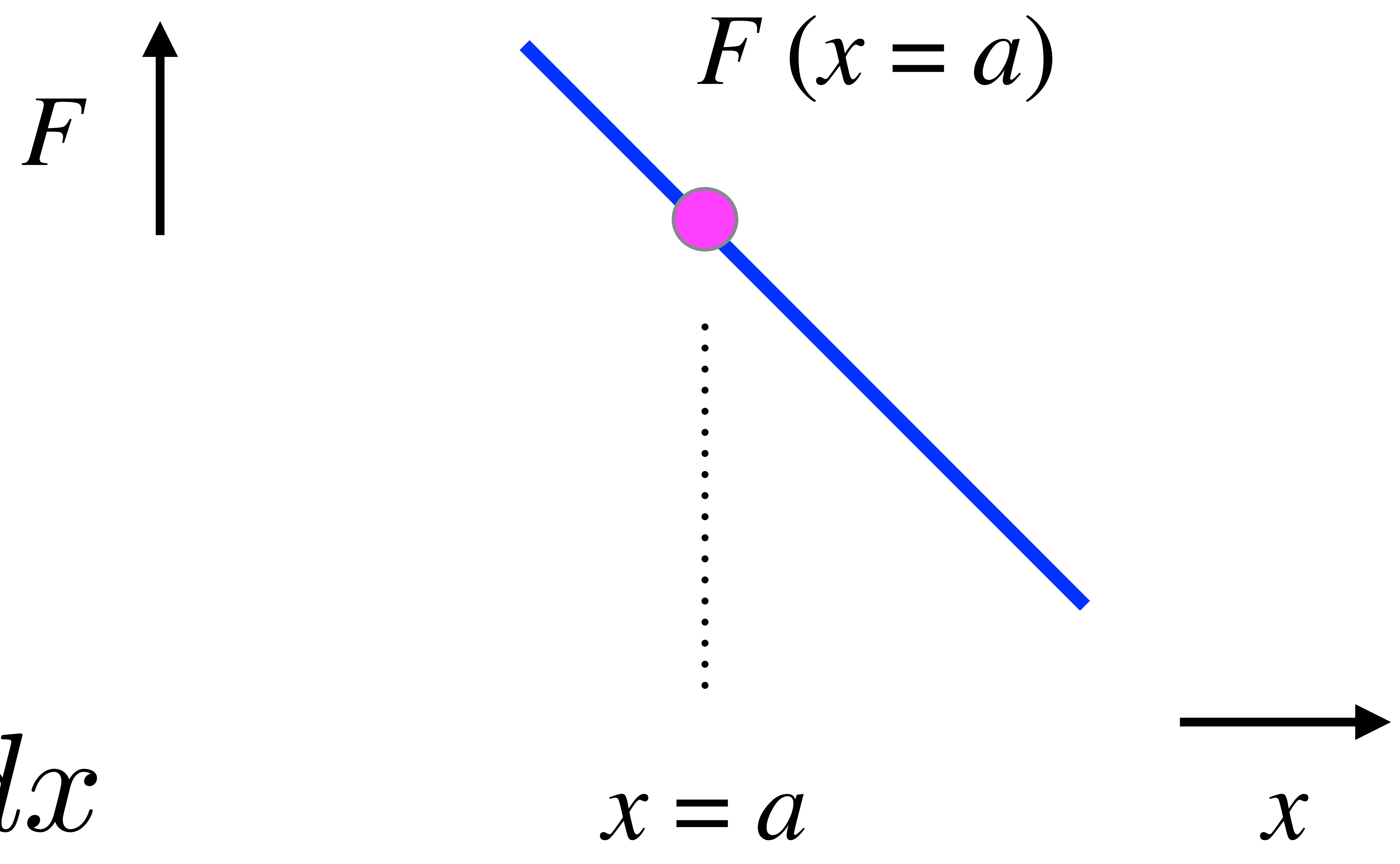
$$\frac{dF}{dx} = -1$$

$$\int_{1}^{2} dF = -1 \int_{1}^{2} dx$$

$$F(x = a) - F(1) = -1 \int_{x_1}^{x=a} dx$$



$$\underline{F(x \overset{2}{=} a)} - \underline{F(1)} = -1 \int_{\underline{x_{1}}}^{x=a} dx$$



One equation, three unknowns

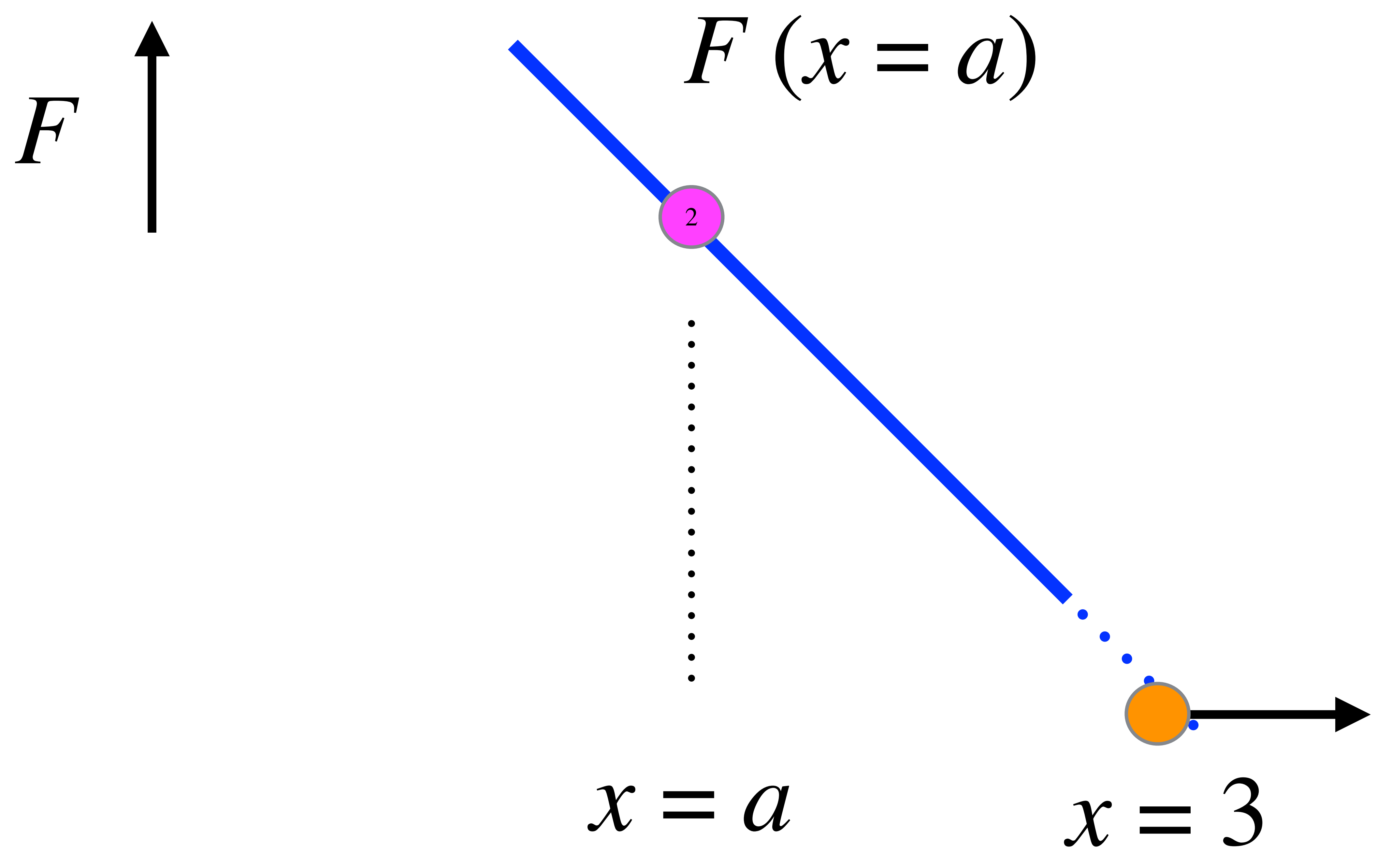
$$\underline{F(x \overset{2}{=} a)} - \underline{F(1)} = -1\int_{\underline{x_{1}}}^{x \overset{2}{=} a} dx$$

$$F(x \overset{2}{=} a) - \overset{1}{0} = -1\int_{\underset{1}{x=3}}^{x \overset{2}{=} a} dx$$

$$F(x) - 0 = -1\int_{3}^{x} dx'$$

$$F(x) = -1\,[x]_3^x = -x + 3$$

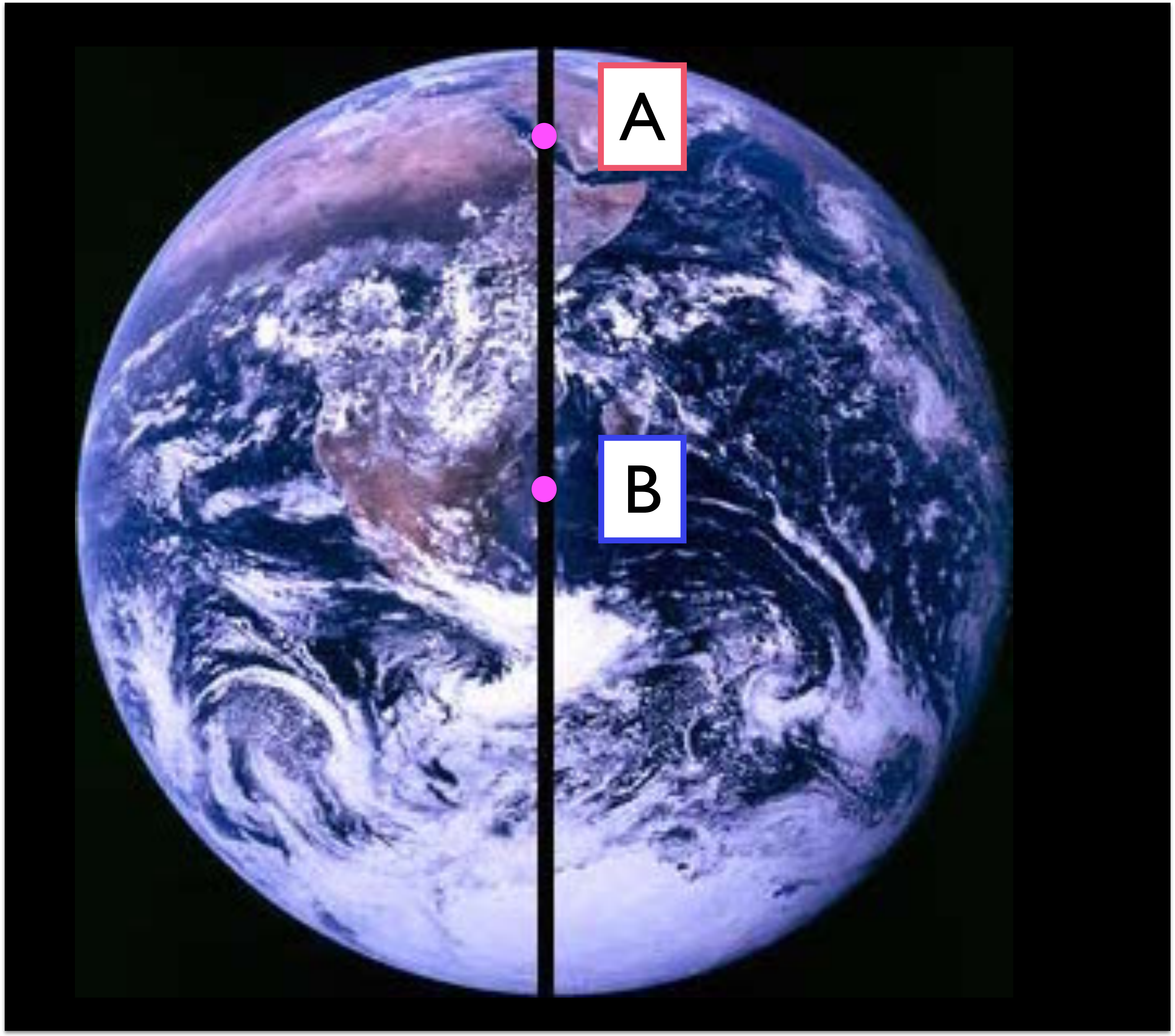
The lower bound of an integral is not always zero!!



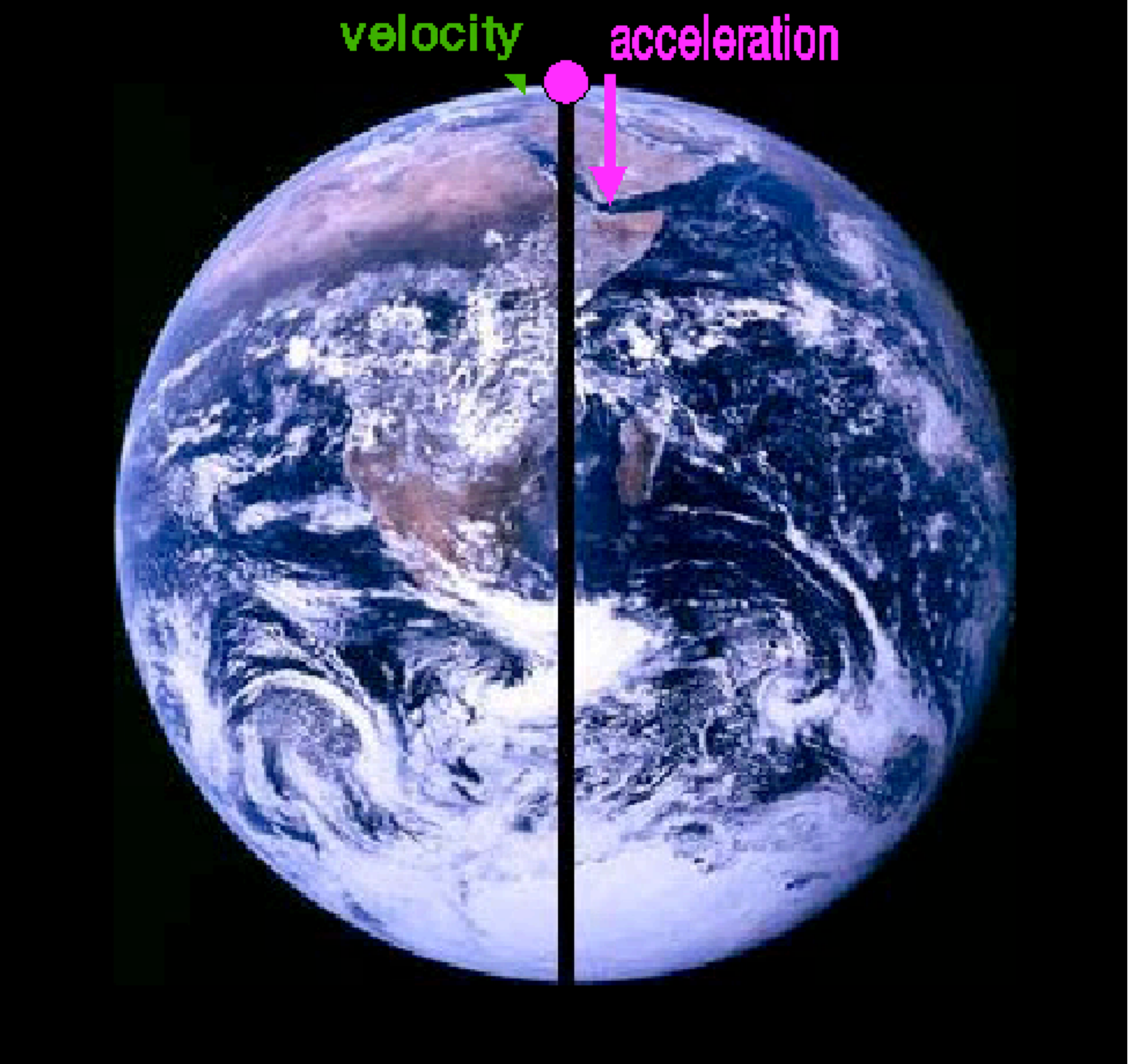
A star is a massive ball of gas => there has to be gravity!

You are throwing a ball in a the tunnel through the center of the Earth…

Where is the magnitude of the gravitational acceleration of the ball the smallest?



C :The same everywhere


velocity
acceleration

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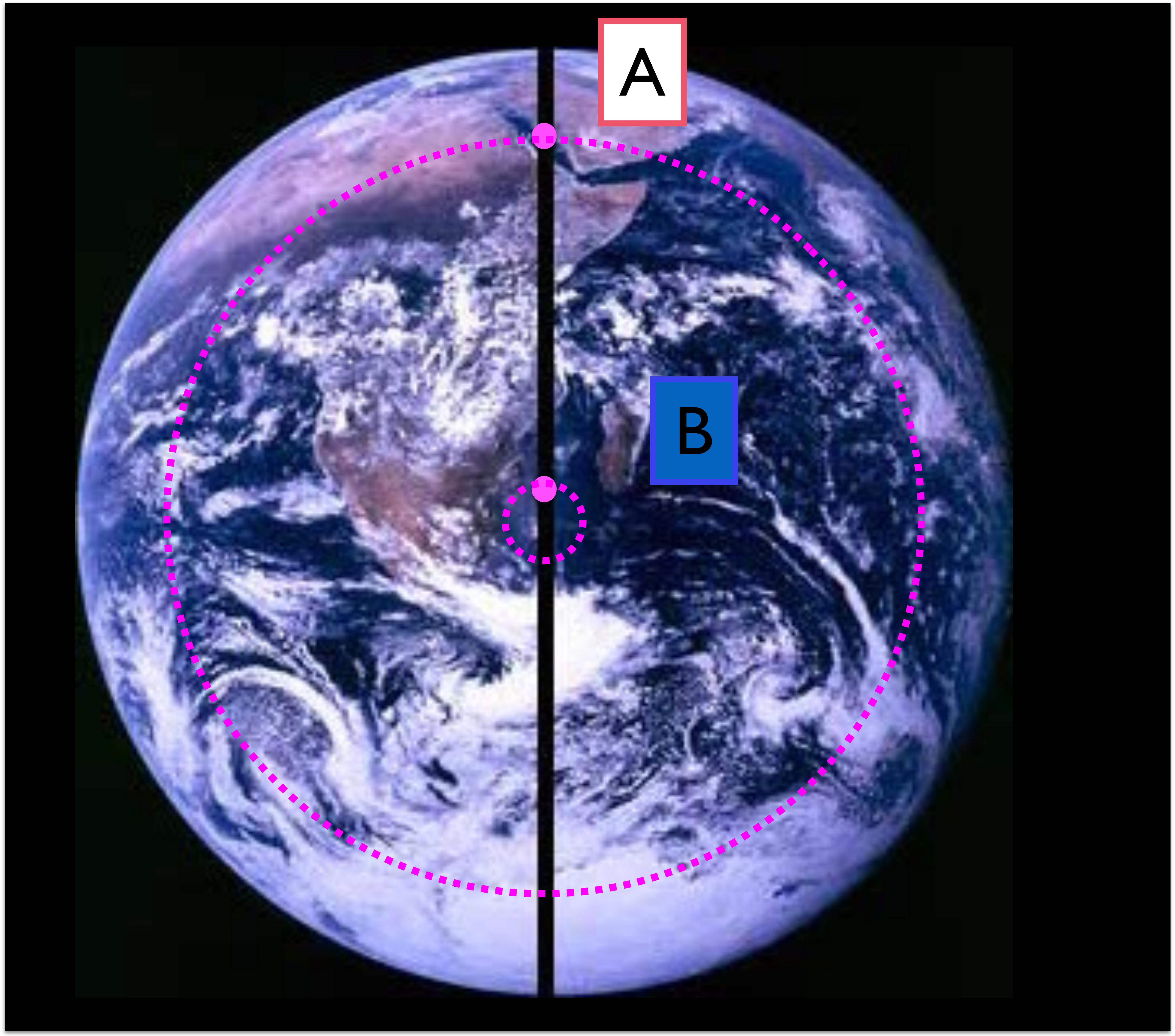

C :The same everywhere

Inside of an object, the gravitational acceleration only depends on the enclosed mass of the object M_r at the location of the test particle.

r : radial coordinate

At the star's center $r = 0$

At the star's surface $r = R_\star$

$$|\vec{F}_g(r)| = \frac{GM_r(r)m}{r^2}$$

$g(r)$

On the board: derivation of the Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

Set of equations

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

$M_r(r)$

$\rho(r)$

Boundary condition:
Center: $M_r(r=0) = 0$
Surface: $M_r(r = R_\star) = M_\star$

We cannot solve for the structure of a star with only one equation (and two unknown) — we will need more physics to make this work.

But before we move on with the physics:

We will explore what $M_r(r)$ and $g(r)$ would look like if we guess what the density looks like in a star (so we will guess a function for $\rho(r)$

WORKSHEET

What if $\rho(r) = \rho_0$

What would the enclosed mass $M_r(r)$ and the gravitational acceleration $g(r)$ look like if we made a guess for how the density varies inside of the star.

1. If the density is $\rho_r = \rho_o$ (if $r \leq R_\star$), what is
   a. The density at the center of the star:
   b. The density at the surface of the star:

2. Our differential equation can be integrated on both sides:

$$\int_{M_r=}^{M_r} dM_r = \int_{r'=}^{r=r'} 4\pi\rho(r')r'^2 dr'$$

We need a boundary condition (a location r in the star at which we know the value of M_r).

Which of these two possibility is the most useful?

[ ] At the center: $M_r(r=0) = 0$
[ ] At the surface: $M_r(r = R_\star) = M_\star$

3. Fill in the left side, and on the right-side (1) replace $\rho(r)$ with its equation and (2) make a change of variable for $x = r/R_\star$.

$\quad - \quad =$

AT HOME

What if $\rho(r) = \rho_0 \left(1 - \frac{r}{R_*}\right)$

What would the enclosed mass $M_r(r)$ and the gravitational acceleration $g(r)$ look like if we made a guess for how the density varies inside of the star.

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PLOT IN NOTEBOOK TOGETHER