Problem 1

The following problem is taken from the book *'Numerical Methods for Engineers with Python 3'* written by *Jaan Kiusalaas edition one* at page 59.

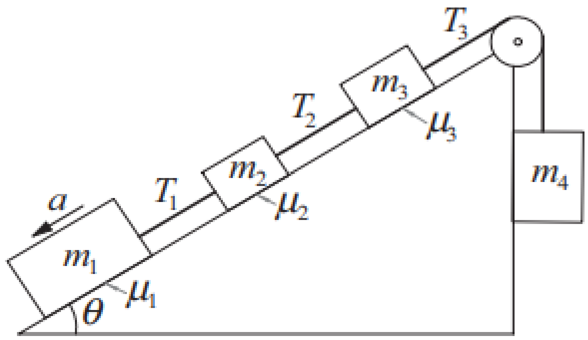


Fig 1

The four blocks of different masses mi are connected by ropes of negligible mass. Three of the blocks lie on a inclined plane, the coefficients of friction between the blocks and the plane being μ_i. The equations of motion for the blocks can be shown to be

$$T_1 + m_1 a = m_1 g(\sin\theta - \mu_1 \cos\theta)$$

$$-T_1 + T_2 + m_2 a = m_2 g(\sin\theta - \mu_2 \cos\theta)$$

$$-T_2 + T_3 + m_3 a = m_3 g(\sin\theta - \mu_3 \cos\theta)$$

$$-T_3 + m_4 a = -m_4 g$$

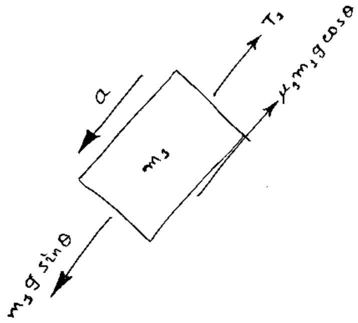
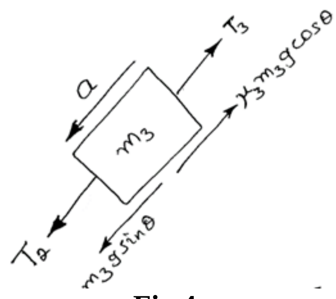
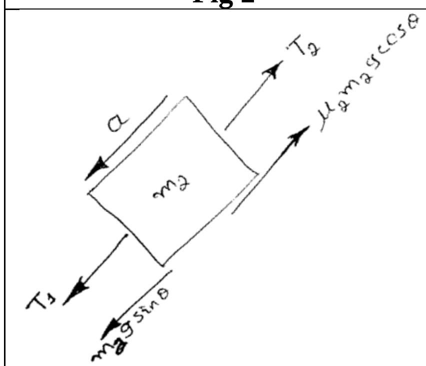
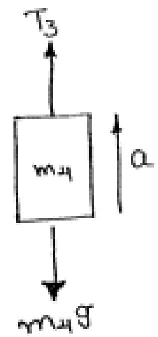
where Ti denotes the tensile forces in the ropes and a is the acceleration of the system. Determine a and Ti if $\theta = 45^\circ$, g = 9.82 m/s2 and

$$\mathbf{m} = \begin{bmatrix} 10 & 4 & 5 & 6 \end{bmatrix}^T \text{ kg}$$

$$\boldsymbol{\mu} = \begin{bmatrix} 0.25 & 0.3 & 0.2 \end{bmatrix}^T$$

In order to solve this problem, first we need to break it down and investigate every element of the system separately. The three consecutive masses that are laying over the inclined surface are subjected to two types of forces in common. These are ***mgsinϑ*** (gravity) and ***μmgcosϑ*** (frictional force). Thus, let's start with the first body in the system.

Assumption: The direction of the system acceleration is considered to be the same as the one indicated in the problem(Fig 1) i.e. down the inclined plane.

 **Fig 2**	$m_1a + m_1gsin\vartheta - \mu_1m_1gcos\vartheta - T_1 = 0$ $- T_1 + m_1a = - m_1gsin\vartheta + \mu_1m_1gcos\vartheta$ $- T_1 + m_1a = - m_1g(sin\vartheta - \mu_1cos\vartheta)$ $\boldsymbol{T_1 - m_1a = m_1g(sin\vartheta - \mu_1cos\vartheta)}$ ***… eq (1)***	 **Fig 4**	$T_2 + m_3a + m_3gsin\vartheta - \mu_3m_3gcos\vartheta - T_3 = 0$ $T_2 - T_3 + m_3a + m_3gsin\vartheta - \mu_3m_3gcos\vartheta = 0$ $T_2 - T_3 + m_3a = - m_3gsin\vartheta + \mu_3m_3gcos\vartheta$ $T_2 - T_3 + m_3a = - m_3g\ (sin\vartheta - \mu_3cos\vartheta)$ $\boldsymbol{- T_2 + T_3 - m_3a = m_3g\ (sin\vartheta - \mu_3cos\vartheta)}$ ***…eq(3)***
 **Fig 3**	$T_1 + m_2a + m_2gsin\vartheta - \mu_2m_2gcos\vartheta - T_2 = 0$ $T_1 - T_2 + m_2a + m_2gsin\vartheta - \mu_2m_2gcos\vartheta = 0$ $T_1 - T_2 + m_2a = - m_2gsin\vartheta + \mu_2m_2gcos\vartheta$ $T_1 - T_2 + m_2a = - m_2g(sin\vartheta - \mu_2gcos\vartheta)$ $\boldsymbol{- T_1 + T_2 - m_2a = m_2g(sin\vartheta - \mu_2gcos\vartheta)}$***…eq(2)***	 **Fig 5**	$T_3 + m_3a - m_4g = 0$ $\boldsymbol{T_3 + m_3a = m_4g}$ ***…eq(4)***

The above four equations can be collected into one, as follows:

$$T_1 - m_1a = m_1g\,(\sin\theta - \mu_1\cos\theta)$$

$$-T_1 + T_2 - m_2a = m_2g\,(\sin\theta - \mu_2\cos\theta)$$

$$-T_2 + T_3 - m_3a = m_3g\,(\sin\theta - \mu_3\cos\theta)$$

$$T_3 + m_4a = m_4g$$

$$\begin{bmatrix} 1 & 0 & 0 & -m1 \\ -1 & 1 & 0 & -m2 \\ 0 & -1 & 1 & -m3 \\ 0 & 0 & 1 & m4 \end{bmatrix} \begin{pmatrix} T1 \\ T2 \\ T3 \\ a \end{pmatrix} = \begin{pmatrix} m1g(\sin\theta - \mu1\cos\theta) \\ m2g\,(\sin\theta - \mu2\cos\theta) \\ m3g\,(\sin\theta - \mu3\cos\theta) \\ m4g \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -10 \\ -1 & 1 & 0 & -4 \\ 0 & -1 & 1 & -5 \\ 0 & 0 & 1 & 6 \end{bmatrix} \begin{pmatrix} T1 \\ T2 \\ T3 \\ a \end{pmatrix} = \begin{pmatrix} 10 * 9.82 * (\sin45 - 0.25\cos45) \\ 4 * 9.82 * (\sin45 - 0.3\cos45) \\ 5 * 9.82 * (\sin45 - 0.2\cos45) \\ 6 * 9.82 \end{pmatrix}$$

In this repository, the mg($\sin\theta - \mu\cos\theta$) equation, which totally is a constant value, is calculated by importing a math module and iterating using for loop. Next, the matrix was solved by the Gauss elimination function which I have uploaded last time(SamuelGetachew2121/Gauss-Elimination-Solver-for-Linear-Equations (github.com)***). The solutions are exactly the same as the solution of the book, except the (-) sign in the acceleration. The reason for this is that, the assumption we made in the beginning regarding the direction of acceleration. If we had the direction of acceleration in the opposite way to what we just considered, the (-) sign wouldn't have appeared. In more technical terms, the body of mass m_4 in this system is falling down and the remaining masses are accelerating upward the inclined plane.***

Answer

T_1= 36.0 N

T_2= 48.6 N

T_3= 67.6 N

a = -1.6 ms^{-2}

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