From the previous exercises, it follows that any graph with largest eigenvalue at most two is either a cycle, one of the trees above, or possibly a tree that contains none of the above as a subgraph. It is not difficult to see that any such tree must be a proper subgraph of the ones above.

1.8   References

Here is the set of references used to write the past few pages.

I used Chapter 8 of Godsil and Royle to write about the spectral decomposition of a symmetric matrix. This was also my reference for the basics and some exercises on the adjacency matrix.

(a) Chris Godsil and Gordon Royle. *Algebraic Graph Theory*. Springer-Verlag, New York, 2001.

Exercise 1.28 comes from Chan and Godsil "Symmetry and Eigenvectors".

I looked extensively for a nice intuitive proof of Perron-Frobenius in its full form, but the best I could do relied on using fixed point theorems. I then came up with the simplified version assuming matrices in question are symmetric. A good reference is Brouwer and Haemers, Chapter 2.

(b) Andries E Brouwer and Willem H Haemers. *Spectra of Graphs*. Universitext. Springer, New York, 2012

I also used the reference above for the spectrum of paths and cycles.