Problem 1

Let V be a finite-dimensional vector space. For any subspace $W \subset V$, put

$$W^{\perp} = \{\phi \in V^* \mid \phi|_W = 0\}.$$

Construct natural isomorphisms $W^* \cong (V^*/W^{\perp})$ and $W^{\perp} \cong (V/W)^*$. (The assumption that V is finite-dimensional is not actually required, but it makes the problem easier.)

Problem 2

Let V be a finite-dimesional vector space, $\dim(V) = n$. For $k \geq 0$, let $G(V, k)$ be the set of all the k-dimensional subspaces of V (it is called the Grassmannian of V). Show that the correspondence

$$W \mapsto W^{\perp}$$

is a bijection

$$G(V, k) \cong G(V^*, n-k).$$