Exercise 1.5.14

If $N_1 \lhd G_1$, $N_2 \lhd G_2$, then $(N_1 \times N_2) \lhd (G_1 \times G_2)$ and $(G_1 \times G_2)/(N_1 \times N_2) \cong (G_1/N_1) \times (G_2/N_2)$.

1.6.11

Find all normal subgroups of D_n.

1.8.2

Give an example of groups H_i, K_j such that $H_1 \times H_2 \cong K_1 \times K_2$ and no H_i is isomorphic to any K_j.

1.8.3

Let G be an (additive) abelian group with subgroups H and K. Show that $G \cong H \oplus K$ if and only if there are homomorphisms $H \leftrightarrows^{\pi_1}_{\iota_1} G \leftrightarrows^{\pi_2}_{\iota_2} K$ such that $\pi_1\iota_1 = 1_H, \pi_2\iota_2 = 1_K, \pi_1\iota_2 = 0$, and $\pi_2\iota_1 = 0$, where 0 is the map sending every element onto the zero (identity) element, and $\iota_1\pi_1(x) + \iota_2\pi_2(x) = x$ for all $x \in G$.

1.8.5

Let G, H be finite cyclic groups. Then $G \times H$ is cyclic if and only if $(|G|, |H|) = 1$.

1.8.9

If a group G is the (internal) direct product of its subgroups H, K, then $H \cong G/K$ and $G/H \cong K$.

1.9.1

Every nonidentity element in a free group F has infinite order.

1.9.4

Let F be the free group on the set X, and let $Y \subset X$. If H is the smallest normal subgroup of F containing Y, then F/H is a free group.