Problem 1

a) A space W is called a quotient space of V modulo U, denoted $W = V/U$, if there exists a surjective linear transformation $e : V \to W$ such that $\ker(e) = U$.

b) For any $v \in V$, consider the left coset $v + U = \{v + u : u \in U\}$. Define $q : V \to V/U$ by $v \mapsto v + U$. Clearly q is surjective. Recall that $\tilde{v} \mapsto \tilde{v} + U = U$ if and only if $\tilde{v} \in U$. Then we have that $\ker(q)$ is precisely the set of all $\tilde{v} \in V$ such that $\tilde{v} + U = U$, which is exactly U. Thus, $q : V \to V/U$ is a surjective linear transformation with $\ker(q) = U$, so V/U is a quotient space of V modulo U.

c)

Problem 2

Problem 3