Seoul National University
Graduate School of Data Science

Structural Causal Bandits under Markov Equivalence 🍕

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Causality Lab

Causality Lab

- Causal Representation Learning
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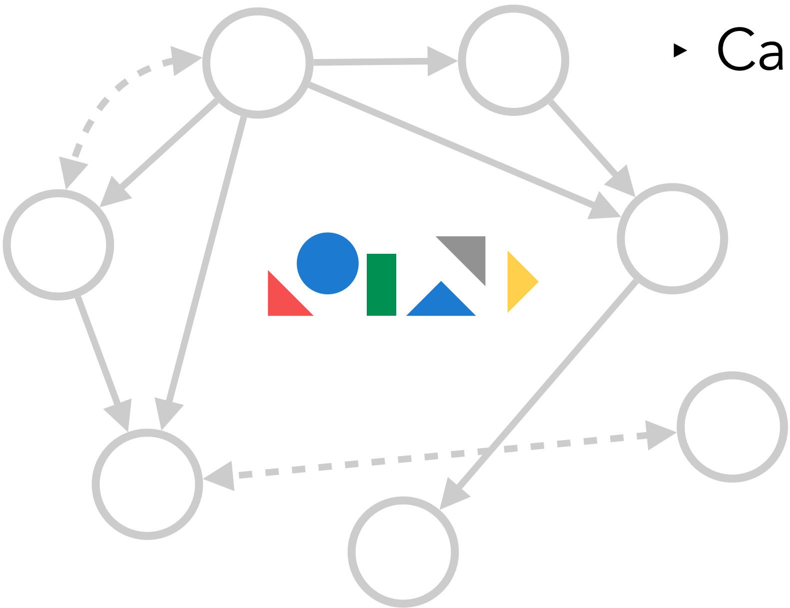
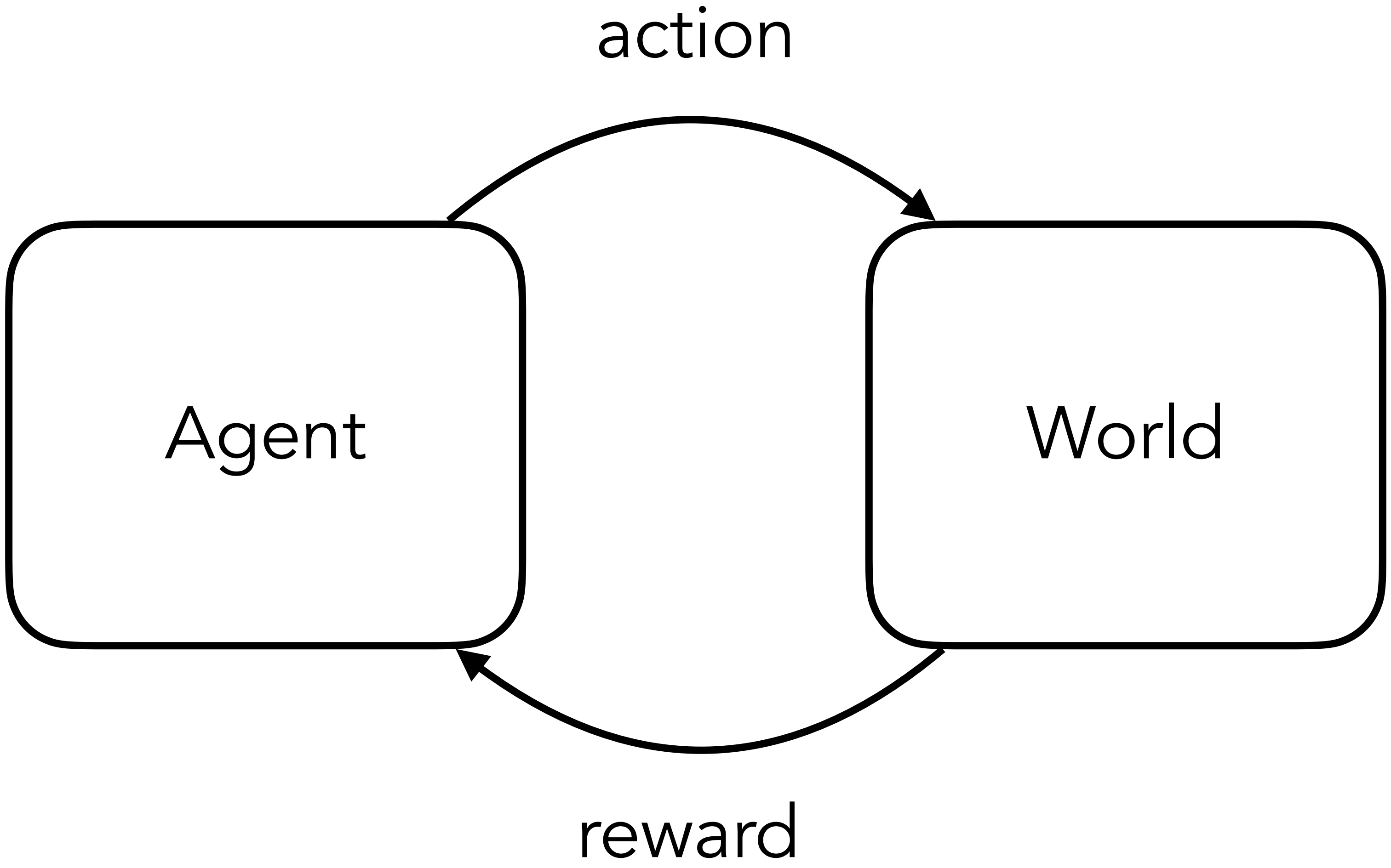


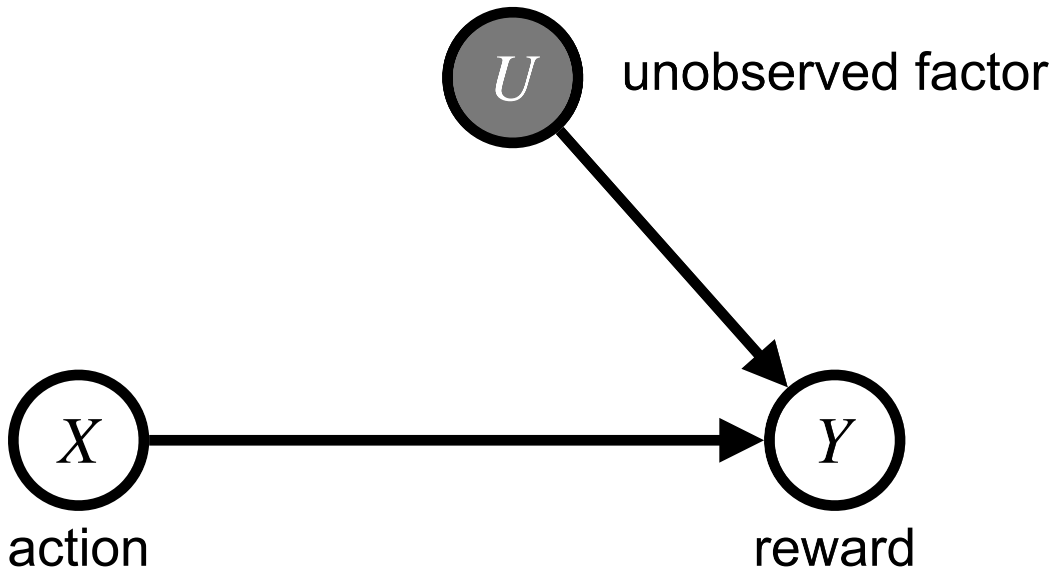
- Causal Discovery
- Causal Recommendation
- Causal Identification
- Causal Fairness
- Causal Estimation
- Causal Explainability

Background

Multi-Armed Bandits

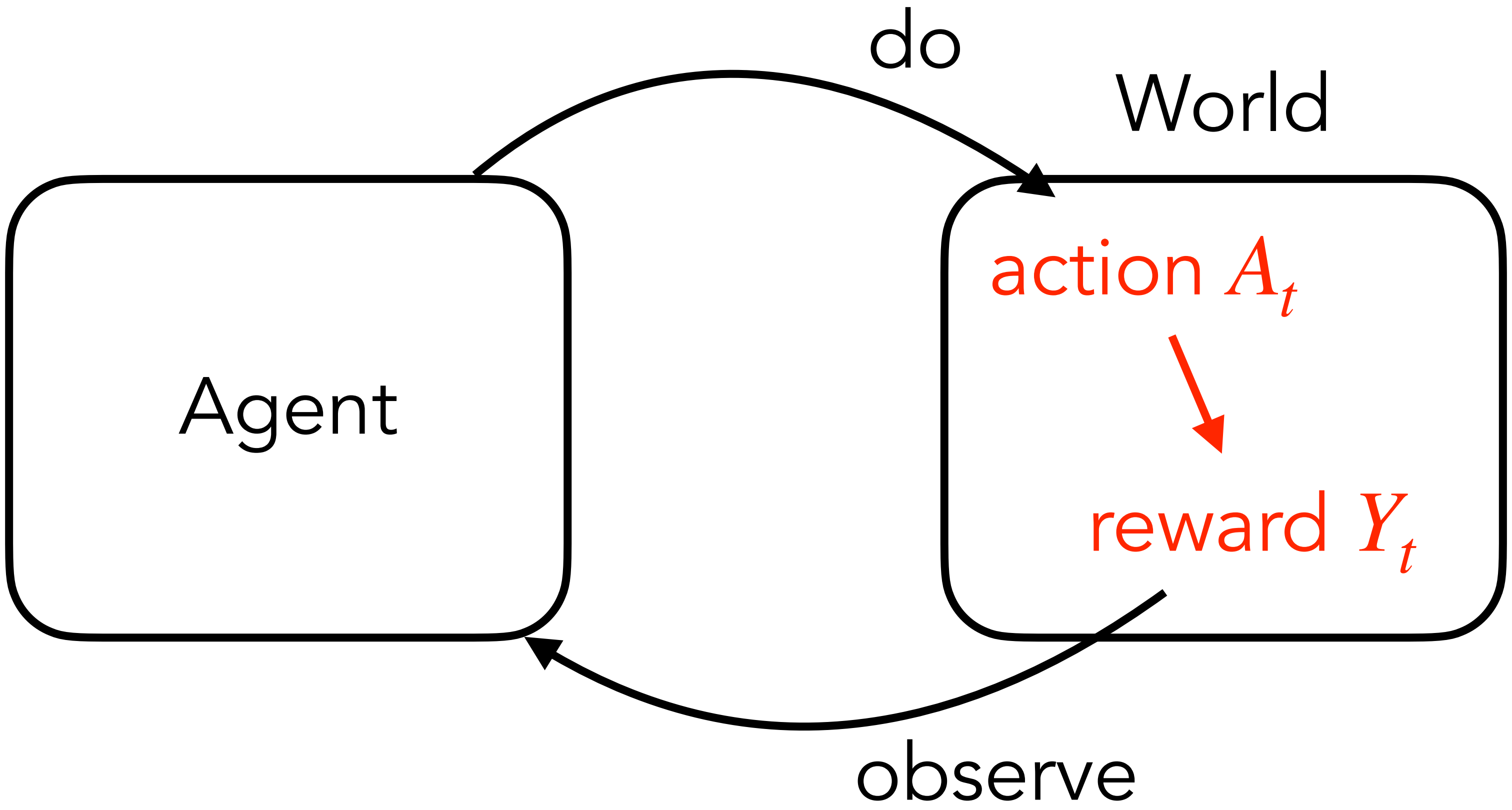


Graphical Understanding of Standard MAB



$$X \in \{1, \cdots, K\}$$

Multi-Armed Bandits through Causal Lens



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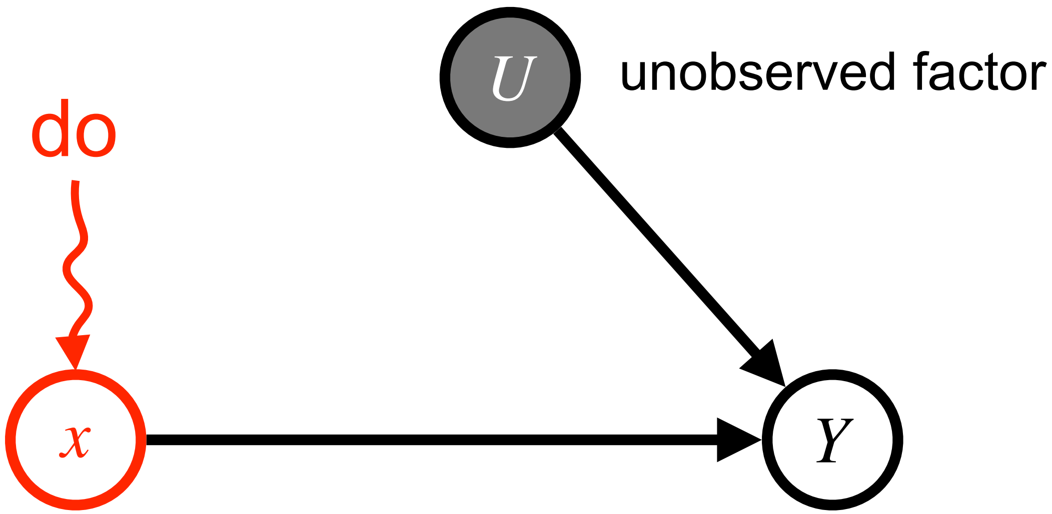
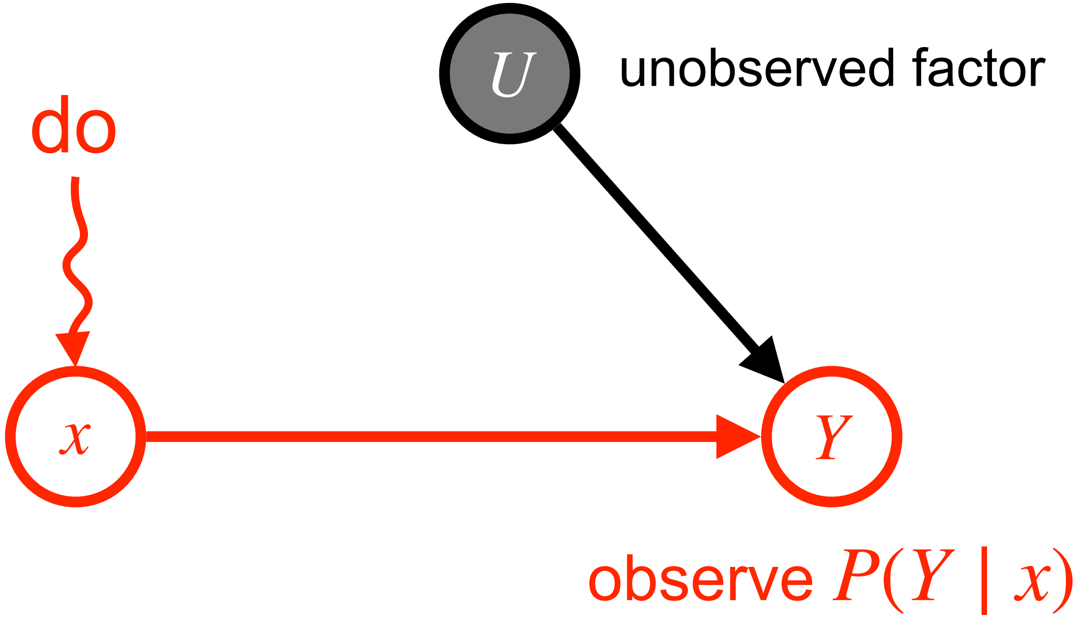


Playing an arm A_t is setting X to x (called do), and observing Y.

Graphical Understanding of Standard MAB



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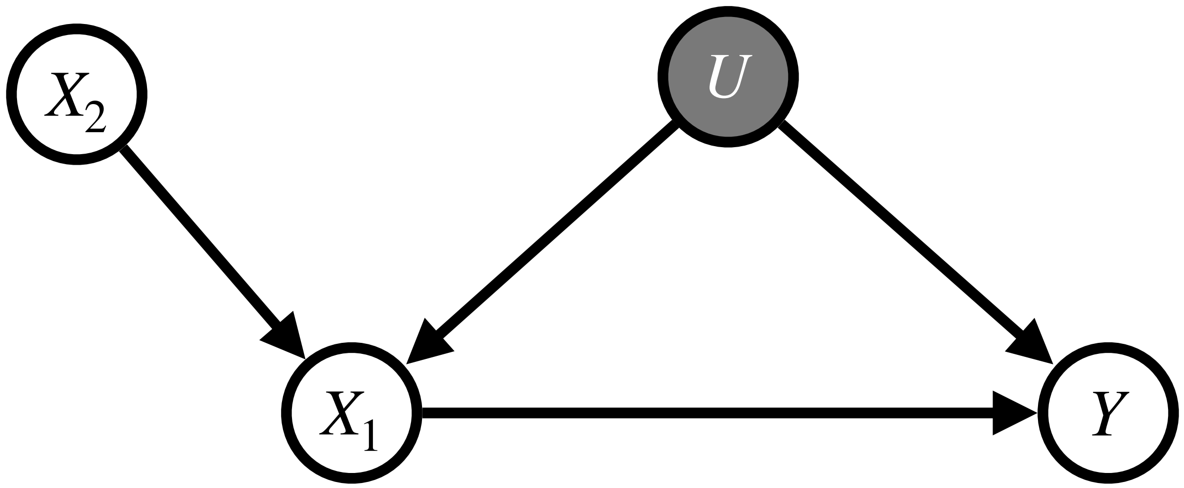
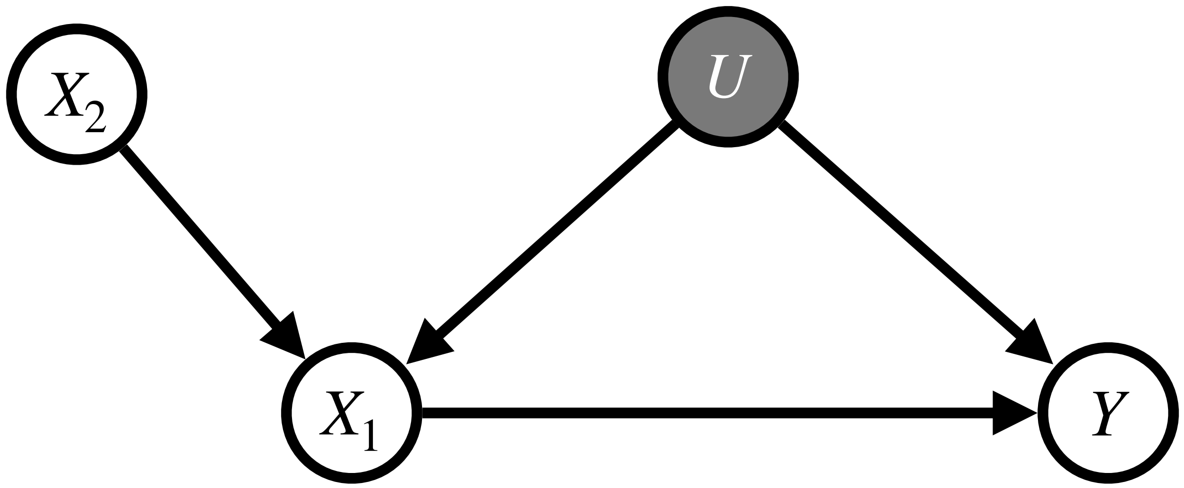


Q. How many **arms** are there? (We can control 2 binary variables, X_1 and X_2).

Graphical Understanding of Causal MAB



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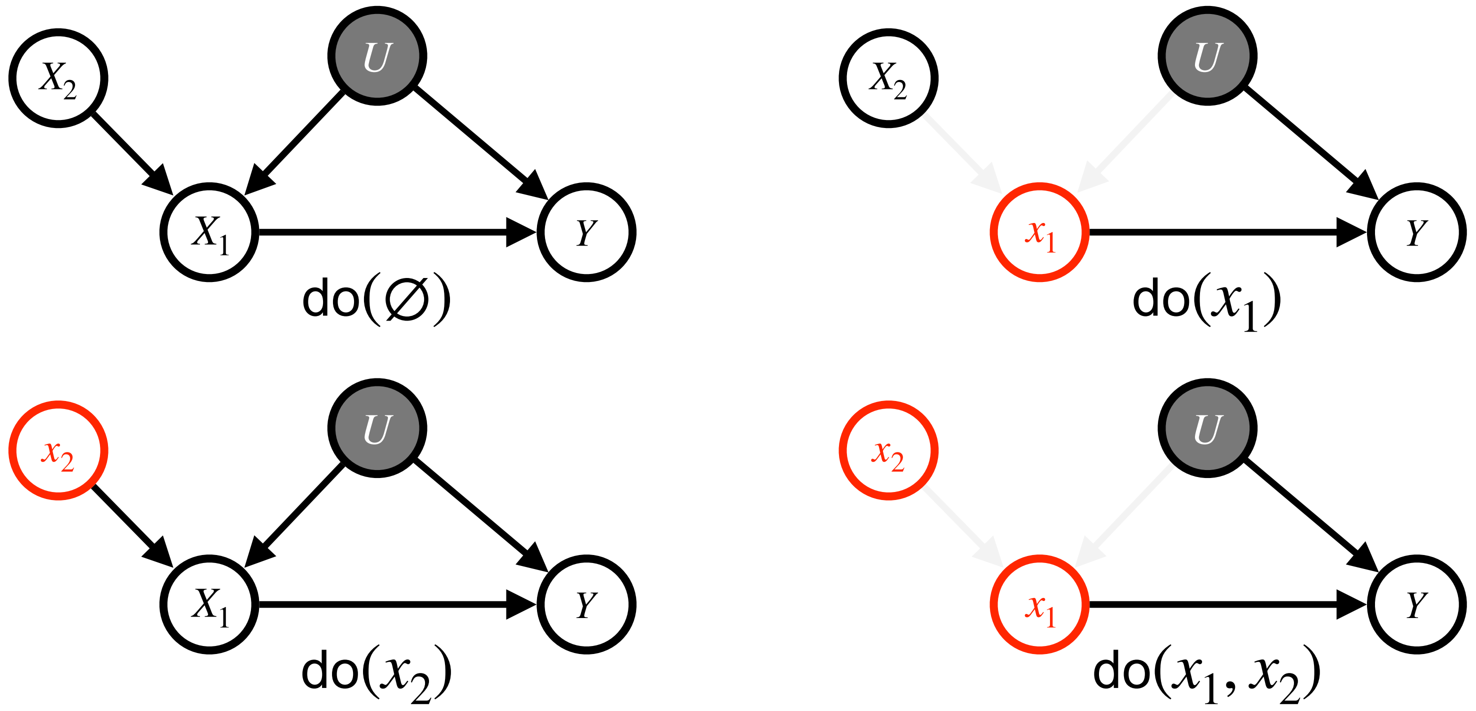
A. **Nine**. We need to choose a set among $\{\emptyset, \{X_1\}, \{X_2\}, \{X_1, X_2\}\}$.

Graphical Understanding of Causal MAB

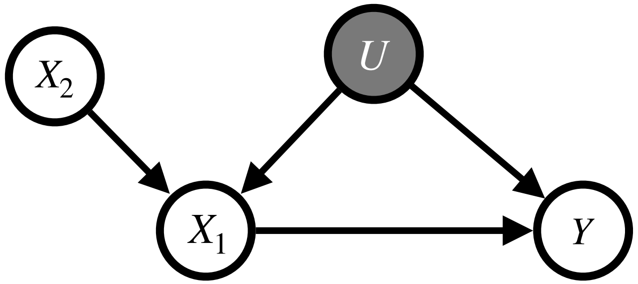


Q. How many **arms** are there? (We can control 2 binary variables, X_1 and X_2).

A. **Nine**. We need to choose a set among $\{\emptyset, \{X_1\}, \{X_2\}, \{X_1, X_2\}\}$.

∵ 1 + 2 + 2 + 4 = 9

Structural Causal Bandits



Intervention Sets — all subsets of $\mathbf{V}$ except Y.

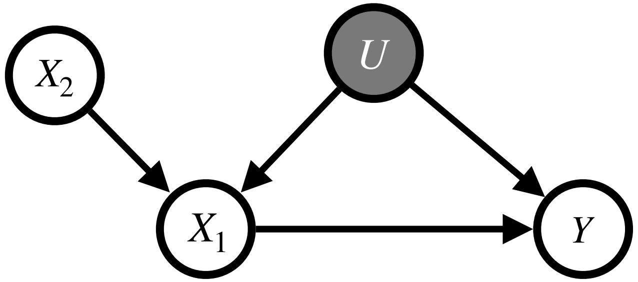
$\varnothing, \{X_1\}, \{X_2\}, \{X_1, X_2\}$

Arms — all possible values for intervention sets

$do(\varnothing), do(X_1 = 0), do(X_1 = 1), \cdots$

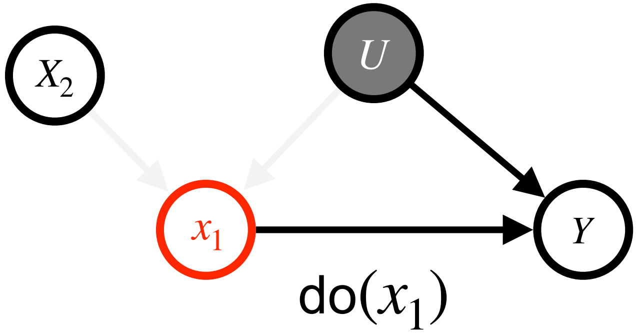
Reward — $\mu_{\mathbf{x}} \triangleq \mathbb{E}[Y \mid do(\mathbf{x})] = \sum_y yP(y \mid do(\mathbf{x}))$

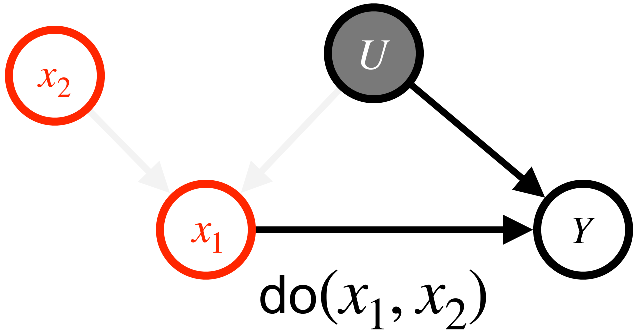
Structural Causal Bandits



Goal: Remove actions that is (1) redundant or (2) cannot be optimal based on given causal diagram.

Structural Property 1: Equivalence





$$\mu_{x_1} = \mu_{x_1,x_2}$$

Implication: prefer playing $\text{do}(x_1)$ to playing $\text{do}(x_1, x_2)$.

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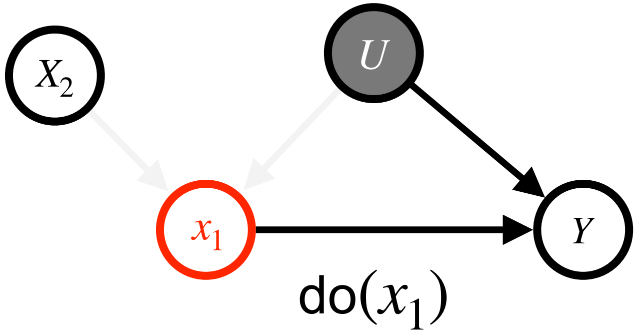

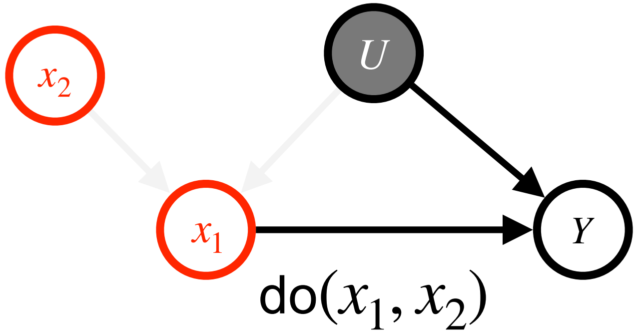

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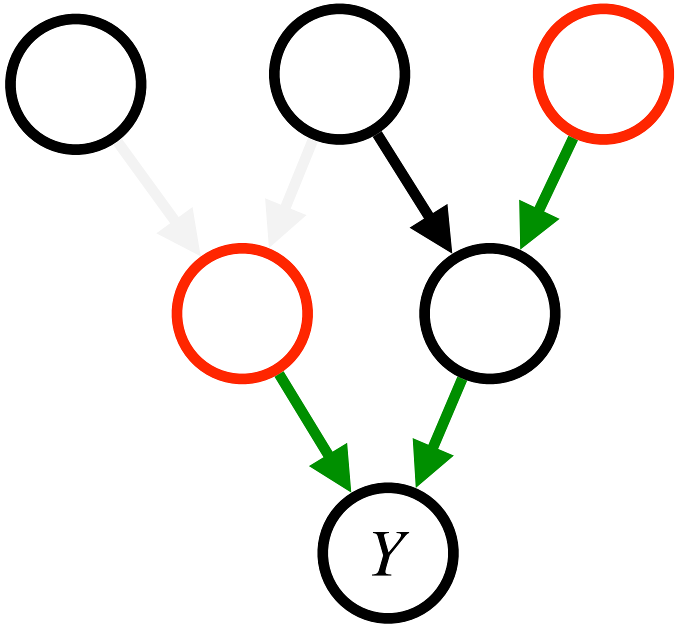
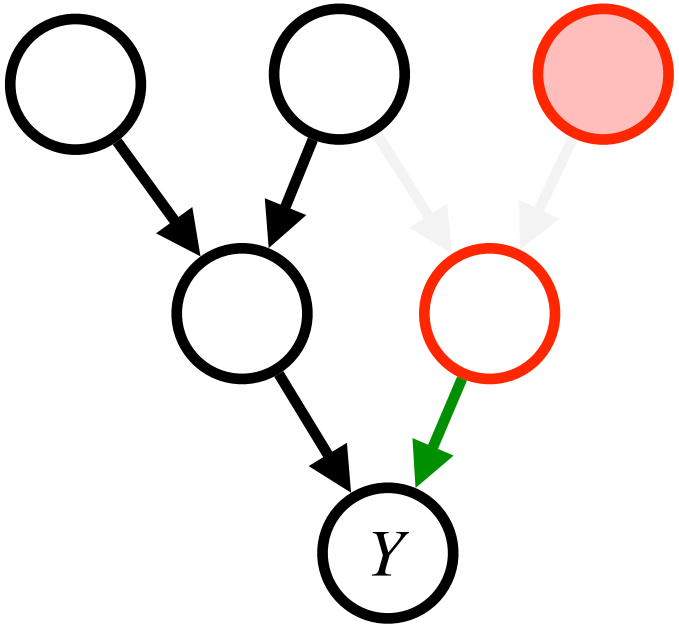
Definition: *Minimal* Intervention Set (MIS)

Graphical condition: All variables in $\mathbf{X}$ are ancesters of Y.

Minimal Intervention Set: Metal Picture



MIS



non-MIS

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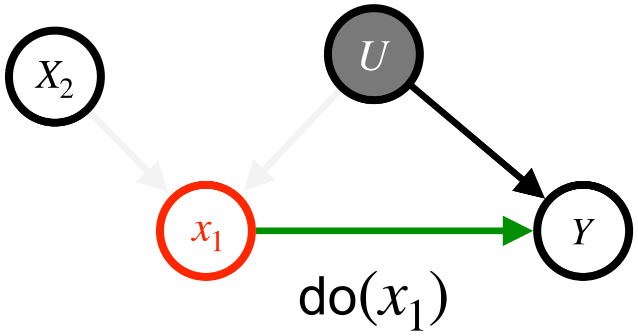

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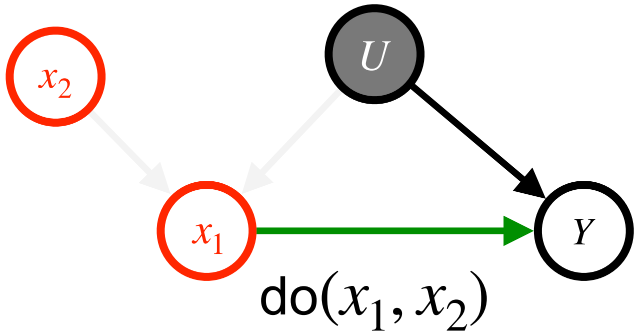
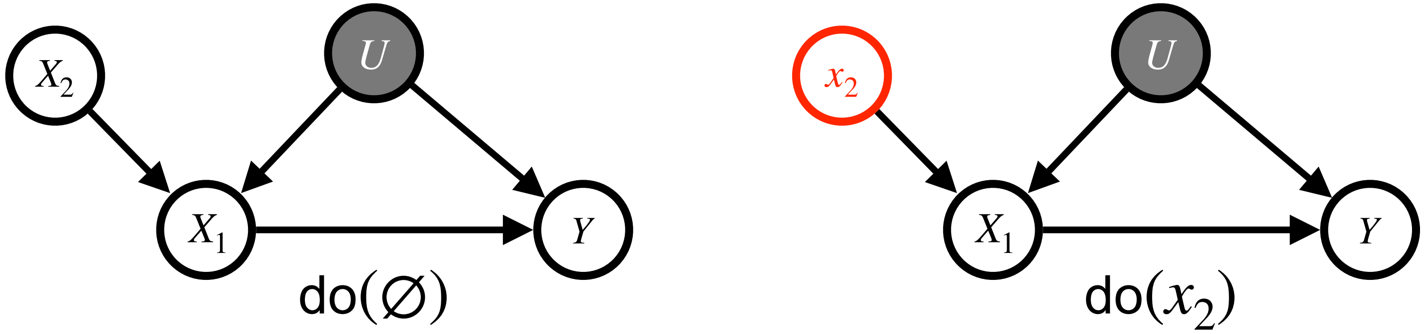


non-MIS

Structural Property 2: Partial-orderedness



$$\mu_{\varnothing} = \sum_{x_2} \mu_{x_2} P(x_2) \leq \sum_{x_2} \mu_{x_2^*} P(x_2) = \mu_{x_2^*}$$

Implication: prefer playing do(x_2) to playing do(∅)

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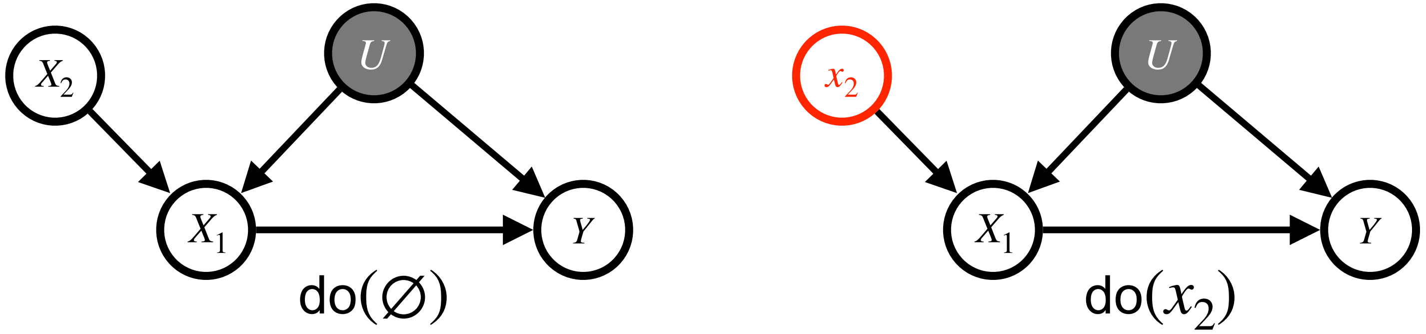

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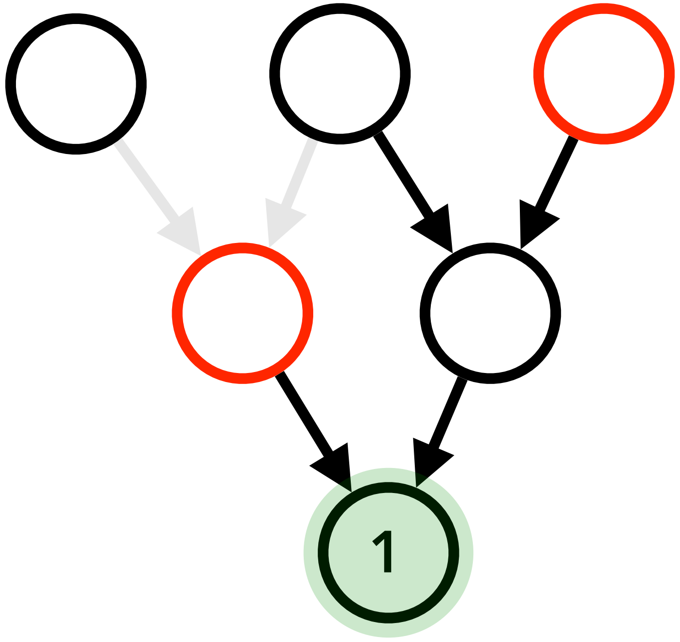
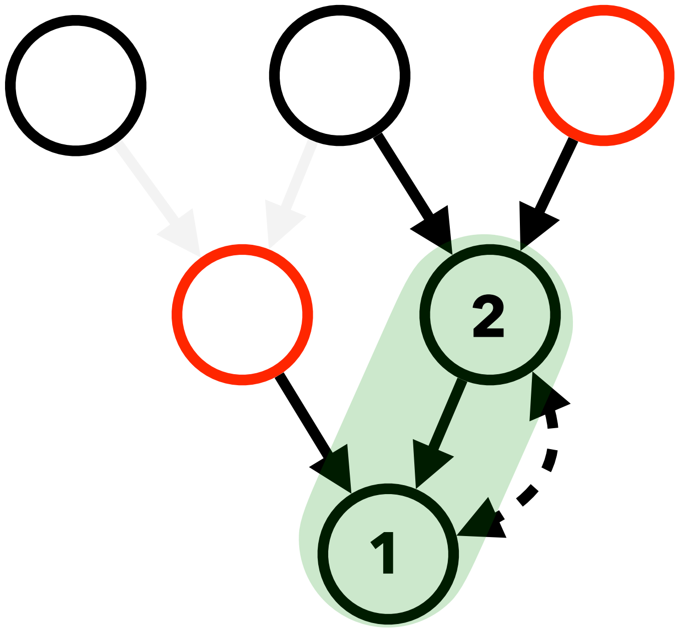
Definition: *possibly-optimal* Minimal Intervention Set (POMIS)

Graphical condition: All variables in $\mathbf{X}$ are parent of minimal closed mechanism under (1) descendant and (2) confounded.

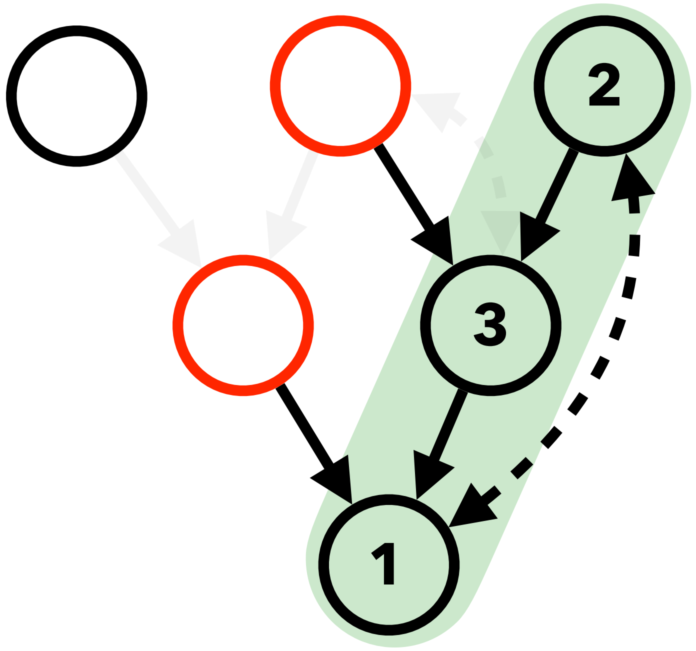
Possibly-Optimal Minimal Intervention Set: Metal Picture



non-POMIS

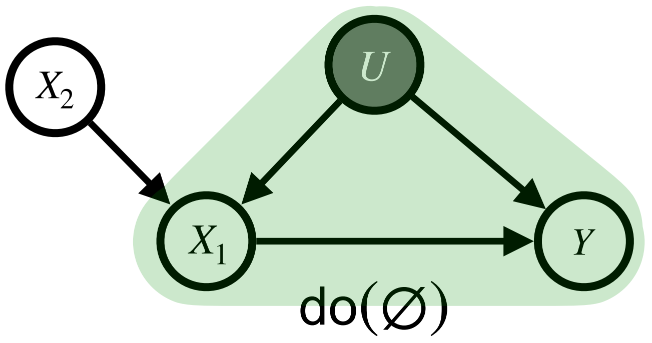


POMIS



POMIS

Minimal Intervention Set: Metal Picture



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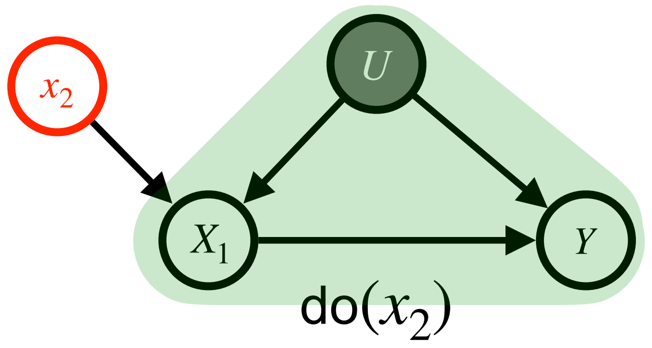
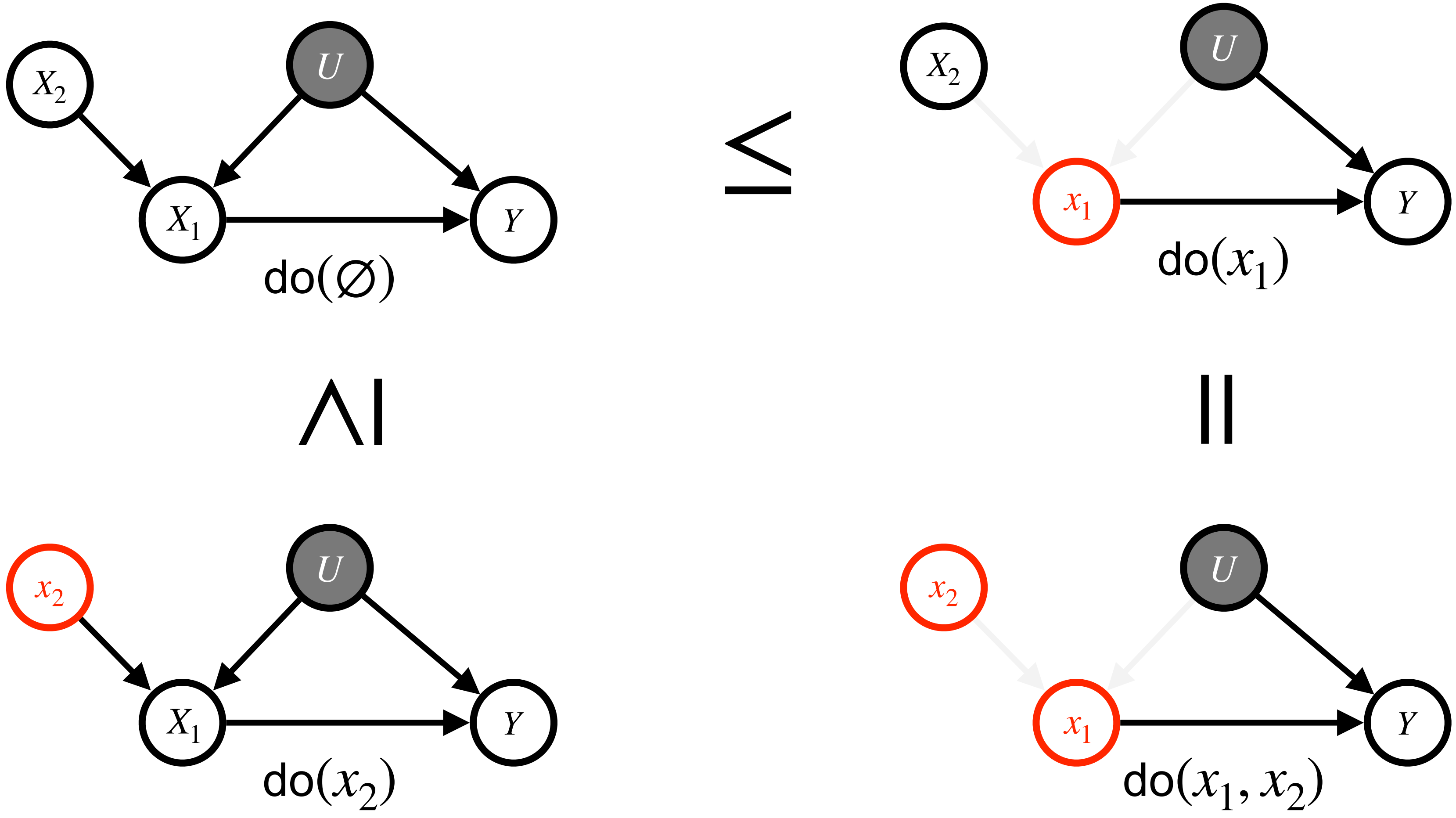


POMIS

Structural Relationships between Intervention Sets



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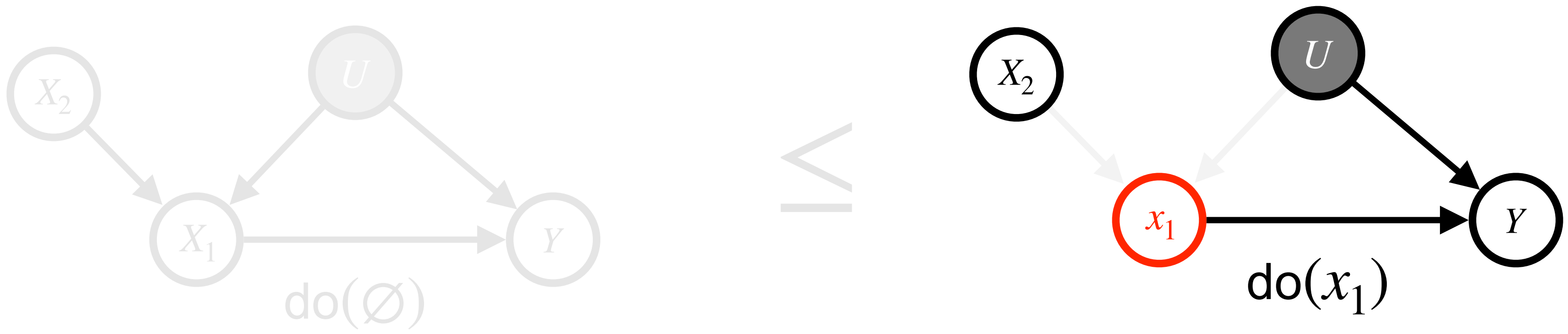
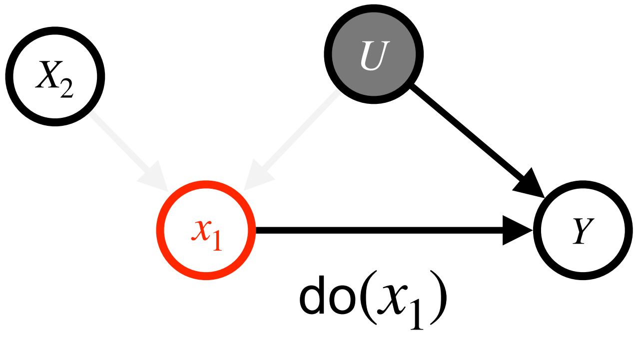


Playing an arms do(x_1) and do(x_2) is sufficient!

Structural Relationships between Intervention Sets



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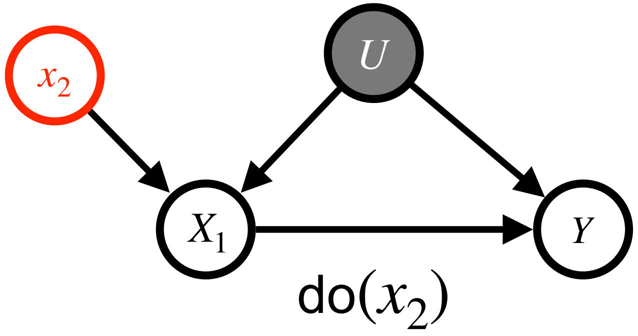
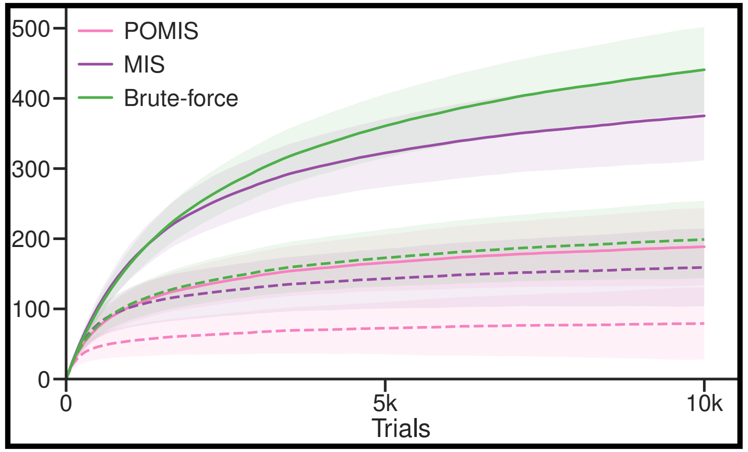
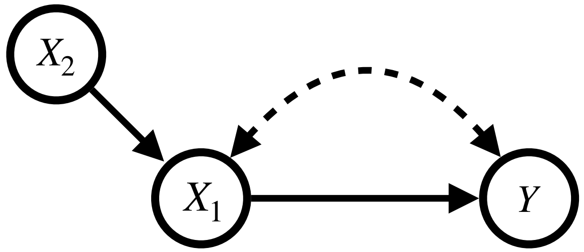


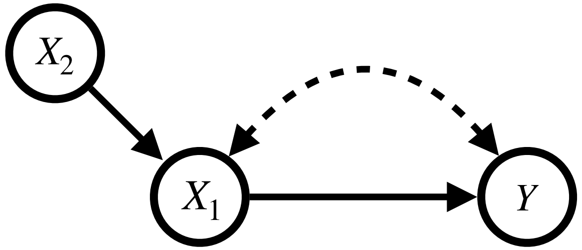


Motivation



A key assumption is that the agent has access to a causal diagram representing the target system. **However**, this is often violated.

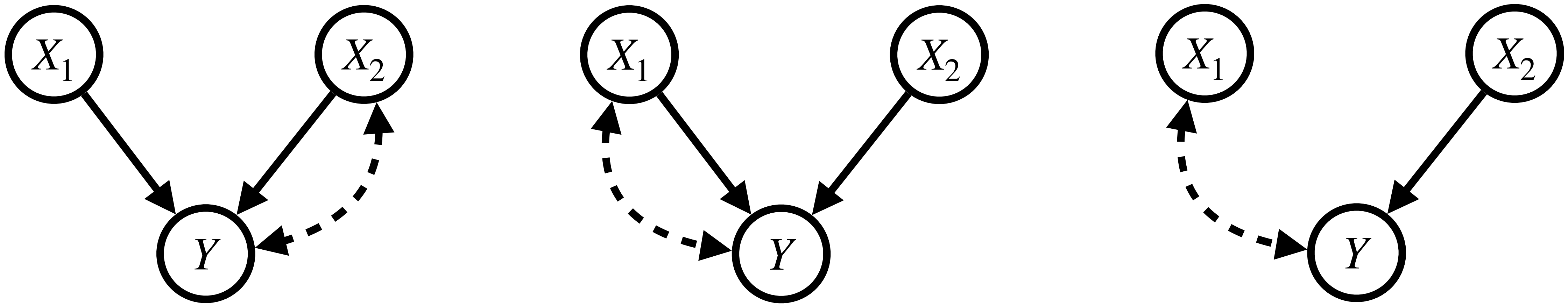
Contribution



A key assumption is that the agent has access to a causal diagram representing the target system. **However**, this is often violated.

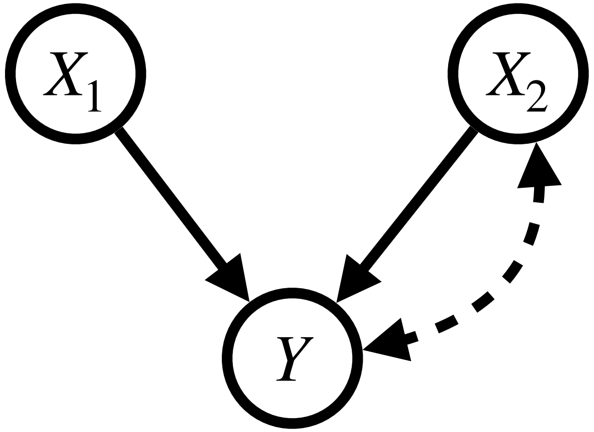
We assume access to a graph represening a **Markov Equivalence Class**, called a **PAG (Partial Ancestral Graph)** rather then a causal diagram.

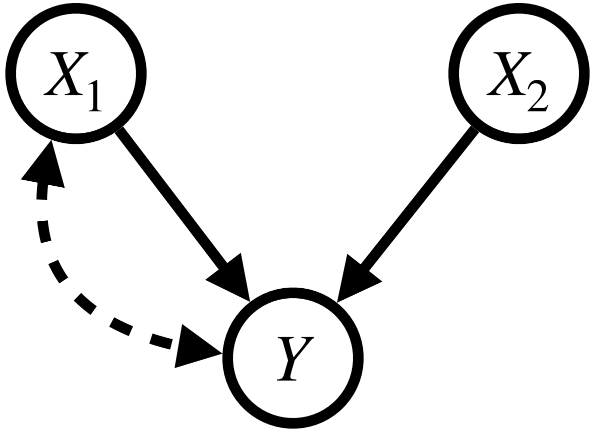
Markov Equivalence Class

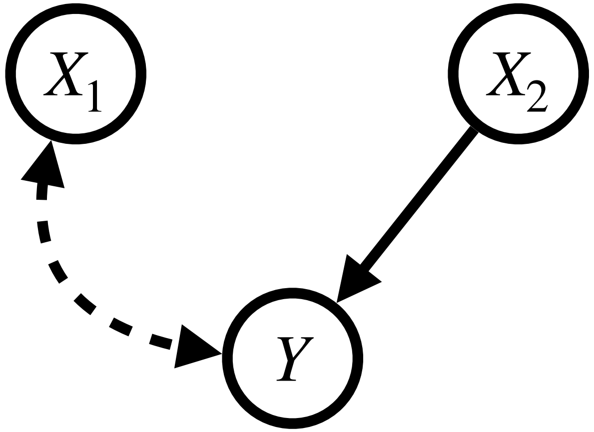


They share (1) the same independence statement $X_1 \perp\!\!\!\perp_d X_2$.

Markov Equivalence Class







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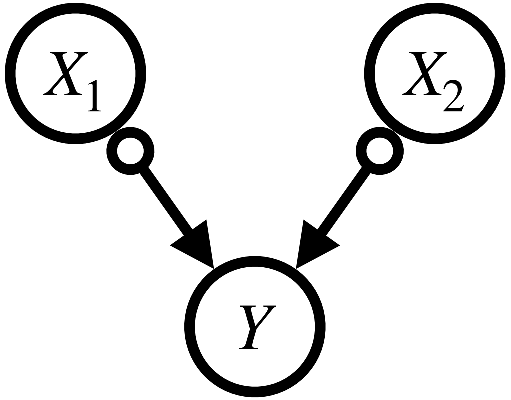
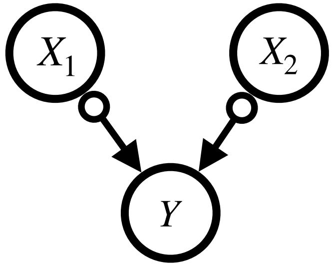


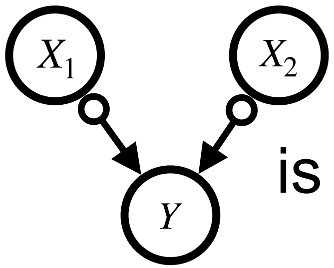
The graph is called as a **PAG (Partial Ancestral Graph)**.

Structural Causal Bandits under Markov Equivalence

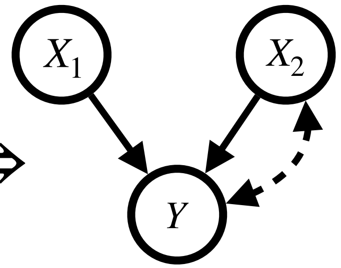


Goal: Remove unnecessary actions that cannot be optimal (i.e., non-POMIS) under any underlying causal diagram.

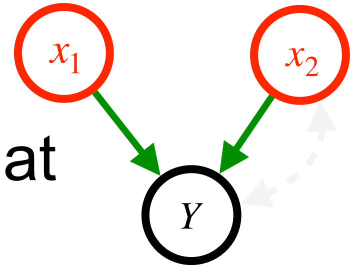
Definitely Minimal Intervention Sets for PAG



is a DMIS $\Leftrightarrow$ $\exists$



such that



Definition: *A set is a Definitely Minimal* Intervention Set (DMIS) if there exists a causal diagram under which it is an MIS.

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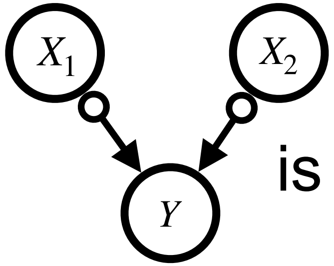

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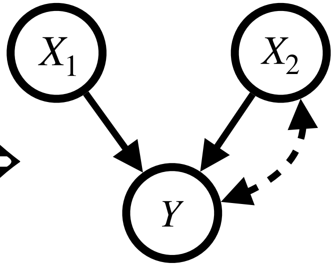

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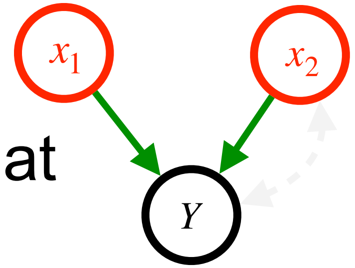
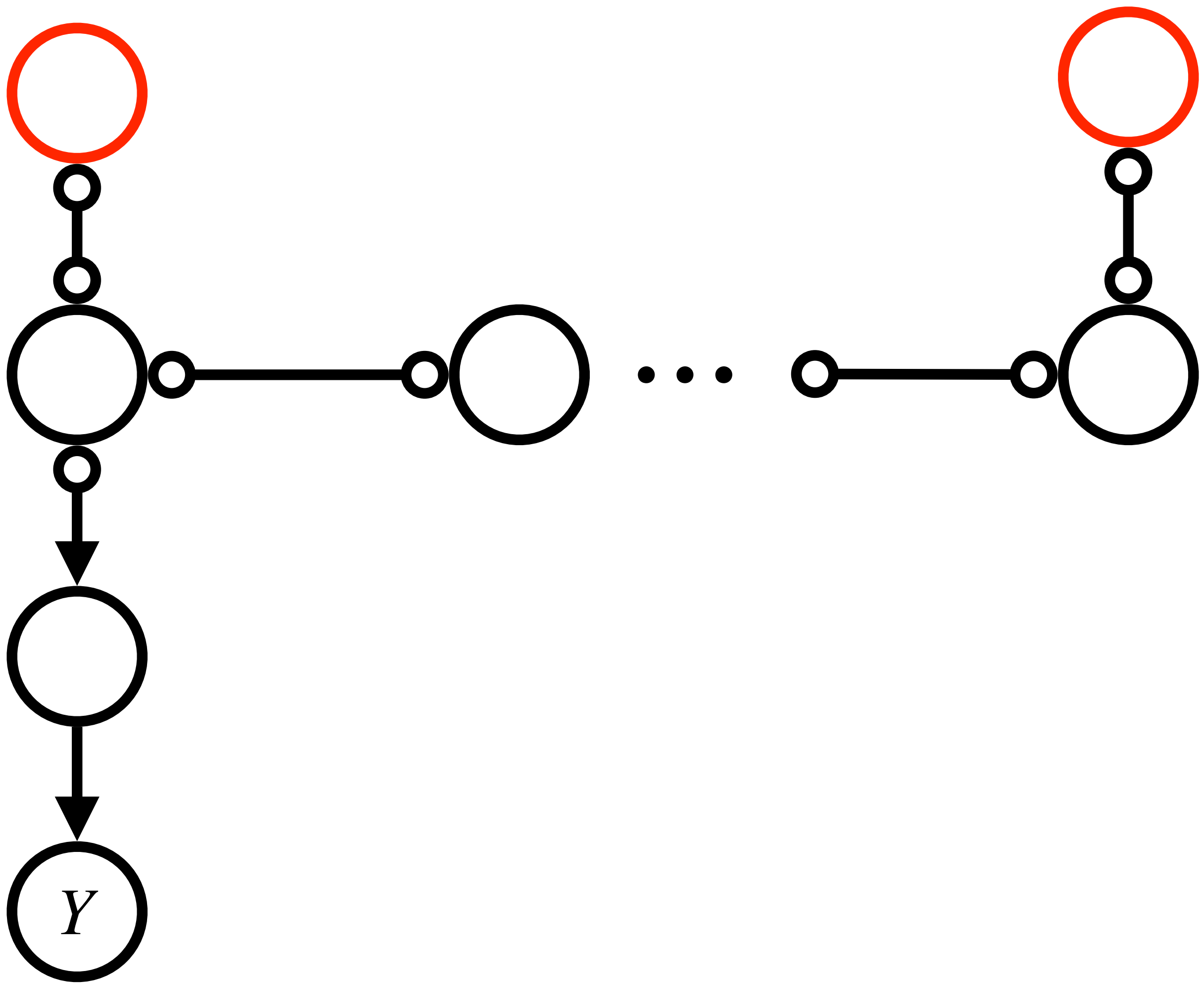


Definition: *A set is a Definitely Minimal* Intervention Set (DMIS) if there exists a causal diagram under which it is an MIS.

Graphical condition: All variables in $\mathbf{X}$ are (1) possibly ancesters of Y. and (2) not relevant.

Definitely Minimal Intervention Set: Metal Picture



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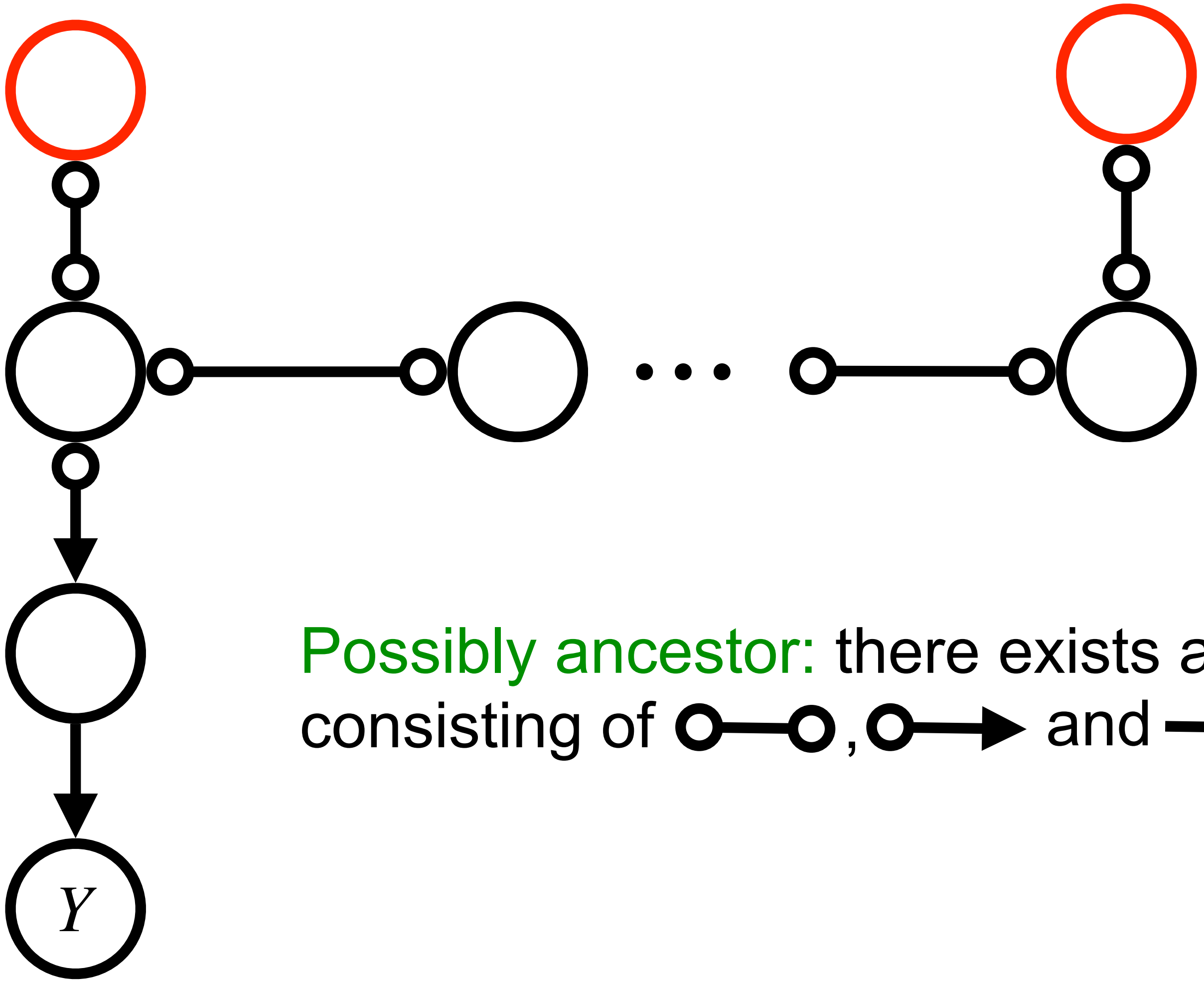
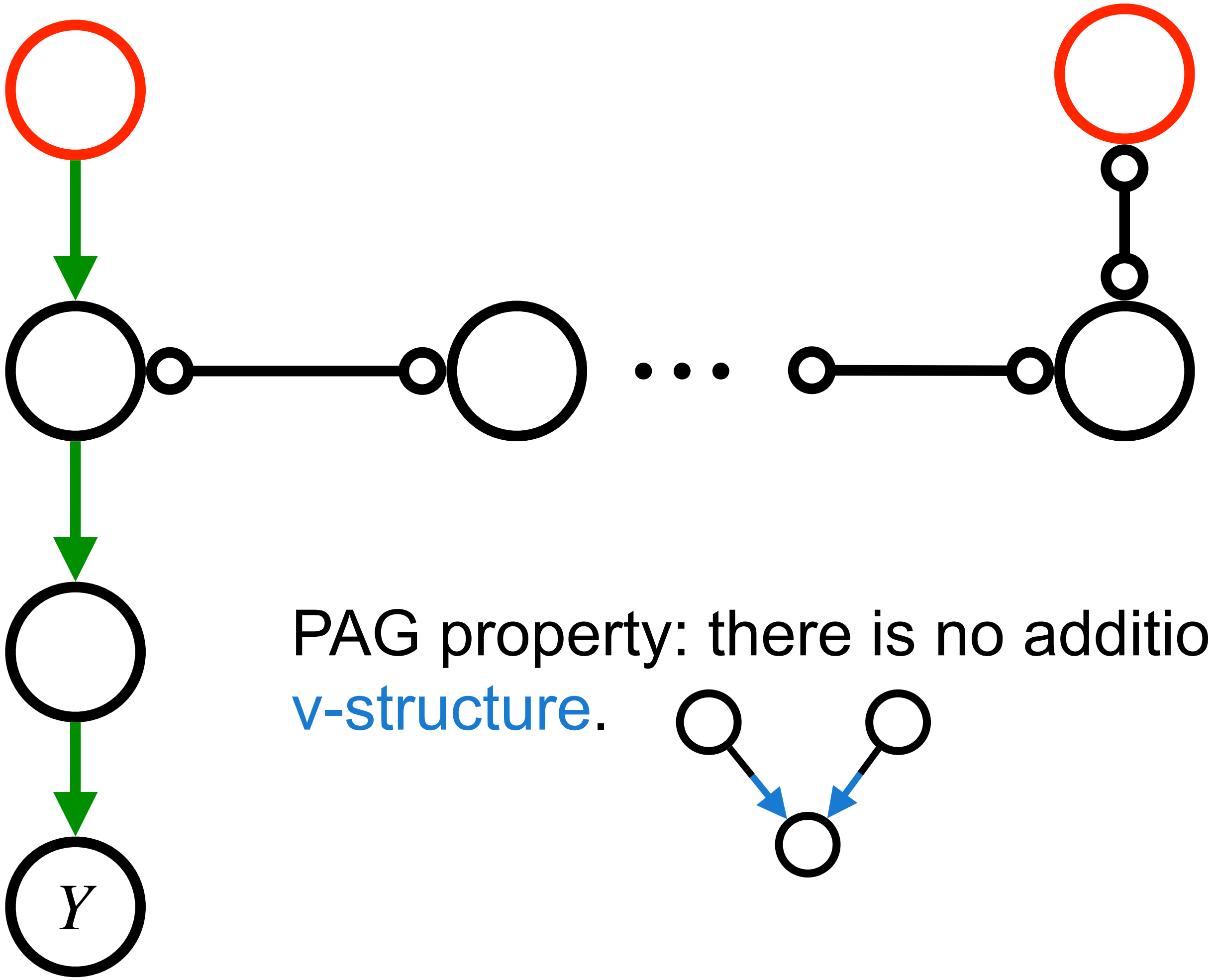


Possibly ancestor: there exists a path consisting of ∘—∘, ∘→ and →.

Definitely Minimal Intervention Set: Metal Picture



PAG property: there is no additional v-structure.

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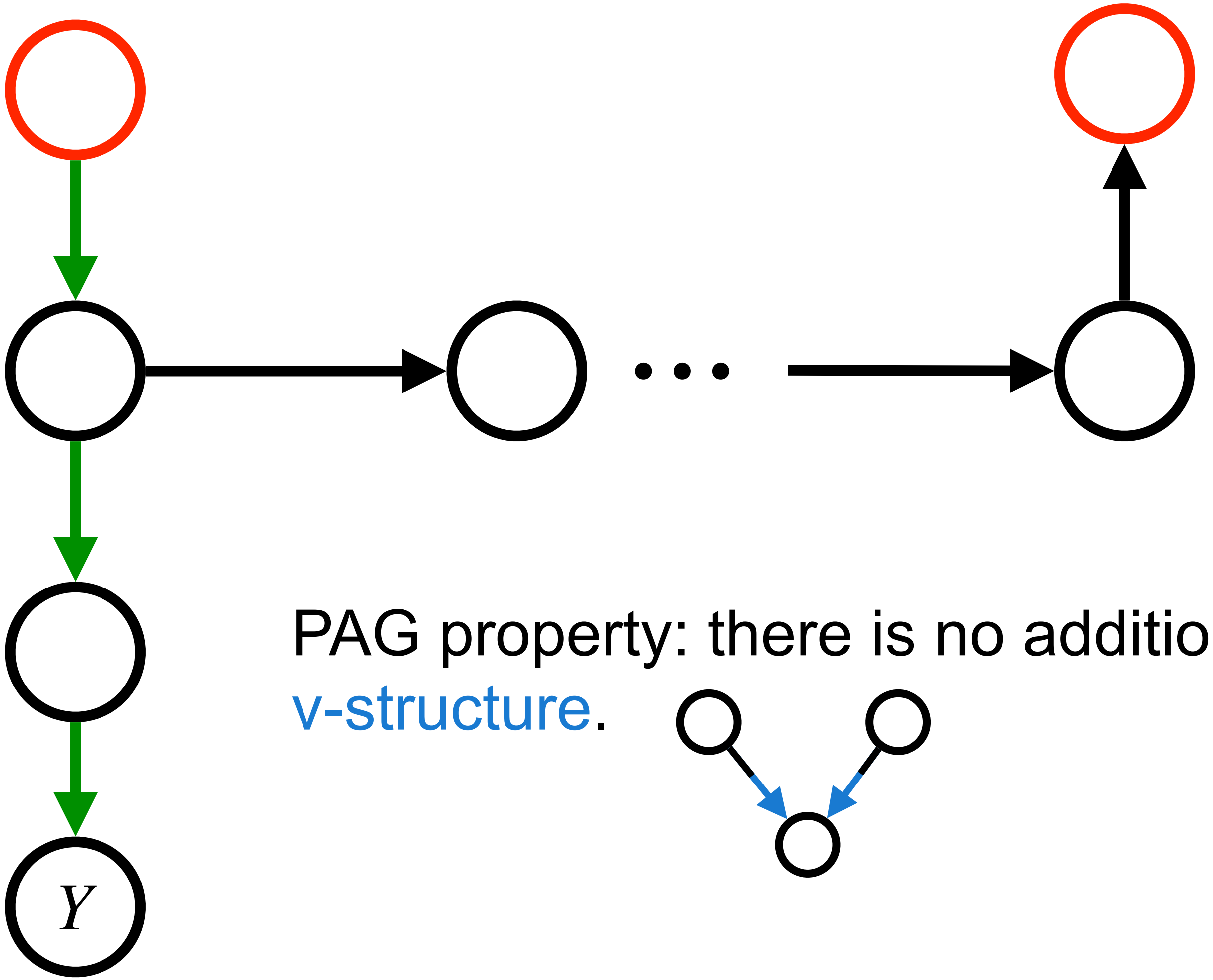

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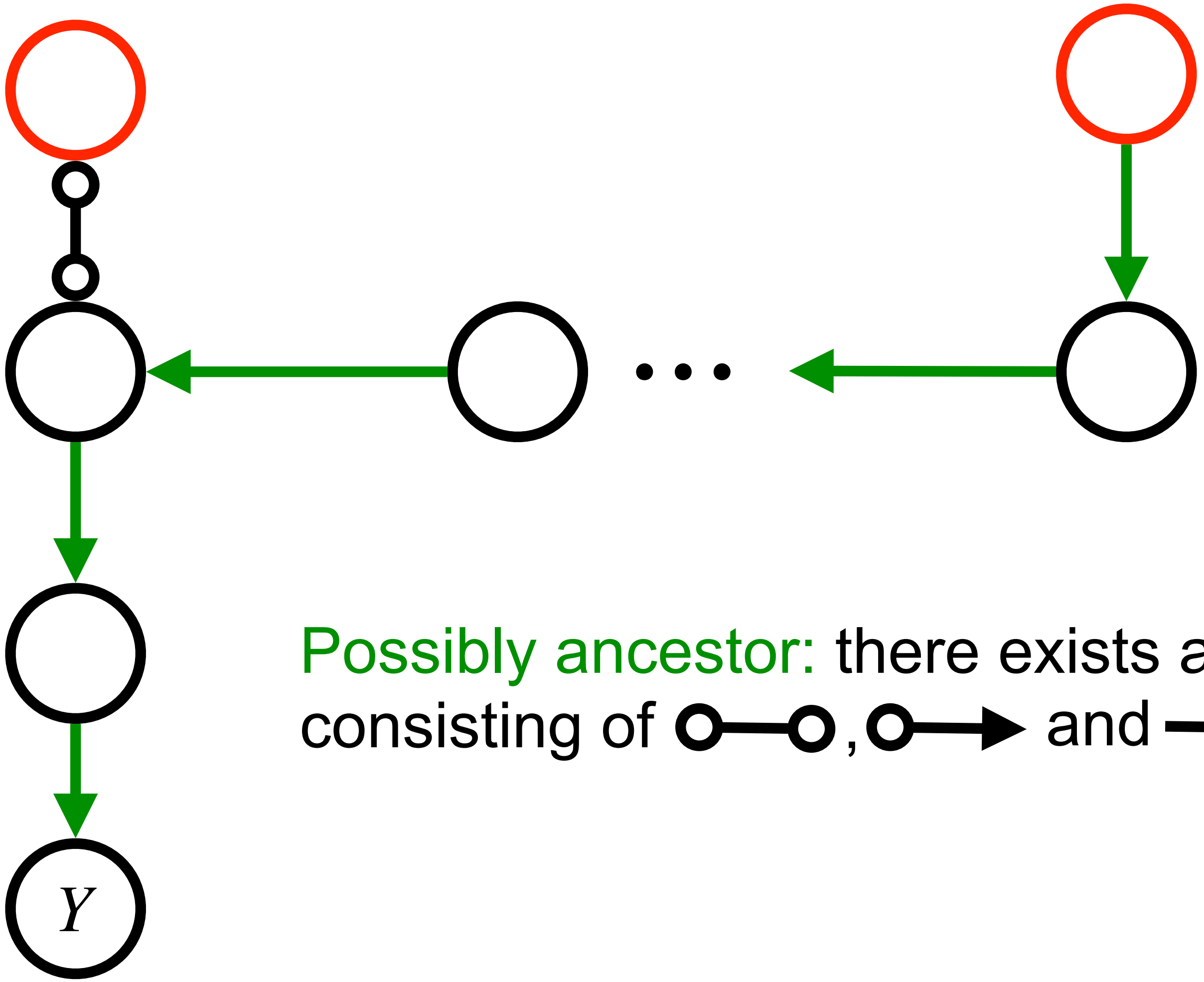

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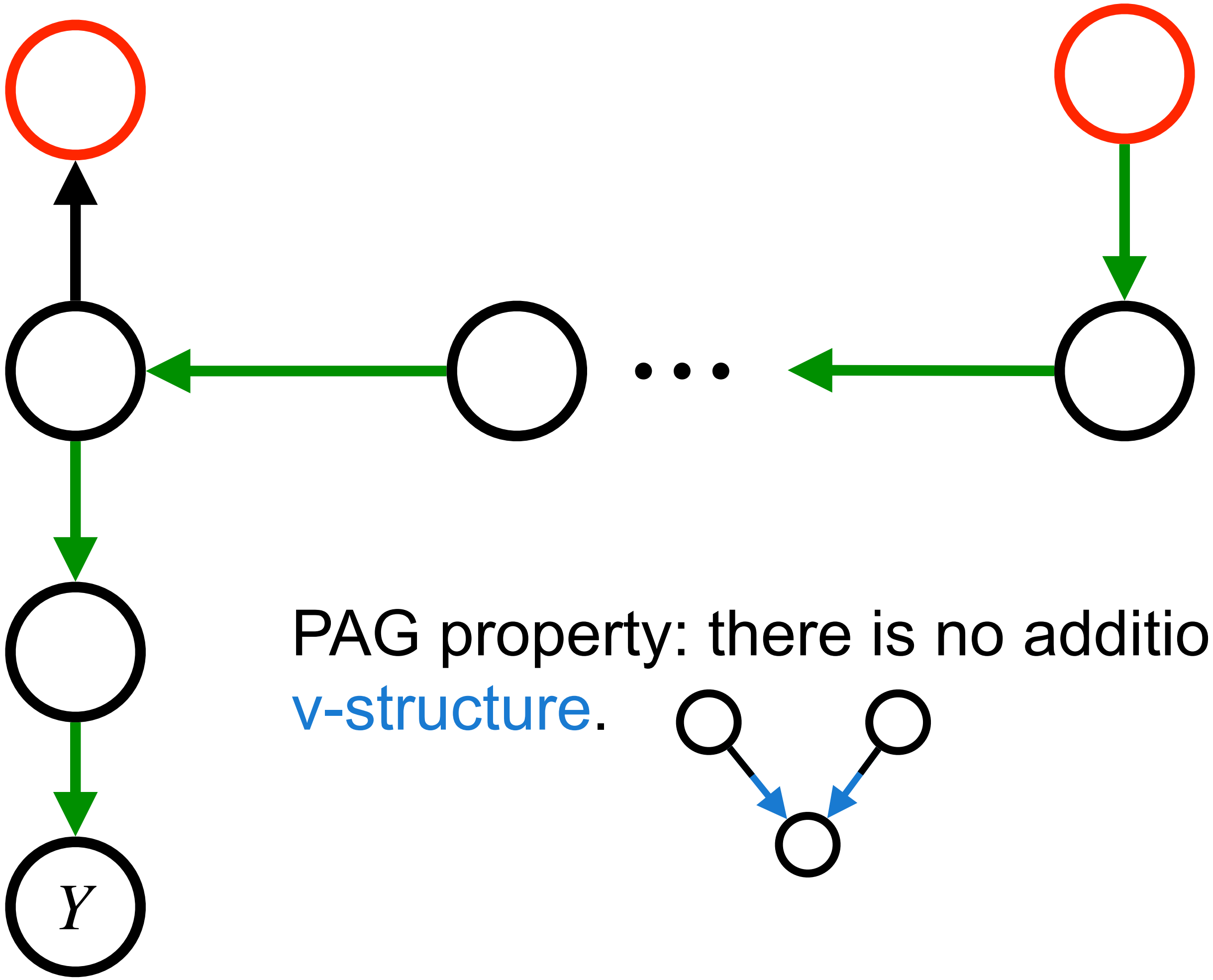

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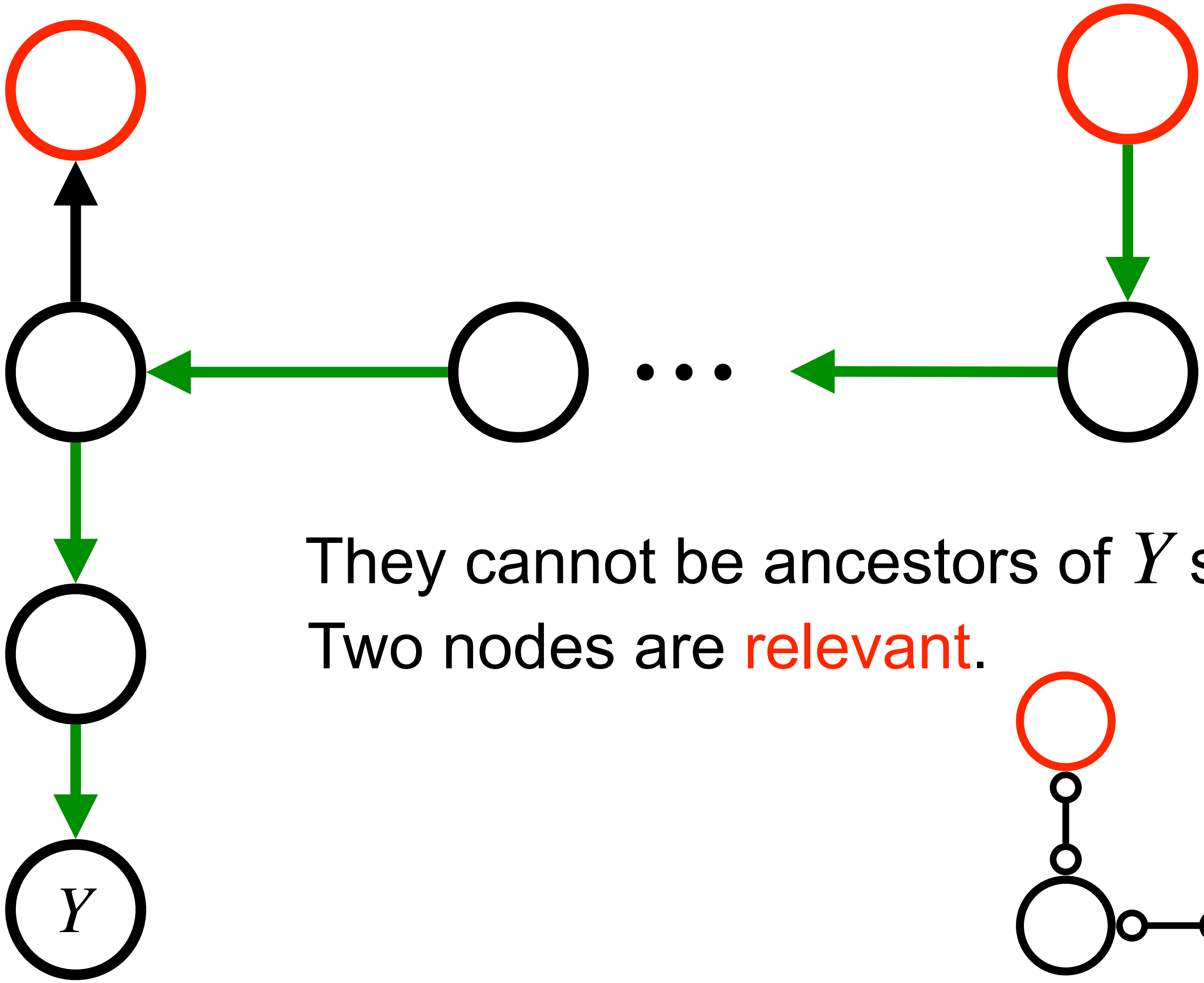
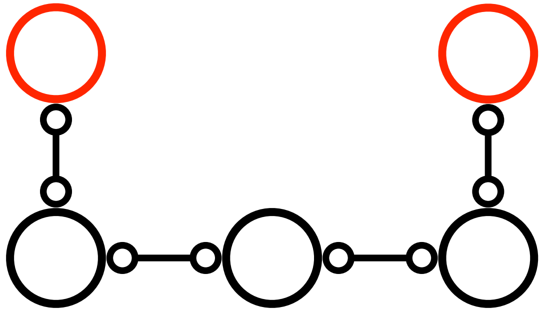
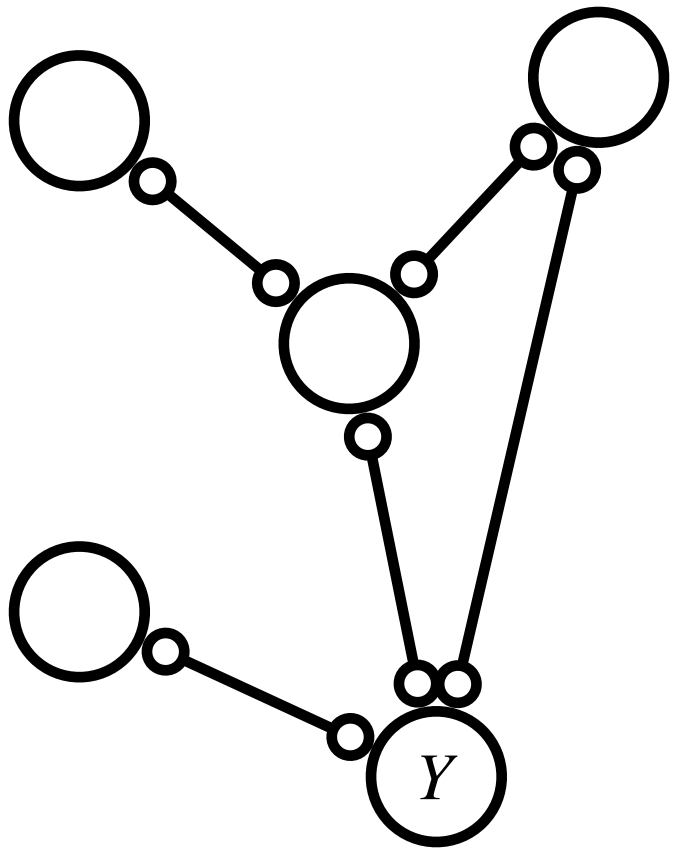


They cannot be ancestors of Y simultaneously.

Two nodes are relevant.



Definitely Minimal Intervention Set: Example



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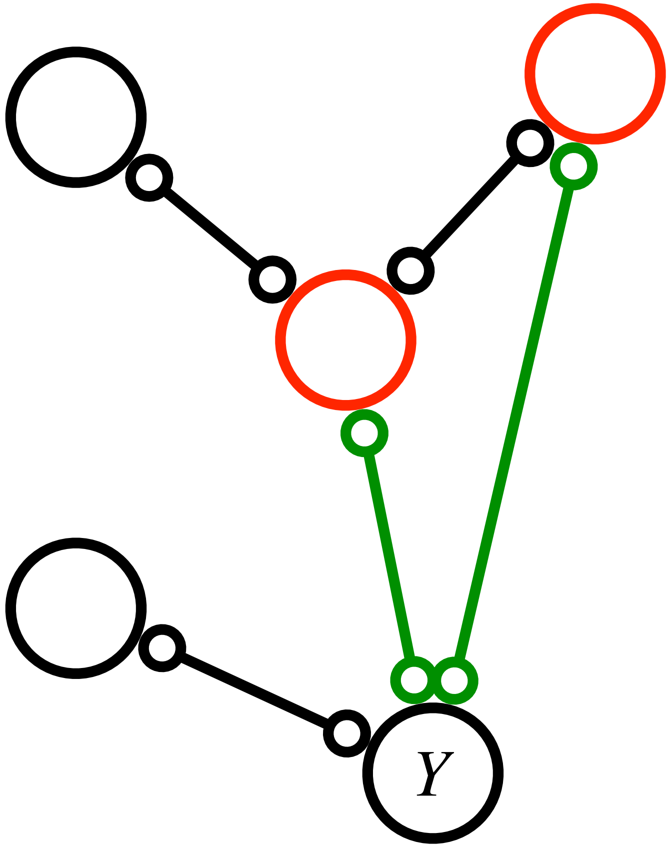
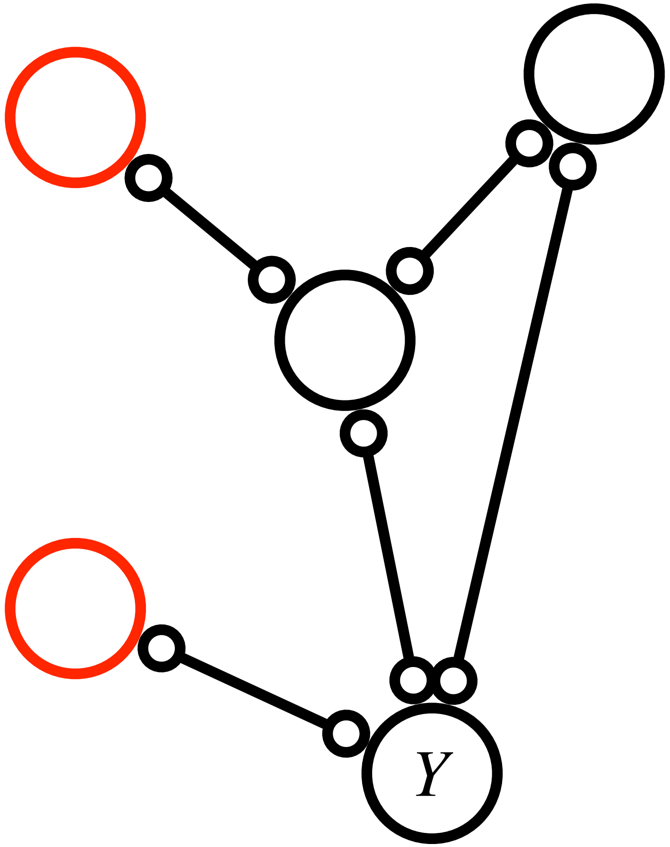


DMIS

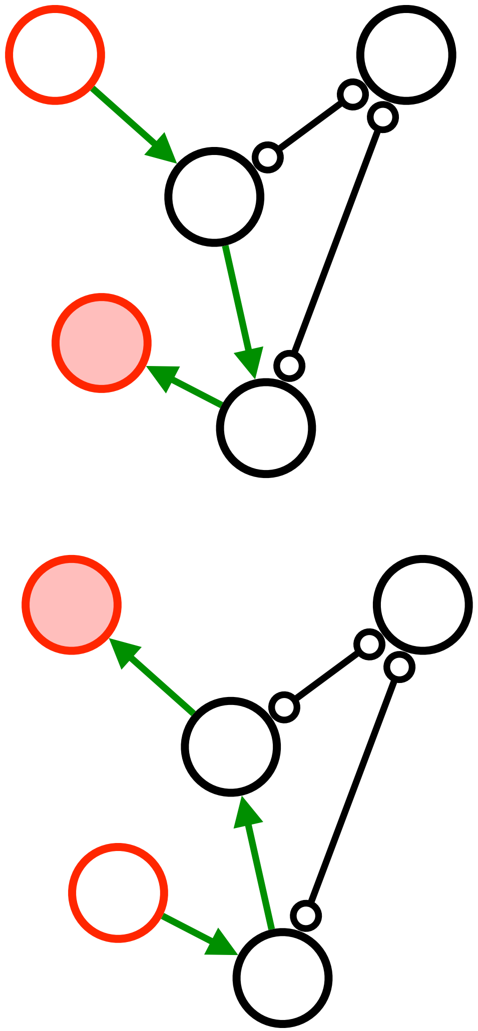
Possibly ancestor: there exists a path consisting of o—o, o→ and →.

Definitely Minimal Intervention Set: Example

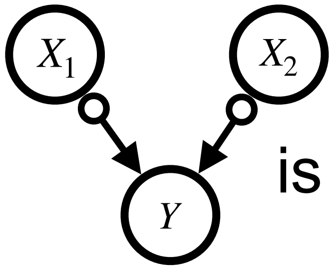


Two nodes are relevant.

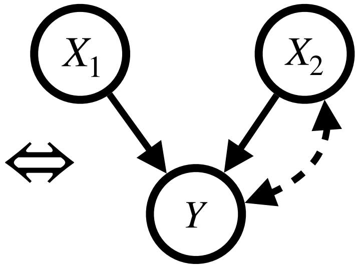
non-DMIS



Possibly-Optimal Minimal Intervention Sets for PAG



is a POMIS ⇔ ∃



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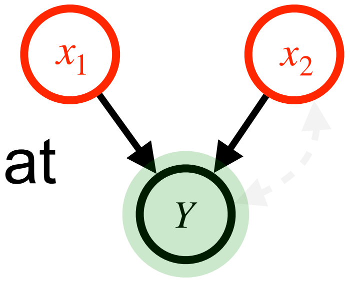
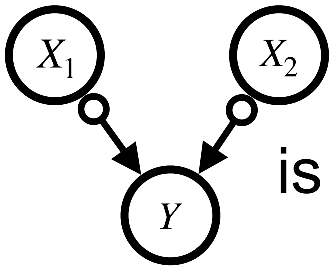
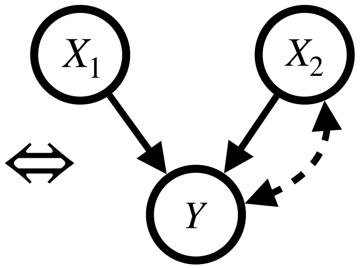


Definition: *A set is a Possibly-Opimal Minimal* Intervention Set (POMIS) if there exists a causal diagram under which it is an POMIS.

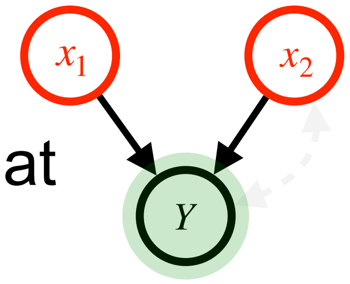
Possibly-Optimal Minimal Intervention Sets for PAG



is a POMIS $\Leftrightarrow \exists$



such that



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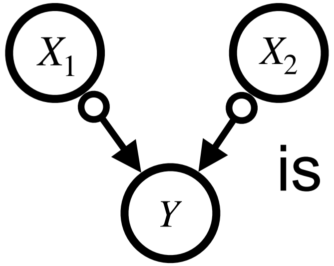
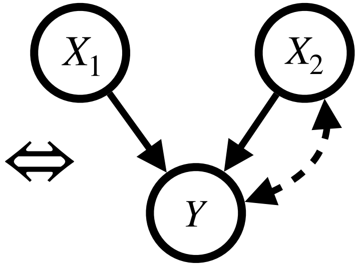
Graphical condition: All variables in $\mathbf{X}$ are parent of minimal closed mechanism under (1) possibly descendant and (2) possibly confounded in a local transformed graph (around $\mathbf{X} \cup \{Y\}$).

Possibly-Optimal Minimal Intervention Sets for PAG



is a POMIS $\Leftrightarrow$ $\exists$



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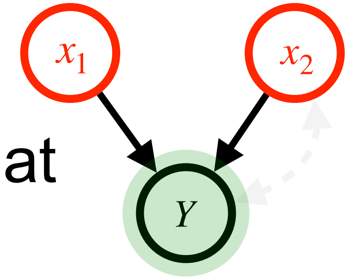

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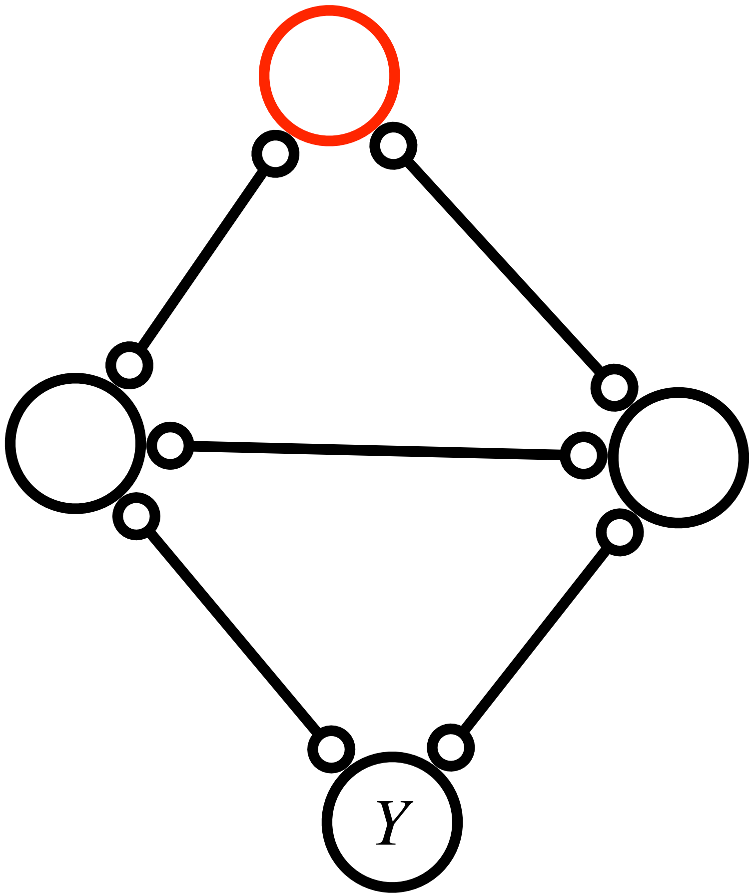
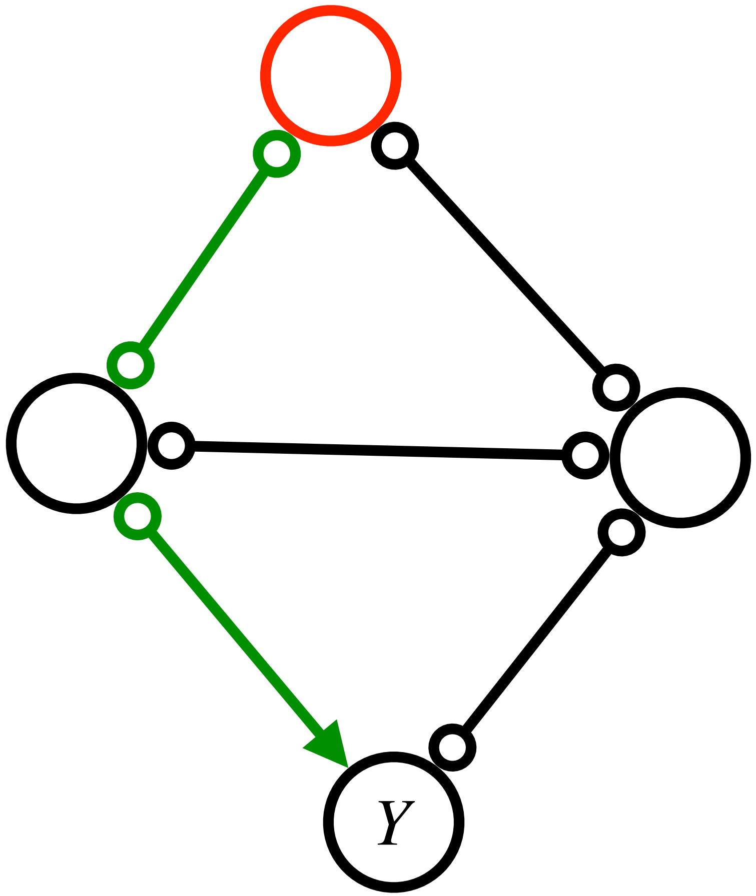
i.e., a graph in which all represented causal diagrams have $\mathbf{X}$ as a MIS.

Possibly-Optimal Minimal Intervention Set: Metal Picture

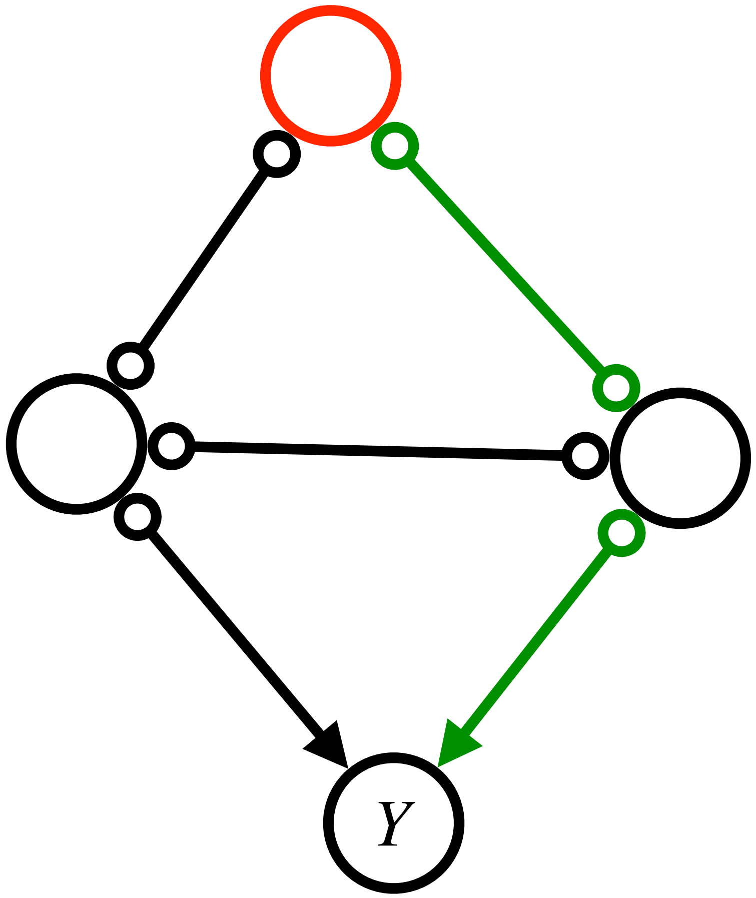


Possibly-Optimal Minimal Intervention Set: Metal Picture

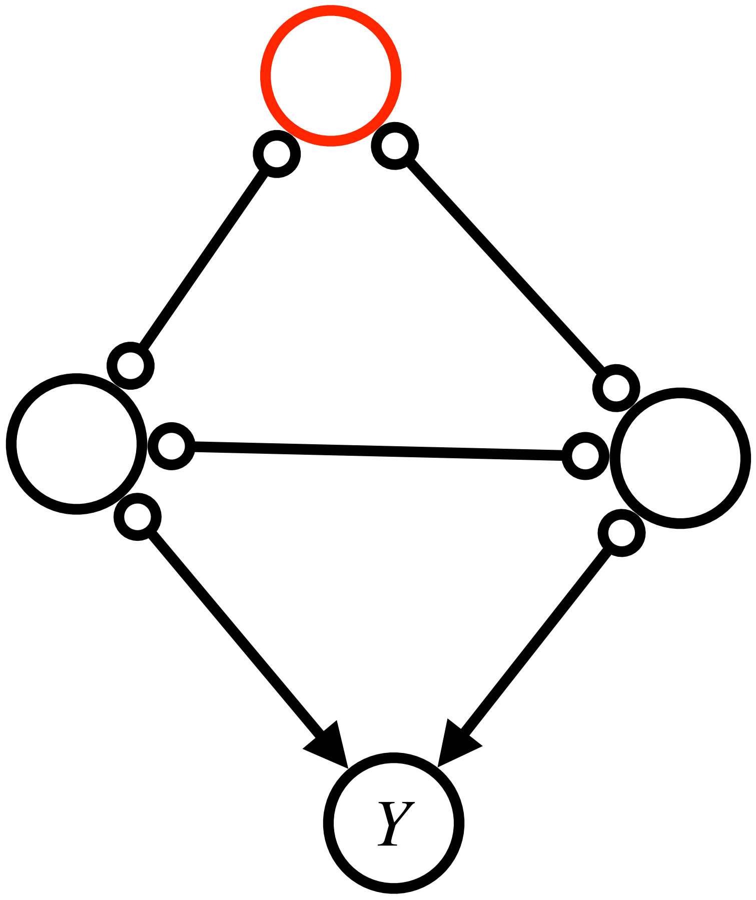


Proposition: Every uncovered proper possibly-directed path ends with an arrowhead  .

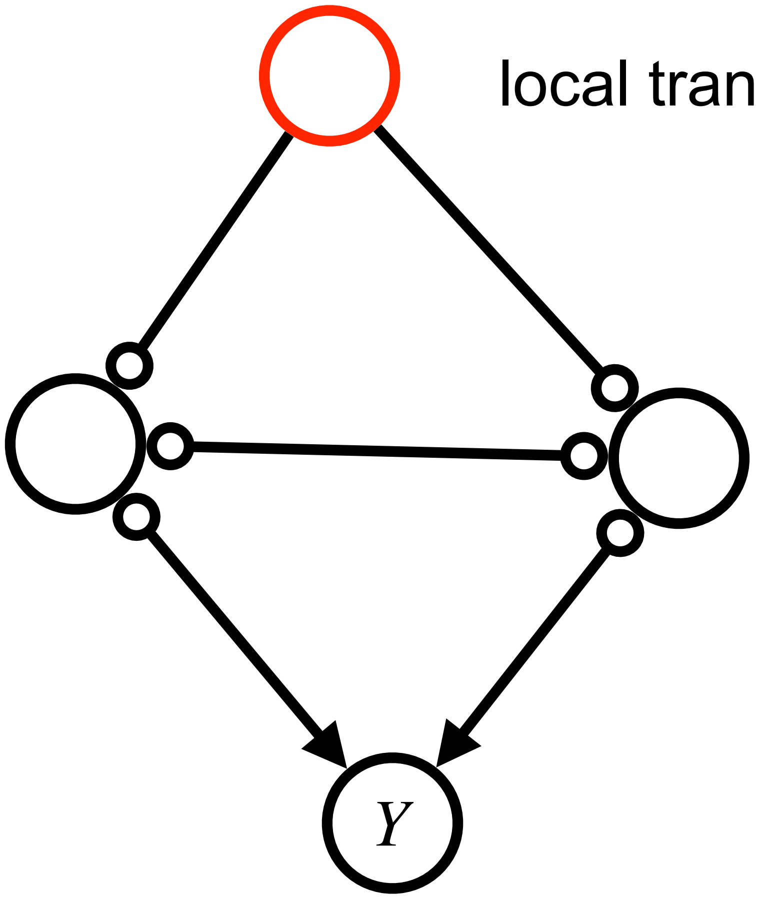
Possibly-Optimal Minimal Intervention Set: Metal Picture



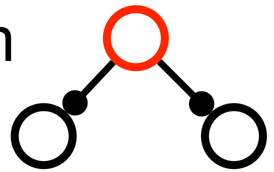
Possibly-Optimal Minimal Intervention Set: Metal Picture



Possibly-Optimal Minimal Intervention Set: Metal Picture



local transform



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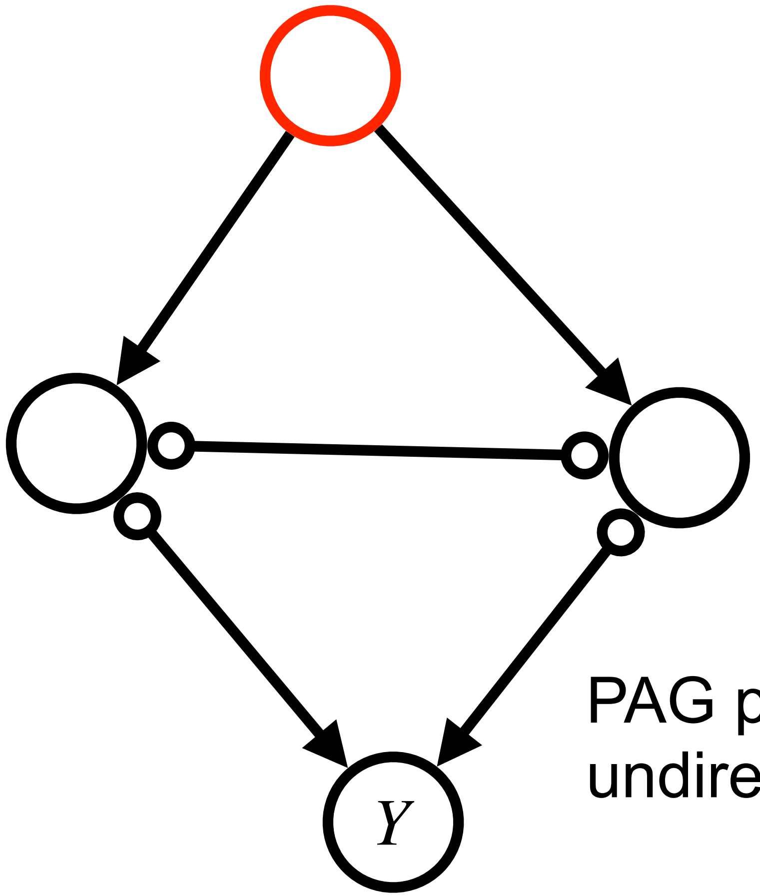
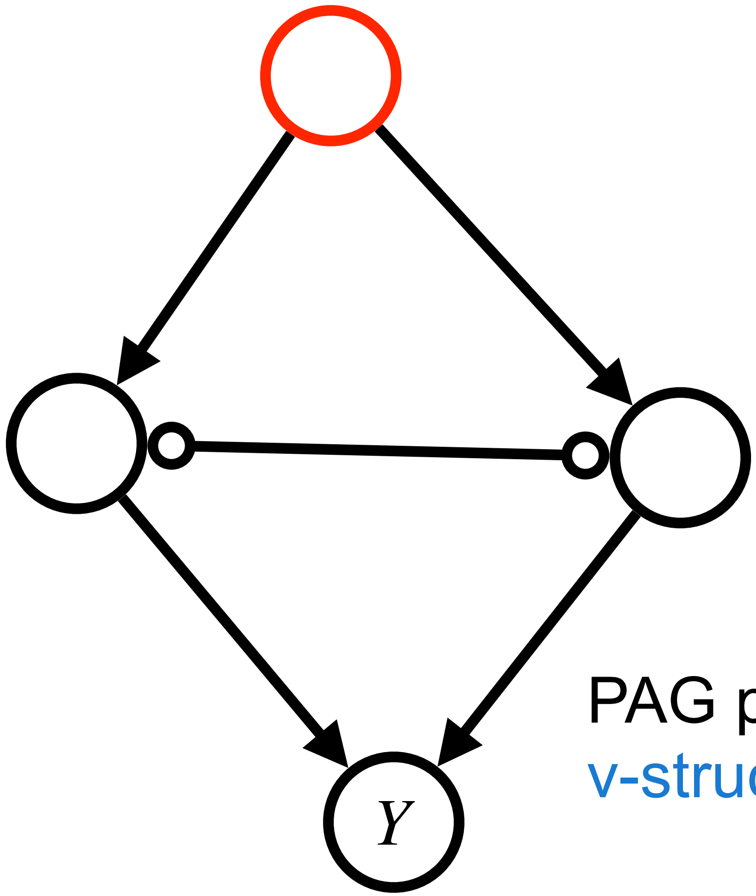
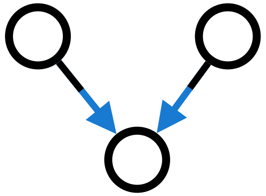


PAG property: there is no undirected edge.

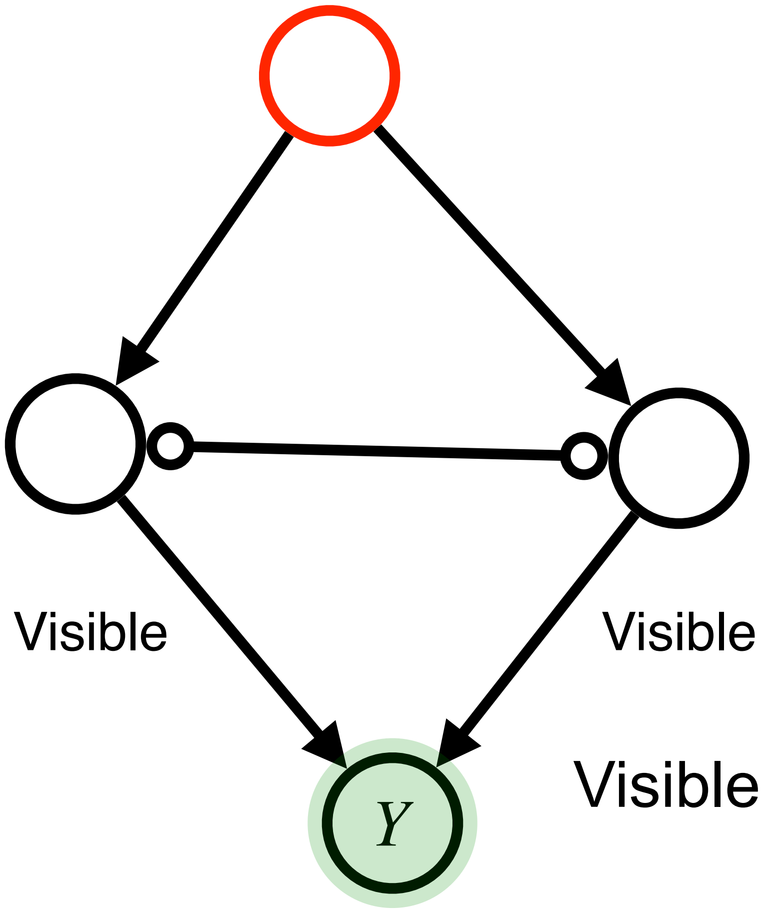
Possibly-Optimal Minimal Intervention Set: Metal Picture



PAG property: there is no additional v-structure.

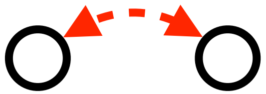


Possibly-Optimal Minimal Intervention Set: Metal Picture

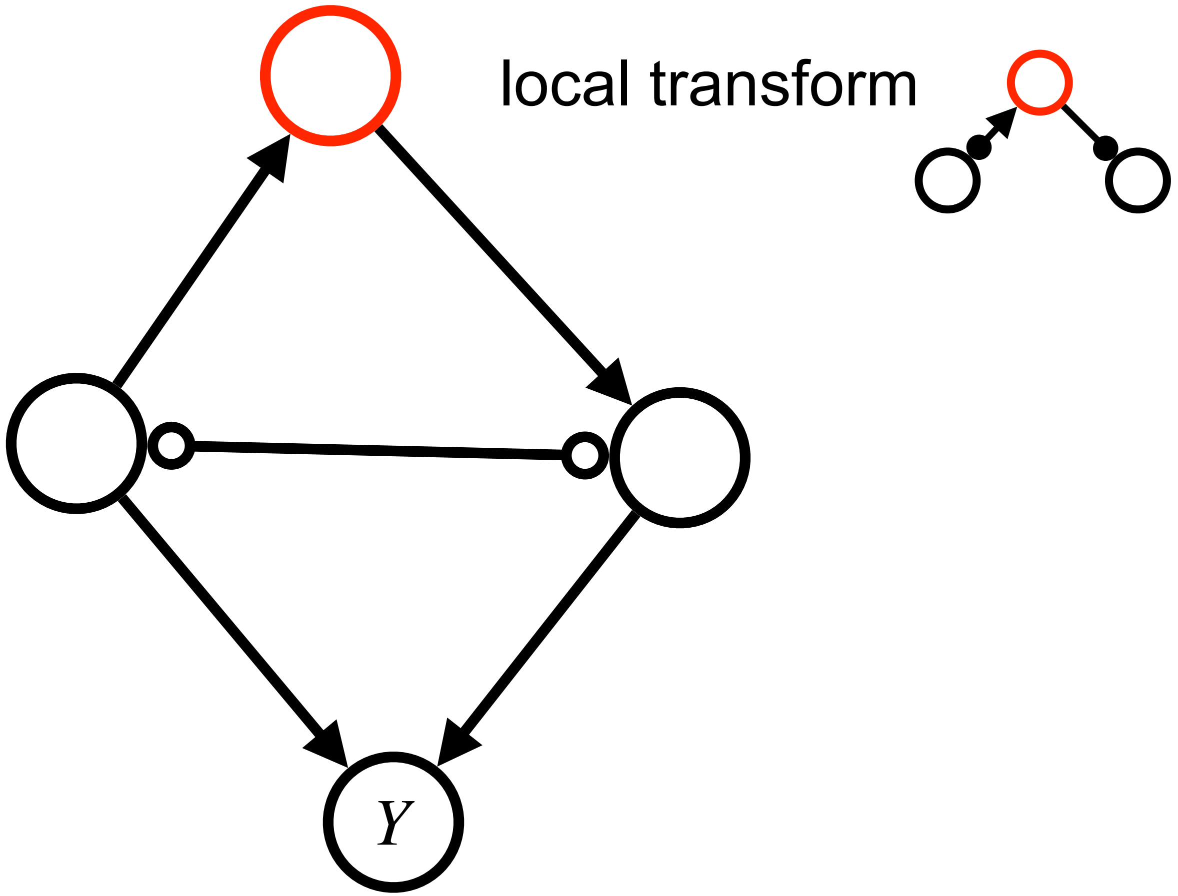


non-POMIS

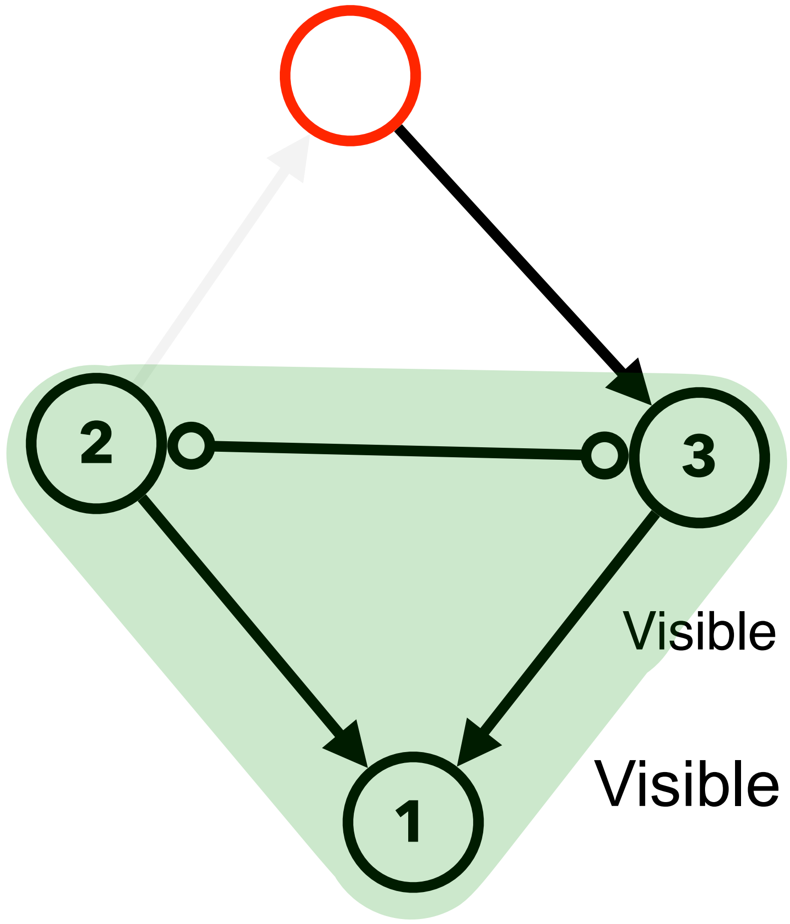
Visible edge: there is no confounder.



Possibly-Optimal Minimal Intervention Set: Metal Picture

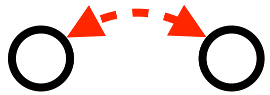


Possibly-Optimal Minimal Intervention Set: Metal Picture

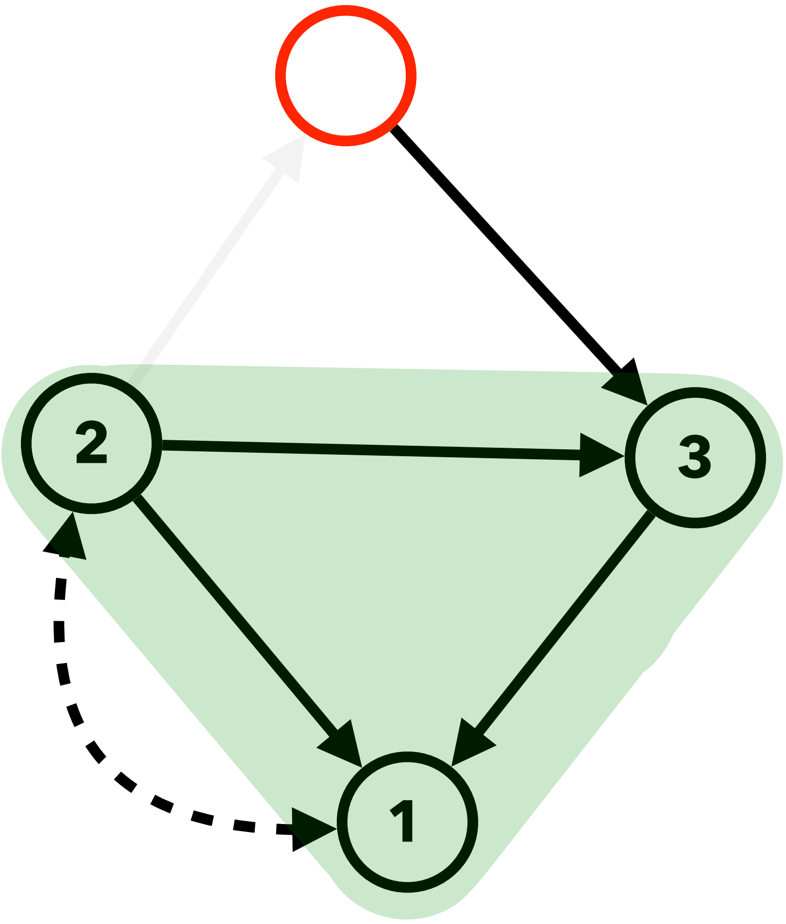


POMIS

Visible edge: there is no confounder.

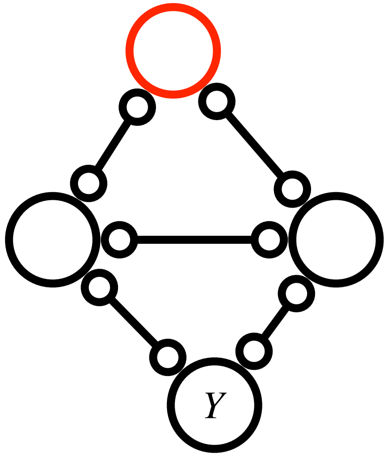


Possibly-Optimal Minimal Intervention Set: Metal Picture



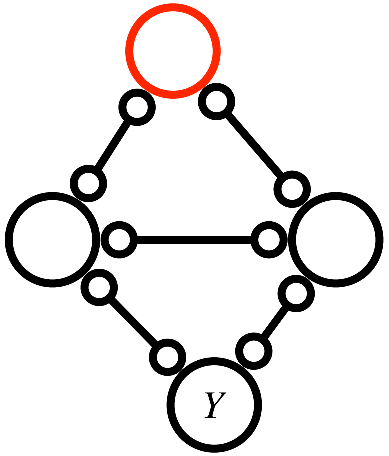
POMIS

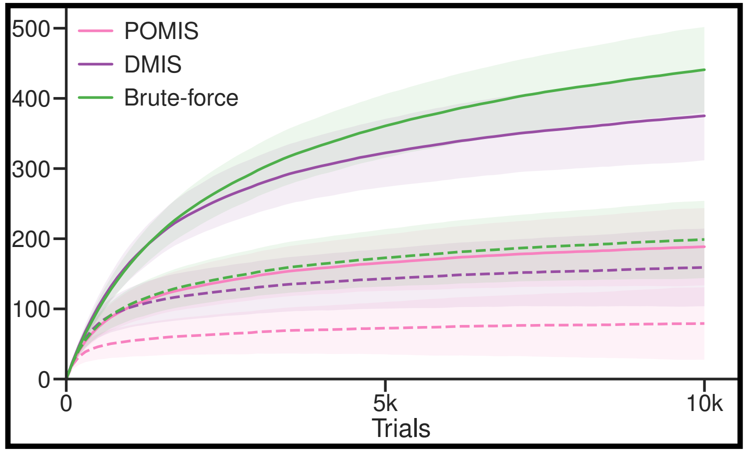
Conclusion



Given a PAG, you do not need to enumerate *all* causal diagrams conforming the PAG to compute POMIS!

Conclusion





Given a PAG, you do not need to enumerate *all* causal diagrams conforming the PAG to compute POMIS!

Playing *only* the arms corresponding to these POMISs is sufficient.

Reference

Structural Causal Bandits: Where to Intervene?
Sanghack Lee and Elias Bareinboim
NeurIPS 2018, https://causalai.net/r36.pdf

Structural Causal Bandits under Markov Equivalence
Min Woo Park, Andy Ardity, Elias Bareinboim and Sanghack Lee
https://causalai.net/r122.pdf