1. (a) $P(W_h \mid g_a) = \dfrac{P(g_a \mid W_h) \cdot P(W_h)}{P(g_a)} = \dfrac{0.99 \times 0.85}{0.95} = 88.6\%$

(b) $P(g_a \mid \sim W_h) = \dfrac{P(\sim W_h \mid g_a) \cdot P(g_a)}{P(\sim W_h)} = \dfrac{(1 - P(W_h \mid g_a)) \cdot P(g_a)}{1 - P(W_h)} = \dfrac{(1 - 0.886) \cdot 0.95}{1 - 0.85} = 72.3\%$

2. $P(gt) = 0.9 \quad P(bt) = 0.1 \quad P(b \mid bt) = 0.75 \quad P(b \mid \sim bt) = 1 - 0.75 = 0.25$

$$P(bt \mid b) = \frac{P(b \mid bt) \cdot P(bt)}{P(b \mid bt) \cdot P(bt) + P(b \mid \sim bt) \cdot P(\sim bt)} = \frac{0.75 \times 0.1}{0.75 \times 0.1 + 0.25 \times 0.9} = 0.25$$

$$P(\sim bt \mid b) = \frac{P(b \mid \sim bt) \cdot P(\sim bt)}{P(b \mid bt) \cdot P(bt) + P(b \mid \sim bt) \cdot P(\sim bt)} = \frac{0.25 \times 0.9}{0.75 \times 0.1 + 0.25 \times 0.9} = 0.75$$

75% chance it is a green taxi

3. (a) $P(H) = \frac{1}{3} \times \frac{3}{3} \times \frac{1}{3} \times \frac{3}{8} = \frac{1}{24} \quad P(NH) = \frac{3}{5} \times \frac{1}{5} \times \frac{3}{5} \times \frac{5}{8} = \frac{9}{200}$ Not happy

(b) $P(H) = \frac{1}{4} \times \frac{1}{2} + \frac{3}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2} = \frac{5}{8}$ Happy

4. $P(C{+}{+}) = 0.5 \quad P(Java) = 0.4 \quad P(C{+}{+} \mid ms) = 0.99$

$P(Java \mid ms) = 0.98 \quad P(ms) = 0.01$

$$P(ms \mid C{+}{+} \wedge Java) = \frac{P(C{+}{+} \wedge Java \mid ms) \cdot P(ms)}{P(C{+}{+} \wedge Java)} = \frac{P(C{+}{+} \mid ms) \cdot P(Java \mid ms)}{P(C{+}{+} \wedge Java)}$$

$$\cdot P(ms) = \frac{0.99 \times 0.98 \times 0.01}{0.5 \times 0.4} = 48.5\%$$

5. from the material we know that $P(Y=1|X) = \dfrac{1}{1+\exp\left(\ln \frac{P(X|Y=0)\,P(Y=0)}{P(X|Y=1)\,P(Y=1)}\right)}$

$$= \frac{1}{1+\exp\left(\ln \frac{P(Y=0)}{P(Y=1)} + \sum_i \ln \frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right)}$$

the prior for $P(Y=1)=\pi$ and $P(Y=0)=1-\pi$

$$P(Y=1|X) = \frac{1}{1+\exp\left(\ln\frac{1-\pi}{\pi} + \sum_i \ln \frac{\theta_{i0}^{X_i}(1-\theta_{i0})^{(1-X_i)}}{\theta_{i1}^{X_i}(1-\theta_{i1})^{(1-X_i)}}\right)}$$

$$= \frac{1}{1+\exp\left(\ln\frac{1-\pi}{\pi} + \sum_i X_i \ln\frac{\theta_{i0}}{\theta_{i1}} + (1-X_i)\ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right)}$$

$$= \frac{1}{1+\exp\left(\left(\ln\frac{1-\pi}{\pi} + \left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right) + \sum_i X_i\left(\ln\frac{\theta_{i0}}{\theta_{i1}} - \ln\frac{(1-\theta_{i0})}{(1-\theta_{i1})}\right)\right)\right)}$$

$w_0 = \ln\frac{1-\pi}{\pi} + \sum_i \ln\frac{(1-\theta_{i0})}{(1-\theta_{i1})}$ and $w_1 = \ln\frac{\theta_{i0}}{\theta_{i1}} - \ln\frac{(1-\theta_{i0})}{(1-\theta_{i1})}$

$$P(Y=1|X) = \frac{1}{1+\exp\left(\sum_i w_i X_i\right)}$$

6. Based on Tom M. Mitchell's new book

$$P(Y=1|X) = \frac{1}{1+\exp\left(\ln\frac{P(Y=0)}{P(Y=1)} + \sum_i \ln\frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right)}$$

$$= \frac{1}{1+\exp\left(\ln\frac{1-\pi}{\pi} + \sum_i \ln\frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right)}$$

$$\sum_i \ln\frac{P(X_i|Y=0)}{P(X_i|Y=1)} = \sum_i \ln\left(\frac{h(X_i,\theta_{i0})\exp(-\theta_{i0}^T X_i + C)}{h(X_i,\theta_{i1})\exp(-\theta_{i1}^T X_i + C)}\right) = \sum_i\left(\ln\frac{h(X_i,\theta_{i0})}{h(X_i,\theta_{i1})} + (-\theta_{i0}^T X_i + C) - (-\theta_{i1}^T X_i + C)\right) = \sum_i\left(\ln\frac{h(X_i,\theta_{i0})}{h(X_i,\theta_{i1})} + \theta_{i1}^T X_i - \theta_{i0}^T X_i\right)$$

$$P(Y=1|X) = \frac{1}{1+\exp\left(\ln\frac{1-\pi}{\pi} + \sum_i \ln\frac{h(X_i,\theta_{i0})}{h(X_i,\theta_{i1})} + \sum_i X_i(\theta_{i1}^T - \theta_{i0}^T)\right)}$$

$$= \frac{1}{1+\exp(-w_0 - \sum_i w_i X_i)} \qquad w_0 = -\ln\frac{1-\pi}{\pi} - \sum_i \ln\frac{h(X_i,\theta_{i0})}{h(X_i,\theta_{i1})}$$

$$w_1 = \theta_{i0}^T - \theta_{i1}^T$$