

$$\begin{aligned}
 E_{\text{avg}} &= E\left[\left\{\frac{1}{M} \sum_{i=1}^M \epsilon_i(x)\right\}^2\right] = E\left[\frac{1}{M^2} \left(\sum_{i=1}^M \epsilon_i(x)\right)^2\right] \\
 &= \frac{1}{M^2} E\left[\sum_{i=1}^M \epsilon_i(x)^2\right] = \frac{1}{M^2} E\left(\sum_{i=1}^M \epsilon_i(x)^2 + \sum_{\substack{i=1 \\ i \neq j}}^M \sum_{j=1}^M \epsilon_i(x) \cdot \epsilon_j(x)\right) \\
 &= \frac{1}{M^2} \left(E\left(\sum_{i=1}^M \epsilon_i(x)^2\right) + E\left(\sum_{\substack{i=1 \\ i \neq j}}^M \sum_{j=1}^M \epsilon_i(x) \epsilon_j(x)\right)\right) \\
 E(\epsilon_i(x) \epsilon_j(x)) &= 0, \quad i \neq j
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{M^2} E\left(\sum_{i=1}^M \epsilon_i(x)^2\right) &= \frac{1}{M} \cdot \frac{1}{M} E\left(\sum_{i=1}^M \epsilon_i^2(x)\right) \\
 &= \frac{1}{M} E_{\text{avg}}
 \end{aligned}$$