Other methods for constructing the blocks

- Defining contrast

Linear combination

$$L = a_1 x_1 + a_2 x_2 + \dots + a_k x_k$$

x_i is the level of the ith factor in a treatment combination.

$$x_i = \begin{cases} 0 & \text{low level} \\ 1 & \text{high level} \end{cases}$$

a_i exponent appearing on the ith factor in the effect to be confounded.

$$a_i = \begin{cases} 0 \\ 1 \end{cases}$$

Ex: 2^3 ABC confounded.

x_1 — A, x_2 — B, x_3 — C

$a_1 = 1$, $a_2 = 1$, $a_3 = 1$

The defining contrast corresponding to ABC is

$$x_1 + x_2 + x_3$$

For treatment (1) $(0, 0, 0)$

$$L = 0 + 0 + 0 = 0$$

For treatment a

$$L = 1 + 0 + 0 = 1$$

For treatment b, c,

$$L = 1$$

For treatment ab

$$L = 1 + 1 + 0 = 0 \pmod 2$$

For treatment ac, bc

$$L = 0 \pmod 2$$

For abc

$$L = 1 + 1 + 1 = 1 \pmod 2$$

Thus (1), ab, ac, bc in block 1

a b c abc in block 2

b b a abc c

2^4 (ABCD)

$L = x_1 + x_2 + x_3 + x_4$

block

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array} \quad \binom{4}{2}$$

principle ~~compor~~ block

(1), ab, ac, ad, bc, bd, cd, abcd

use $a \times$ a b c d abc abd acd bcd

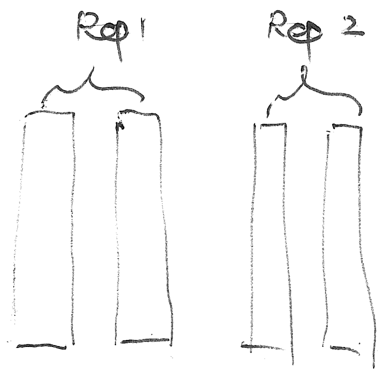
$a^2 = I$

· Outline of ANOVA

Source	DF	SS
A	1	1870
B	1	39
C	1	390
D	1	850
AB	1	0.9
AC	1	1300
AD	1	1105
BC	1	.5
BD	1	2.1
CD	1	108
ABC	1	5
ABD	1	14
ACD	1	68
BCD	1	10
block (ABCD)	1	850
	15	

ANOVA

A	1
C	1
D	1
CD	1
AC	1
AD	1
ACD	1
block	1
Error	15-7



2^4 Replicate, so 32 EU.

	DF
Main effect	4
Two-factor interaction	6
Three-factor interaction	4
block (ABCD confounded)	3

block 1
rep 1
block × rep 1

Error $31-17=14$

Ex: ABCDEF in 4 blocks

ABC DEF

$L_1 = x_1 + x_2 + x_3$

$L_2 = x_4 + x_5 + x_6$

$(L_1, L_2) = (0,0), (0,1), (1,0), (1,1)$

L_1: 000, 110, 101, 011

L_2: 000, 110, 101, 011

(1), de, df, ef, ~~ab, ac, bc~~

ab	abde	abdf	abef
ac	acde	acdf	acef
bc	bcde	bcdf	bcef

a× a ade adf aef

b bde bdf bef

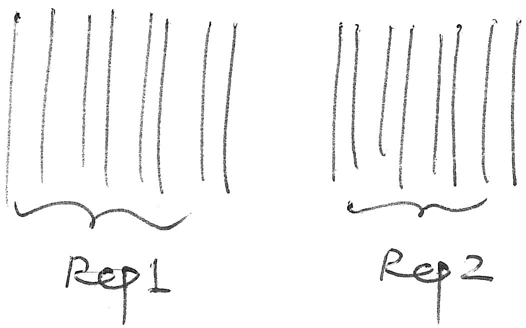
c cde cdf cef

abc abcde abcdf abcef

d×

2^6 Replicate 2 replicate

Source	Df	
Main	6	
2-factor	$\binom{6}{2}$	
3-factor	$\binom{6}{3}-2$	(exclude ABC, DEF)
4-factor	$\binom{6}{4}$	
5-factor	$\binom{6}{5}$	
~~6-factor~~		
block	7	3; 1; 3x1
Error		



- ABCDE

  ABC

  CDE

  → ABDE

2^5 factorial in 4 blocks, 64 ells

ABE CDE $\Rightarrow$ ABCD confounded

$L_1 = x_1 + x_2 + x_5$

$L_2 = x_3 + x_4 + x_5$

$(L_1, L_2) = (0,0), (0,1), (1,0), (1,1)$

a b e / c d	b1	b2	b3	b4
0 0 0	1 1 0	1 0 1	1 0 1	
0 0 0	0 0 0	1 0 1	0 1 1	
	0 0 0	0 1 1	0 1 1	
	1 1 0	1 0 1	0 1 1	
	1 1 0			
	1 1 0			

	df
main factor	5
2-inters	10
3-interactions	8
4-interaction	4
5-interaction	1
~~Error 63~~	
block	7
Error	$63 - \cancel{28}\,35 = 28$

principle block	(1)	ab	cd	abcd	ace	bce	ade	bde
a ×	a	b	acd	bcd	ce	abce	de	abde
c ×	c	abc	d	abd	ae	be	acde	bcde
e ×	e	abe	cde	abcde	ac	bc	ad	bd

Fractional factorials

- Don't have EUs for complete factorial design
  - EX: 9 main factors, but can only run 2^7 experiments.
    - assume high order interactions insignificant
      - 6&7 order interactions

- Two-level fractional factorial
  - Assume higher-order interactions negligible, there more information on lower level effects.
  - Example Latin square (2^3 factorial)

Let A - Block factor 1
B - Block factor 2
C - treatment

A	B	C	AB	AC	BC	ABC	symbol
−	−	−				−	(1)
+	−	−				+	a
−	+	−				+	b
+	+	−				−	ab
−	−	+				+	c
+	−	+				−	ac
−	+	+				−	bc
+	+	+				+	abc

4 observations instead of 8

A \ B	−	+
−	+	−
+	−	+

A \ B	−	+
−	−	+
+	+	−

observed combinations associated with ABC column

Chapter 22. Analysis of covariance

"ANOVA with covariates"

- Combination of ANOVA and regression when one has categorical factors and some quantitative predictors
- The coviates is not primary interest, but it can help reduce the error variance.

Model: $Y_{ij} = \mu + \tau_i + \gamma (X_{ij} - \bar{X}_{..}) + \varepsilon_{ij}$

$i = 1, \dots r$

$j = 1, \dots n_i$

$\sum_{i=1}^{r} \tau_i = 0$

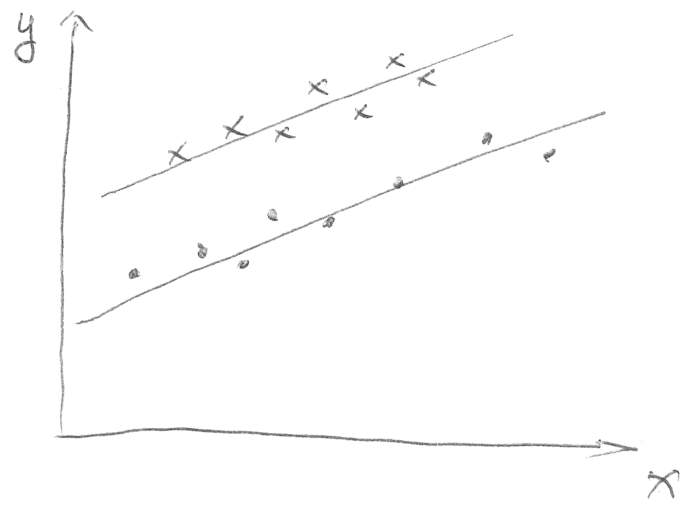
γ : regression coeff.

$\varepsilon_{ij} \sim \text{Normal}(0, \sigma^2)$

Basic ideas

- Covariates can reduce the MSE, increase the power for testing. Baseline or pretest values are often used as covariates.
- Assume linear relationship to the response, and the relationship is the same for all levels of the factor. no interaction

For each i, we have a simple linear regression, in which the slopes are the same, but the intercepts may differ



parameters and estimates

- μ, τ_i, γ, σ^2
- regression methods.

$$\hat{\gamma} = \frac{\sum_i \sum_j (Y_{ij} - \bar{Y}_{i\cdot})(X_{ij} - \bar{X}_{i\cdot})}{\sum_i \sum_j (X_{ij} - \bar{X}_{i\cdot})^2}$$

$$\hat{\mu} = \bar{Y}_{\cdot\cdot}$$

$$\hat{\tau}_i = \bar{Y}_{i\cdot} - \bar{Y}_{\cdot\cdot} - \hat{\gamma}(\bar{X}_{i\cdot} - \bar{X}_{\cdot\cdot})$$

Diagnostics.

1. Exam the data & the residuals
2. Check the same-slope assumption