Deviance Goodness of fit test.

H_0 : fitted model

$$E\,y_{\cdot j} = \frac{1}{1+e^{-x_j \beta}} \qquad \text{est by MLE}$$

H_A : Saturated model

$$E\,y_{\cdot j} = \pi_j \quad \text{estimated by} \quad \hat{\pi}_j = \frac{y_{\cdot j}}{n_j}$$

$$j = 1, \cdots J$$

Deviance

$$Dev = -2 \log \frac{\mathcal{L}(\hat{\beta} \text{ fitted model})}{\mathcal{L}(\hat{\pi}_1, \cdots \hat{\pi}_J)}$$

$$= -2 \sum_{j=1}^{J} y_{\cdot j} \log \frac{(1+e^{-x_j \beta})^{-1}}{y_{\cdot j}/n_j} + (n_j - y_{\cdot j}) \log \frac{1-(1+e^{-x_j\beta})^{-1}}{1-\frac{y_{\cdot j}}{n_j}}$$

Under regularity conditions.

$$Dev \longrightarrow \chi^2_h$$

h: degree of freedom

difference of # of parameters.

Deviance for GLM exponential family

H_0: fitted model

H_A: saturated model (the most complete model)

$$\mathrm{Dev} = 2\left[\log \mathcal{L}(\underbrace{y}_{\text{under saturated model}}, \phi \mid y) - \log \mathcal{L}(\underbrace{\hat{\mu}}_{\text{under the fitted model}}, \phi \mid y)\right] \quad (*)$$

for exponential family when $a_i(\phi) = \frac{\phi}{w_i}$

$$(*) = \sum_{i=1}^{n} 2 w_i \{ y_i(\tilde{\theta}_i - \hat{\theta}_i) - b(\tilde{\theta}_i) + b(\hat{\theta}_i) \} / \phi$$

$\tilde{\theta}_i$: estimates under saturated model

$\hat{\theta}_i$: . . . . fitted . . .

$= D(y, \hat{\mu}) / \phi \longrightarrow$ scaled deviance

$D(y, \hat{\mu}) \longrightarrow$ deviance

Ex: poisson $f(y \mid \theta, \phi) = \exp(y \log\lambda - \lambda - y!)$

$$\theta = \log\lambda, \quad b(\theta) = \exp(\theta), \quad \phi \equiv 1, \quad a(\phi) \equiv 1$$

$$D(y, \hat{\mu}) = \sum_{i=1}^{n} 2\{y_i(\tilde{\theta}_i - \hat{\theta}_i) - b(\tilde{\theta}_i) + b(\hat{\theta}_i)\}$$

$$= 2\sum_{i=1}^{n} \{y_i \log(\frac{y_i}{\hat{\mu}_i}) - (y_i - \hat{\mu}_i)\}$$

Normal. $f(y \mid \theta, \phi) = \exp\left[\frac{y\mu - \frac{\mu^2}{2}}{\sigma^2} - \frac{1}{2}\{\frac{y^2}{\sigma^2} + \log(2\pi\sigma^2)\}\right]$

$$\theta = \mu, \quad b(\theta) = \frac{\theta^2}{2}, \quad \phi = \sigma^2, \quad a(\phi) = \phi$$

$$D(y, \hat{\mu}) = 2\sum_{i=1}^{n} \{y_i(y_i - \hat{\mu}_i) - (\frac{y_i^2}{2} - \frac{\hat{\mu}_i^2}{2})\}$$

$$= \sum_{i=1}^{n} (y_i - \hat{\mu}_i)^2$$

ANOVA (Chapter 16)

Regression : stat relation between
— quantitative predictors X
— quantitative response Y

ANOVA : relation between
— any predictors X
— quantitative response Y.

Concepts.

1. Factor and factor level
   A factor is a predictor variable.
   A factor level is a particular form of the factor. Mostly, the levels cannot be compared.

2. Single factor and multi factor studies
   only 1 factor → 1-way ANOVA
   more than one factors → eg. 2-way ANOVA

3. experimental and observational studies

experimental study: the levels of all the factors can be controlled.

observational study: - - - - - - - - - - cannot be controlled.

4. treatment and block

The factor can be called either treatment or block variable.

The interesting variable is treatment,

The uninteresting - - - - - - - - .

5. ANOVA models

fixed effects model, random effects model, mixed effects model.

fixed effects: all levels of interest are selected by a nonrandom process and are all included in the study.

: Inferences are to be made only to those levels

Ex: Compare the effect of 3 medicines A, B, C.

Random effects. level consists of a random sample of levels from a population of all possible levels.

- Inference is about the population of levels, not just the subset in the study.

Ex: four clinics are randomly selected from a population of clinics in a region.

Mixed models

Some factors are fixed effects and other factors are random effects.

One Way ANOVA

fixed model (cell means model)

Assume that factor A has r levels, and in each level there are n_i repeated observations, $i=1, \cdots r$.

expression : $y_{ij} = \mu_i + \varepsilon_{ij}$, $i=1, \cdots r$
$j=1, \cdots n_i$

y_{ij} = response in the jth trial for the ith factor level

$\mu_i = E(y_{ij})$ = Constant parameter.

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$

$n_{total} = n_T = \sum_{i=1}^{r} n_i$, r : number of levels

$\Rightarrow Var(y_{ij}) = \sigma^2$

$y_{ij} \sim Normal(\mu_i, \sigma^2)$

can be written in a matrix term.

Ex. $r=2$, $n_1=2$, $n_2=3$, $n_T=5$.

$$\begin{pmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \\ y_{23} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_1 \\ \mu_2 \\ \mu_2 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{21} \\ \varepsilon_{22} \\ \varepsilon_{23} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{21} \\ \varepsilon_{22} \\ \varepsilon_{23} \end{pmatrix}$$

Generally,

$$Y = \begin{pmatrix} y_{11} \\ \vdots \\ y_{1n_1} \\ \vdots \\ y_{r1} \\ \vdots \\ y_{rn_r} \end{pmatrix} \qquad \varepsilon = \begin{pmatrix} \varepsilon_{11} \\ \vdots \\ \varepsilon_{1n_1} \\ \vdots \\ \varepsilon_{r1} \\ \vdots \\ \varepsilon_{rn_r} \end{pmatrix} \qquad \beta = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_r \end{pmatrix} \qquad X = \begin{pmatrix} 1_{n_1} & \cdots & 0_{n_1} \\ 0_{n_2} & 1_{n_2} & \\ \vdots & & \ddots \\ 0_{n_r} & & 1_{n_r} \end{pmatrix}$$

1_{n_i}: column vector of length n_i with all elements 1