Stat 6338 Homework #1

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1. A medical substance is shipped by airfreight in cartons of 1000 ampules. The data below, involving 10 shipments, contains

   X_i = the number of times carton i was transferred from one aircraft to another

   Y_i = the number of ampules broken upon arrival

i	1	2	3	4	5	6	7	8	9	10
X_i	1	0	2	0	3	1	0	1	2	0
Y_i	16	9	17	12	22	13	8	15	19	11

   Part 1

   (a) Fit a traditional linear regression model. Estimate parameters, plot the estimated regression function. Is this a good fit?

   (b) Obtain a point estimate of the expected number of broken ampules when $X = 0, 1, 2, 3$ transfers are made.

   (c) Estimate the increase in the expected number of ampules broken when there are 2 transfers as compared to 1 transfer.

   Part 2

   (a) Fit the Poisson regression model assuming $Y_i \sim Poisson(\lambda_i)$ where $\lambda_i = \lambda(\mathbf{x}, \boldsymbol{\beta}) = e^{\beta_0 + \beta_1 X_i}$ is the response function.

   (b) Estimate of the expected number of broken ampules when $X = 0, 1, 2, 3$ and compare your results with Part 1(b).

   (c) Plot the Poisson and linear regression functions, together with the data. WHich model appears to be a better fit?

   (d) Management wishes to estimate the probability that 10 or fewer ampules are broken when there is no transfer of the shipment. Use the fitted Poisson regression function to obtain the estimate.

   (e) Obtain an approximate 95% confidence interval for the slope β_1 and interpret it.

2. Data is generated from the exponential distribution with density $f(y) = \lambda e^{-\lambda y}$ where $\lambda, y > 0$.

   (a) Identify the specific form of $\theta, \phi, a(), b()$ and $c()$ for the exponential distribution.

   (b) What is the canonical link and variance function for a GLM with a response following the exponential distribution?

   (c) Identify a practical difficulty that may arise when using the canonical link in the instance.

   (d) Express the deviance in this case in terms of the response y_i and the fitted values $\hat{\mu}_i$.