

# NLP3 PROJECT

LAB02

January 2023



nelson.vicel-farah antoine.zellmeyer karen.kaspar romain1.brand maxence.plantard

EPITA SCIA - Promo 2023

# Contents

1	Lan	guage Detection (24 points) – Guided coding	1
	1.1	Question 1 (1 point)	1
	1.2	Question 2 (1 point)	1
	1.3	Question 3 (1 point)	2
	1.4	Question 4 (1 point)	2
	1.5	Question 5 (4 point)	2
	1.6	Question 6 (3 points)	3
	1.7	Question 7 (3 points)	3
	1.8	Question 8 (1 point)	4
	1.9	Question 9 (2 points)	5
	1.10	Question 10 (1 point)	5
		Question 11 (2 points)	5
		Question 12 (4 points)	6
<b>2</b>	Rot	tate two semantic spaces (23 points) – Not guided coding	6
	2.1	Question 1 (1 point)	6
	2.2	Question 2 (2 points)	
	2.3	Question 3 (2 points)	7
	2.4	Question 4 (5 points)	8
	2.5	Question 5 (2 points)	8
	2.6		9
	2.7	Question 7 (3 points)	
	2.8	Question 8 (4 points)	
	2.9	Question 9 (2 points)	
		Question 10 (5 points)	
ก	<b>A</b> 44.		10
3	3.1	${f ention \ Exploration \ (22 \ points)}$ ${f Question \ a \ (2 \ points) \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	12
	3.2	Question b (4 points)	
	3.3	Question c (5 points)	
		3.3.1 i (2 points)	
	0.4	3.3.2 ii (3 points)	
	3.4	Question d (3 points)	
		3.4.1 i (1 points)	
		3.4.2 ii (2 points)	15

# 1 Language Detection (24 points) – Guided coding

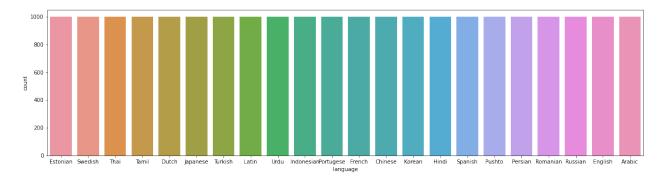
#### Question 0 (1 point)

Try out a translation of a French sentence in Google Translate (or Bing Translate) to English. What happens if you select the wrong source language as follows? Explain in a few sentences what is happening in the backend.

By selecting the wrong language in Google Translate, the end result will be the exact input in most cases and a wrong translation in others. The backend of the translator will process the text as if it were written in the selected language. It will then consider unrecognized word as if they were names or verbatim quotes, thus restoring them as is.

#### 1.1 Question 1 (1 point)

Describe the distribution of languages and give at least two comments about the dataset. (1 point)



We count 22 languages that are equally distributed in the dataset. We can also note the sparsity of chosen languages for this dataset as there are really common languages (e.g English, Spanish) as much as less common and more vernacular languages (e.g Tamil, Urdu).

# 1.2 Question 2 (1 point)

Do the appropriate pre-processing to maximise the accuracy of language detection. What is your strategy?

Our pre-processing includes a few steps. The first step transforms uppercase characters into lowercase characters. We then remove punctuation. For specific languages that use logograms such as Japanse and Chinese, we have chosen to apply a specific

pre-processing step that adds spaces between each character to allow the tokenization of words based on space-separated occurrences. That same treatment is applied for the Thai language.

#### 1.3 Question 3 (1 point)

What would be the problem if your dataset was unbalanced? (1 point)

The model would be highly biased toward languages that outnumber the others. We can also note that some languages would suffer from a lack of samples, which would make them harder to identify for a machine learning model that had been fed with these data.

#### 1.4 Question 4 (1 point)

What techniques could you use to solve that?

- Oversampling: This involves increasing the number of samples in the minority class by duplicating existing samples or generating new synthetic samples.
- Undersampling: This involves reducing the number of samples in the majority class by removing some of the samples.
- Weighted loss function: We can use a weighted loss function to give more importance to the samples in the minority class during training.

# 1.5 Question 5 (4 point)

Train a model of your choice and describe the accuracy across languages. Use an 80%, 20% train-test split. Performance is not key but explain thoroughly the process and the metric(s) you are tracking.

We chose the Multinomial Naive Bayes algorithm to classify the texts. The input are first tokenized with CountVectorizer which gives a bag-of-words representation of word, then we feed the Naive Bayes algorithm with these data. This gives us an accuracy of 98% on the test set.

precision	recall	f1-score	support
0.99	0.99	0.99	202
0.99	0.98	0.99	204
0.98	0.99	0.98	198
1.00	0.78	0.88	257
0.96	1.00	0.98	192
0.99	0.96	0.98	208
0.96	1.00	0.98	193
0.97	1.00	0.98	194
0.99	1.00	0.99	198
0.97	1.00	0.98	194
0.93	0.99	0.96	188
0.99	1.00	0.99	198
0.96	0.99	0.98	195
0.96	0.99	0.98	193
0.98	0.99	0.99	197
0.99	0.99	0.99	200
0.99	0.99	0.99	199
1.00	1.00	1.00	201
0.98	1.00	0.99	196
0.99	1.00	1.00	199
0.99	1.00	0.99	198
0.98	1.00	0.99	196
		0.98	4400
0.98	0.98	0.98	4400
0.98	0.98	0.98	4400
	0.99 0.99 0.98 1.00 0.96 0.99 0.96 0.97 0.99 0.99 0.96 0.98 0.99 0.99 0.99	0.99 0.99 0.99 0.98 0.98 0.99 1.00 0.78 0.96 1.00 0.99 0.96 0.96 1.00 0.97 1.00 0.97 1.00 0.99 1.00 0.97 1.00 0.99 0.99 0.99 0.99 0.99 0.96 0.99 0.96 0.99 0.98 0.99 0.99 0.99 0.99 0.99 0.99 0.99 0.99 1.00 0.98 1.00 0.99 1.00 0.99 1.00	0.99       0.99       0.99         0.99       0.98       0.99         0.98       0.99       0.98         1.00       0.78       0.88         0.96       1.00       0.98         0.99       0.96       0.98         0.96       1.00       0.98         0.97       1.00       0.98         0.99       1.00       0.99         0.97       1.00       0.98         0.93       0.99       0.96         0.99       1.00       0.99         0.96       0.99       0.99         0.96       0.99       0.98         0.98       0.99       0.99         0.99       0.99       0.99         0.99       0.99       0.99         0.99       1.00       1.00         0.98       1.00       0.99         0.99       1.00       0.99         0.99       1.00       0.99         0.99       1.00       0.99         0.99       1.00       0.99         0.99       1.00       0.99         0.99       1.00       0.99         0.99       1.00

# 1.6 Question 6 (3 points)

Train a fasttext model on Tatoeba parallel corpus and check that performance is good.

The accuracy of fasttext is 0.958 on Tatoeba after 2 hours of training. The performance is therefore good.

# 1.7 Question 7 (3 points)

Test your fasttext model on the same dataset as in question 1-5. Compare with your custom model (make sure you use the exact same data for testing). How can you explain the difference in performance between the two models?

The accuracy of fasttext on the previous dataset is way lower than our Naive Bayes model. In this case, the accuracy of fasttext is **0.80** while the Naive Bayes accuracy is

#### 0.98.

This difference in performance can be explained mainly due to the distribution shift between the datasets these two models have been trained on. The custom model have been trained on the same dataset it has been tested on, while fasttext has only been trained on Tatoeba.

A second hypothesis could be that fasttext tries to embody the meaning of words and makes inference based on subwords, which could cause inconsistencies due to the fact that some languages may have phonemes in common, while MNB relies on the frequency of individual words to identify a language. This means that the MNB model focuses on more important language-specific cues: the alphabet and the lexicon. It can then make more accurate inferences.

#### 1.8 Question 8 (1 point)

Compute your performance metrics yourself and compare with sklearn.

The performance metric we rely on to compare models for this kind of task is the **accuracy** which is computed by.

```
\frac{\text{Right answers}}{\text{Wrong answers} + \text{Right answers}}
```

We eventually get the same results of the accuracy\_score function from scikit-learn, which lets us think that our implementation is on point.

We could also compute the recall and precision scores to identify what languages Fasttext is struggling to recognize.

#### 1.9 Question 9 (2 points)

How could you improve the fasttext model performance from the previous question? Explain in a few sentences.

There are multiple ways of improving the performance of fasttext:

- One way would be to train the fasttext model on the data it is actually tested on.
- We could also perform hyperparameter tuning. i.e we try different parameters and keep the ones that get the best performance.
- Applying pre-processing on the Tatoeba dataset.

#### 1.10 Question 10 (1 point)

Which method would you use for language detection and why?

The most efficient method we tried for this task seems to be the Multinomial Naive Bayes. While fasttext could perform better in some cases, Naive Bayes is way faster and less complex to train for a similar result, which may be a better choice when having limited computational resources or high computational costs.

#### 1.11 Question 11 (2 points)

Given a sentence with  $N_1$  tokens in English and  $N_2$  token in French, what would be your strategy to assign a language to such sentence?

To determine the language of a sentence with  $N_1$  tokens in English and  $N_2$  tokens in French, we can follow the following steps:

- Identify all the words that are enclosed in quotation marks as they might indicate expressions or verbatim quotes.
- Exclude the quoted words from the count of  $N_1$  and  $N_2$
- We determine the language of the sentence by identifying the language with the most remaining tokens after the exclusion of the words between quotation. Therefore if  $N_1$  has more tokens than  $N_2$ , we consider that the sentence is written in English. Alternatively, if  $N_2$  has more tokens than  $N_1$ , we assume that the sentence is written in French.

#### 1.12 Question 12 (4 points)

Would a multilingual architecture be robust to multiple languages in a single sentence? Elaborate your answer accordingly.

A multilingual model may be able to effectively handle multiple languages within a single sentence, depending on how it was trained and how it processes input.

There are several methods the model could use, such as identifying and processing each language separately or using a shared representation to process the entire sentence.

The model's ability to handle multiple languages in a single sentence will depend on its language identification and processing abilities, the complexity and structure of the sentence, and the diversity and representativeness of the data used to train the model.

# 2 Rotate two semantic spaces (23 points) – Not guided coding

#### 2.1 Question 1 (1 point)

Explain in a few sentences how MUSE is doing the alignment of the semantic spaces in the supervised way.

MUSE is a tool that allows for the creation of a mapping from one language's word embeddings to another. In a supervised setting, MUSE uses a pre-existing dictionary that translates words between the two languages and the embeddings of both languages to learn the mapping. MUSE uses the Orthogonal Procrustes problem to find the optimal mapping, which involves finding an orthogonal matrix that can most accurately transform one set of embeddings into another. To do this, MUSE computes the matrix M by multiplying the matrices of the two sets of embeddings, and then uses Singular Value Decomposition to find the closest orthogonal matrix to M, which is equivalent to the desired mapping matrix.

# 2.2 Question 2 (2 points)

What is the limit of doing that alignment based on the approach taken in the supervised way?

There are several limitations to the supervised approach to aligning semantic spaces using MUSE:

- The approach requires a large amount of labeled parallel data, which consists of texts in both languages that are translations of one another. This can be difficult to obtain, especially for low-resource languages or specialized domains.
- The approach is limited by the quality of the parallel data. If the parallel data is noisy or contains errors, it can negatively impact the quality of the learned transformation matrix and the alignment of the semantic spaces.
- The approach is sensitive to the choice of supervised learning algorithm and hyperparameters. Different algorithms and hyperparameter choices can lead to different results, and it may be difficult to determine the optimal settings without a large amount of trial and error.
- Finally, the approach is limited to aligning the semantic spaces of two languages. If more than two languages are involved, additional transformation matrices would need to be learned and combined, which can be a complex and time-consuming process.

#### 2.3 Question 3 (2 points)

How can we align two semantic spaces in a domain specific field, e.g., in a tech company?

To perform this task, we can follow the following steps:

- Collect a large amount of parallel data that is relevant to the specific domain (in this case the tech company). The data must consist of texts in both languages that are translations of one another.
- Preprocess the data to remove noise and errors, such as typos and non-standard language.
- Create word embeddings for each language using an unsupervised approach, such as Skip-Gram or GloVe.
- Use a supervised learning algorithm, such as Procrustes analysis or Orthogonal Procrustes, to learn a transformation matrix that maps the embeddings of one language to the other using the parallel data.
- Use the transformation matrix to align the semantic spaces of the two languages by applying it to the word embeddings of one language.

It may also be helpful to use domain-specific data or techniques to fine-tune the alignment for the tech domain. This could include using additional parallel data that is specific to the tech domain or incorporating domain-specific knowledge into the transformation matrix.

#### 2.4 Question 4 (5 points)

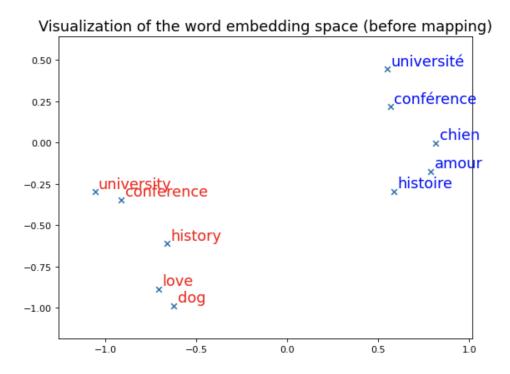
Align the French space and the English space together, with the method of your choice.

Firstly, we load the pre-trained monolingual word embeddings from fastText Wikipedia embeddings. Then, to align French and English embeddings, we use the unsupervised method of MUSE. This gives us a linear mapping of 300x300 weights that we can use to align the French embeddings.

#### 2.5 Question 5 (2 points)

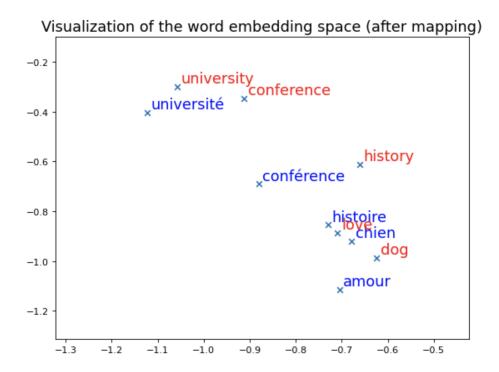
Visualize the output on a few words of your choice. Comment on the performance of the alignment based on the output.

To render a good visualization of the 300-dimensional embeddings we use the Principal Component Analysis to project the data on 2 dimensions. Despite the obvious information loss, we obtain a decent representation of the semantic space :



Before applying the mapping, there is no alignment with relation to the meaning whatsoever. It lumps together the words from the same language, creating two groups of words, one in each language, with nothing indicating that one word is a translation of the other in French.

Lil Clowns 8 8/16



After applying the mapping, we notice a considerable increase in performance. The words from one language aren't lumped together anymore and most translations are close to each other. We also notice that words that have a common context (e.g university and conference) are closer to each other than words with unrelated meaning.

# 2.6 Question 6 (2 points)

How can you find the translation of a word with this approach? Explain your method and the distance metric you choose in a few sentences.

To find a translation of a word from aligned embeddings, the process is to look for the nearest neighbor of the embedding on the other language semantic space. To make this operation succeed, we need a metric to compute the distance between two data points. For this step we chose the **cosine similarity**. We could also have used the **inner product** if we had chosen to normalize the embeddings beforehand.

# 2.7 Question 7 (3 points)

Apply your approach and comment on the performance of the translation.

```
Nearest neighbors of "université":
0.8894 - university
0.7790 - faculty
0.7130 - universityuniversity
0.6767 - professor
0.6703 - wisconsin-madison
Nearest neighbors of "amour":
0.7338 - love
0.6439 - unrequited
0.6349 - longing
0.6126 - lover
0.6063 - affection
Nearest neighbors of "histoire":
0.7363 - history
0.5442 - prehistory
0.5373 - histories
0.5295 - historiography
0.5231 - historical
Nearest neighbors of "conférence":
0.7318 - conference
0.6318 - conferences
0.5806 - meeting
0.5565 - plenary
0.5462 - convened
Nearest neighbors of "chien":
0.7598 - dog
0.6263 - dogs
0.6057 - poodle
0.5823 - puppies
0.5797 - puppy
```

We observe that the translations are rather accurate although some irrelevant results may appear in the nearest neighbors of some word (e.g "wisconsin-madison" as a translation of *université*).

# 2.8 Question 8 (4 points)

What is the limit of aligning two spaces at a sentence level? What do you suggest to improve the alignment, at a sentence level?

When aligning two spaces at the sentence level, there are a few potential challenges that may arise:

- Sentence structure: Sentences can have different structures and may use different syntactic constructs, which can make aligning the spaces more difficult.
- Semantic meaning: Sentences can convey different meanings, even if they use

the same words. This can make it hard to align the spaces, as the words in the sentences may be in different contexts and have different connotations.

• Language differences: If the two spaces are in different languages, aligning them at the sentence level may be more challenging due to differences in grammar, syntax, and idiomatic expressions.

To improve alignment at the sentence level, one approach is to use supervised methods that rely on labeled data. For example, we can use parallel data to train a model to align the spaces. Additionally, we can use cross-lingual transfer learning, where we pre-train a model on a large corpus of one language and then fine-tune it on a smaller dataset in another language. These methods can help the model learn the structural and semantic similarities and differences between the two languages.

Other techniques that can be helpful to improve alignment at the sentence level include the use of bilingual dictionaries, machine translation techniques, or use of multilingual models with alignments within their sub-spaces.

#### 2.9 Question 9 (2 points)

Someone, in your company, asked you to do sentiment analysis on their dataset. Given a set of sentences  $s_1, \ldots, s_N$ , where  $s_i$  can be written in any language, what architecture would you use to have a vector representation of  $s_i$ ? Motivate in 2-3 bullet points.

Even though we can use pre-trained multilingual language model such as mBERT or XLM-RoBERTa as they can handle input text in many languages without the need to fine-tune the model for each individual language, we suggest a less computationally intensive solution:

- Language Detection: We can use language detection techniques to identify the language of the text like we did in the previous part. It could be achieved by using our Naive Bayes model or fasttext.
- Word embeddings: Use fastText's pre-trained word embeddings for each language to get a vector representation of each word in the sentence. FastText has pre-trained embeddings for more than 300 languages.
- Aligning Embeddings: After language detection, we embed the words in the same semantic space using techniques like MUSE. We should for this step align the languages to the english space as a lot of pre-trained model exist for english sentiment analysis. We could also consider to normalize the embeddings which would allow faster inference and better alignment with language too far from English (e.g Chinese).

• Sentence Embedding : Once the embeddings are aligned, we use the average or pooling techniques to generate a sentence embedding from the word embeddings.

By following this process, we end up with vector representations of sentences that can be used as input to a classification model or any other downstream task.

#### 2.10 Question 10 (5 points)

How would you do sentiment analysis across multiple languages in a domain specific context? Justify your approach step by step.

Doing sentiment analysis across multiple languages in a domain-specific context can be challenging, as it requires both a good understanding of the domain-specific language and the ability to perform sentiment analysis in multiple languages. Here's the approach we came up with:

- Data collection: We start by collecting a large dataset of domain-specific text in multiple languages. This dataset should include both positive and negative examples of sentiment in the domain of interest.
- Preprocessing: Perform text preprocessing on the collected data, such as lower-casing, removing punctuation, tokenizing or removing stop words.
- Aligned Word Embeddings: We use the same process as in the previous question: detecting language, retrieving embeddings from fastText and aligning all languages to English.
- Sentiment Analysis: Next, we apply sentiment analysis techniques on the aligned data. We could either translate word in English by finding the nearest english neighbor of each vector, feed the output to a pre-trained transformer such as BERT, RoBERTa, or ALBERT and fine-tune these models on the domain-specific dataset, OR train a smaller classifier model on these embeddings as an LSTM, a BiLSTM or Fasttext.
- Evaluation: Finally, evaluate the model's performance on a test dataset to see how well it is able to detect sentiment in the domain-specific text.

# 3 Attention Exploration (22 points)

# 3.1 Question a (2 points)

Describe (in one sentence) what properties of the inputs to the attention operation would result in the output c being approximately equal to  $v_i$  for

some  $j \in \{1, ..., n\}$ . Specifically, what must be true about the query q, the values  $\{v_1, ..., v_n\}$  and/or the keys  $\{k_1, ..., k_n\}$ ?

with 
$$c = \sum_{i=1}^{n} a_i v_i$$

and 
$$a_i = \frac{\exp(k_i^T q)}{\sum_{j=1}^n \exp(k_j^T q)}$$

c tends to  $v_j$  when  $a_j$  tends to 1 while every other  $a_i$  tends to 0, which happens when  $k_i^T q$  is substantially higher than every other  $k_i^T q$ .

#### 3.2 Question b (4 points)

Give an expression for a query vector q such that the output c is approximately equal to the average of  $v_a$  and  $v_b$ , that is,  $\frac{1}{2}(v_a + v_b)$ .

To achieve this, we need  $a_a$  and  $a_b$  to tend to  $\frac{1}{2}$  while every other  $a_i$  tend to 0. This occurs when

$$q = \lim_{C \to \infty} C * (k_a + k_b)$$

In a numerical setting, C can be an arbitrary large scalar. In this case, q is neither perpendicular to  $k_a$  nor  $k_b$  but stays perpendicular to every other vector  $k_i$ . Which gives us:

$$a_{a} = \frac{\exp(C * k_{a}^{T}(k_{a} + k_{b}))}{\exp(C * k_{a}^{T}(k_{a} + k_{b})) + \exp(C * k_{b}^{T}(k_{a} + k_{b})) + \sum_{i \notin \{a,b\}} \exp(C * k_{i}^{T}(k_{a} + k_{b}))}$$

$$= \frac{\exp(C * k_{a}^{T}(k_{a} + k_{b}))}{\exp(C * k_{a}^{T}(k_{a} + k_{b})) + \exp(C * k_{b}^{T}(k_{a} + k_{b})) + (n - 2)}$$

$$= \frac{\exp(C * (k_{a}^{T}k_{a} + k_{a}^{T}k_{b})))}{\exp(C * (k_{a}^{T}k_{a} + k_{a}^{T}k_{b}))) + \exp(C * (k_{b}^{T}k_{a} + k_{b}^{T}k_{b}))) + (n - 2)}$$

$$= \frac{\exp(C * ||k_{a}||^{2} + 0))}{\exp(C * ||k_{a}||^{2} + 0)) + \exp(0 + C * ||k_{b}||^{2})) + (n - 2)}$$

$$= \frac{\exp(C)}{2\exp(C) + n - 2} \text{ (the norm of key vectors is 1)}$$

The same goes for  $a_b$ . Every other attention weight  $a_i$  for  $i \notin \{a, b\}$  can be described as follow:

$$a_i = \frac{1}{2exp(C) + n - 2}$$

which means that C has to largely outweigh  $\log(n-2)$  in order to make n-2 insignificant and allow  $a_a$  and  $a_b$  to approximate  $\frac{1}{2}$ .

#### 3.3 Question c (5 points)

#### 3.3.1 i (2 points)

Design a query q in terms of the  $\mu_i$  such that as before,  $c \approx \frac{1}{2}(v_a + v_b)$ , and provide a brief argument as to why it works.

We can adapt the query from the previous question for this case, which gives the following query :

$$q = \lim_{C \to \infty} C * (\mu_a + \mu_b)$$

This happens due to a being vanishingly small in the covariance matrix  $\Sigma_i = aI$ , which implies that  $k_i \approx \mu_i$ .

#### 3.3.2 ii (3 points)

When you sample  $\{k_1, ..., k_n\}$  multiple times, and use the q vector that you defined in part i., what qualitatively do you expect the vector c will look like for different samples?

For a vanishingly small a we can assume that  $k_a \sim \mathcal{N}(1, \frac{1}{2}) * \mu_a$ . If we use the results from **question b** for a large C that makes (n-2) insignificant we get:

Let X be defined by 
$$X \sim \mathcal{N}(1, \frac{1}{2})$$

$$c \approx \frac{exp(C * X)}{exp(C * X) + exp(C)} v_a + \frac{exp(C)}{exp(C * X) + exp(C)} v_b$$

$$\approx \frac{1}{exp((1 - X) * C) + 1} v_a + \frac{1}{exp((X - 1) * C) + 1} v_b$$

We can conclude that c fluctuates between  $v_a$  and  $v_b$  as we sample the key vectors multiple times.

#### 3.4 Question d (3 points)

#### 3.4.1 i (1 points)

Design  $q_1$  and  $q_2$  such that c is approximately equal to  $\frac{1}{2}(v_a+v_b)$ 

We assume that the covariance matrix is  $\Sigma_i = \alpha I$ .

We know that the final output of the multi-headed attention is the average of each head:  $c = \frac{1}{2}(c_1 + c_2)$ , therefore we need to have  $c_1 \approx v_a$  and  $c_2 \approx v_b$ . This can be achieved with the following queries:

$$q_1 = C * \mu_a \text{ and } q_2 = C * \mu_b$$

. We hence have:

$$c_1 = \sum_{i=1}^n \alpha_i * v_i$$

if i is different from a we have:

$$\alpha_i \approx \frac{exp(0)}{exp(C)} \approx \frac{1}{exp(C)} \approx 0$$

otherwise:

$$\alpha_a \approx \frac{exp(C)}{exp(C)} \approx 1$$

Therefore all the elements of the sum are cancelled except for the element at the index a: hence  $c_1 \approx v_a$ 

In the same manner  $c_2 \approx v_b$ 

So  $c \approx \frac{1}{2}(v_a + v_b)$ 

#### 3.4.2 ii (2 points)

Take the query vectors  $q_1$  and  $q_2$  that you designed in part i. What, qualitatively, do you expect the output c to look like across different samples of the key vectors? Please briefly explain why. You can ignore cases in which  $q_i^T k_a < 0$ .

For a vanishingly small a we can write that  $k_a \sim \mathcal{N}(1, \frac{1}{2}) * \mu_a$  while for any other i,  $k_i = \mu_i$ .

Let X be defined by  $X \sim \mathcal{N}(1, \frac{1}{2})$ 

The query vectors that we designed before are:

$$q_1 = C * \mu_a \text{ and } q_2 = C * \mu_b$$

We have seen that the value is cancelled for all indexes except when i = a. For i = a, we have:

$$\begin{split} c_1 &\approx \frac{exp(k_a^Tq_1)}{exp(k_a^Tq_1)}v_a \\ c_1 &\approx \frac{exp(X*\mu_a^T*C*\mu_a)}{exp(X*\mu_a^T*C*\mu_a)} \approx \frac{exp(X*C)}{exp(X*C)}v_a \\ &\approx v_a \text{ because C outweigh X and X} > 0 \end{split}$$

Therefore  $c_1 \approx v_a$  and in the same manner  $c_2 \approx v_b$ 

For i = b, and  $c_2$  we have

$$c_2 \approx \frac{exp(k_b^T q_2)}{exp(k_b^T q_2) + exp(k_a^T q_2)} v_b + \frac{exp(k_a^T q_2)}{exp(k_b^T q_2) + exp(k_a^T q_2)} v_a$$
$$c_2 \approx \frac{exp(\mu_b^T * C * \mu_b)}{exp(\mu_b^T * C * \mu_b)} \approx \frac{exp(C)}{exp(C)} v_b \approx v_b$$

We end up with the same result as in the first question with  $c \approx \frac{1}{2}(v_a + v_b)$  because the magnitude of  $k_a$  does not influence the result enough.