

# Flux computation: Brightness Temperature, Optical Depth, Column Density and Mass

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In the python code `flux_to_mc.py`, we aim at converting flux of certain molecular line emission (in unit of Jy/beam) to various physical quantities of interest. Up to now, only some transitions of  $C^{18}O$ ,  $^{13}CO$ , and  $^{12}CO$  lines are available in the code (please refer to `flux_to_mc_usage.txt`). The following paragraphs will be dedicated to introduce the formulas used in the code.

## 1 Introduction

The physical quantities to be computed are:

1. **Flux (Jy/beam)**: In a usual interferometer molecular line observation, intensity is reported in unit of Jy/beam. So in this step, we may simply sum up the total flux.
2. **Flux Density (Jy)**: By the definition of flux density (or spectral irradiance), 1 Jy is equivalent to  $10^{-26}$  watts per square metre per hertz. Since it's the energy flux received, it depends on the distance between the source and the observer.
3. **Spectral Radiance (Jy/sr)**: It describes the amount of energy the source gives off. It is measured in terms of the power emitted per unit area of the source, per unit solid angle that the radiation is measured over, per unit frequency. Thus, it's an intrinsic property of the source, independent of anything about the observer.
4. **Brightness Temperature (K)**: Due to Planck's black body radiation formula, one is tempted to convert spectral radiance into a more convenient unit, which is temperature. In other words, brightness temperature

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is defined to be the temperature of a black body which emits the spectral radiance we observe. However, maybe because of its simplicity, the Planck's law is conventionally replaced by Rayleigh-Jeans approximation. In single-dish observations, what an observer obtains is this quantity, corrected by the efficiency of the antenna. The unit of intensity in this regime is thus described as  $T_a^*$ , where  $a$  stands for *antenna*.

5. **Optical Depth:** Assume all molecules along the line-of-sight are in thermal equilibrium (i.e. have the same temperature), then the more the amount of molecule, the brighter the line emission we would see. Due to self absorption, however, the emission will also be reduced according to how many molecules are lying along the line-of-sight. The quantity describing this "suppression" is called optical depth.
6. **Column Density ( $cm^{-2}$ ):** The real amount of the molecules is translated from optical depth into column density. From here to mass, the quantities are no longer directly obtained. Instead, they're estimated based on several assumptions and they rely greatly on quantum mechanics, which shows how these molecules emit lights, and how much.
7. **Mass ( $M_\odot$ ):** Suppose that we know the composition of gases in the region we are observing, we may therefore calculate the total amount (total column density times area of region) of gases. Then we know how much mass is there.

## 2 Formulas

I will not derive the formulas here. I shall simply give the results that are adopted in the code. All formulas below can be found in e.g. (Steven W. & Pallà, 2004; Mangum & Shirley, 2015).

### 2.1 Flux to flux density

These two quantities are converted by

$$flux = S \times beam\ area \quad (1)$$

where  $S$  is the flux density, and the beam area (dimensionless) is

$$beam\ area = \frac{beam\ size}{cell\ size} \quad (2)$$

### 2.2 Flux density to brightness temperature

The definition of brightness temperature  $T_B$  is

$$T_B \equiv \frac{c^2}{2k_B\nu_o^2} \cdot I \quad (3)$$

where  $c$  is the speed of light,  $k_B$  is the Boltzmann constant, and  $\nu_o$  is the rest frequency of the line species. The spectral radiance  $I$  is related to flux density by

$$S = I \times \text{area of the region counted in flux} \quad (4)$$

This is because spectral radiance has a unit of "per solid angle", so we need to divide the flux density by the area (in radians squared). (Note: in regions where sources are small, solid angle can be approximated by radians squared provided  $\sin \theta \approx \theta$  is small).

### 2.3 Brightness temperature to optical depth

The relation between the brightness temperature (or main beam temperature), excitation temperature ( $T_{ex}$ ), and optical depth ( $\tau$ ) is expressed by

$$T_B \equiv T_{mb} = \frac{h\nu_o}{k_B} \left[ \left( e^{\frac{h\nu_o}{k_B T_{ex}}} - 1 \right)^{-1} - \left( e^{\frac{h\nu_o}{k_B T_{bg}}} - 1 \right)^{-1} \right] \cdot (1 - e^{-\tau}) \quad (5)$$

where  $h$  is Planck's constant,  $T_{bg}$  is the back ground temperature, i.e., the CMB temperature. Inverting this, we get

$$\tau = -\ln \left( 1 - \frac{T_B}{T_o} [f(T_{ex} - f(T_{bg}))^{-1}] \right) \quad (6)$$

in which I've abbreviated  $T_o \equiv h\nu_o/k_B$  and  $f(T) \equiv (\exp(T_o/T) - 1)^{-1}$ .

### 2.4 Optical depth to column density

The optical depth is given by

$$\tau = \frac{h\nu_o}{4\pi} (n_J B_{J,J+1} - n_{J+1} B_{J+1,J}) \phi(\nu_o) \Delta s \quad (7)$$

$$= \frac{c^2 A_{J+1,J}}{8\pi\nu_o^2} \cdot \frac{g_{J+1}}{g_J} \left( 1 - e^{-\frac{h\nu_o}{k_B T_{ex}}} \right) \cdot N_J \phi(\nu_o) \quad (8)$$

where  $n$  are the number density at states ( $J+1$  and  $J$ ),  $A, B$  are the corresponding Einstein A, B coefficient,  $\phi(\nu)$  is the relative probability of emission into a photon frequency  $\nu$  near the line center frequency  $\nu_o$ ,  $\Delta s$  is the depth (length) along the l.o.s.,  $g$  are the degeneracies, and  $N_J$  is the column density in the lower state ( $\equiv n_J \Delta s$ ).

One can further get rid of the degeneracies, resulting in

$$\tau = \frac{c^2 A_{J+1,J}}{8\pi\nu_o^2} \cdot \left( e^{\frac{h\nu_o}{k_B T_{ex}}} - 1 \right) \cdot N_{J+1} \phi(\nu_o) \quad (9)$$

Notice that  $N_J$  becomes  $N_{J+1}$ . Finally, the total column density can be calculated from the column density in the upper state (i.e.  $N_{J+1}$ ) via

$$N_{tot} = N_{J+1} \times \frac{Z}{2J+1} \cdot e^{-hB_e J(J+1)/k_B T_{ex}} \quad (10)$$

$$= N_{J+1} \times \frac{(2J+1)e^{-hB_e J(J+1)/k_B T_{ex}}}{k_B T_{ex}/hB_e} \quad (11)$$

where  $B_e$  is the rotational constant for the molecule, and  $Z$  is the partition function, defined as

$$Z \equiv \sum_{J=0}^{\infty} (2J+1) \cdot e^{(-hB_e J(J+1)/k_B T_{ex})} / (2J+1) \quad (12)$$

The second line of equation 10 comes from the following approximation of  $Z$ :

$$Z = \sum_{J=0}^{\infty} (\dots) \approx \int_0^{\infty} dJ \quad (\dots) \quad (13)$$

$$= \frac{k_B T_{ex}}{hB_e} \quad (14)$$

Thus, upon integrating (summing up) all channels of  $\tau$  in equation 9, we obtain the desired column density.

## 2.5 Column density to mass

Finally, we can estimate total gas mass, based on the following assumptions:

- Known abundance ratios between  $CO$  isotopes and  $H_2$ . The X-factor for  $C^{18}O$  is set to be  $3 \times 10^{-7}$  (Frerking et al., 1982), and the relative abundance between the three  $CO$  isotopes are  $\frac{[^{13}CO]}{[C^{18}O]} = 7.3$ ,  $\frac{[^{12}CO]}{[C^{18}O]} = 560$  (Curtis et al., 2010). In other words, we have  $[C^{18}O] = X_{18} \cdot N_{H_2}$  and  $[^{13}CO] = X_{13} \cdot N_{H_2} = 7.3 \times X_{18} \cdot N_{H_2}$ , etc., where  $N_{H_2}$  is the column density of  $H_2$ .
- The mean molecular weight of gas per  $H_2$  is 2.7 (Nishimura et al., 2015). This value is estimated by the fact that  $H_2$  composes roughly 3/4 of elements in the universe.

The mass ( $M$ ) is

$$M = (2.7 \times m_{H_2}) \cdot D^2 N_{H_2} \cdot \text{area of the region} \quad (15)$$

where  $m_{H_2}$  is the mass of  $H_2$  and  $D$  is the distance to the source. The estimate comes from calculating total number through the last three factor, and under the approximation of small angle.

Constant	Value	Reference
$\nu_{^{13}CO,0}$	331 GHz	-
$\nu_{C^{18}O,0}$	220 GHz	-
$A_{13}$	$2.181 \times 10^{-6} \text{ (s}^{-1}\text{)}$	<a href="#">Yıldız et al. (2013)</a>
$A_{C^{18}O}$	$6.011 \times 10^{-7} \text{ (s}^{-1}\text{)}$	<a href="#">Yıldız et al. (2013)</a>
$B_{e,^{13}CO}$	55.10 GHz	<a href="#">Pickett et al. (1998)</a>
$B_{e,C^{18}O}$	54.89 GHz	<a href="#">Pickett et al. (1998)</a>

Table 1: Constants.

### 3 Constants

Some constants used are listed in table 1.

### 4 References

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