

$$e) \alpha \in \mathbb{R}, (\alpha f) \otimes g = \alpha (f \otimes g)$$

*Ejemplos:

$$\mathbb{R}^2 \quad B = \{e_1, e_2\}, \quad B^* = \{e_1^*, e_2^*\}$$

$$T^1(E) = \{f: E^* \rightarrow \mathbb{R} = \mathbb{R} \text{ lineales}\} = \boxed{E^{**} = E}$$

$$T^2(E) \ni e_1 \otimes e_2$$

$$(e_1 \otimes e_2)(e_1^*, e_2^*) = e_1(e_1^*) \cdot e_2(e_2^*) = 1$$

$$(e_1^*, e_2^*) = 1$$

$$(e_2^*, e_1^*) = 0$$

$$(e_2^*, e_2^*) = 0$$

→ Quan tenim $f \in E^*$ ($E = \mathbb{R}^3$):

$$(\mathbb{R}^3)^* \quad B = \mathbb{R}^3$$

$$(2, 1, 5) \cdot \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = x \in \mathbb{R} = \mathbb{R}$$

És el mateix per a A que a B
(a(b)) o que B opera a A (b(a)) i per
tant la matriu $B \in E^{**} \rightarrow E = E^{**}$

→ Obs:

$$1) E \text{ K-es. dim } n, B = \{e_1, \dots, e_n\}$$

Los índices

$$\underbrace{(e_{i_1}^* \otimes \dots \otimes e_{i_p}^*)}_{I = \{i_1, \dots, i_p\}} \otimes \underbrace{(e_{j_1} \otimes \dots \otimes e_{j_q})}_{J = \{j_1, \dots, j_q\}} = \begin{cases} 1 & \text{si } I=J, J=M \\ 0 & \text{otherwise} \end{cases}$$

$$L = \{l_1, \dots, l_p\} \quad M = \{m_1, \dots, m_q\}$$

$$2) f, g \in T_p^q(E)$$

$$f = g \Leftrightarrow \forall e_{i_1}, \dots, e_{i_p} \in B \quad f(e_{i_1}, \dots, e_{i_p}) = g(e_{i_1}, \dots, e_{i_p})$$

$$\forall e_{j_1}^*, \dots, e_{j_q}^* \in B^*$$

↑ per multilinearitat de f i g.