Oscar Benedito

Àlgebra multilineal i geometria

3. Dimensión y bases de $T_p^q(E)$

*Rec: $T_p^q(E)$ és K-ev.

*Teorema: Sea E K-ev. de dim n y $B=\{e_1,\dots,e_n\}$

(1) $\dim_K T_p^q(E) = n^{(p+q)}$

(2) Una base de $T_p^q(E)$ es:

$$B_p^q = \left\{ e_{i_1}^* \otimes \dots \otimes e_{i_p}^* \otimes e_{j_1} \otimes \dots \otimes e_{j_q} \;\middle|\; \begin{array}{l} i_1,\dots,i_p \in \{1,\dots,n\} \\ j_1,\dots,j_q \in \{1,\dots,n\} \end{array} \right\}$$

(3) Si $f \in T_p^q(E)$ $\underbrace{f_{B_p^q} = (f(e_{i_1},\dots,e_{i_p},e_{j_1}^*,\dots,e_{j_q}^*))}_{\text{vector escrito en la base } B_p^q}$

• Demo: 1) consequencia de (2)

2) L.I. Sea:

$$\omega = \sum \alpha_{IJ} (e_{i_1}^* \otimes \dots \otimes e_{i_p}^* \otimes e_{j_1} \otimes \dots \otimes e_{j_q}) = 0$$

Sean I_0, J_0 conjuntos de índices cualesquiera.

$$0 \underset{\text{hipótesis}}{=} \omega(e_{i_1},\dots,e_{i_p},e_{j_1}^*,\dots,e_{j_q}^*) \underset{\text{Obs (1) final de la pg 2 G}}{=} \alpha_{I_0 J_0}$$

Generadores: Sea $f \in T_p^q(E)$, definimos $g \in T_p^q(E)$

$$g = \sum_{\forall I,J} (f(e_{i_1},\dots,e_{i_p},e_{j_1}^*,\dots,e_{j_q}^*))(e_{i_1}^* \otimes \dots \otimes e_{i_p}^* \otimes e_{j_1} \otimes \dots \otimes e_{j_q})$$

Si $f=g$ $\Rightarrow$ B_p^q son generadores de $T_p^q(E)$

$\Rightarrow$ (3) del teorema

$$g(e_{i_1^0},\dots,e_{i_p^0},e_{j_1^0}^*,\dots,e_{j_q^0}^*) \underset{\text{Obs (1) final pg 2 G}}{=} f(e_{i_1^0},\dots,e_{i_p^0},e_{j_1^0}^*,\dots,e_{j_q^0}^*) \;//$$

*Ejemplos:

1) $E = \mathbb{R}^n$ $B = \{e_1, \dots, e_n\}$, $B^* = \{e_1^*, \dots, e_n^*\}$

(A) Sea $u \in \mathbb{R}^n$, $E = E^{**} = T_0^1(E)$

$u = u(e_1^*)e_1 + \dots + u(e_n^*)e_n$ $(B_0^1 = B)$

(B) $T_1^0(E) = E^*$

$B_1^0 = \{e_1^*, \dots, e_n^*\} = B^*$

$\omega \in T_1(E) = E^*$

$\omega = \omega(e_1)e_1^* + \dots + \omega(e_n)e_n^*$

(C) $f \in T_2(E)$ $\underline{n=3}$

$B_2^0 = \{e_1 \otimes e_1, e_1 \otimes e_2, e_1 \otimes e_3, e_2 \otimes e_1, \dots, e_3 \otimes e_3\}$

$f \underset{\substack{\uparrow \\ (3)}}{=} f(e_1, e_1)\, e_1^* \otimes e_1^* + \dots + f(e_3, e_3) \cdot e_3^* \otimes e_3^*$