Solution Quiz 1,

1.

$Y_n = \max\{X_1, \dots, X_n\}$

$Z_n = \min\{X_1, \dots, X_n\}$

$$F_{Y_n}(y) = P(Y_n < y) = P(X_1 < y, X_2 < y, \dots, X_n < y)$$

$$\overset{\text{indep}}{=} P(X_1 < y) \cdots P(X_n < y)$$

$$= F_X(y) \cdots F_X(y) = (F_X(y))^n$$

$$f(x) = 4x^3 \to F_X = x^4 \longrightarrow F_{Y_n} = x^{4n}, \quad 0 \leq x \leq 1$$

$$F_{Z_n}(z) = P(Z_n < z) = 1 - P(Z_n > z)$$

$$\overset{\text{indep}}{=} 1 - P(X_1 > z)\, P(X_2 > z) \cdots P(X_n > z)$$

$$= 1 - (1 - P(X_1 < z)) \cdots (1 - P(X_n < z))$$

$$= 1 - (1 - F_X(z))^n$$

$$F_X = x^4 \longrightarrow F_{Z_n} = 1 - (1 - x^4)^n$$

$$f_{Z_n} = F_{Z_n}'$$

2. $x \longrightarrow$ randomly selected number less than $mn$

$A$ : $x$ divisible by $m$

$B$ : $x$ divisible by $n$

we want : $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B)$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$m = ad$ $\qquad d = (m,n)$

$n = bd$ $\qquad 1 = (a,b)$

$$\longrightarrow P(A \cap B) = \frac{d-1}{mn-1}$$

$$\longrightarrow P(A^c \cap B^c) = 1 - \left[ \frac{n-1}{mn-1} + \frac{m-1}{mn-1} - \frac{(m,n)-1}{mn-1} \right]$$