Stochastic Processes
Sharif University of Technology
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CE 40-695

Time: 20 mins

Name: Std. Number:

Quiz 1 Solutions (Stationary Stochastic Processes)

Questions

1. Specify whether the following statement is true or false and give a brief justification or counterexample.

   The random process x(t) = A where A is a continuous random variable with a pdf uniform between 0 and 1 is strict sense stationary.

   Solution:
   True: you can conclude that the process x(t) is strict sense stationary because there is no way to tell where on the time axis you are by looking at any set of samples, no matter at what times they are taken. Mathematically, this intuition is captured by saying that PDF's of all orders are shift invariant, or $f_{x(t_1),x(t_2),\ldots,x(t_N)} = f_{x(t_1+\tau),x(t_2+\tau),\ldots,x(t_N+\tau)}$ for all N and τ.

2. $X(t)$ is a wide sense stationary stochastic process with autocorrelation function :
   $R_X(\tau) = 10sin(2\pi 1000\tau)/(2\pi 1000\tau)$. The process $Y(t)$ is a version of X(t) delayed by t_0 seconds: $Y(t) = X(t - t_0)$.

   (a) Derive the autocorrelation function of $Y(t)$.
   (b) Is $Y(t)$ wide sense stationary?

   Solution:
   (a) $Y(t)$ has autocorrelation function:
   $R_Y(t,\tau) = E[Y(t)Y(t+\tau)]$
   $= E[X(t-t_0)X(t+\tau-t_0)]$
   $= R_X(\tau)$.

   (b) We have already verified that $R_Y(t,\tau)$ depends only on the time difference τ. Since $E[Y(t)] = E[X(t-t_0)] = \mu_X$, we have verified that Y (t) is wide sense stationary.