Введение в стохастику: Моделирование кривых процентных ставок и стоимости облигаций.

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Disclaimer

Agenda

1. Interest rate fundamentals
   - Libor
   - FedBunds
   - Base rate

2. Yield curve
   - Types and economic interpretation

3. Using Yield curve
   - pricing zero coupon bond

4. Yield curve interpolation
   - Pricing a simple swap

5. Looking at data for RUB -> Short rate model
6. Homework

What is an interest rate? (1/3) Interest rate is contract specific.

- From economic perspective It is the price of capital: if you want to use my capital for a year you have to pay me something.
- Think about renting a flat. The bigger the flat the higher the rent, the longer you're renting for the more you pay. Think about rent per square metre per year – that's the equivalent of interest rate in the world of real estate.

- How is interest rate determined?
  - In general it's contract specific – there is no single interest rate.
  - Determined by the lender and borrower. Depends on a range of things
    - The state of the market: supply/demand
    - The quality of the borrower: how likely are you to repay?
    - The term of the contract: borrowing for a month vs borrowing for 10 years.
    - The start date of the contract: we can agree today that I borrow from you in a year's time for 3 months.
    - Any other conditions: you give me something else as a collateral, capital guarantee
  - Does interest rate have to be positive?

- How to talk about interest rates if they are so contract specific?
  - Same thing we did with stocks: standardize and take the average!

What is an interest rate? (2/3) Libor, Euribor and other Prime rates.

- How to talk about interest rates if they are contract specific?
  - Well, we need some standardization here:
    - Standard lenders and borrowers – major global banks operating in London (around 17-18)
    - Standard start date – today before 11am
    - Standard end date: 1D, 1W, 2W, 1M, 3M, 6M, 12M
    - Standard currencies: USD, GBP, EUR, JPY, CHF
    - Standard size – reasonable as per prevailing market conditions
    - Standard clearing/collateral: unsecured
    - Take the average, publish at 11:30am
    - That is **L**ondon **I**nter **B**ank **O**ffered **R**ate or *Libor*.

  - Similarly there is *Euribor*, just taking the average across eurozone banks.
  - Please notice Eur Libor ≠ Euribor
  - There are equivalents for most currencies, for RUB there is MosPrime rate.
  - https://www.cbr.ru/eng/hd_base/mosprime/

What is an interest rate? (3/3) FedFunds/Eonia/Ruonia… remove credit risk.

- 6M Libor GBP is 0.6%... Why? It's supply/demand but there are 3 consideration:
    - State of the economy: primarily current base rate, reserve requirements
    - Future of the economy: if the economy does well the rate will go up…
    - Credit risk: I give money to JPM, I'm a bit worried they can go bust (like Leman) so I'll raise my rate a little bit because of that.

- To remove credit risk there is FedFunds (in US) or Eonia (in eurozone), Ruonia in Russia etc
    - Similar idea as Libor but minor tweaks
    - Overnight rate only
    - Leading banks with minimal credit risk
    - Actual transactions, not just banks' estimates
    - In US operates through Fed Reserves to eliminate credit risk

- Which rate is higher FedFunds or USD Libor overnight?
- The difference is called FedFunds spread, it represents the credit risk of the banking system.
- Usually it's around 1-2 bps. In 2008 it was > 80 bps – liquidity crisis.
- http://www.ruonia.ru/eng/

Let us look at the data!

Key observations:
- Rates are highly correlated, they basically all move together, as if there was a single factor that drives them
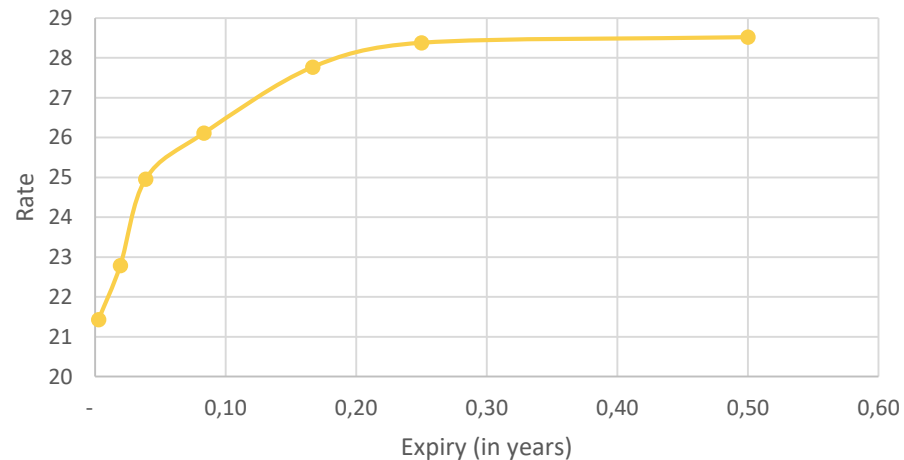- We see that rates are sensitive to macro-economic events

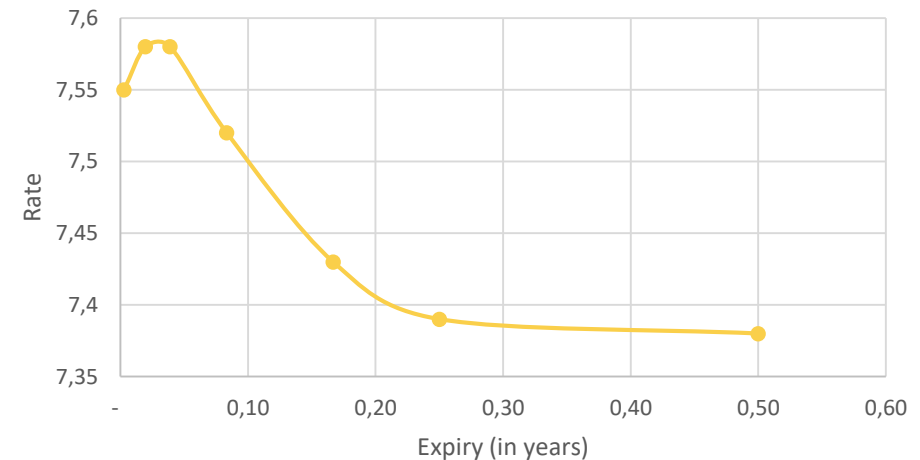MosPrime Rate
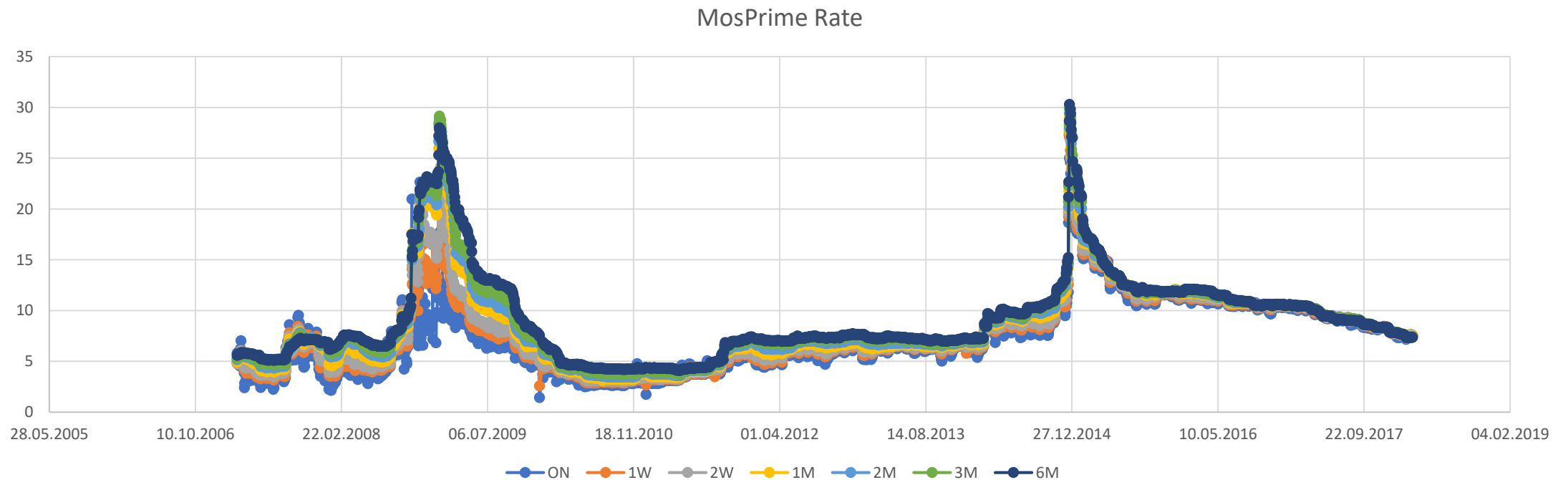
Let us look at the data!

Key observations:
- Curve shape changes depending on the economy
- Rate is generally positive and tends to return in the long-run to some reasonable level

RUB Yield curve as of 24/12/2014, typical shape

RUB Yield curve as of 07/03/2018, inverted shape

Rate dynamics: short rate model

Say you want to borrow for 1 week. You can either borrow using 1W rate or borrow/repay every day using the overnight rate:

$$1+5/365 \times r_t^{1W} = (1 + 1/365 \times r_t^{ON})(1 + 1/365 \times r_{t+1}^{ON}) \dots (1 + 1/365 \times r_{t+5}^{ON})$$

So 1W rate should be equal to "averaged" ON rate over that week.

Let's use continuous-time rates i.e instead of $(1 + 1/365 \times r_t^{ON})$ we'll write $e^{1/365 \times r_t^{ON}}$

Then we can say that

$$e^{\frac{5}{365} \times r_t^{1W}} = e^{1/365 \times r_t^{ON} + 1/365 \times r_{t+1}^{ON} + \dots + 1/365 \times r_{t+5}^{ON}}$$

More generally can say that

$$r_t^T = \frac{1}{T} \sum_{k=t}^{k=T} r_k \Delta t$$

Rate dynamics: short rate model

Now all I need is to understand how to model the short rate.
Let's assume for now that the short rate is just a function of time i.e. $r_t = r(t)$

I know yield curve for a set of points... Let's assume that the function $r(t)$ is linear between those points.
Then the rate should be interpolated linearly between the observable dates.

Finally I notice that daily changes in rate seem to have Normal distribution...

I can propose the following model

$$r_t - r_{t-1} = \Delta r_t = \alpha_t(\theta_t - r_t)\,\Delta t + \sigma_t \Delta W_t$$

Where
α_t is the speed of mean-reversion
θ_t is the long-term rate level
σ_t is rate volatility

This is the Hull-White model!

Home work

1. Create a video of MosPrime YieldCurve evolution and post it on youtube:
   1. Read data into python
   2. For each date plot a yield curve and save it into a file as .jpg
   3. Use *def make_video(...)* to turn it into a video
   4. Post your video into the SF chat / youtube!
   5. Try cubic interpolation of the curve

2. Swap trader
   1. Price your swap for a period of dates using the corresponding yield curve for each date.
   2. Plot swap price as a function of time. Can you explain the behaviour?

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