# Моделирование свопа. Моделирование кредитного портфеля.

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## Agenda

- Swap modelling using yield curve
- Writing a realistic swap pricing model
- Credit portfolio, what is that?
  - Two types of credit risk
- Credit risky bond model
- Credit Default Swap model
- Credit portfolio modelling Using Monte-Carlo
- One day from at Valuation Control Group

## Interest rate Swap modelling

What is a swap?

- Two parties
- Two legs
- Each leg is a stream of cashflows
- Cash flows can be fixed (fixed leg)
- Or floating i.e. linked to something
    - To interest rates for interest rate swaps
    - To equity for equity swaps
- Can be Exchange traded (e.g. 2Y fixed rate against 6M floating rate swap)
- Or OTC (can be as custom as the client wants)

## Interest rate Swap modelling

- Two legs can be modelled and priced separately
- Fixed leg can be priced just using the yield curve like we did the previous rates lecture
- For the floating leg we usually need a model to forecast the future rate

- Let's consider a deterministic short rate model i.e. there is short (think overnight) rate

$r$ which is changes deterministically with time

$$r = r(t)$$

- We can calibrate this model to the observed yield curve
- How to obtain let's say the 6M rate that will be observed in 6M?
- In our model any future rate is deterministic and can be obtained from the short rate function:

$$1 + rate(t_1, t_2) = e^{\int_{t_1}^{t_2} r(\tau)d\tau} = e^{\int_0^{t_2} r(\tau)d\tau - \int_0^{t_1} r(\tau)d\tau} = \frac{1 + yc(t_2)}{1 + yc(t_1)}$$

- We now can write a pricing function for any interest rate swap!

## Credit portfolio, what is that?

- Any portfolio with substantial credit risk

- Example: portfolio containing
  - credit risky bonds i.e. bonds where the issuer has substantial credit risk e.g. a corporation/sovereign with a low credit rank

  - Mortgages or mortgage linked derivatives

  - credit risky derivatives, e.g.
    - Credit Default Swap or CDS linked derivatives

  - Credit indices

- Such portfolio typically requires special attention to assess credit risk

## Credit Risky Bond Model

- You want to buy a credit risky bond of company XXX
- The bond will pay you $100 in 1Y if company XXX is alive
- The bond will pay you $40 if company XXX defaults
- Say you can buy such bond today for $90.
- What does that mean if risk free bond can be bought for $99?

Risk free bond:

$$\$99 = \frac{\$100}{1+r}$$

Risky bond:

$$\$90 = (1-\boldsymbol{\lambda})\frac{\$100}{1+r} + \boldsymbol{\lambda}\frac{\$40}{1+r}$$

I can conclude that $\boldsymbol{\lambda} = 15.2\%$ - the default probability over 1Y. So credit risky bonds can be used to imply the default probability that can be used for the rest of the portfolio credit analysis.

## Credit Default Swap

- Buyer pays the seller constant coupon payments every quarter until the reference entity defaults
- Seller pays Notional (1- residual reference value) to the buyer when the entity defaults.
- Limited maturity, typically 2-10 years.

Example:
Notional $10M
Coupon payment $25k each quarter.
So you pay per year $100k or 1% (100bps) of the Notional.
Percentage payment per year is called ***Credit Spread*** and is quoted on the exchange.

*What can we say about market implied default probability?*
Assume default probability per annum is $\boldsymbol{\lambda}$

$$s = \boldsymbol{\lambda}(\mathbf{1} - \mathbf{R})$$

So

$$\boldsymbol{\lambda} = \frac{s}{1 - R}$$

R is the residual reference value; Typically if CDS is linked to a company that would be the residual value of the bond after default.

## Credit Portfolio Risk Assessment (e.g. Credit VAR)

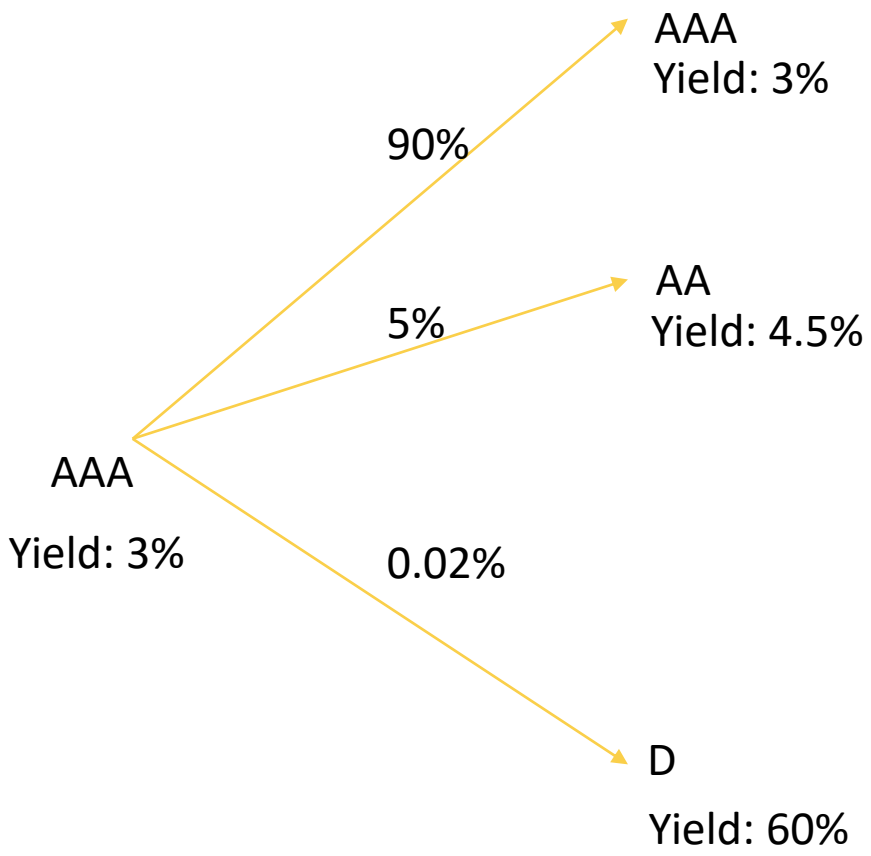
- Portfolio Value depends on default probabilities of its reference entities
- We can imply the default probabilities for certain reference entities using prices of Credit Risky Bonds or CDS
- The data is sparse and is only available for a limited number of reference entities
- Besides we would be interested in the evolution of default probabilities to perform risk assessment

- What shall we do? Let's focus on bonds. For bonds there is ranking, which is supposed to be linked to credit worthiness.
- Ranking agencies publish some statistics about the rank transition probabilities:
  e.g. they that AAA bond will remain AAA with probability 90%, will downgrade to AA with probability 5%... Default with probability 0.02%.
- Next, we can find statistics on how much a typical AAA bond loses in value when it is downgraded to e.g. BBB.

- Combine these two pieces of data: we now know some transition probabilities and the corresponding bond values in each state!

## Credit Portfolio Risk Assessment (e.g. Credit VAR)

"AAA bond remains AAA with prob 90%, downgrades to AA with prob 5%, … defaults with prob 0.02%"
"AAA bond gets 1.5% in yield when it's downgraded to AA,…"



# Credit Portfolio Risk Assessment (e.g. Credit VAR)

(percentages)

| From \ To | AAA | AA | A | BBB | BB | B | CCC/C | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 90.79 | 8.30 | 0.70 | 0.10 | 0.10 | - | - | 0.01 |
| AA | 0.70 | 90.76 | 7.70 | 0.60 | 0.10 | 0.10 | - | 0.04 |
| A | 0.10 | 2.40 | 91.30 | 5.20 | 0.70 | 0.20 | - | 0.10 |
| BBB | - | 0.30 | 5.90 | 87.40 | 5.00 | 1.10 | 0.10 | 0.20 |
| BB | - | 0.10 | 0.60 | 7.70 | 81.20 | 8.40 | 1.00 | 1.00 |
| B | - | 0.10 | 0.20 | 0.50 | 6.90 | 83.50 | 3.90 | 4.90 |
| CCC/C | 0.20 | - | 0.40 | 1.20 | 2.70 | 11.70 | 64.50 | 19.30 |
| D | - | - | - | - | - | - | - | 100.00 |

## Credit Portfolio Risk Assessment (e.g. Credit VAR)

Ok, now we can calculate all the risk metrics we want for a single bond!

- How about other instruments? Swaps, options? For this instruments it's really not obvious how their price depends on the credit ranking.

- We can do tier scenarios though: calculate portfolio value for any given scenario e.g. spot of the underlying equity goes down, or vol goes up (see previous lectures).

- Notice that default probability is highly correlated with the stock price. How about we "map" ranking transition probabilities onto the stock price levels? E.g. say we calculated that stock loses loses less than 10% with prob 90% and less than 50% with probability 5% etc. Then we can say that
  - If stock stays above 90% the ranking remains AAA
  - If it is between 70% - 90% it's downgraded to AA
  - …
  - If it goes below 20% the company is going to default

- The idea is to link stock price, which is observable and relatively easy to model, to default probabilities, which are unobservable and are difficult to model.

## Credit Portfolio Risk Assessment (e.g. Credit VAR)

Once this link is established the rest is technical. Consider example of a portfolio of financial instruments on 2 underlying stocks:

- Choose time horizon, e.g. 1Y
- Simulate levels of both stocks up to 1Y using MC
- Look up default probabilities corresponding to the simulated levels
- Calculate the portfolio value in the simulated scenario taking into account the default probabilities you looked up
- Repeat many times
- build histogram of the simulated values

- Using the histogram you start answering risk questions, e.g. look up Credit VAR.

# One day from at Valuation Control Group

- Quants write model to price instruments which are in the bank's portfolio
- Each model has
  - inputs:
    - e.g. volatilities, correlations
    - default probabilities
    - Yield curves
  - assumptions:
    - stock returns are distributed normally
    - short deterministic rate model
    - Etc

  - Your job is to control valuation of the bank's portfolio
    - Make sure traders use correct inputs for their models
    - Make sure correct appropriate model has been used for valuation
    - If model has limitations -> calculate necessary reserves (uncertainty reserves)
    - Opine on approximate methodologies used in the bank

# Home work



1. Write tests for the swap pricer from the lecture.
2. Your boss asks you to use cubic interpolation of the Yield Curve. How would you do that?
3. How does modular design using a hierarchy of classes helps speed up business development?
4. Write Tests for the Credit Portfolio Model from the lecture. If you didn't know the model how would you sanity check it?
5. Have a look at https://www.ecb.europa.eu/pub/pdf/scpops/ecbocp64.pdf?c9adda20c5ec1c53935f8dc6b3a57fef

# Контакты

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