Введение в стохастику: Моделирование акций и портфеля финансовых инструментов.

Евгений Рыскин
evgeny.ryskin@gmail.com

Disclaimer

The views and opinions expressed in this presentation are strictly those of the author and do not reflect the official policy or position of their employer. Examples of analysis performed within this work are only examples designed for educational purposes. They should not be utilised in real-world analytic products as they are based only on very limited, dated open source information under simplified assumptions made for tractability. The information contained in this presentation is provided for educational purposes only on an “as is” basis with no guarantees of completeness or accuracy. The potential errors and omissions in this presentation are those of the author.

Module principles

- Business casual style
- Interactive: we ask each other lots of questions, we keep feedback loops strong
- Practitioner’s point of view
- Strive to develop “intuitive” understanding

What is a stock/equity?

- a way to participate in/share a company's business
- Gives some rights to future profits in a form of dividends
- Gives some voting rights
- Gives some default risk (If everything is bad you will probably get your money last)
- Spot price is defined by supply/demand at stock exchanges
- In theory, stock price should be equal to the market estimate of the discounted sum of future dividends
- This is called price discovery. It is never perfect.

What is an equity index?

- Some number calculated by a trusted calculation agent
- Usually a financial services company e.g. Standard and Poor's, JPM
- E.g. just a market cap of some *N* companies (S&P 500) or MSCI World
- Dividends could be paid or assumed reinvested
- Spot is not tradable - you can not "buy" S&P500 index
- There is forward market or "synthetic" market – ETFs or trackers
- Index can be replicable - in theory, it's possible to fully replicate the index by buying individual stocks – or non replicable

ETF – Electronically Traded Fund
Tracker – A fund trying to replicate a given index

Let's have a look at SPX historical returns

Go to: Excel

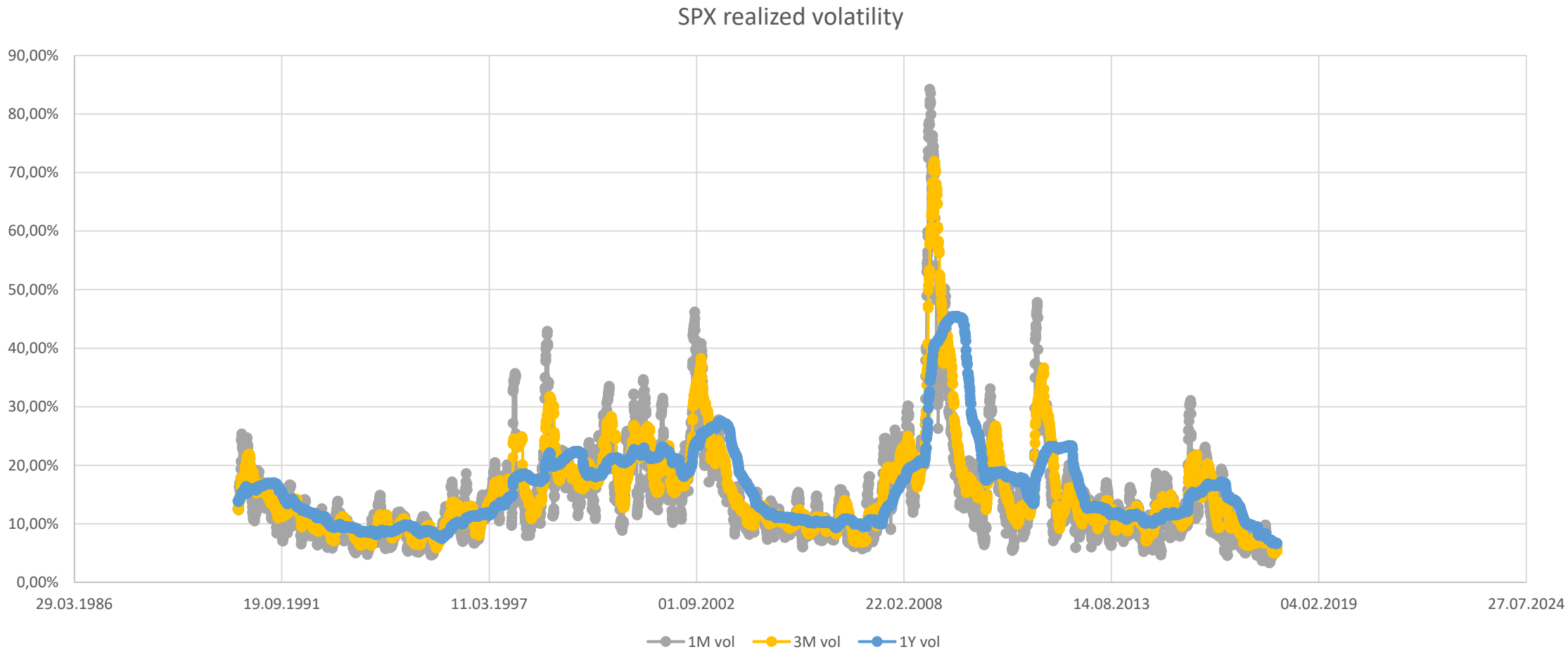
Go to: python

SPX – Bloomberg ticker of S&P500 index

Key observations

- Daily Returns tend to fluctuate around some level
- Economics tells me that one should not be able to reliably(!) predict the direction of the move
- I conclude the returns are independent, like coin flips
- This level and the "width" of the fluctuations change in time
- In the short run, say 1-2 years, I can roughly say the "width" is constant
- In the short run, daily returns seem to have a distribution close to a Normal one
- In the long run, the distribution is clearly not normal – the tails are "heavier" than normal

Equity dynamics: realized volatility



Equity dynamics: daily returns in the short-term seem to be normally distributed

$$\frac{S_{t+1}}{S_t} - 1 = \frac{S_{t+1} - S_t}{S_t} = \frac{\Delta S_t}{S_t} = \mu_t \Delta t + \sigma_t \sqrt{\Delta t} N_t$$

μ_t is drift

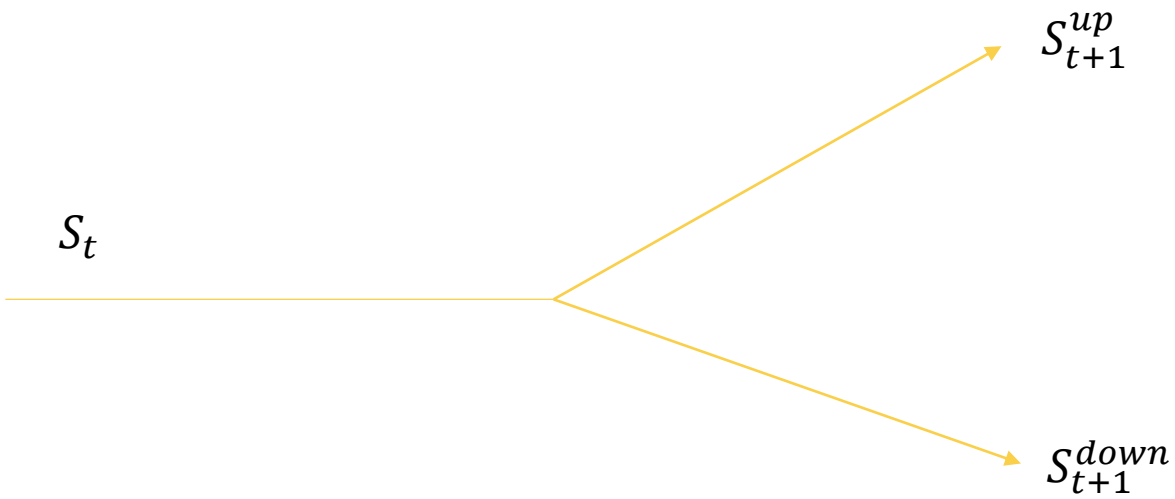
σ_t is volatility

N_t is a realization of a standard normal random variable (usual notation $\sqrt{\Delta t} N_t = \Delta W_t$)

$\frac{\Delta S_t}{S_t}$ is a daily return

What is risk? It's Uncertainty + Exposure

You hold Apple stock -> you hold equity risk that Apple price can go up or down



What do you need a model for? What kind of questions can you answer?(1/2)

I have $100 invested in SPX.
What is the probability to lose > $10 after 1 year?
What is the probability to lose > $20 after 1 year? **Go to: python**
….

Alternative question could be: what's the max loss for a given probability level?
E.g. I want to know that with probability 95% I am not going to lose more than $X.
This is the idea of VAR – Value At Risk. It is a relatively easy to understand risk measure.
VAR is closely related to the concept of percentile which is the mathematical equivalent of the above **Go to: python**

The answers are needed to conduct business in a predictable and sustainable fashion.

What do you need a model for? What kind of questions can you answer?(2/2)

I have $100 invested in SPX. Current volatility is thought to be 20%. My 5%-VAR is 60$.
If tomorrow volatility increases to 30% what will my VAR be?

Go to: python

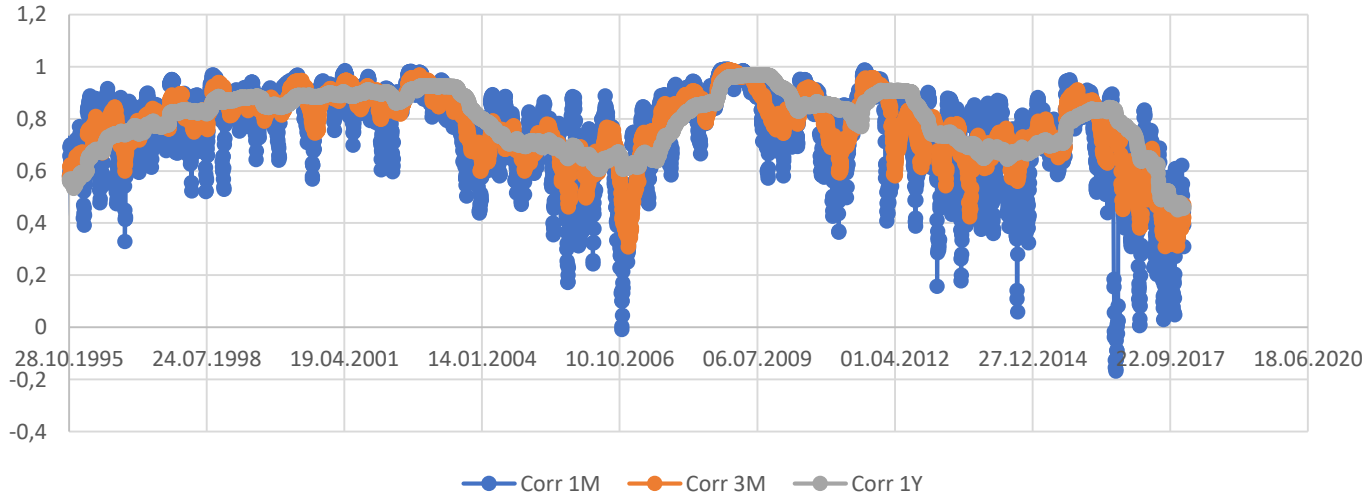
Equity Modelling: what if there are 2 stocks?

We already know how to model individual stocks

$$\frac{\Delta S_t^1}{S_t^1} = \mu_t^1 \Delta t + \sigma_t^1 \Delta W_t^1$$

$$\frac{\Delta S_t^2}{S_t^2} = \mu_t^2 \Delta t + \sigma_t^2 \Delta W_t^2$$

We just said that for both stocks daily returns are distributed normally….
Can we say anything else about the relationship between daily returns of the two stocks?
Let's look at the data again.



Let’s have a look at SPX & SX5E historical returns

Go to: python

SX5E– Bloomberg ticker of EuroStoxx 50 Index

Key observations

- Stocks are related for economic reasons. The relationship is manifested e.g. by the correlation of daily returns.
- Relationship can be assessed using realized correlation.
- Correlation is a very simplistic way to describe the relationship between two stocks - that’s why it’s constantly changing. The economic world is more complex.
- Describing relationship using correlation is as approximative as describing single stock returns using the Normal distribution.
- Correlation is an intuitive and easy to understand way to describe the relationship between stock returns.

Equity Modelling: what if there are 2 stocks?

We already know how to model individual stocks

$$\frac{\Delta S_t^1}{S_t^1} = \mu_t^1 \Delta t + \sigma_t^1 \Delta W_t^1$$
$$\frac{\Delta S_t^2}{S_t^2} = \mu_t^2 \Delta t + \sigma_t^2 \Delta W_t^2$$

We just said that for both stocks daily returns are distributed normally….
Can we say anything else about the relationship between daily returns of the two stocks?
Let's look at the data again… Let's say that returns are correlated with some correlation ρ_t which can be assumed constant in the short run:

$$\rho_t = Corr(\Delta W_t^1, \Delta W_t^2)$$

We can now simulate 2 stocks and e.g. calculate VAR for a portfolio of 2 stocks.

Go to: python

What is a financial product (a.k.a. financial instrument)?

- Usually two parties: seller (e.g. JPM) and buyer
  - e.g. an individual,
  - a pension/hedge fund
  - an industry client
  - a commercial/investment/central bank
  - Etc
- Term Sheet (basically just a contract saying who pays what and when)
- Clearing method / collateralization agreement

- Examples
  - Simple loan / Savings account / Bond (*)
  - Simple future/forward contract (*)
  - Simple interest rate swap (*)
  - Cross currency interest rate swap (**)
  - Call option (**)
  - Variance Swap (***)
  - Mortgage (***)
  - Credit card (***)
  - 1st generation derivatives (****)
  - 2nd generation derivatives (******)

Modelling a portfolio of financial instruments

- Need to be able to model the underlyings
- Need to have a pricing function for each type of financial instruments you hold

- Let's look at an example:
  - You hold a stock, spot is $100
  - You also hold a Put option on the same stock, strike is K=$80, maturity is T=1Y.
  - You are looking at 1Y horizon.
  - You know the value of the put at maturity: if the stock price is S_T the put value is
$$(K - S_T)^+ = K - S_T \; if \; K > S_T \; and \; 0 \; otherwise$$
  - You know how to model S
  - Let's calculate 5% VAR! Can we make an educated guess?

Go to: python

Underlying – asset that is referenced by a derivative contract e.g. for a put option on SPX, SPX is the underlying

Put option - you can sell the underlying at the strike price at maturity. Useful if the market price at maturity is lower than strike.

Conclusion: one day at a Market Risk department

- You have a portfolio of financial instruments
- You need to assess Risk
- You do it using a range of Risk Metrics e.g. Delta/Vega risk, VAR, Tier Scenarios
- You typically have access to an automated system/toolbox for calculation of the Risk Metrics
- Your job is to
  - Understand which risks metrics are relevant for a given product type
  - Understand how to combine risks metrics
  - Have some understanding of the models and how they compute risks
  - Understand complex/approximate/temporary bookings
  - Prepare risk reports
  - Monitor the risk limits

- Market Risk department has to comply with internal and external regulatory requirements
- It makes sure the desk is taking appropriate, measurable and limited risks thus contributing to the stability of the firm and more broadly the entire financial system.

Home work

1. Rerun the lecture code using period 2008-2009. Any anomalies? Does our modelling approach work?

2. Download historical data for AT&T, you can use https://investors.att.com/stock-information/historical-stock-information/historical-quote/att-inc
    1. Compare realized volatilities of AT&T and SPX:
    2. What are the key observations?
    3. Can you explain the results? What are economic differences between AT&T and SPX?
    4. Can you see any seasonality in AT&T data? Why would that be?
    5. Compare drift and vol? Can you explain the results?
    6. Look at the correlation, does the sign make sense to you?
    7. Run the lecture code using AT&T data.

3. Calculate 1Y 5% Var for 3 portfolios with the same initial value ($10M)
    1. 100% SPX
    2. 100% AT&T
    3. 50% AT&T+50% SPX

    Do the results make sense to you?

4. Rewrite the MC engine using numpy to speed it up. Can you speed it up by the factor of x25?

Контакты

По всем вопросам:

corporate@sflearning.org

t.me/sftelegram

vk.com/sfeducation

societe_financiers

fb.com/societefinanciers

seekingalpha.com/author/societe-financiers

youtube.com/c/societefinanciers