

Derivative of $\cos x$.

What is the specific formula for the derivative of the function $\cos x$?

This calculation is very similar to that of the derivative of $\sin(x)$. If you get stuck on a step here it may help to go back and review the corresponding step there.

As in the calculation of $\frac{d}{dx} \sin x$, we begin with the definition of the derivative:

$$\frac{d}{dx} \cos x = \lim_{\Delta x \rightarrow 0} \frac{\cos(x + \Delta x) - \cos(x)}{\Delta x}$$

Use the angle sum formula $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$ and then simplify:

$$\begin{aligned} \frac{d}{dx} \cos x &= \lim_{\Delta x \rightarrow 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \left[\frac{\cos x \cos \Delta x - \cos x}{\Delta x} + \frac{-\sin x \sin \Delta x}{\Delta x} \right] \\ &= \lim_{\Delta x \rightarrow 0} \left[\frac{\cos x (\cos \Delta x - 1)}{\Delta x} + \frac{-\sin x \sin \Delta x}{\Delta x} \right] \\ &= \lim_{\Delta x \rightarrow 0} \left[\cos x \left(\frac{\cos \Delta x - 1}{\Delta x} \right) + (-\sin x) \left(\frac{\sin \Delta x}{\Delta x} \right) \right] \\ \frac{d}{dx} \cos x &= \lim_{\Delta x \rightarrow 0} \cos x \left(\frac{\cos \Delta x - 1}{\Delta x} \right) + \lim_{\Delta x \rightarrow 0} (-\sin x) \left(\frac{\sin \Delta x}{\Delta x} \right) \end{aligned}$$

Once again we use the following (unproven) facts:

$$\begin{aligned} \lim_{\Delta x \rightarrow 0} \frac{\cos \Delta x - 1}{\Delta x} &= 0 \text{ (A)} \\ \lim_{\Delta x \rightarrow 0} \frac{\sin \Delta x}{\Delta x} &= 1. \text{ (B)} \end{aligned}$$

And we conclude:

$$\begin{aligned} \frac{d}{dx} \cos x &= \lim_{\Delta x \rightarrow 0} \cos x \left(\frac{\cos \Delta x - 1}{\Delta x} \right) + \lim_{\Delta x \rightarrow 0} (-\sin x) \left(\frac{\sin \Delta x}{\Delta x} \right) \\ &= \cos x \cdot 0 + (-\sin x) \cdot 1 \\ \frac{d}{dx} \cos x &= -\sin(x). \end{aligned}$$