Removable Discontinuities

At a *removable* discontinuity, the left-hand and right-hand limits are equal but either the function is not defined or not equal to these limits:

$$\lim_{x\to x_0^+} f(x) = \lim_{x\to x_0^-} f(x) \neq f(x_0)$$

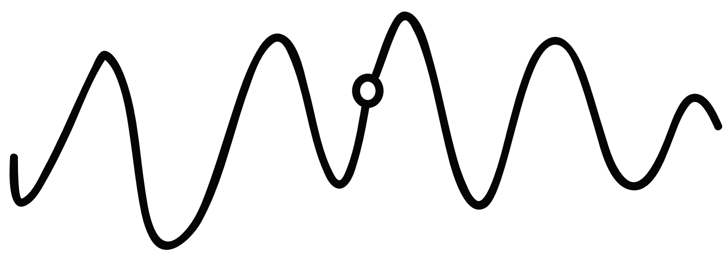

Figure 1: A removable discontinuity: the function is continuous everywhere except one point

For example, $g(x) = \dfrac{\sin(x)}{x}$ and $h(x) = \frac{1-\cos x}{x}$ are defined for $x \neq 0$, but both functions have removable discontinuities. This is not obvious at all, but we will learn later that:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \ \text{ and } \ \lim_{x\to 0} \frac{1-\cos x}{x} = 0.$$

So both of these functions have removable discontinuities at $x = 0$ despite the fact that the fractions defining them have a denominator of 0 when $x = 0$.