

## Physical Interpretation of Derivatives

You can think of the derivative as representing a rate of change (speed is one example of this). This makes it very useful for solving physics problems.

Here's one example from physics: If  $q$  is an amount of electric charge, the derivative  $\frac{dq}{dt}$  is the change in that charge over time, or the electric current.

A second, more tangible example is to let  $s$  stand for distance; then the rate of change  $\frac{ds}{dt}$  is what we call speed. Let's investigate this second example in more detail to get a visceral sense of what instantaneous speed means.

On Halloween, MIT students have a tradition of dropping pumpkins from the roof of the building this lecture was given in. Let's say that the building is about 300 feet tall. We'll use a slightly smaller value of 80 meters for the height because it makes the problem easier to solve.

The equation of motion for objects near the earth's surface (which we will just accept for now) says that the height above the ground  $h$  of the pumpkin  $t$  seconds after it's dropped from the building is roughly:

$$h = 80 - 5t^2 \text{ meters}$$

Let's think about this. The instant the pumpkin is dropped,  $t = 0$  and  $h = 80$  meters. When  $t = 4$  seconds,  $h = 80 - 5(4^2) = 0$ , and the pumpkin has reached the ground.

The average speed of the pumpkin over the time it's falling is

$$\frac{\Delta h}{\Delta t} = \frac{\text{distance traveled}}{\text{time elapsed}} = \frac{0 - 80 \text{ meters}}{4 - 0 \text{ seconds}} = -20 \text{ m/s}.$$

(The numerator is  $0 - 80$  and not  $80 - 0$  because we must subtract the initial position from the final position, not the other way around.)

The people watching the pumpkin drop probably don't care about the *average* speed. They want to know how fast the pumpkin is going when it slams into the ground. That's known as the instantaneous speed, and is the derivative  $h'(t) = \frac{d}{dt}h$ . To find the instantaneous velocity at  $t = 5$ , we evaluate  $\frac{d}{dt}h$ .

$$\frac{d}{dt}h = 0 - 10t = -10t$$

If you've had calculus before, you're probably able to find the derivative of the polynomial  $80 - 5t^2$  on your own. If not, you'll have to take a few things on faith here. First, the derivative of  $80 - 5t^2$  is just the derivative of 80 minus the derivative of  $5t^2$ . Next, the derivative of 80 is the slope of the graph of  $y = 80$  when  $x = 0$ ; that graph is a horizontal line! And finally, since we know from last class that the derivative of  $t^2$  is  $2t^{2-1} = 2t$  it should not surprise you that the derivative of  $5t^2$  is  $10t$ .

We know that the pumpkin hits the pavement 4 seconds after it's dropped, at time  $t = 4$ , so the pumpkin's speed is:

$$h'(4) = (-10)(4) = -40 \text{ m/s} \quad (\text{about } 90 \text{ mph or } 145 \text{ kph}).$$

The value of  $\frac{d}{dt}h$  is negative because the pumpkin's height is decreasing; it is moving downward.

In actuality, the building is a little taller than 80 meters and there is air resistance. You may do a much more thorough study on your own if you wish.