Smoothing a Piecewise Polynomial

For each of the following, find all values of a and b for which $f(x)$ is differentiable.

a) $f(x) = \begin{cases} ax^2 + bx + 6, & x \leq 0; \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 0. \end{cases}$

b) $f(x) = \begin{cases} ax^2 + bx + 6, & x \leq 1; \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 1. \end{cases}$

$f(x) =$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} ax^2 + bx + b = a + b + b$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2x^5 + 3x^4 + 4x^2 + 5x + 6$

$= 2 + 3 + 4 + 5 + 6$

$= 20$

$\Rightarrow a + b + b = 20$

$\Rightarrow a + b = 14$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} =$

$2ax + b$

$= 10x^4 + 12x^3 + 8x + 5$

$3 \quad 5$

$2a + b = 10 + 12 + 8 + 5$

$a + b = 14$

$a = 21$

$b = -7$

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