

Smoothing a Piecewise Polynomial

For each of the following, find all values of a and b for which $f(x)$ is differentiable.

$$\text{a) } f(x) = \begin{cases} ax^2 + bx + 6, & x \leq 0; \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 0. \end{cases}$$

$$\text{b) } f(x) = \begin{cases} ax^2 + bx + 6, & x \leq 1; \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 1. \end{cases}$$

$$f(x) =$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} ax^2 + bx + 6 = a + b + 6$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2x^5 + 3x^4 + 4x^2 + 5x + 6$$

$$= 2 + 3 + 4 + 5 + 6$$

$$= 20$$

$$\Rightarrow a + b + 6 = 20$$

$$\Rightarrow a + b = 14$$

$$2ax + b$$

$$= 10x^4$$

$$+ 2x^3$$

$$+ 8x + 5$$

$$35$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} =$$

$$\boxed{2a + b = 10 + 12 + 8 + 5}$$

$$a + b = 14$$

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$$\begin{aligned} a &= 2 \\ b &= -7 \end{aligned}$$

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