Network Theorems

Current Sources

The current source is described as the dual of the voltage source. Just as a battery provides a fixed voltage to a network, a current source establishes a fixed current in the branch where it is located. Further, the current through a battery is a function of the network to which it is applied, just as the voltage across a current source is a function of the connected network. The term dual is applied to any two elements in which the traits of one variable can be interchanged with the traits of another. This is certainly true for the current and voltage of the two types of sources.

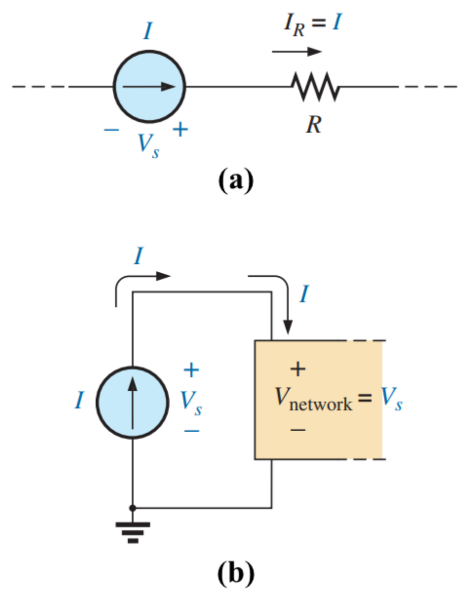


Figure 1: Introducing the current source symbol.

The symbol for a current source appears in Figure 1(a). The arrow indicates the direction in which it is supplying current to the branch where it is located. The result is a current equal to the source current through the series resistor. In Fig. 1(b), we find that the voltage across a current source is determined by the polarity of the voltage drop caused by the current source. For single-source networks, it always has the polarity of Fig. 1(b), but for multisource networks it can have either polarity. In general, therefore,

A current source determines the direction and magnitude of the current in the branch where it is located.

Furthermore,

The magnitude and the polarity of the voltage across a current source are each a function of the network to which the voltage is applied.

Problem 1: Find the source voltage, the voltage V_1, and current I_1 for the circuit in Fig. 2.

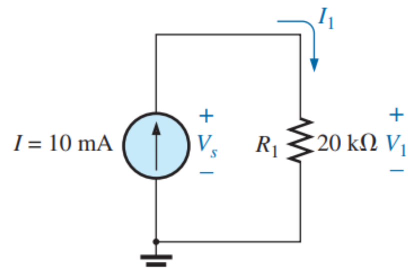


Figure 2.

Solution:

Since the current source establishes the current in the branch in which it is located, the current I_1 must equal I, and

$$I_1 = I = 10\ mA$$

The voltage across R_1 is then determined by Ohm's law:

$$V_1 = I_1 R_1 = (20\ mA)(20\Omega) = 200\ V$$

Since resistor R_1 and the current source are in parallel, the voltage across each must be the same, and

$$V_s = V_1 = 200V$$

with the polarity shown.

Problem 2: Determine the current I_1 and the voltage V_s for the network in Fig. 3.

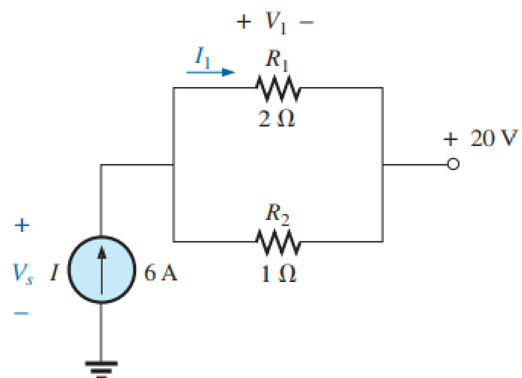


Figure 3.

Solution:

Using the current divider rule:

$$I_1 = \frac{R_T}{R_1} I = \frac{R_2}{R_1 + R_2} I = \frac{1\Omega}{1\Omega + 2\Omega} \times (6A) = 2A$$

The voltage V_1:

$$V_1 = I_1 R_1 = (2A)(2\Omega) = 4V$$

Applying Kirchhoff's voltage rule to determine V_s:

$$V_s = V_1 + 20V = 4V + 20V = 24V$$

Nodal Analysis

Nodal analysis is based on Kirchhoff's Current Law (KCL). It provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

Procedure

1. Identify the total number of nodes within the network.
2. One node is taken as a reference node, and assign the voltage at each node like V_1, V_2, and so on.
3. Develop the Kirchhoff's current law equation for each non-reference node. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.
4. Solve the KCL equations to get the nodal voltages.

Note:

- Leaving current is positive.
- Entering current is negative.
- Node voltage is highest in its' own node equation.

Problem 3: Determine the node voltages for the network in Fig 4. And calculate the current flow through 6 Ω and 12 Ω resistor.

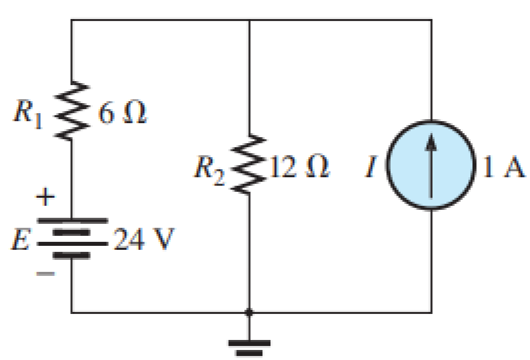
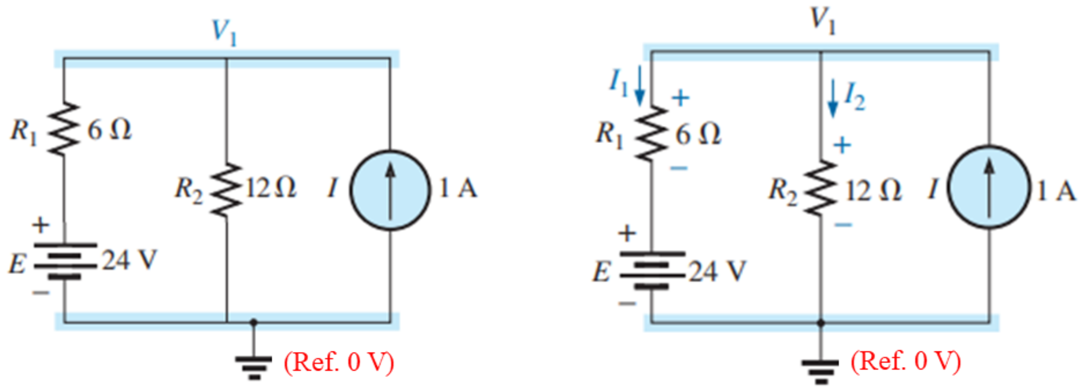


Figure 4.

Solution:



(a) (b)

Figure 5: (a) Network with assigned nodes, (b) Applying KCL to the node V_1.

Applying KCL at node V_1

$$\frac{V_1 - 24}{6} + \frac{V_1 - 0}{12} - 1 = 0$$

$$Or, \qquad \frac{2\,V_1 - 48 + V_1 - 12}{12} = 0$$

$$Or, \qquad 3V_1 - 60 = 0$$

$$Or, V_1 = 20\ V$$

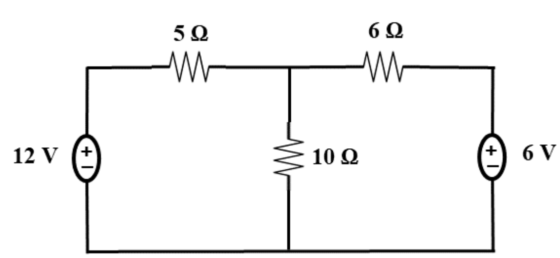
The currents I_1 and I_2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\ V - 24\ V}{6\ \Omega} = \frac{-4\ V}{6\ \Omega} = -0.67\ A$$

And,

$$I_2 = \frac{V_1 - 0}{R_2} = \frac{20\ V - 0\ V}{12\ \Omega} = \frac{20\ V}{12\ \Omega} = 1.67\ A$$

Problem 4: Determine the current through 10 Ω resistor using nodal analysis.



Solution: Try it yourself.

Answer: 0.728 A

Problem 5: Determine the node voltages for the network in Fig 6. And calculate the current flow through 8 Ω, 4 Ω and 10 Ω resistor.

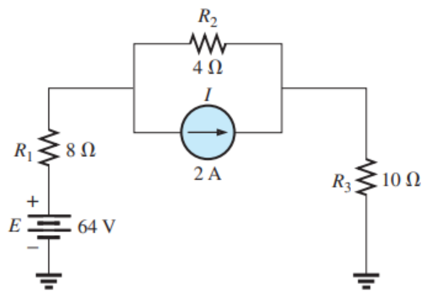
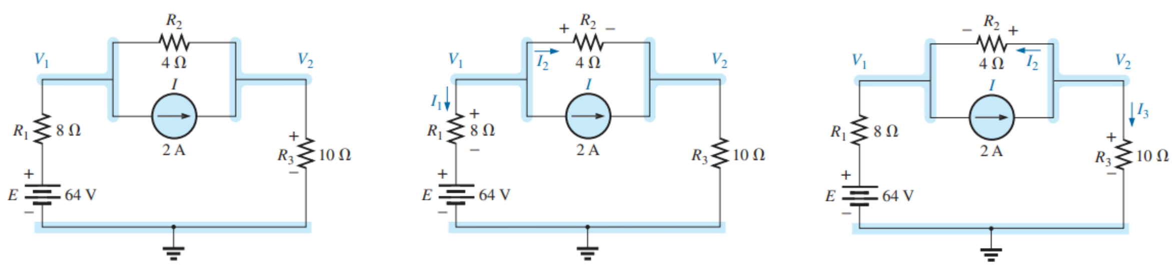


Figure 6.

Solution:



(a) (b) (c)

Apply KCL at node V_1

$$\frac{V_1 - 64}{8} + \frac{V_1 - V_2}{4} + 2 = 0$$

$$Or, \frac{V_1 - 64 + 2V_1 - 2V_2 + 16}{8} = 0$$

$$Or, 3V_1 - 2V_2 - 48 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Apply KCL at node V_2

$$\frac{V_2 - V_1}{4} - 2 + \frac{V_2 - 0}{10} = 0$$

$$Or, \frac{5V_2 - 5V_1 - 40 + 2V_2}{20} = 0$$

$$Or, -5V_1 + 7V_2 - 40 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving equation (1) and (2),

V_1 = 37.82 V

V_2 = 32.73 V

Current flow through 8 Ω resistor

$$I_{R_1} = \frac{64\ V - 37.82\ V}{8\ \Omega} = 3.27\ A$$

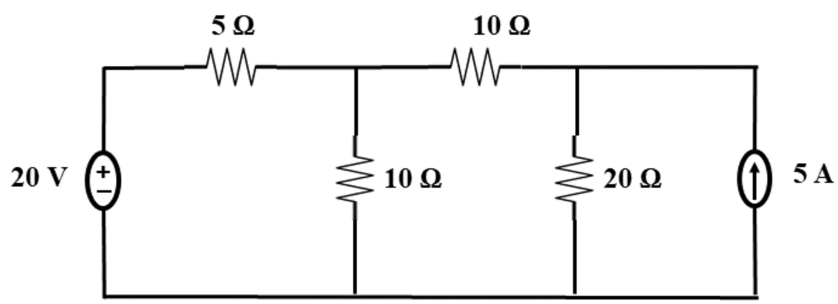
Current flow through 4 Ω resistor

$$I_{R_2} = \frac{37.82\ V - 32.73\ V}{4\ \Omega} = 1.27\ A$$

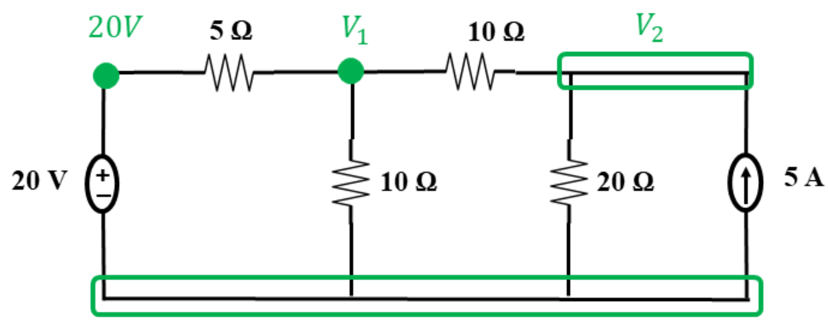
Current flow through 10 Ω resistor

$$I_{R_3} = \frac{32.73\ V - 0\ V}{10\ \Omega} = 3.27\ A$$

Problem 6: Find the node voltages in the following circuit and determine the current flow through 20 Ω resistor.



Solution:



Apply KCL at node V_1

$$\frac{V_1 - 20}{5} + \frac{V_1 - 0}{10} + \frac{V_1 - V_2}{10} = 0$$

$$Or, \frac{2\,V_1 - 40 + V_1 + V_1 - V_2}{10} = 0$$

$$Or, 4\,V_1 - V_2 = 40 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Apply KCL at node V_2

$$\frac{V_2 - V_1}{10} - 5 + \frac{V_2 - 0}{20} = 0$$

$$Or, \frac{2V_2 - 2V_1 - 100 + V_2}{20} = 0$$

$$Or, -2\,V_1 + 3\,V_2 = 100 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving equation (1) and (2),

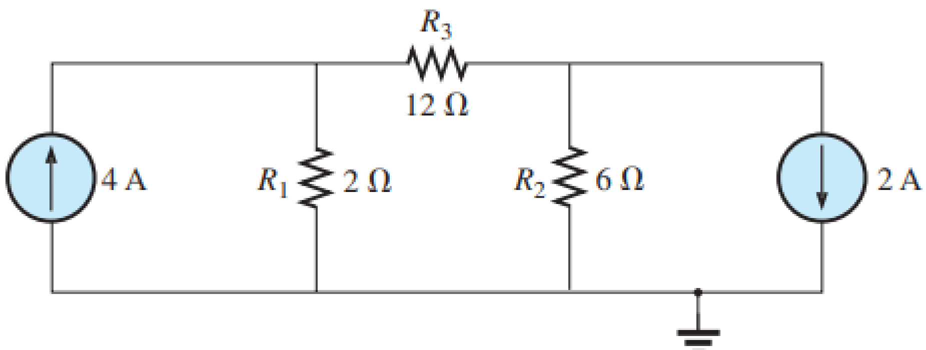
$V_1 = 22$ V

$V_2 = 48$ V

Current flow through 8 Ω resistor

$$I = \frac{(48 - 0)V}{20\ \Omega} = 2.4\ A$$

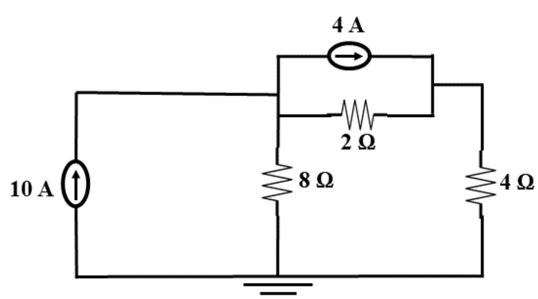
Problem 7: Determine the nodal voltages for the network shown in below.



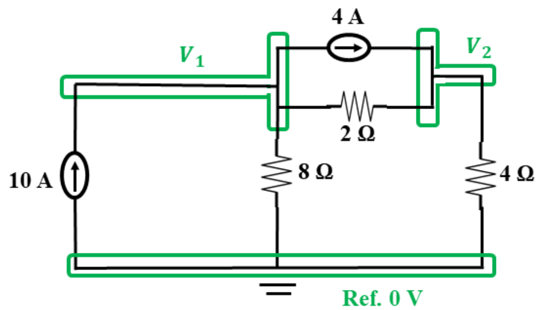
Solution: Try it yourself.

Answer: V_1 = 6 V and V_2 = - 6 V.

Problem 8: Determine the node voltages in the following circuit shown in below.



Solution:



Apply KCL at node V_1

$$-10 + \frac{V_1 - 0}{8} + 4 + \frac{V_1 - V_2}{2} = 0$$

$$Or, \frac{-80 + V_1 + 32 + 4V_1 - 4V_2}{8} = 0$$

$$Or, 5\,V_1 - 4\,V_2 = 48 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Apply KCL at node V_2

$$-4 + \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{4} = 0$$

$$Or, \frac{-16 + 2V_2 - 2V_1 + V_2}{4} = 0$$

$$Or, -2\,V_1 + 3\,V_2 = 16 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving equation (1) and (2),

$V_1 = 29.71$ V

$V_2 = 25.14$ V

Supernode Analysis

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.

Note the following properties of a supernode:

1. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
2. A supernode has no voltage for its own.
3. A supernode requires the application of both KCL and KVL.

Problem 9: Find the currents in the two resistors in the circuit of Figure 7.

Answer: 1.67 mA; 0.33 mA

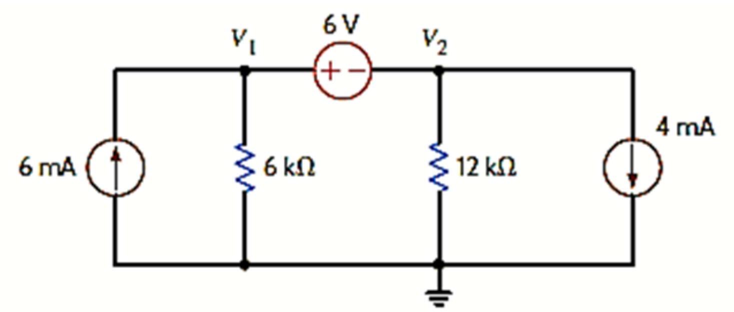


Figure 7.

Problem 10: Determine the current I_0 in the network in Figure 8.

Answer: - 0.4285 mA

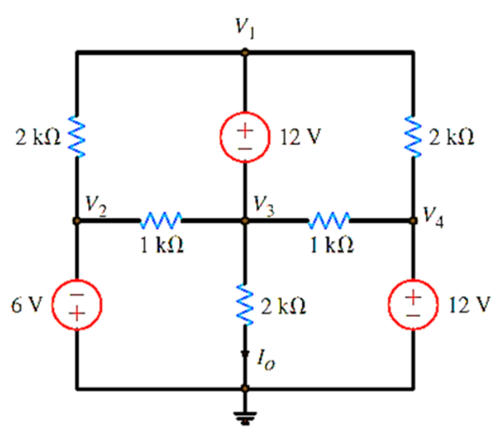


Figure 8.

Problem 11: Use nodal analysis to find I_0 in the network in Figure 9.

Answer: 3.8 mA

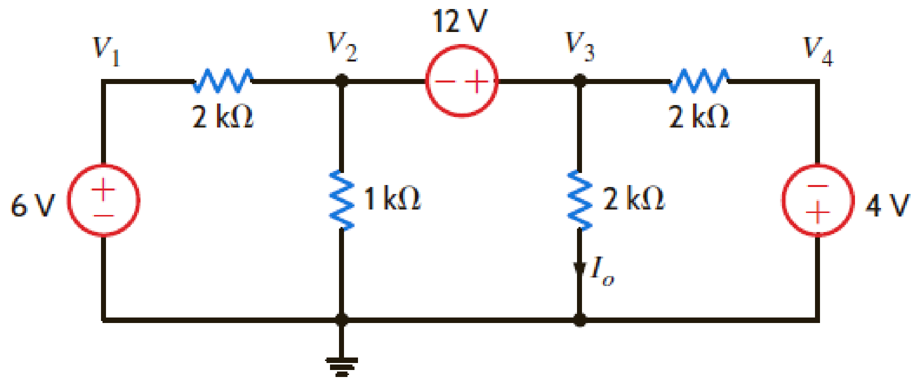


Figure 9.

Problem 12: Find V_0 in Figure 10 using nodal analysis.

Answer: 5.6 V

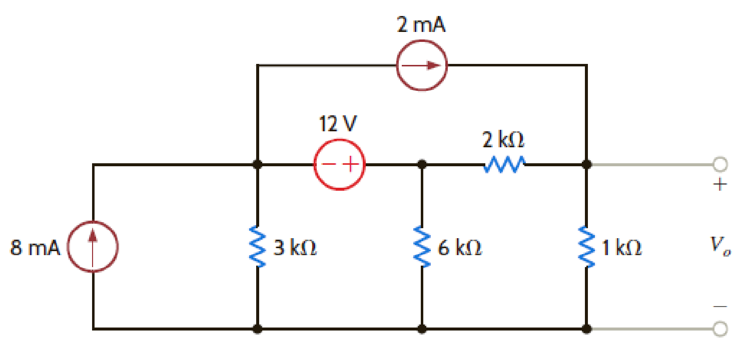


Figure 10.

Problem 13: For the circuit shown in Figure 11, find the node voltages.

Answer: -7.333 V; -5.333 V

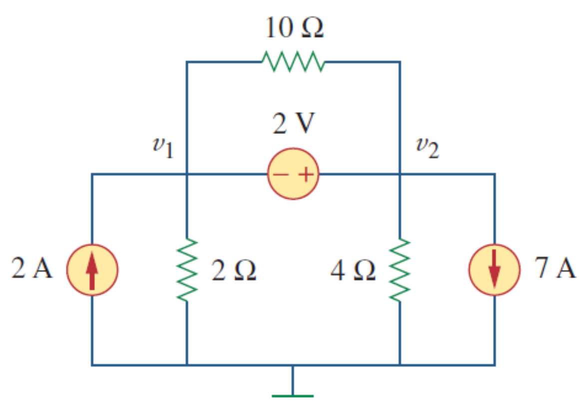


Figure 11.

Problem 14: Find v and I in the circuit of Figure 12.

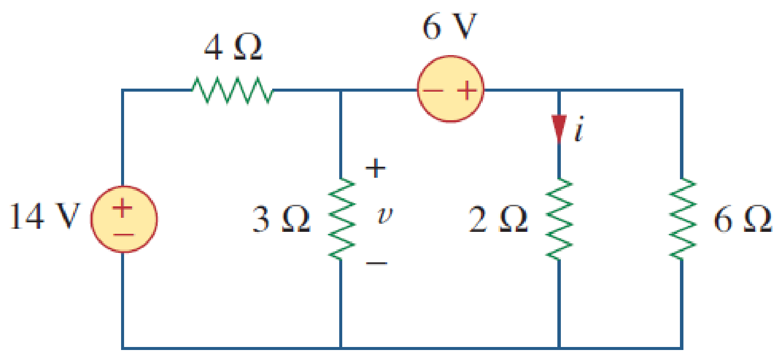


Figure 12.

Thevenin Theorem (Open Circuit Voltage)

Any linear two-terminal circuit containing several voltages, currents, and resistances, can be replaced by just one single voltage source, V_{Th}, and a single resistance, R_{Th}, connected in series with it.

Where, V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

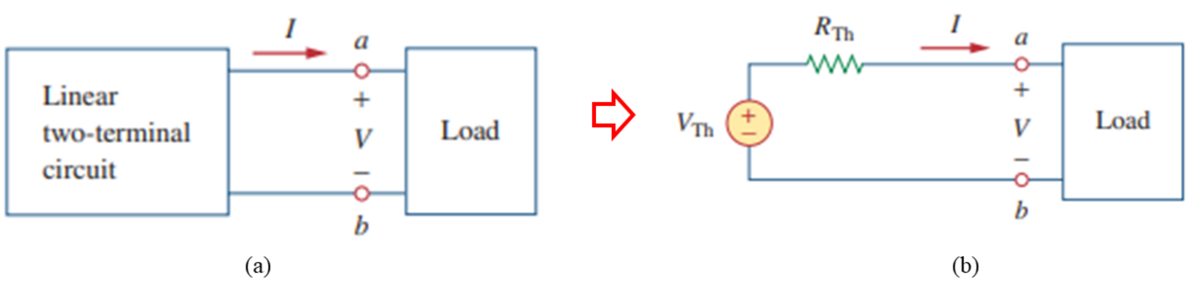


Figure 13:Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

Turned off: It means all the independent sources are replaced by their internal resistances, i.e. we replace every voltage source by 0 V and every current source by 0 A.

Voltage Source → Short Circuited

Current Source → Open Circuited

Question: Explain Thevenin Theorem with appropriate diagram.

Thevenin Theorem Procedure

Preliminary:

1. Remove that portion of the network where the Thévenin equivalent circuit is found. In Fig. 14(a), this requires that the load resistor R_L be temporarily removed from the network.
2. Mark the terminals of the remaining two-terminal network. (Fig. 14 (b))

R_{Th} :

3. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) (Fig. 14(c))

V_{Th} :

4. Calculate V_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (Fig. 14 (d)).

Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor R_L between the terminals of the Thévenin equivalent circuit as shown in Fig. 14(e).

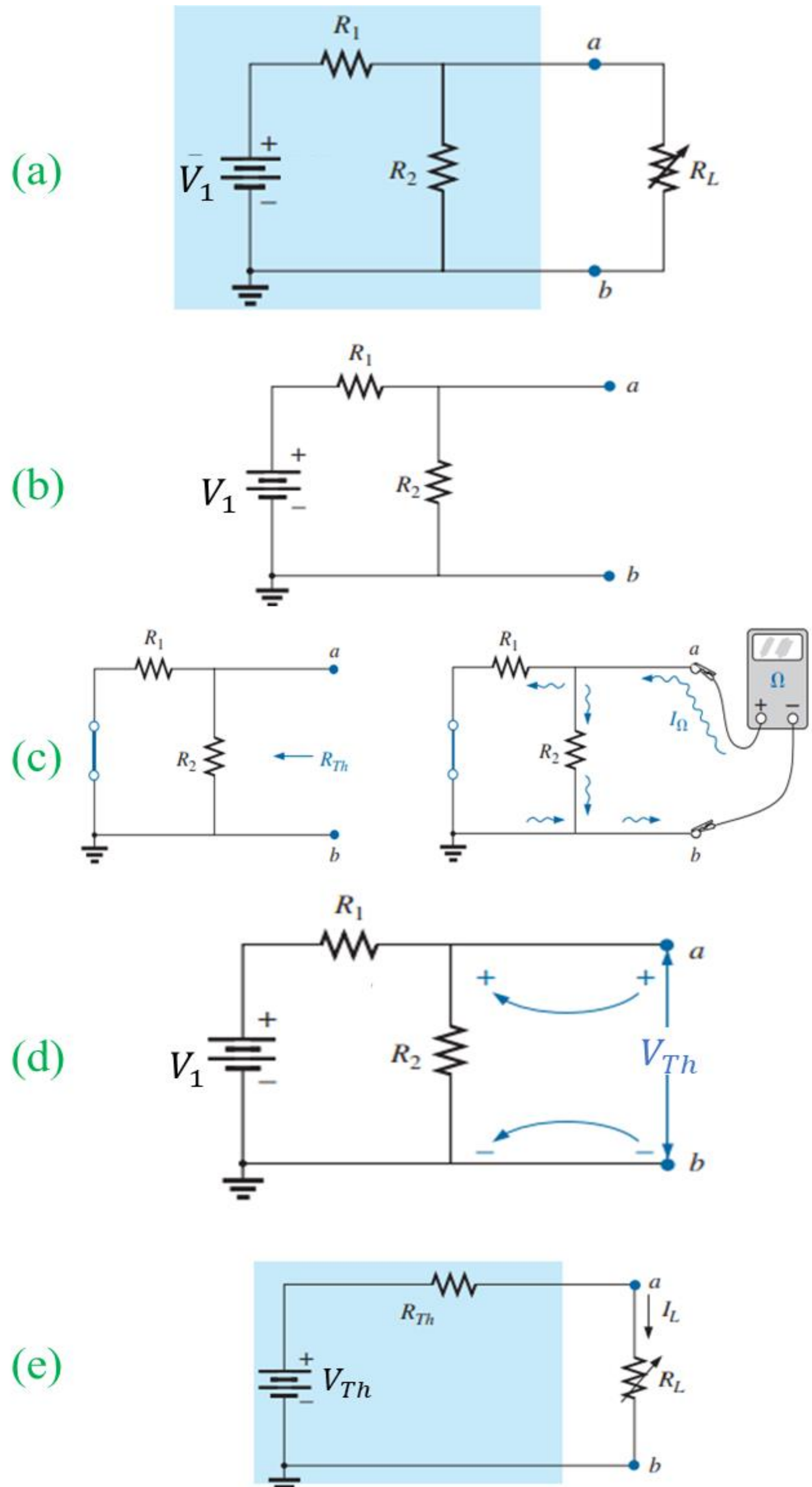
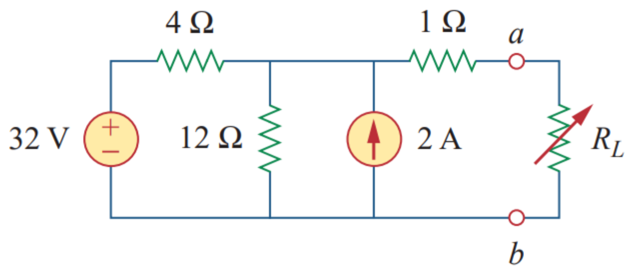


Figure 14: (a) Example circuit, (b) Identifying the terminals of particular importance, (c) Determining R_{Th} for the network, and (d) Determining V_{Th} for the network, and (e) Substituting the Thévenin equivalent circuit for the network external to R_L

Problem 15: Find the Thevenin equivalent circuit of the circuit shown in Figure below, to the left of the terminals Then find the current through $R_L = 6\ \Omega$, $16\ \Omega$, and $36\ \Omega$.



Solution:

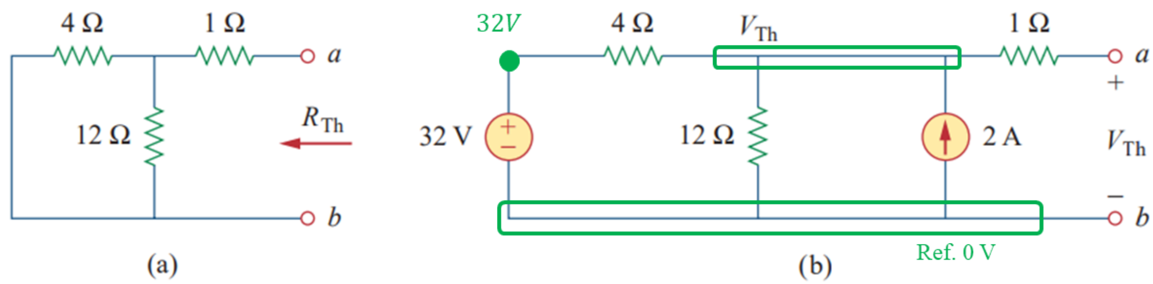


Figure 15: (a) finding R_{Th}, (b) finding V_{Th} .

Find R_{Th} :

We find by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 15(a). Thus,

$$R_{Th} = (4\ \Omega \text{ II } 12\ \Omega) + 1\ \Omega = 4\ \Omega$$

Find V_{Th} :

We ignore 1 Ω the resistor since no current flows through it. At the V_{Th} node, KCL gives

$$\frac{V_{Th} - 32}{4} + \frac{V_{Th} - 0}{12} - 2 = 0$$

$$Or, \frac{3\, V_{Th} - 96 + V_{Th} - 24}{12} = 0$$

$$Or, 4\, V_{Th} = \ 120$$

$$Or, V_{Th} = \ 30\ V$$

The Thevenin equivalent circuit is shown in Fig. 16. The current through R_L is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

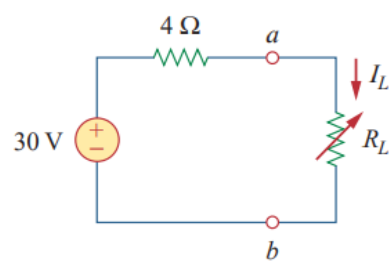


Figure 16: Thevenin Equivalent Circuit.

When $R_L = 6\ \Omega$,

$$I_L = \frac{30}{10} = 3\ A$$

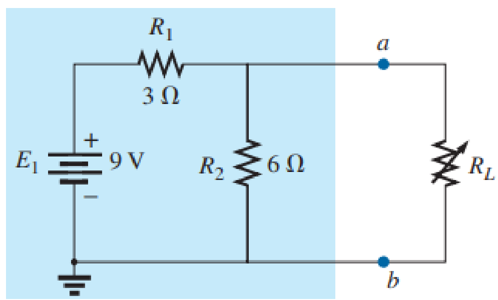
When $R_L = 6\ \Omega$,

$$I_L = \frac{30}{20} = 1.5\ A$$

When $R_L = 6\ \Omega$,

$$I_L = \frac{30}{40} = 0.75\ A$$

Problem 16: Find the Thévenin equivalent circuit for the network in the shaded area of the network in Figure below. Then determine the current through R_L for values of 2 Ω, 10 Ω, and 100 Ω.



Solution:

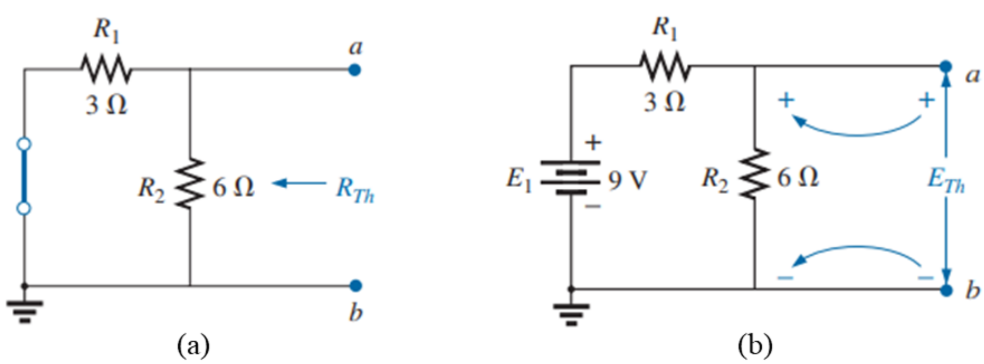


Figure 17: (a) finding R_{Th}, (b) finding V_{Th} .

Find R_{Th}:

Replacing the voltage source E_1 with a short-circuit equivalent yields the network in Fig. 17(a), where

$$R_{Th} = (3\ \Omega \text{ II } 6\ \Omega) = 2\ \Omega$$

Find E_{Th} :

For this case, the open circuit voltage E_{Th} is the same as the voltage drop across the 6 Ω resistor (Figure 17(b)). Applying the voltage divider rule,

$$E_{Th} = \frac{R_2}{R_T} E_1 = \frac{R_2}{R_1 + R_2} E_1 = \frac{6\Omega}{6\Omega + 3\Omega}(9V) = 6V$$

The Thevenin equivalent circuit is shown in Fig. 18. The current through R_L is

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{6}{2 + R_L}$$

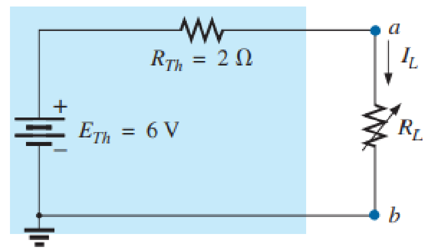


Figure 18: Thevenin Equivalent Circuit.

When R_L = 2 Ω,

$$I_L = \frac{6}{4} = 1.5\ A$$

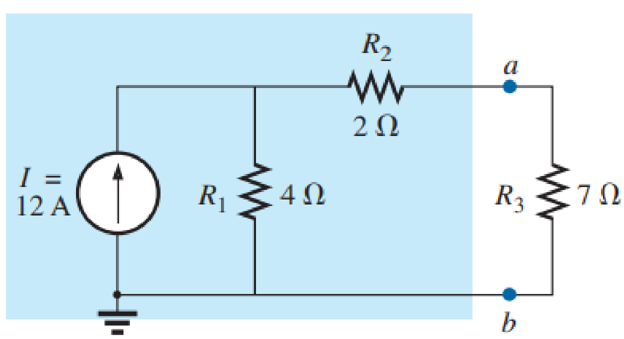
When R_L = 10 Ω,

$$I_L = \frac{6}{12} = 0.5\ A$$

When R_L = 100 Ω,

$$I_L = \frac{6}{102} = 0.06\ A$$

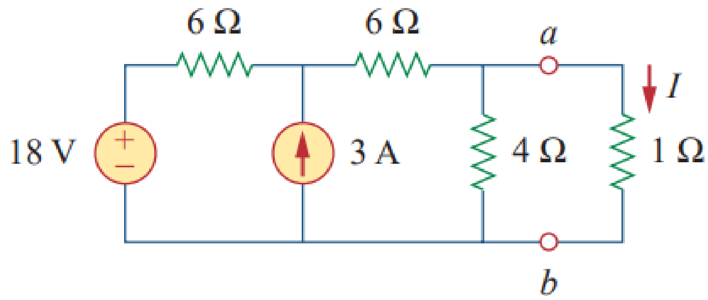
Problem 17: Find the Thévenin equivalent circuit for the network in the shaded area of the network in Figure below. Then determine the current through R_L.



Solution: Try it yourself.

Answer: R_{Th} = 6 Ω, V_{Th} = 48 V, $I_{7\Omega}$ = 3.69 A.

Problem 18: Determine the Thevenin equivalent circuit at terminal a-b. Then find the current flow through the 1 Ω resistor.



Solution:

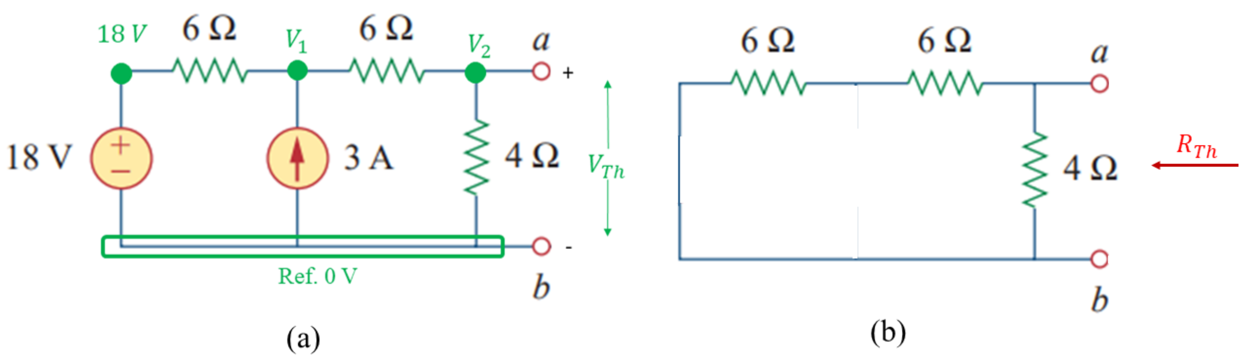


Figure 19: (a) finding R_{Th}, (b) finding V_{Th} .

Find V_{Th}: (Figure 19 (a))

Apply KCL at node V_1

$$\frac{V_1 - 18}{6} - 3 + \frac{V_1 - V_2}{6} = 0$$

$$Or, \frac{V_1 - 18 - 18 + V_1 - V_2}{6} = 0$$

$$Or, 2\,V_1 - V_2 = \ 36 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Apply KCL at node V_2

$$\frac{V_2 - V_1}{6} + \frac{V_2 - 0}{4} = 0$$

$$Or, \frac{2V_2 - 2V_1 + 3\,V_2}{12} = 0$$

$$Or, -2\,V_1 + 5\,V_2 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving equation (1) and (2),

$$V_1 = 22.5 \text{ V}$$

$$V_2 = 9 \text{ V}$$

Therefore, $V_{Th} = V_2 - 0 = 9$ V

Find R_{Th}: (Figure 19 (b))

$$R_{Th} = (6\,\Omega + 6\,\Omega) \text{II } 4\,\Omega = 12\,\Omega \text{ II } 4\,\Omega = 3\,\Omega$$

Thevenin Equivalent Circuit:

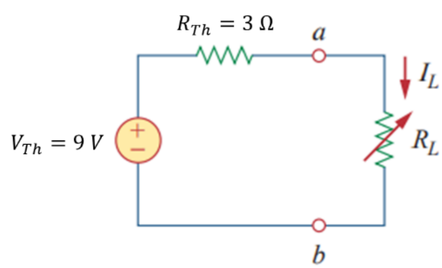
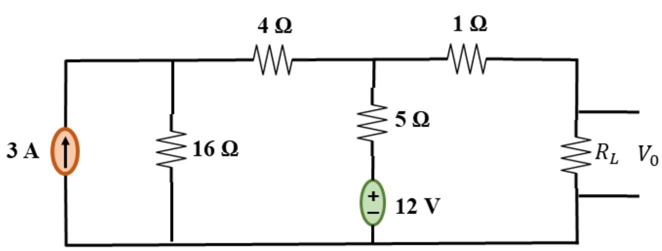


Figure 20: Thevenin equivalent circuit for the network.

The Thevenin equivalent circuit is shown in Fig. 20. The current through R_L =1 **Ω** is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{9}{3+1} = 2.25\,A$$

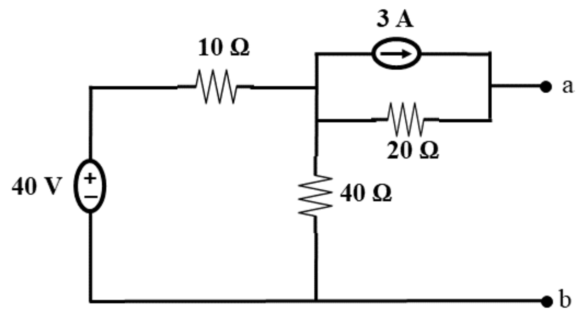
Problem 19: Apply Thevenin theorem to find V_0 in the circuit below when R_L = 10 **Ω**.



Solution: Try it yourself.

Answer: V_0 = 12.8 V.

Problem 20: Design the Thevenin equivalent circuit at terminal a-b of the circuit in the figure below.



Solution:

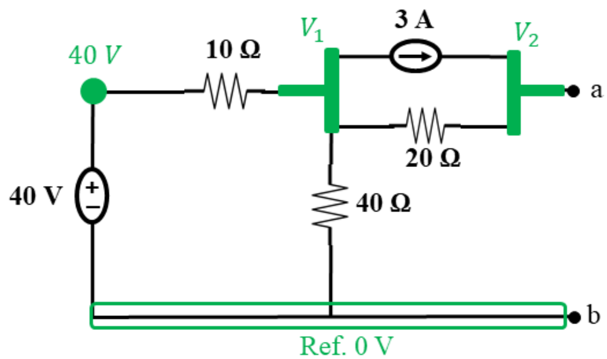


Figure 21: Finding V_{Th} for the network.

Find V_{Th}: (Figure 21)

Apply KCL at node V_1

$$\frac{V_1 - 40}{10} + \frac{V_1 - 0}{40} + \frac{V_1 - V_2}{20} + 3 = 0$$

$$Or, \frac{4\,V_1 - 160 + V_1 + 2\,V_1 - 2V_2 + 120}{40} = 0$$

$$Or, 7\,V_1 - 2\,V_2 = 40 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Apply KCL at node V_2

$$\frac{V_2 - V_1}{20} - 3 = 0$$

$$Or, \frac{V_2 - V_1 - 60}{20} = 0$$

$$Or, V_2 - V_1 = 60 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving equation (1) and (2),

$$V_1 = 32\ V$$

$$V_2 = 92\ V$$

Therefore, $V_{Th} = V_2 - 0 = 92$ V

Find R_{Th}: (Figure 22)

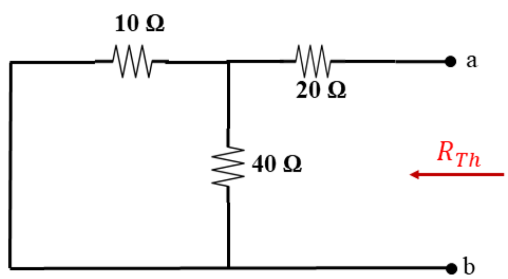


Figure 22: Finding R_{Th} for the network.

$$R_{Th} = (10\ \Omega\ \text{II}\ 40\ \Omega) + 20\Omega = 8\ \Omega + 20\ \Omega = 28\ \Omega$$

Thevenin Equivalent Circuit:

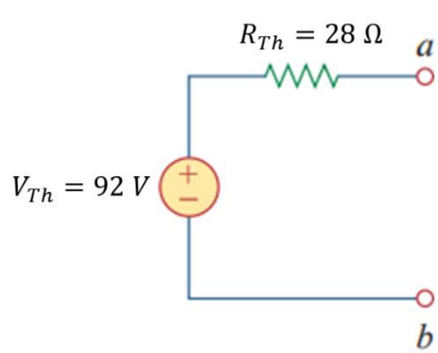
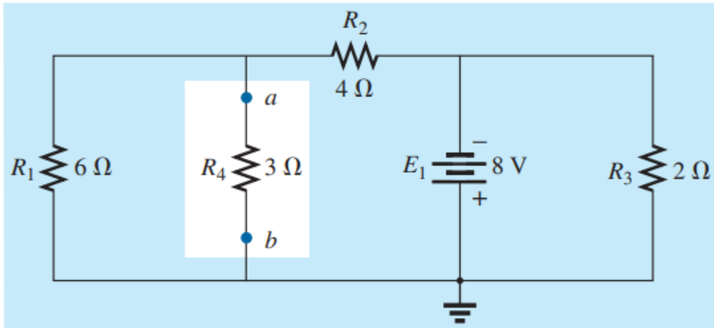


Figure 23: Thevenin Equivalent Circuit for the network.

Problem 21: Find the Thevenin equivalent circuit at terminal a-b for the network shown in Figure below.



Solution:

Find E_{Th}: (Figure 24)

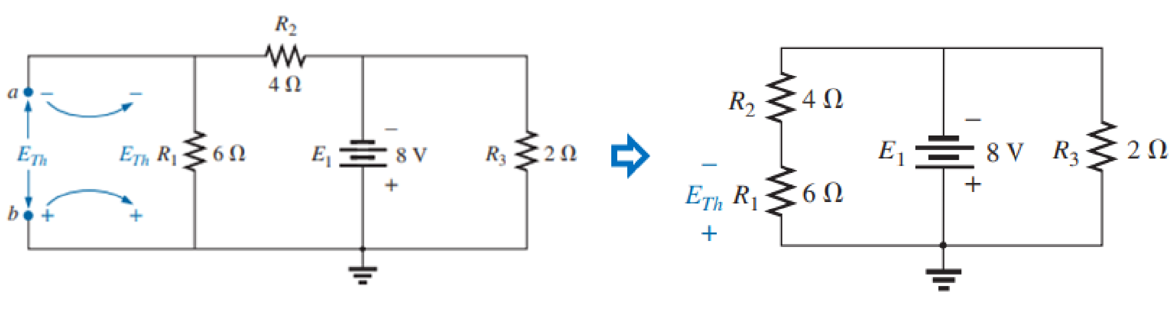


Figure 24: Finding E_{Th} for the network.

The network can be redrawn as shown in Fig. 24. Since the voltage is the same across parallel elements, the voltage across the series resistors R_1 and R_2 is E_1, or 8 V. Applying the voltage divider rule,

$$E_{Th} = \frac{R_1}{R_T} E_1 = \frac{R_1}{R_1 + R_2} E_1 = \frac{6\Omega}{6\Omega + 4\Omega}(8V) = 4.8\ V$$

Find R_{Th}: (Figure 25)

$$R_{Th} = R_1 \text{ II } R_2 = 6\ \Omega \text{ II } 4\ \Omega = 2.4\ \Omega$$

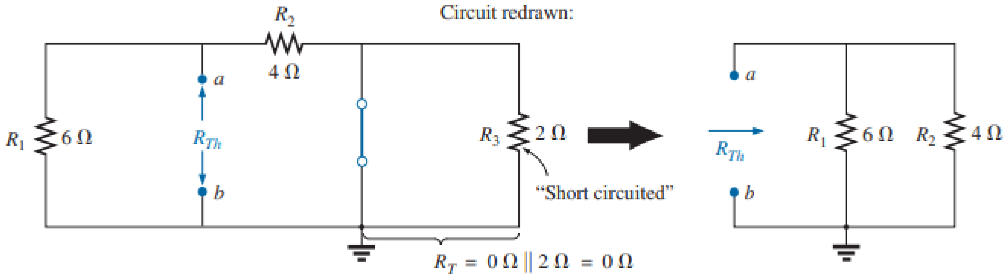


Figure 25: Finding R_{Th} for the network.

Thevenin Equivalent Circuit:

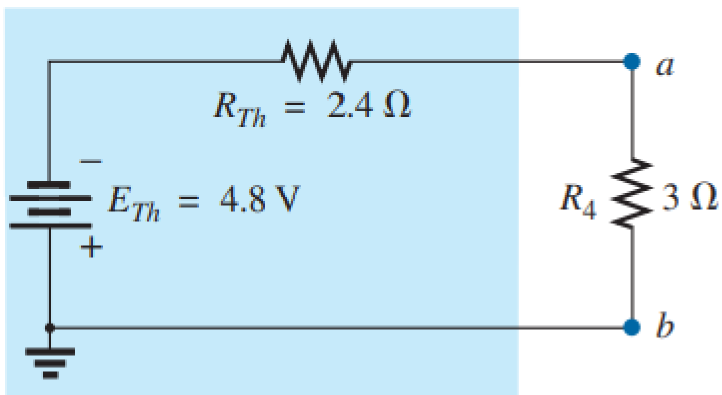
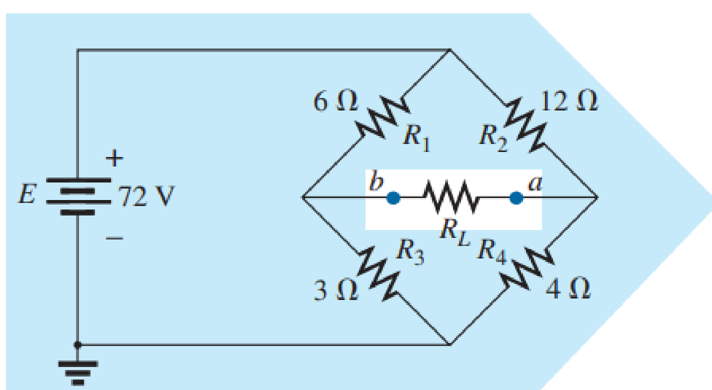


Figure 26: Thevenin equivalent circuit for the network.

Problem 22: Find the Thevenin equivalent circuit at terminal a-b for the network shown in Figure below.



Find E_{Th}: (Figure 27 (b))

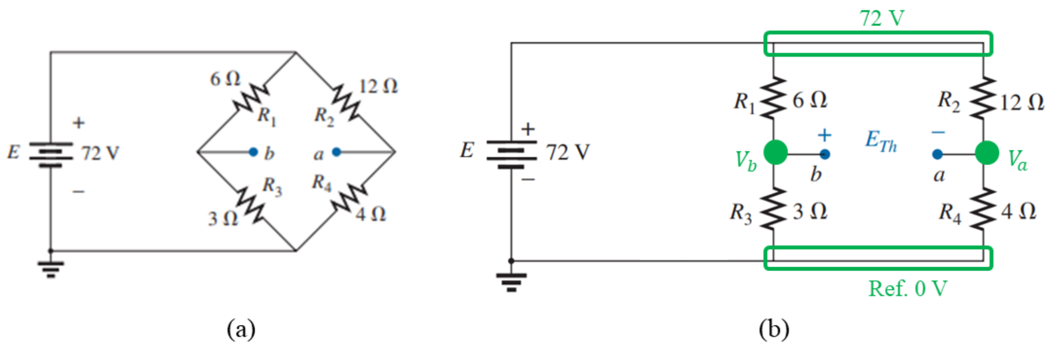


Figure 27: (a) Identifying the terminals of particular interest for the network, (b) Determining E_{Th} for the network.

Apply KCL at node V_b,

$$\frac{V_b - 72}{6} + \frac{V_b - 0}{3} = 0$$

$$Or, \frac{V_b - 72 + 2\,V_b}{6} = 0$$

$$Or, 3\,V_b = 72$$

$$Or, V_b = 24\ V$$

Apply KCL at node V_a,

$$\frac{V_a - 72}{12} + \frac{V_a - 0}{4} = 0$$

$$Or, \frac{V_a - 72 + 3\,V_a}{12} = 0$$

$$Or, 4\,V_a = 72$$

$$Or, V_a = 18\ V$$

Therefore, $E_{Th} = V_b - V_a = 24\text{ V} - 18\text{ V} = 6\text{ V}$

Find R_{Th}: (Figure 28)

$$R_{Th} = (R_1 \text{ II } R_3) + (R_2 \text{ II } R_4) = (6\,\Omega \text{ II } 3\,\Omega) + (4\,\Omega \text{ II } 12\,\Omega) = 2\,\Omega + 3\,\Omega = 5\,\Omega$$

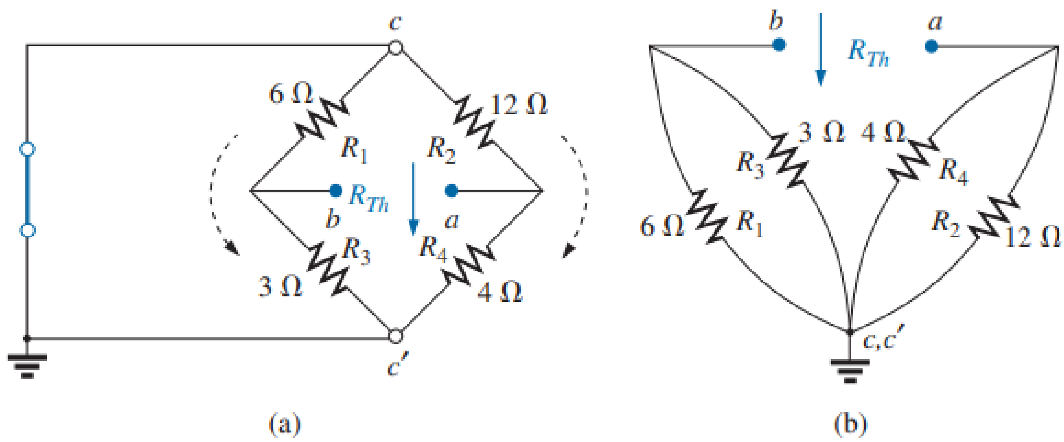


Figure 28: Finding R_{Th} for the network.

Thevenin Equivalent Circuit:

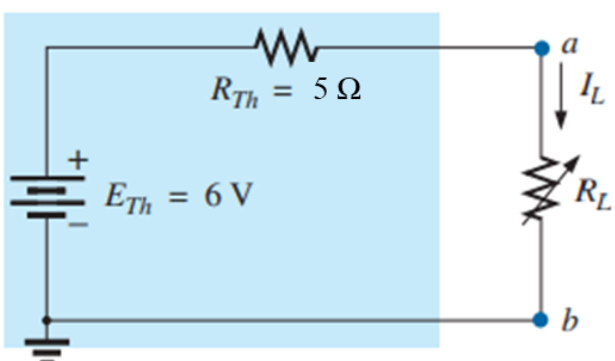


Figure 29: Thevenin equivalent circuit for the network.

Superposition Theorem

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

To apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are turned off. This implies that when removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network. When removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network. This way we obtain a simpler and more manageable circuit. The above statements are illustrated in Figure 30.
2. Dependent sources are left intact because they are controlled by circuit variables.

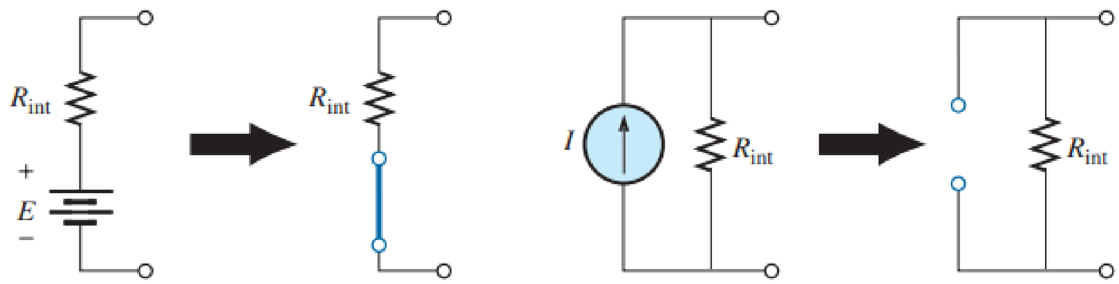
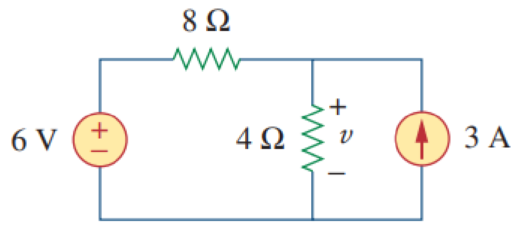


Figure 30: Removing a voltage source and a current source to permit the application of the superposition theorem.

Procedure to apply Superposition Theorem

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using network analysis techniques.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Problem 23: Use the superposition theorem to find v in the circuit of figure below.



Solution:

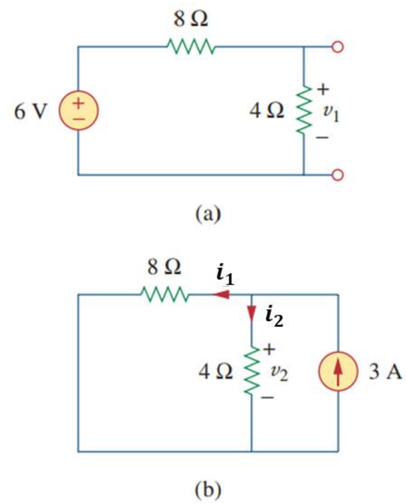


Figure 31: (a) Calculating v_1, (b) calculating v_2,.

Since there are two sources, let

$$v = v_1 + v_2$$

To obtain v_1, we replace the 3A-current source by an open circuit, as shown in Fig. 31 (a). Using voltage divider rule,

$$v_1 = \frac{4}{R_T} V = \frac{4}{4+8}(6) = 2\ V$$

To get v_2, we replace the 6V-voltage source by a short circuit, as shown in Fig. 31(b). Using current divider rule,

$$i_2 = \frac{R_T}{4} I$$

Where, $R_T = (8\ \Omega \text{ II } 4\ \Omega) = 2.67\ \Omega$

$$i_2 = \frac{2.67}{4}(3) = 2\ A$$

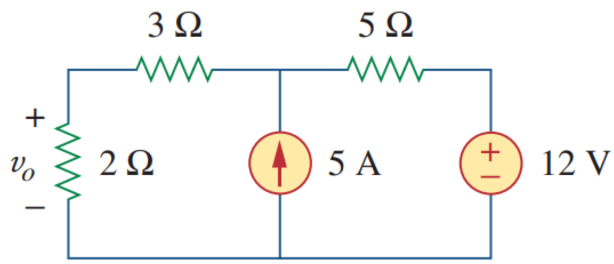
Hence,

$$v_2 = 4\ i_2 = 4 \times 2 = 8V$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10\ V$$

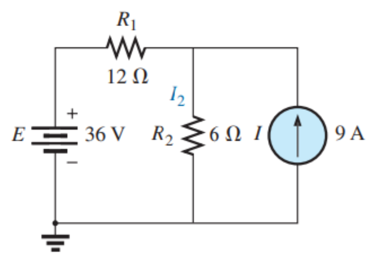
Problem 24: Using the superposition theorem, find v_0 in the circuit of Figure below.



Solution: Try it yourself.

Answer: 7.4 V.

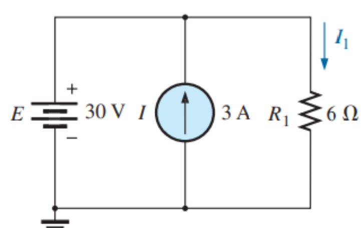
Problem 25: Using the superposition theorem, determine the current through resistor R_2 for the network in Figure below.



Solution: Try it yourself.

Answer: 8 A.

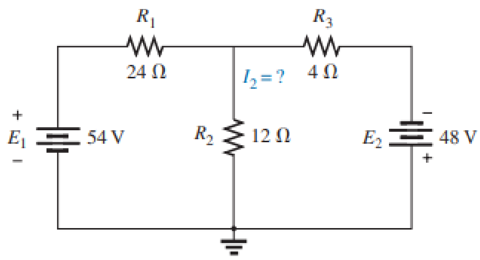
Problem 26: Using the superposition theorem, determine current I_1 for the network in Figure below.



Solution: Try it yourself.

Answer: 5 A.

Problem 27: Using the superposition theorem, determine the current through the 12 Ω resistor in Figure below.



Solution:

Considering the effects of the 54 V source requires replacing the 48 V source by a short-circuit

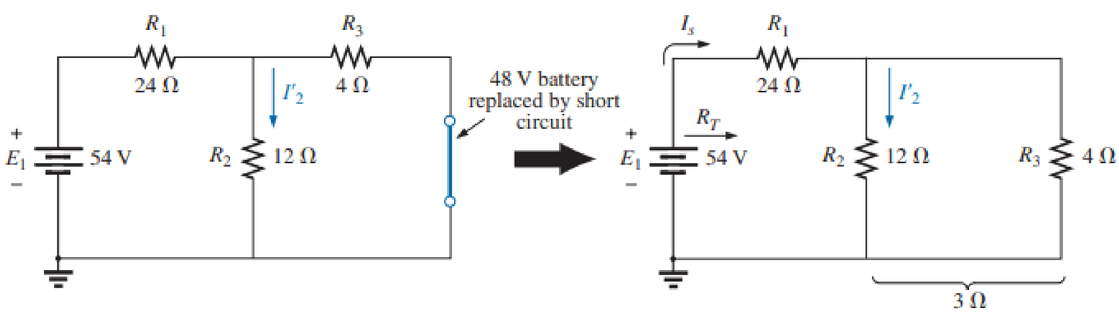


Figure 32: To determine the effect of the 54 V voltage source on current I_2.

equivalent as shown in Fig. 32.

The result is that the 12 Ω and 4 Ω resistors are in parallel.

The total resistance seen by the 54 V source is therefore

$$R_T = R_1 + (R_2 \text{ II } R_3) = 24\ \Omega + (12\ \Omega \text{ II } 4\ \Omega) = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

and the source current is

$$I_s = \frac{E_1}{R_T} = \frac{54\ V}{27\ \Omega} = 2A$$

Using the current divider rule results in the contribution to I_2 due to the 54 V source:

$$I_2' = \frac{R_{23}}{R_2} I_s = \frac{R_3}{R_2 + R_3} I_s = \frac{4\ \Omega}{4\ \Omega + 12\ \Omega}(2A) = 0.5\ A$$

If we now replace the 54 V source by a short-circuit equivalent, the network in Fig. 33 results. The result is a parallel connection for the 12 Ω and 24 Ω resistors.

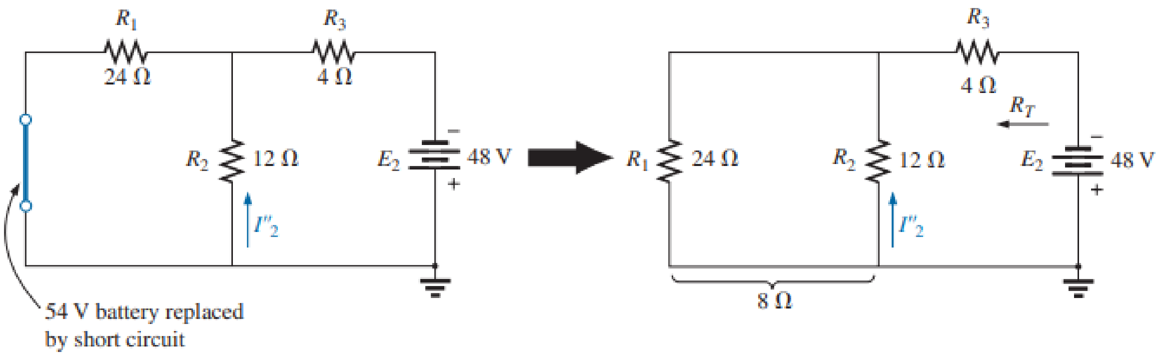


Figure 33: To determine the effect of the 48 V voltage source on current I_2.

Therefore, the total resistance seen by the 48 V source is

$$R_T = R_3 + (R_2 \text{ II } R_1) = 4\ \Omega + (12\ \Omega \text{ II } 24\ \Omega) = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

and the source current is

$$I_s = \frac{E_2}{R_T} = \frac{48\ V}{12\ \Omega} = 4\ A$$

Using the current divider rule results in the contribution to I_2 due to the 48 V source:

$$I_2'' = \frac{R_{12}}{R_2} I_s = \frac{R_1}{R_1 + R_2} I_s = \frac{24\ \Omega}{24\ \Omega + 12\ \Omega}(4\ A) = 2.67\ A$$

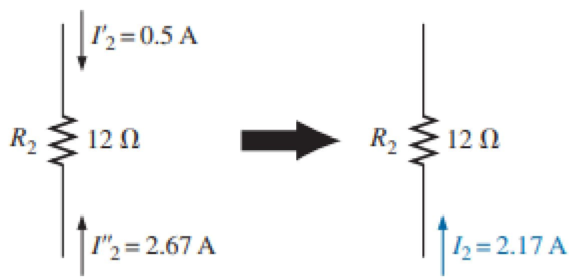
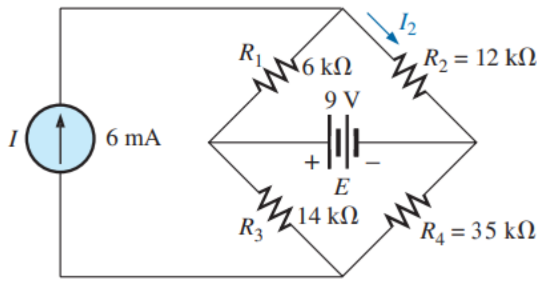


Figure 34: Using the results of Figs. 32 and 33 to determine the current I_2 for the network.

The current I_2 due to each source has a different direction, as shown in Fig. 34. The net current therefore is the difference of the two and the direction of the larger as follows:

$$I_2 = I_2'' - I_2' = 2.67\ A - 0.5\ A = 2.17\ A$$

Problem 28: Using the principle of superposition, find the current I_2 through the 12 kΩ resistor in Figure below.



Solution:

Considering the effect of the 6 mA current source (Figure 35):

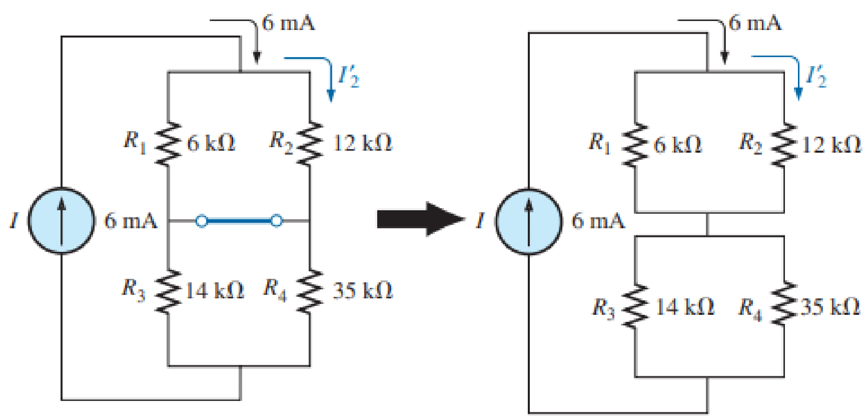


Figure 35: The effect of the current source I on the current I_2.

Current divider rule:

$$I'_2 = \frac{R_{12}}{R_2} I = \frac{R_1}{R_1 + R_2} I = \frac{6\ \text{k}\Omega}{6\ \text{k}\Omega + 12\ \text{k}\Omega}\ (6\ mA) = 2\ mA$$

Considering the effect of the 9 V voltage source (Figure 30):

$$I''_2 = \frac{E}{R_{12}} = \frac{E}{R_1 + R_2} = \frac{9\ \text{V}}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = 0.5\ mA$$

Since I'_2 and I''_2 have the same direction through R_2, the desired current is the sum of the two:

$$I_2 = I''_2 + I'_2 = 2\ mA + 0.5\ mA = 2.5\ mA$$

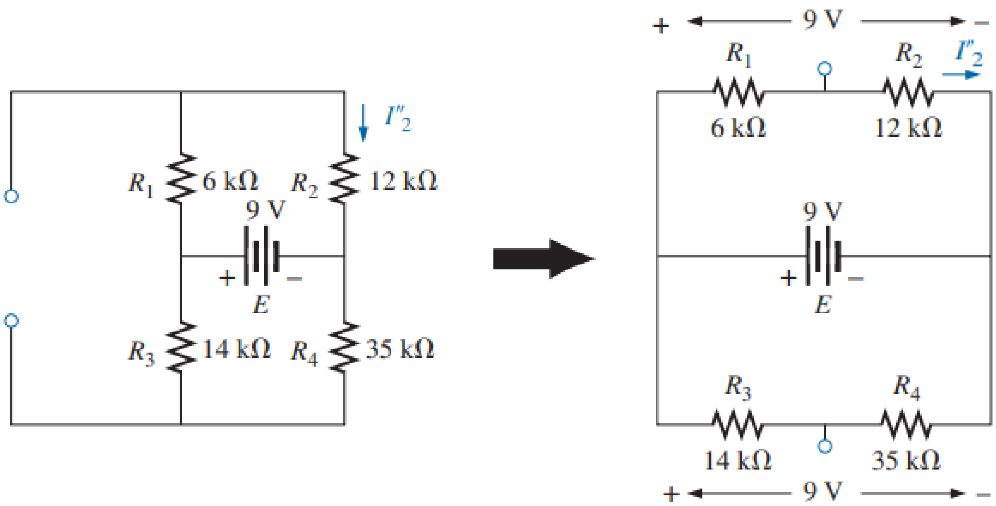
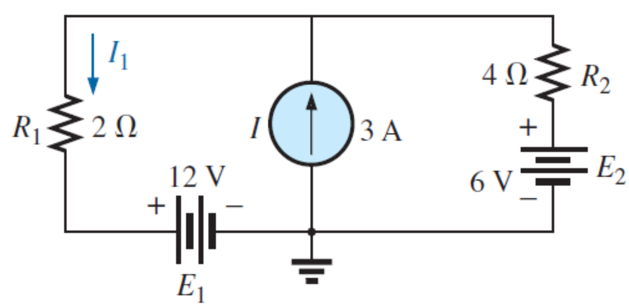


Figure 36:The effect of the voltage source E on the current I_2.

Problem 29: Find the current through the 2Ω resistor of the network in Figure below.



Solution:

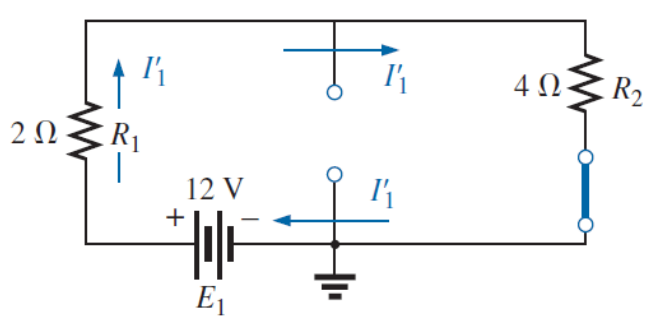


Figure 37: To effect of E_1 on the current I_1.

Considering the effect of the 12 V source (Figure 37):

$$I_1' = \frac{E_1}{R_{12}} = \frac{E_1}{R_1 + R_2} = \frac{12\text{ V}}{2\,\Omega + 4\,\Omega} = 2\,A$$

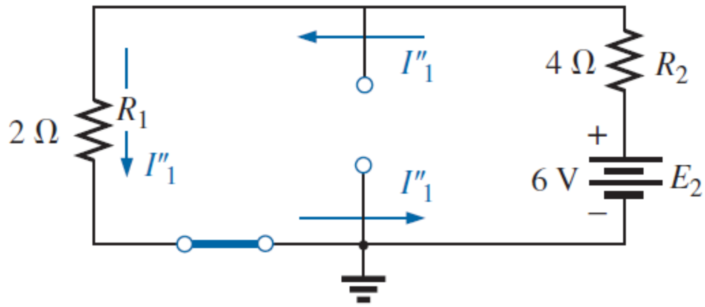


Figure 38: To effect of E_2 on the current I_1.

Considering the effect of the 6 V source (Figure 38):

$$I_1'' = \frac{E_2}{R_{12}} = \frac{E_2}{R_1 + R_2} = \frac{6\text{ V}}{2\,\Omega + 4\,\Omega} = 1\,A$$

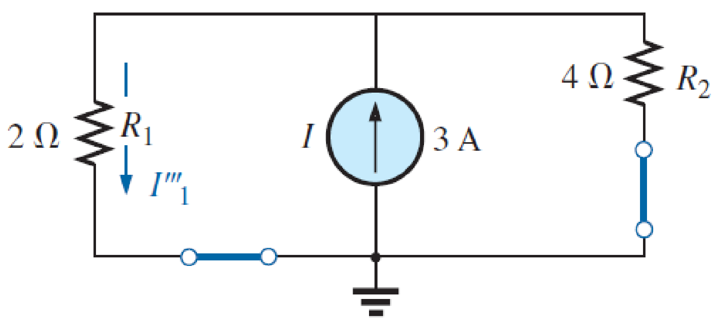


Figure 39: To effect of I on the current I_1.

Considering the effect of the 3 A source (Figure 39):

Applying the current divider rule,

$$I_1''' = \frac{R_{12}}{R_1} I = \frac{R_2}{R_1 + R_2} I = \frac{4\,\Omega}{2\,\Omega + 4\,\Omega}\,(3\,A) = 2\,A$$

The total current through the 2 Ω resistor appears in Figure 40 and

$$I_1 = I_1'' + I_1''' - I_1' = 1\,A + 2\,A - 2A = 1\,A$$

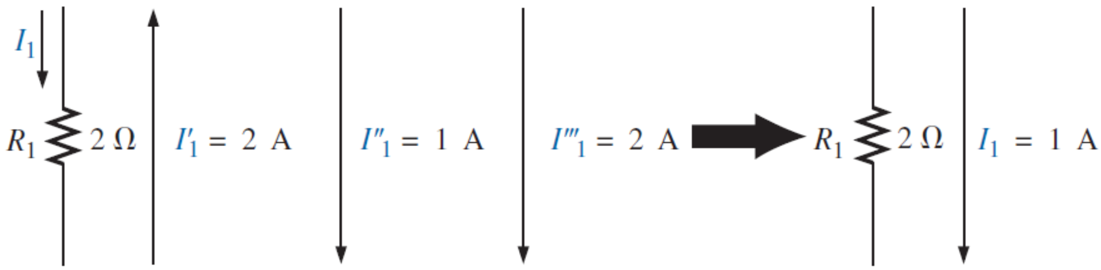


Figure 40: The resultant current I_1.

Mesh Analysis

Mesh Analysis is also known as loop analysis or the mesh-current method.

To understand mesh analysis, we should first explain more about what we mean by a mesh.

A mesh is a loop which does not contain any other loops within it.

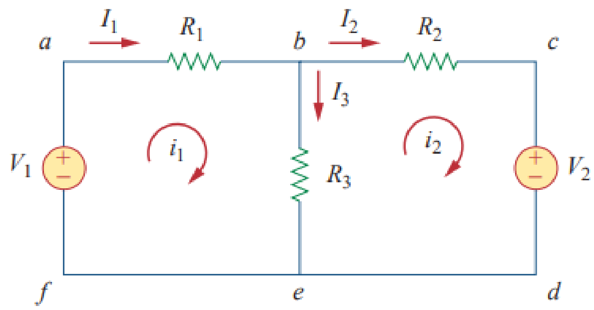
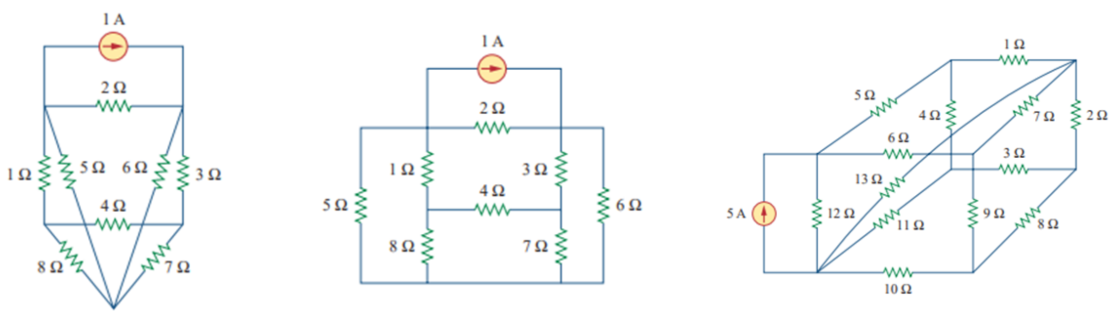


Figure 41: A circuit with two meshes.

In Figure 35, for example, paths *abefa* and *bcdeb* are meshes, but path *abcdefa* is not a mesh.

- Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.
- The current through a mesh is known as mesh current. Here, i_1 and i_2 are the mesh current.
- Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar. A planar circuit is one that can be drawn in plane with no branches crossing one another; otherwise it is nonplanar.
- A circuit may have crossing branches and still be planar if it can redrawn such that it has no crossing branches.



(a) (b) (c)

Figure 42: (a) A planar with crossing branches, (b) the same circuit redrawn with no crossing branches, (c) nonplanar circuit.

Steps to Determine Mesh Currents

1. Make a clear diagram.
2. Assign mesh currents i_1 , i_2, i_3, …. i_n to the N meshes.
3. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages i terms of the mesh currents.
4. Solve the resulting n simultaneous equations to get the mesh currents.

N.B. The direction of the mesh current is arbitrary (clockwise or counterclockwise).

Problem 30: For the circuit in Figure 43, find the branch currents I_1, I_2, and I_3 using mesh analysis.

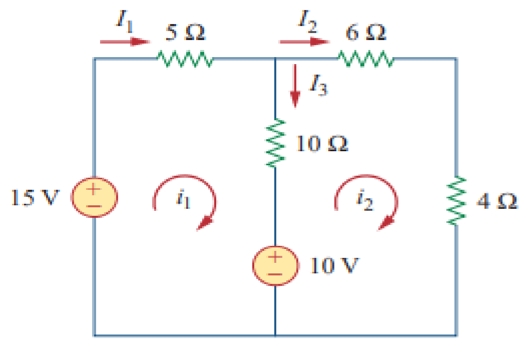


Figure 43.

Solution:

We first obtain the mesh currents using KVL.

For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

Or, $15i_1 - 10i_2 - 5 = 0$

Or, $3i_1 - 2i_2 = 1$ (1)

For mesh 2,

$$6i_2 + 4i_2 - 10 + 10(i_2 - i_1) = 0$$

Or, $-10i_1 + 20i_2 - 10 = 0$

Or, $-i_1 + 2i_2 = 1$ (2)

By solving equation (1) and equation (2),

$i_1 = 1\ A$ and $i_2 = 1\ A$

Thus,

$I_1 = i_1 = 1A$

$I_2 = i_2 = 1A$

$I_3 = i_1 - i_2 = 0\ A$

Problem 31: Calculate the mesh currents i_1 and i_2 of the circuit of Figure 44.

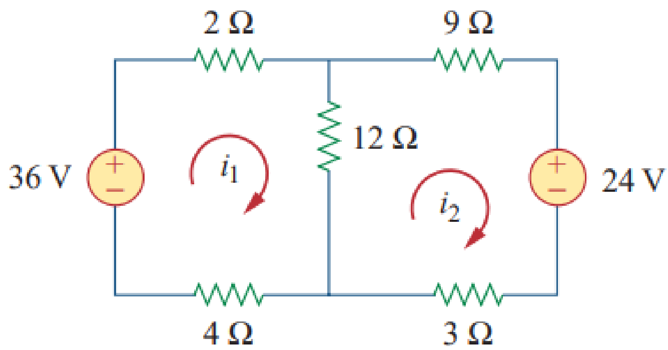


Figure 44.

Solution: Try it yourself.

Answer: $i_1 = 2\ A, i_2 = 0\ A$.

Problem 32: Use mesh analysis to find the current I_0 in the circuit of Figure 45.

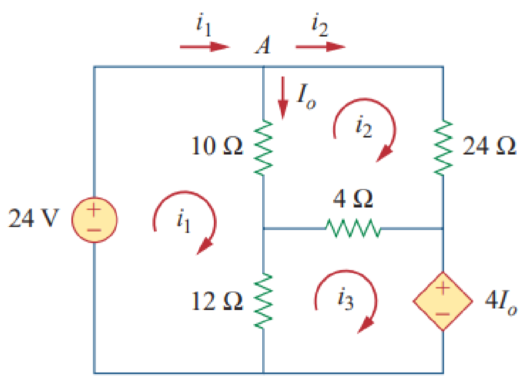


Figure 45.

Solution:

We apply KVL to the three meshes in turn.

For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

Or, $$11i_1 - 5i_2 - 6i_3 = 12 \quad (1)$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

Or, $$-5i_1 + 19i_2 - 2i_3 = 0 \quad (2)$$

For mesh 3,

$$4I_0 + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $I_0 = i_1 - i_2 = 0$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

Or, $$-i_1 - i_2 + 2i_3 = 0 \qquad (3)$$

By solving equation (1), (2) and (3),

$$i_1 = 2.25\ A$$

$$i_2 = 0.75\ A$$

$$i_3 = 1.5\ A$$

Thus, $I_0 = i_1 - i_2 = 1.5\ A$.

Problem 33: Using mesh analysis, find I_0 in the circuit of Figure 46.

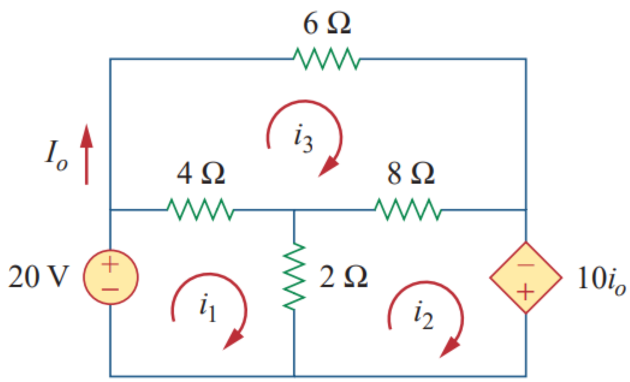


Figure 46.

Solution: Try it yourself.

Answer: -5 A.

Problem 34: Use mesh analysis to determine the three mesh currents in the circuit of Figure 47.

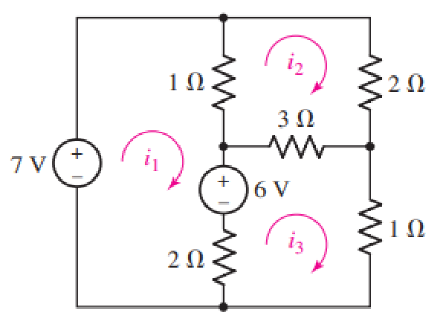


Figure 47.

Solution: Try it yourself.

Answer: $i_1 = 3\ A, i_2 = 2\ A, and\ i_3 = 3\ A.$

Problem 35: Determine the current i_1 in the circuit of Figure 48.

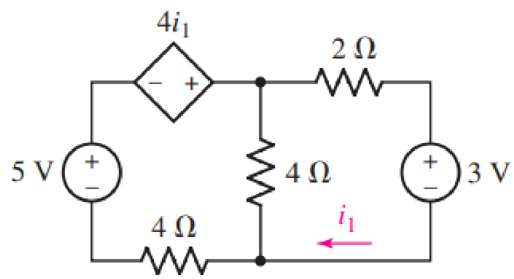


Figure 48.

Solution: Try it yourself.

Answer: $i_i = -250\ mA, i_2 = 375\ mA.$

Mesh Analysis with Current Source

Case 1: When a current source exists only in one mesh: Consider the circuit in Figure 49, for example.

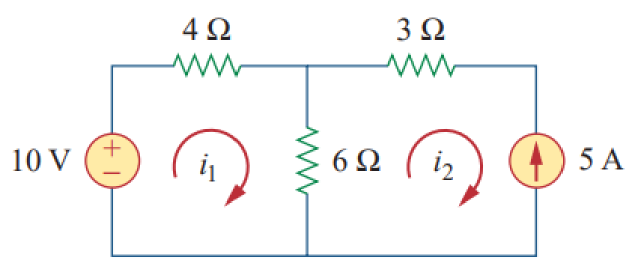


Figure 49.

We set $i_2 = -5A$ and write a mesh equation for the other mesh in the usual way; that is

$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

Or, $$-10 + 4i_1 + 6(i_1 + 5) = 0$$

Or, $$-10 + 10i_1 + 30 = 0$$

Or, $$i_1 = -2A$$

Case 2: When a current source exists between two meshes. Consider the circuit in Figure 50 (a), for example. We create a superrmesh by excluding the current source and any elements connected in series with it, as shown in Figure 50(b). Thus,

A supermesh results when two meshes have a (dependent or independent) current source in common.

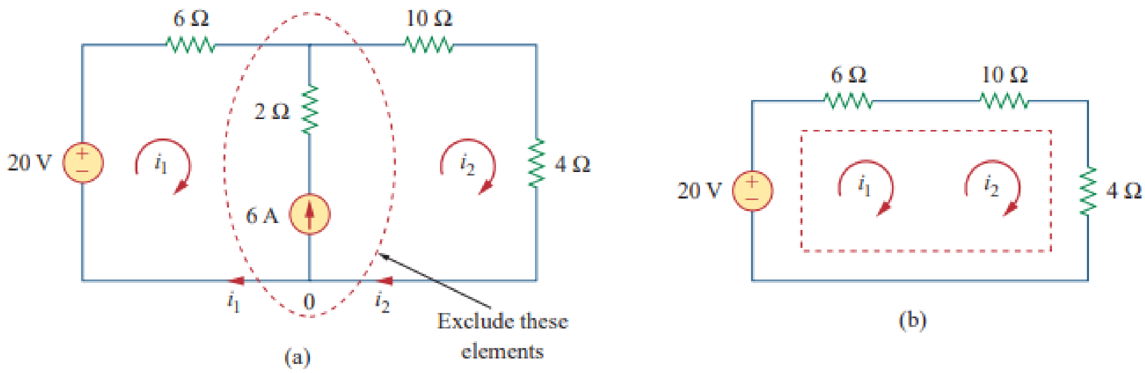


Figure 50: (a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current.

Applying KVL to the supermesh in Figure 50(b) gives

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20 \qquad (1)$$

We apply KCL to a node in the branch where the two meshes intersect. Applying KCL to node 0 in Figure 50(a) gives

$$i_2 = i_1 + 6 \qquad (2)$$

Solving equation (1) and (2), we get

$$i_1 = -3.2\ A, i_2 = 2.8\ A$$

Note the following properties of a supermesh:

1. The current source in the supermesh provides the constraint equation necessary to solve for the mesh currents.
2. A supermesh has no current of its own.
3. A supermesh requires the application of both KVL and KCL.

Problem 36: In the circuit shown in Figure 51 determine the current through 2Ω resistor.

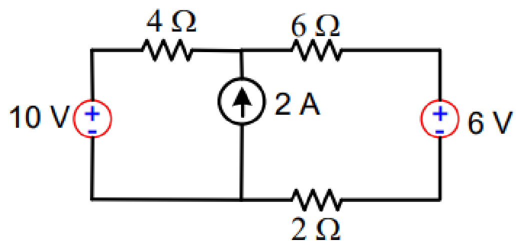


Figure 51.

Soluution:

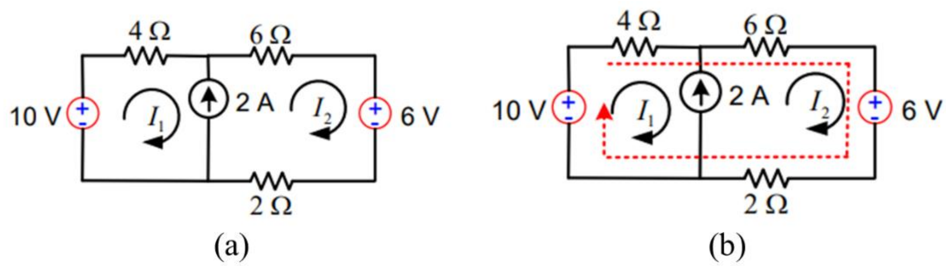


Figure 52.

I_1 and I_2 are the two meshes in which 2 A current source is common to I_1 and I_2 meshes which is as shown in Figure 52 (a).

Draw a supermesh by combining two meshes I_1 and I_2 as shown in Figure 52(b).

Apply KVL to the supermesh as

$$4I_1 + 6I_2 + 6 + 2I_2 - 10 = 0$$

Or, $$4I_1 + 8I_2 = 4 \qquad (1)$$

Apply KCL at the upper node of current source

$$I_2 - I_1 = 2 \qquad (2)$$

Solving the above equations

$$I_1 = -1A \text{ and } I_2 = 1\ A$$

Current through 2Ω resistor is $I_2 = 1\ A$.

Problem 37: In the circuit shown in Figure 53 determine all the loop currents.

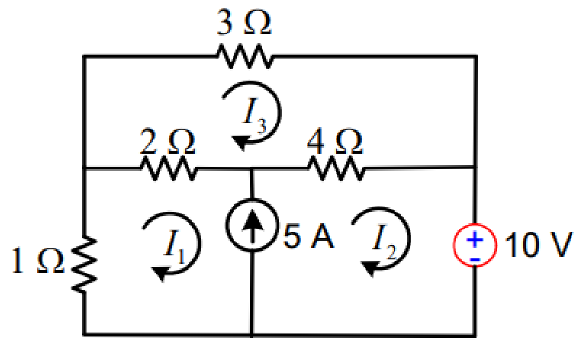


Figure 53.

Solution:

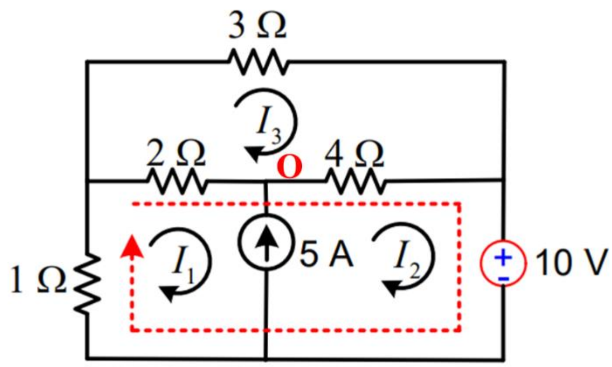


Figure 54.

Apply for Supermesh

$$1i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0$$

Or, $$3i_1 + 4i_2 - 6i_3 = -10 \qquad (1)$$

Apply KVL for i_3

$$3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$$

Or, $$-2i_1 - 4i_2 + 9i_3 = 0 \qquad (2)$$

Apply KCL at node O

$$i_2 - i_1 = 5 \qquad (3)$$

Solving the above equations

$$i_1 = 5.556\ A, i_2 = -0.556\ A, i_3 = -1.4814\ A$$

Problem 38: Use mesh analysis to determine all the mesh currents in Figure 55.

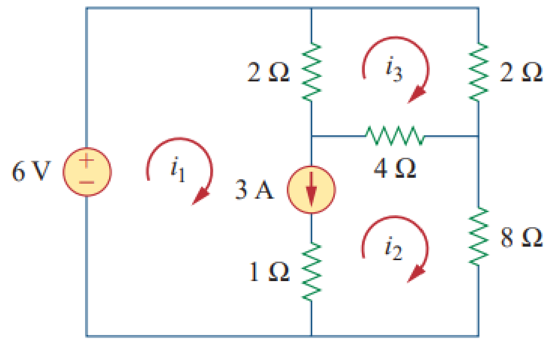


Figure 55.

Solution:

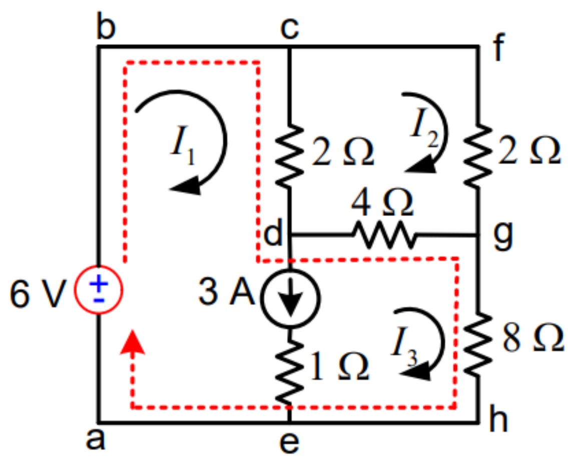


Figure 56.

Apply KVL for I_2 is

$$2I_2 + 4(I_2 - I_3) + 2(I_2 - I_1) = 0$$

Or, $$-2I_1 + 8I_2 - 4I_3 = 0 \qquad (1)$$

Apply KVL for supermesh

$$2(I_1 - I_2) + 4(I_3 - I_2) + 8I_3 - 6 = 0$$

Or, $$2I_1 - 6I_2 + 12I_3 = 6 \qquad (2)$$

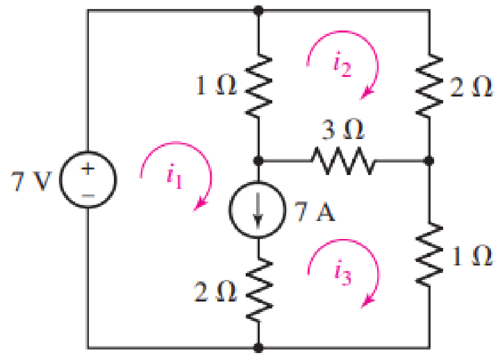
Apply KCL at node e

$$I_1 - I_3 = 3 \qquad (3)$$

By solving equations (1), (2), and (3)

$$I_1 = 3.4736\ A, I_2 = 1.105\ A, I_3 = 0.4736\ A$$

Problem 39: Determine the three mesh currents in Figure below.



Solution: Try it yourself.

Answer: $i_1 = 9\ A, i_2 = 2.5\ A, and\ i_3 = 2A.$

Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals where the independent sources are turned off.

Thus, the circuit in Figure 57(a) can be replaced by the one in Figure 57(b).

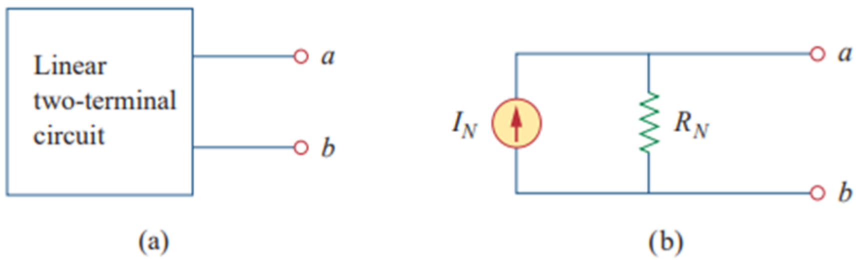


Figure 57: (a) Original circuit, (b) Norton's equivalent circuit.

We find R_N in the same way we find R_{Th}. The Thevenin and Norton resistances are equal i.e.

$$R_N = R_{Th}$$

Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals.

To find the Norton current I_N, we determine the short-circuit current flowing from terminal a to b in both circuits in Figure 57. Thus,

$$I_N = i_{sc}$$

shown in Figure 58. Dependent and independent sources are treated the same way as in Thevenin's theorem.

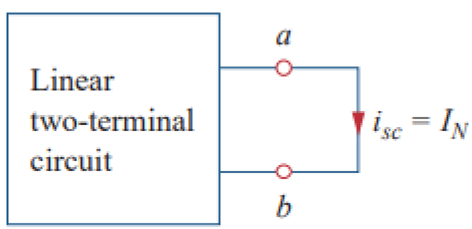


Figure 58: Finding Norton current, I_N.

Observe the close relationship between Norton's and Thevenin's theorems: $R_N = R_{Th}$, and

$$I_N = \frac{V_{Th}}{R_{Th}}$$

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

Since V_{Th}, I_N, and R_{Th} are related to determine the Thevenin or Norton equivalent circuit requires that we find:

The open-circuit voltage v_{oc} across terminals a and b.(i.e. $V_{Th} = v_{oc}$)

The short-circuit current i_{sc} at terminals a and b.(i.e. $I_N = i_{sc}$)

The equivalent or input resistance R_{in} at terminals a and b when all independent sources are turned off. ($R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$)

Problem 40: Find the Norton equivalent circuit of the circuit in Figure 59 at terminals a-b.

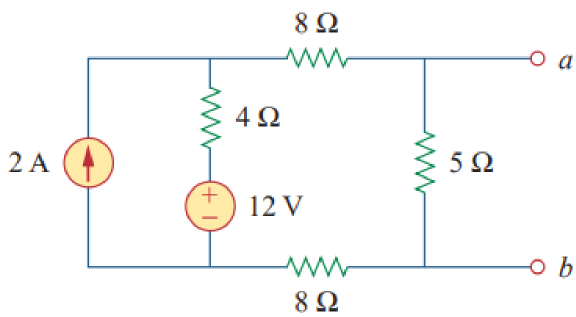


Figure 59.

Solution:

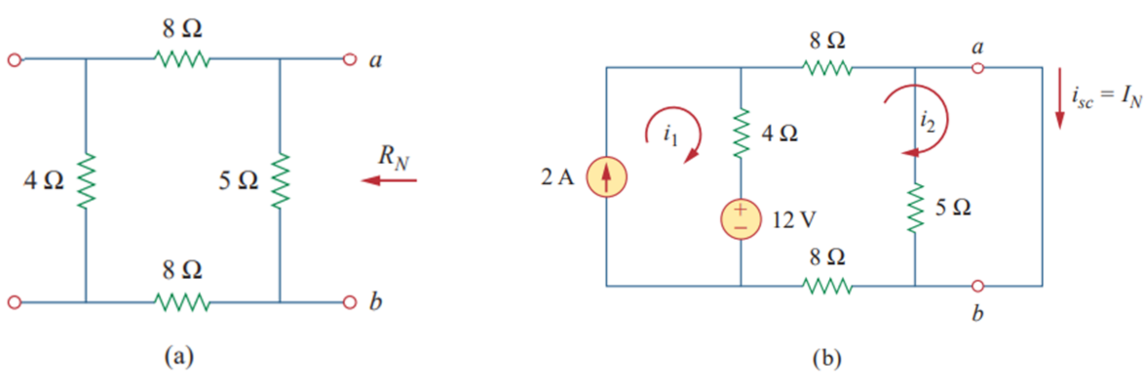


Figure 60: (a) Determine R_N, (b) Determine I_N.

$\boldsymbol{R_N}$:

From Figure 60(a),

$$R_N = 5 \,||(8 + 4 + 8) = 5||20 = 4\Omega$$

$\boldsymbol{I_N}$:

From Figure 60(b)

Apply mesh analysis, we obtain

From i_1,

$$i_1 = 2\,A$$

From i_2,

$$4\,(i_2 - i_1) + 8i_2 + 8i_2 - 12 = 0$$

Or, $20\,i_2 - 4i_1 - 12 = 0$

Or, $20\,i_2 - 4 \times 2 - 12 = 0$

Or, $20\,i_2 = 20$

Or, $i_2 = 1\,A$

Hence, we obtain, , $i_2 = 1\,A = i_{sc} = I_N$

Norton Equivalent Circuit:

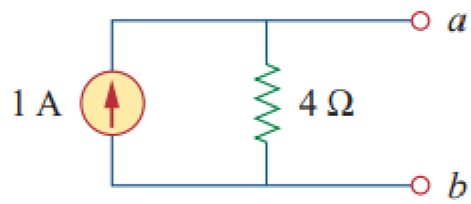


Figure 61: Norton equivalent of the circuit.

Problem 41: Find the Norton equivalent circuit for the circuit in Figure 62, at terminals a-b.

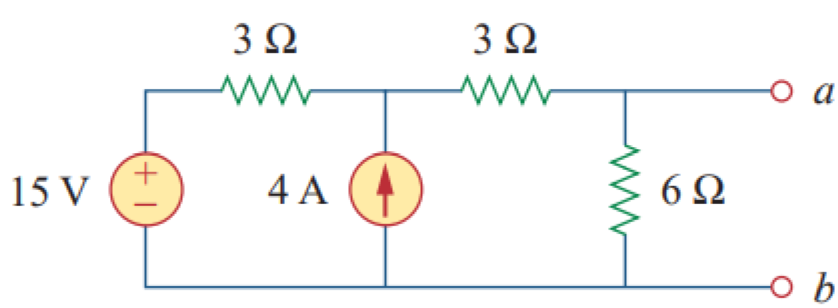


Figure 62.

Solution: Try it yourself.

Answer: $R_N = 3\Omega, I_N = 4.5\,A$.

Problem 42: Find the Norton equivalent circuit for the network in the shaded area of Figure 63.

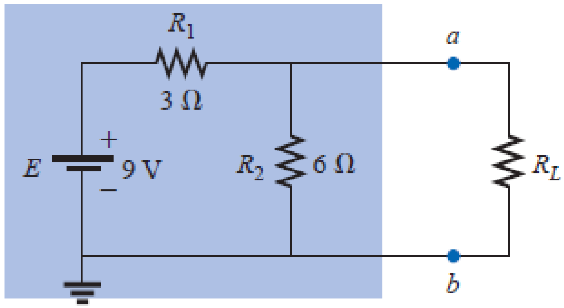


Figure 63.

Solution: Try it yourself.

Answer: $R_N = 2\Omega, I_N = 3\ A.$

Problem 43: Find the Norton equivalent circuit for the network external to the 9Ω resistor in Figure 64.

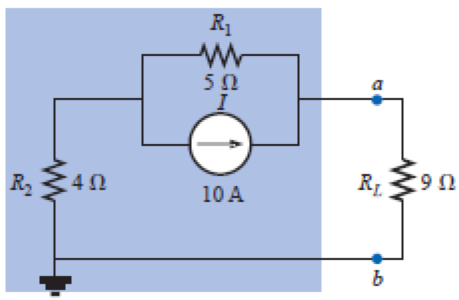


Figure 65.

Solution:

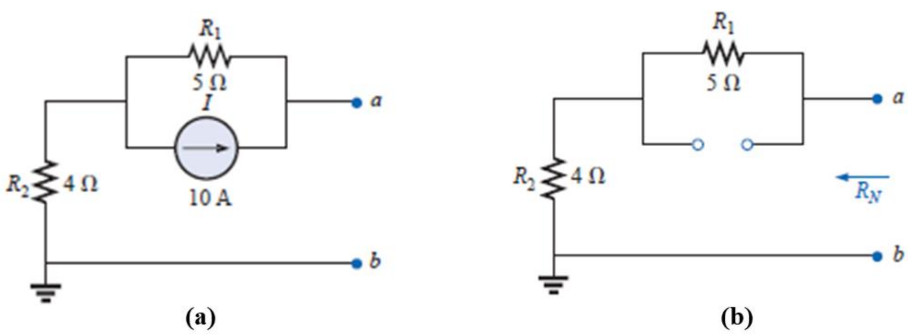


Figure 64: (a) Identifying the terminals of particular interest for the network, (b) Determining R_N.

R_N:

$$R_N = R_1 + R_2 = 5 + 4 = 9\Omega$$

I_N :

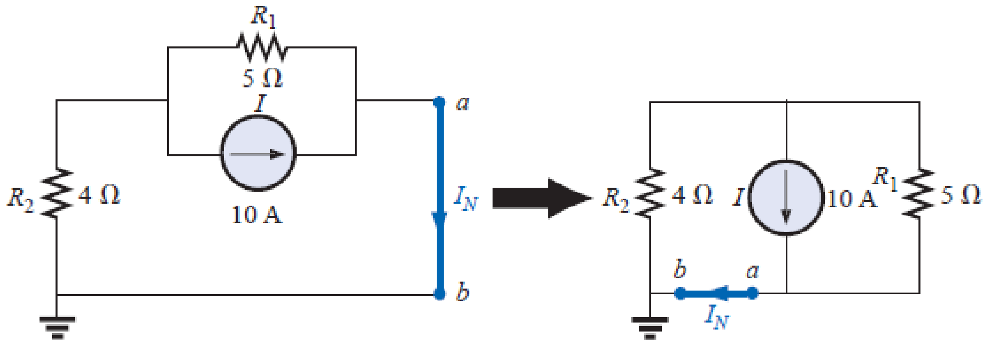


Figure 66: Determining I_N for the network.

$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{5 \times 10}{5 + 4} = 5.556\ A$$

Norton Equivalent Circuit:

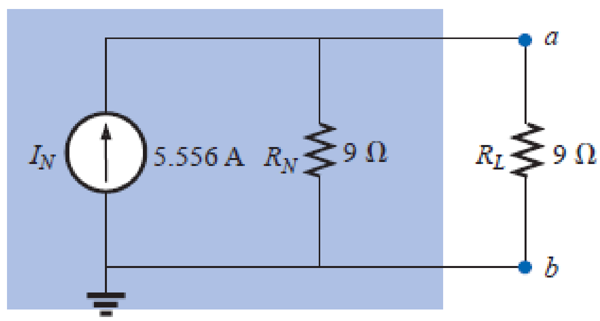


Figure 67: Substituting the Norton equivalent circuit
for the network external to the resistor R_L.

Problem 44: Find the Norton equivalent circuit for the portion of the network to the left of a-b in Figure 68.

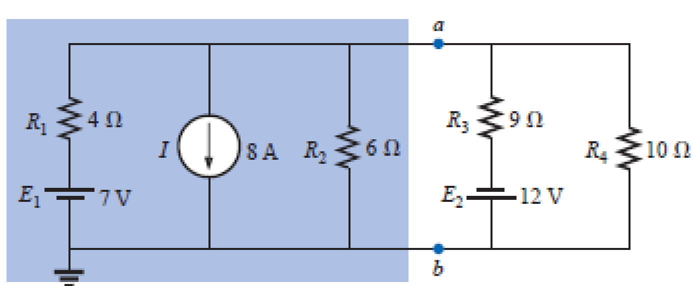


Figure 68.

Solution:

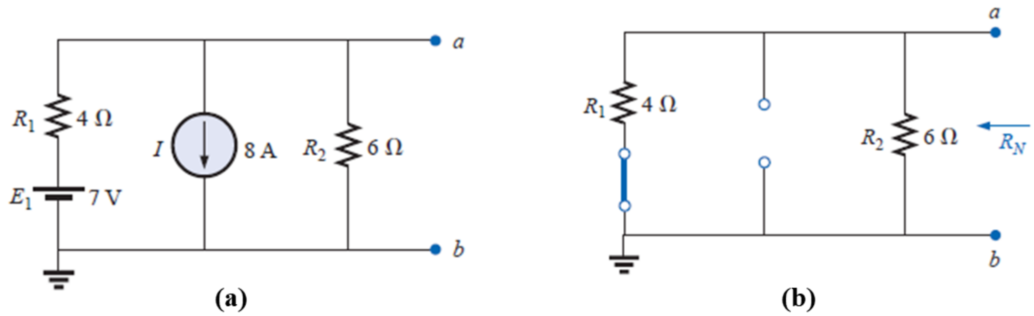


Figure 69: (a) Identifying the particular interest for the network. (b) Determining R_N of the network.

R_N: From Figure 69(b)

$$R_N = 4||6 = 2.4\ \Omega$$

I_N:

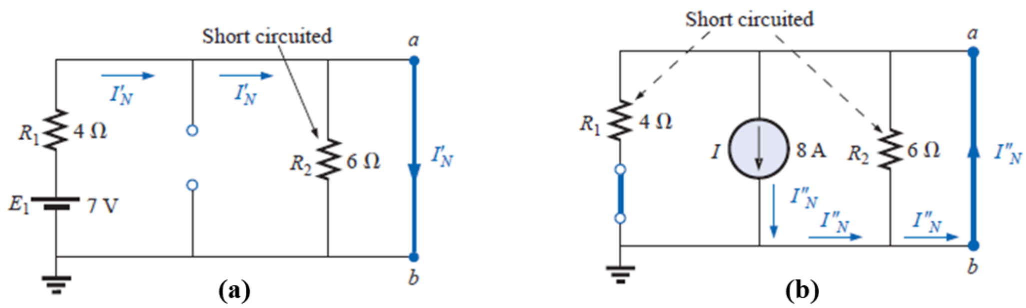


Figure 70.

Using superposition theorem,

For the 7-V battery, determining the contribution to I_N from the voltage source E_1.(Figure 70(a))

$$I_N' = \frac{E_1}{R_1} = \frac{7}{4} = 1.75\ A$$

For the 8-A source (Figure 70(b)),

$$I_N'' = I = 8A$$

The result is

$$I_N = I_N'' - I_N' = 8 - 1.75 = 6.25\ A$$

Norton Equivalent Circuit:

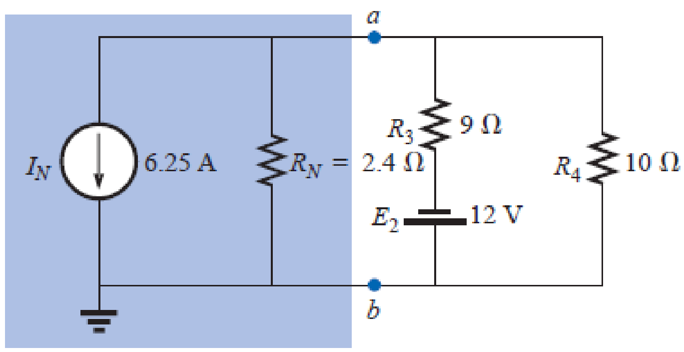


Figure 71: Substituting the Norton equivalent circuit for the network to the left of the terminals a-b.

Problem 45: Determine the Thevenin and Norton equivalents of the circuit of Figure 72.

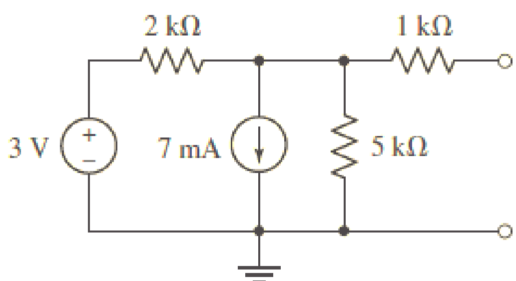


Figure 72.

Solution: Try it yourself.

Answer: -7.857 V,, -3.2235 mA, 2429 kΩ.

Problem 46: Find the Norton equivalent circuit to the left of terminals A-B for the network shown in Figure 73. Connect the Norton equivalent circuit to the load and find the current in the 50**Ω** resistor.

Solution: Try it yourself.

Answer: $R_N = 55\Omega$, $I_N = 10.7$ A, $I_{50\Omega} = 5.6$ A.

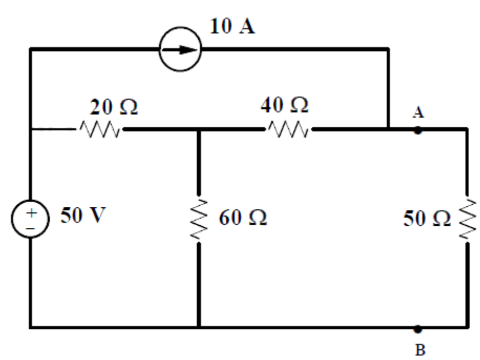


Figure 73.