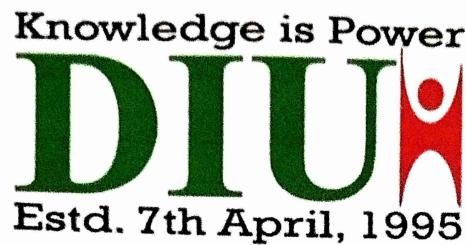


# Dhaka International University



## DEPARTMENT OF CSE

### ASSIGNMENT

**COURSE NAME** : Linear Algebra and Coordinate Geometry

**COURSE CODE** : 0541-101

**ASSIGNMENT NO.** : 01

**ASSIGNMENT NAME** : Assignment on Coordinate Geometry

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## Assignment on Coordinate Geometry

This assignment has explored the foundational principles and practical application of coordinate geometry. The following topic will be discussed on this assignment:

SL	Table of Content (Topic)
01	Verify Invariant Theorem with some example.
02	Show the method how we can eliminate the first-degree term from a second degree equation
03	Explain pair of straight lines, plane, conic and space.
04	$x^2 + y^2 + 3xy - 6x + 8y - 12 = 0$ Evaluate a) Intersection point b) Angle between the lines c) Bisectors equation d) Test the equation represents pair of straight line or not

Topic 1: Verify Invariant Theorem with some example.

Soln:

Theorem: If transformation equation of  $ax^2 + 2hxy + by^2 = 0$  is  $a_1x^2 + 2h_1xy + b_1y^2 = 0$  when the direction of the axes turned through an angle  $\theta$ , without changing of the origin show that  $a+b$  and  $ab-h^2$  are invariant. i.e.  $a+b = a_1+b_1$  and  $ab-h^2 = a_1b_1 - h_1^2$ .

Proof:

Given equation is  $ax^2 + 2hxy + by^2 = 0$

now rotate the axes by angle  $\theta$ ,

$$\therefore x = x_1 \cos \theta - y_1 \sin \theta$$

$$\therefore y = x_1 \sin \theta + y_1 \cos \theta$$

$$\therefore ax^2 + 2hxy + by^2 = 0$$

$$\Rightarrow a(x_1 \cos \theta - y_1 \sin \theta)^2 + 2h(x_1 \cos \theta - y_1 \sin \theta)(x_1 \sin \theta + y_1 \cos \theta) + b(x_1 \sin \theta + y_1 \cos \theta)^2 = 0$$

$$\Rightarrow a(x_1^2 \cos^2 \theta - 2x_1 y_1 \sin \theta \cos \theta + y_1^2 \sin^2 \theta) + 2h(x_1^2 \sin \theta \cos \theta + x_1 y_1 \cos^2 \theta - x_1 y_1 \sin^2 \theta - y_1^2 \sin \theta \cos \theta) + b(x_1^2 \sin^2 \theta + 2x_1 y_1 \sin \theta \cos \theta + y_1^2 \cos^2 \theta) = 0$$

$$\Rightarrow a x_1^2 \cos^2 \theta - 2ax_1 y_1 \sin \theta \cos \theta + ay_1^2 \sin^2 \theta + 2hx_1^2 \sin \theta \cos \theta + 2hx_1 y_1 \cos^2 \theta - 2hy_1^2 \sin^2 \theta - 2hy_1^2 \sin \theta \cos \theta + bx_1^2 \sin^2 \theta + 2bx_1 y_1 \sin \theta \cos \theta + by_1^2 \cos^2 \theta = 0$$

$$\Rightarrow x_1^2(a \cos^2 \theta + 2h \sin \theta \cos \theta + b \sin^2 \theta) + 2x_1 y_1(-a \sin \theta \cos \theta + h \cos^2 \theta - h \sin^2 \theta + b \sin \theta \cos \theta) + y_1^2(a \sin^2 \theta - 2h \sin \theta \cos \theta + b \cos^2 \theta) = 0$$

by comparing with our given equation  $a_1x^2 + 2hxy + b_1y^2 = 0$

$$\therefore a_1 = a \cos^2\theta + 2h \sin\theta \cos\theta + b \sin^2\theta$$

$$\therefore b_1 = a \sin^2\theta - 2h \sin\theta \cos\theta + b \cos^2\theta$$

$$\therefore h_1 = -a \sin\theta \cos\theta + h \cos^2\theta - h \sin^2\theta + b \sin\theta \cos\theta$$

Now,

$$a_1 + b_1 = a \cos^2\theta + 2h \sin\theta \cos\theta + b \sin^2\theta + a \sin^2\theta - 2h \sin\theta \cos\theta + b \cos^2\theta$$

$$= a(\sin^2\theta + \cos^2\theta) + b(\sin^2\theta + \cos^2\theta)$$

$$\therefore a_1 + b_1 = a + b$$

$$\left[ \because \sin^2\theta + \cos^2\theta = 1 \right]$$

Now,

$$2a_1 = 2a \cos^2\theta + 2h \cdot 2 \sin\theta \cos\theta + 2b \sin^2\theta$$

$$= 2a(1 - \sin^2\theta) + 2h \cdot \sin 2\theta + 2b \sin^2\theta$$

$$= 2a - 2a \sin^2\theta + 2b \sin^2\theta + 2h \sin 2\theta$$

$$= 2a + 2 \sin^2\theta(b - a) + 2h \sin 2\theta$$

$$= 2a + (1 - \cos 2\theta)(b - a) + 2h \sin 2\theta$$

$$= 2a + b - a - b \cos 2\theta + a \cos 2\theta + 2h \sin 2\theta$$

$$= (a + b) + \cos 2\theta(a - b) + 2h \sin 2\theta$$

$$\left[ \begin{array}{l} \because \cos^2\theta = 1 - \sin^2\theta \\ \therefore 2 \sin\theta \cos\theta = \sin 2\theta \end{array} \right]$$

$$\left[ \because 2 \sin^2\theta = 1 - \cos 2\theta \right]$$

Similarly,

$$2b_1 = (a + b) - \cos 2\theta(a - b) + 2h \sin 2\theta$$

$$2h_1 = 2h \cos 2\theta - \sin 2\theta(a - b)$$

$$\therefore 4a_1 b_1 = (a+b)^2 - \{ \cos 2\theta (a-b) + 2h \sin 2\theta \}^2$$

$$\therefore 4h^2 = 4h^2 \cos^2 2\theta - 4h(a-b) \sin 2\theta \cos 2\theta + (a-b)^2 \sin^2 2\theta$$

$$\begin{aligned}\therefore 4(a_1 b_1 - h^2) &= (a+b)^2 - (a-b)^2 \cos^2 2\theta - 4h^2 \sin^2 2\theta - \\&\quad 4h(a-b) \sin 2\theta \cos 2\theta - 4h^2 \cos^2 2\theta + \\&\quad 4h(a-b) \sin 2\theta \cos 2\theta - (a-b)^2 \sin^2 2\theta \\&= (a+b)^2 - (a-b)^2 (\sin^2 2\theta + \cos^2 2\theta) - \\&\quad 4h^2 (\sin^2 2\theta + \cos^2 2\theta) \\&= \{(a+b)^2 - (a-b)^2\} - 4h^2\end{aligned}$$

$$\Rightarrow 4(a_1 b_1 - h^2) = 4ab - 4h^2$$

$$\therefore a_1 b_1 - h^2 = ab - h^2$$

(Proved.)

Topic : 02 - Show the method how we can eliminate the first degree term of a second degree equation.

Q. Remove the first degree terms from the equation

$$x^2 + 2xy + 3y^2 + 2x - 4y - 1 = 0.$$

Soln:

Let, first degree terms will be removed if we transform or shift origin in  $(\alpha, \beta)$ .

$$\text{Given equation: } x^2 + 2xy + 3y^2 + 2x - 4y - 1 = 0 \quad \dots \quad (1)$$

Transformed equation of (1) will be,

$$\begin{aligned} & (x_1 + \alpha)^2 + 2(x_1 + \alpha)(y_1 + \beta) + 3(y_1 + \beta)^2 + 2(x_1 + \alpha) - 4(y_1 + \beta) - 1 = 0 \\ \Rightarrow & x^2 + 2x\alpha + \alpha^2 + 2xy + 2x\beta + 2y\alpha + 2\alpha\beta + 3y^2 + 6y\beta + 3\beta^2 \\ & + 2x + 2\alpha - 4y - 4\beta - 1 = 0 \\ \Rightarrow & x^2 + 2xy + y^2 + 2x(\alpha + \beta + 1) + 2y(\alpha + 3\beta - 2) + \\ & (\alpha^2 + 2\alpha\beta + 3\beta^2 + 2\alpha - 4\beta - 1) = 0 \quad \dots \quad (2) \end{aligned}$$

$\therefore$  first degree term will be removed if

$$\alpha + \beta + 1 = 0 \quad \dots \quad (3)$$

$$\alpha + 3\beta - 2 = 0 \quad \dots \quad (4)$$

$$\text{Now, } \{(4) - (3)\} \Rightarrow \alpha + 3\beta - 2 - \alpha - \beta - 1 = 0$$

$$\Rightarrow 2\beta - 3 = 0$$

$$\therefore \beta = \frac{3}{2}$$

putting  $\beta$  in equation (3),

$$\alpha + \frac{3}{2} + 1 = 0$$

$$\therefore \alpha = -\frac{5}{2}$$

Now from equation (2)

$$x^2 + 2xy + y^2 + \frac{25}{4} - \frac{30}{4} + \frac{27}{4} - \frac{10}{2} - \frac{12}{2} - 1 = 0$$
$$\Rightarrow x^2 + 2xy + y^2 = \frac{13}{2}$$

∴ Eliminated first degree term equation is

$$x^2 + 2xy + y^2 = \frac{13}{2}$$

There is another way to remove first degree term from a second degree equation.

Given equation,

$$x^2 + 2xy + 3y^2 + 2x - 4y - 1 = 0$$

Comparing with  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  equation,

$$a = 1, b = 3, c = -1, h = 1, g = 1, f = -2$$

Now the formula of origin shift  $(\alpha, \beta)$  is

$$\begin{aligned}(\alpha, \beta) &= \left( \frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right) \\&= \left( \frac{1 \times (-2) - 3 \times 1}{1 \times 3 - 1^2}, \frac{1 \times 1 - 1 \times (-2)}{1 \times 3 - 1^2} \right) \\&= \left( -\frac{5}{2}, \frac{3}{2} \right)\end{aligned}$$

Now, first degree term removed equation,

$$x^2 + 2xy + 3y^2 = c_1 \quad \text{--- (1)}$$

$$\begin{aligned}\text{where, } c_1 &= -(\alpha g + \beta f + c) \\&= -\left(-\frac{5}{2} \cdot 1 + \frac{3}{2} \cdot (-2) + (-1)\right)\end{aligned}$$

$$\begin{aligned} \Rightarrow c_1 &= -\left(-\frac{5}{2} - 4\right) \\ &= -\left(-\frac{13}{2}\right) \\ &= 13/2 \end{aligned}$$

putting  $c_1 = 13/2$  in equation (1) we get first degree removed equation,

$$x^2 + 2xy + 3y^2 = \frac{13}{2}$$

(Ans.)

**Topic 03:** Explain pair of straight lines, plane, conic and space.

#### ■ Pair of straight lines:

A pair of straight line refers to two straight lines represented by a single second-degree equation in two variable  $x, y$ . This is typically represented as :

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

If  $l_1x + m_1y + n_1 = 0$  and  $l_2x + m_2y + n_2 = 0$  represents two equations of straight lines then the equation of pair of straight line is,

$$(l_1x + m_1y + n_1)(l_2x + m_2y + n_2) = 0$$

... condition: The equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  can be either pair of straight lines or conics. To determine whether this equation represents a pair of straight line or conics, if the determinants is zero.

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$\therefore$  If  $a+b=0$ ; both lines are perpendicular.

$\therefore$  If  $h^2=ab$ ; they are parallel.

### ■ Plane:

Plane is a two-dimensional flat surface that extends infinitely in the all directions. It is characterized by its width and length but has no thickness.

In geometry, a plane is defined by an infinite number of points and can be described using the equation of plane involving variables such as  $x, y, z$  in three dimensional space.

$$ax + by + cz + d = 0$$

## Conics :

Conics are the curves formed by the intersection of a plane with a double-napped cone. Depending on the angle and the position of the plane, the intersection can produce different types of curves: circle, ellipse, parabolla or hyperbolla.

The general equation of second degree is:

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

To determine whether this equation represents a conics or not we have to evaluate the, ' $\Delta$ '

$$\therefore \Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$$

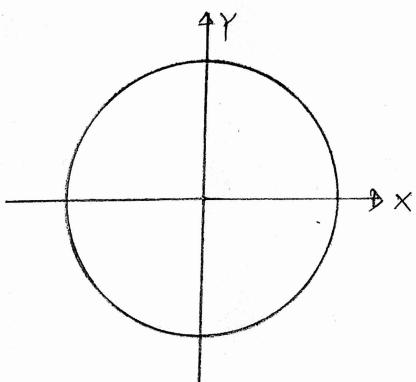
if  $\Delta \neq 0$ , the equation represents conics.

conics can be circle, ellipse, parabolla or hyperbolla. To determine exact type we have to dive deeper.

... circle :

condition -  $a=b$  and  $h=0$

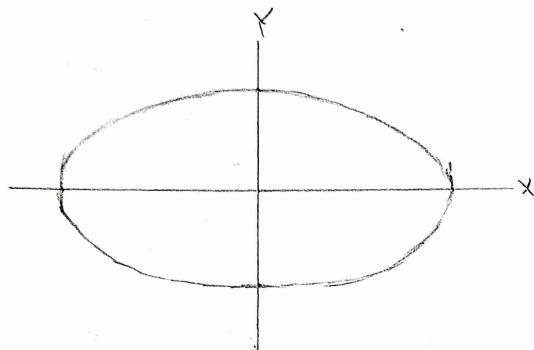
standard equation -  $x^2 + y^2 = a^2$



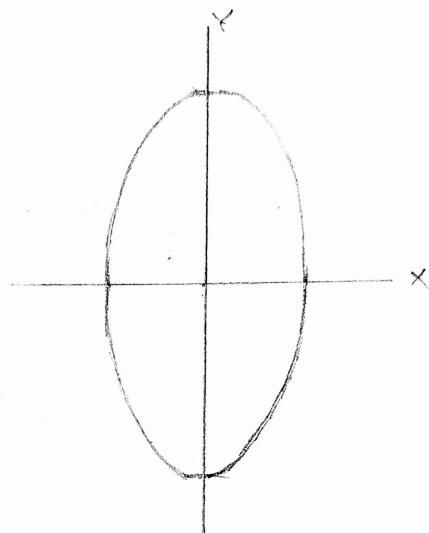
... Ellipse:

condition -  $ab - b^2 > 0$

standard Equation -  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$$

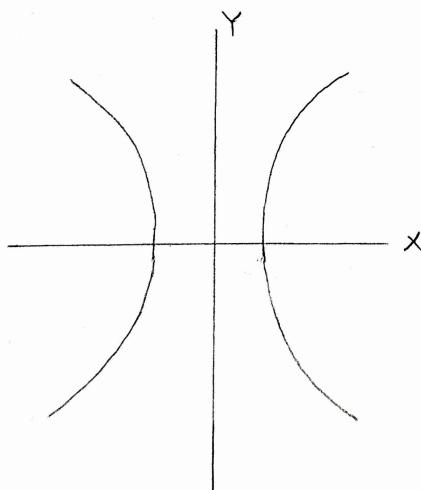


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (b > a)$$

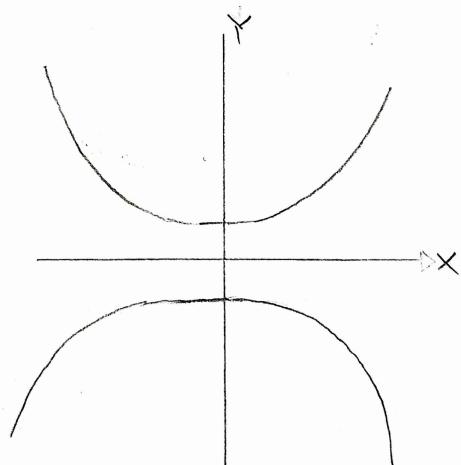
... Hyperbolla:

condition -  $ab - b^2 < 0$

standard Equation -  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  or  $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

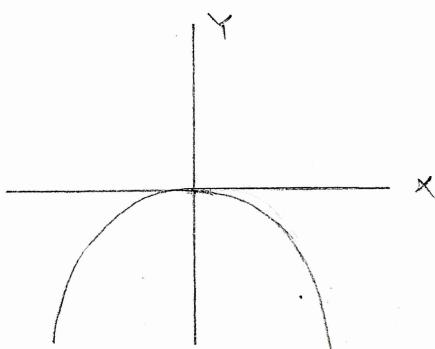
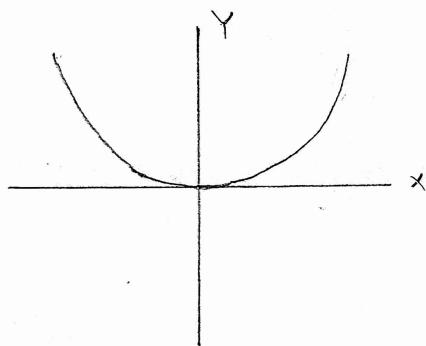
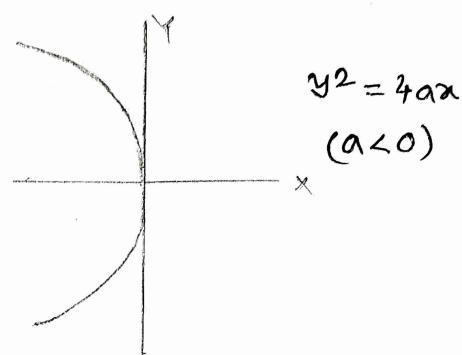
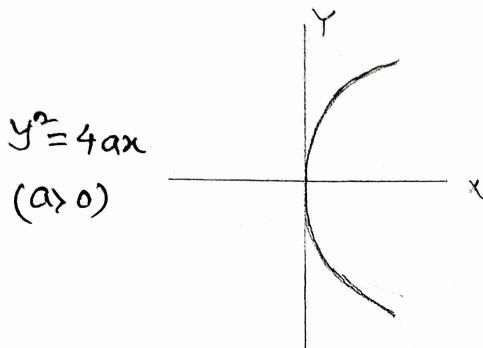


$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

... parabola:

$$\text{condition: } ab - h^2 = 0$$

$$\text{standard equation: } y^2 = 4ax \text{ or } x^2 = 4ay$$



## Space:

Space refers to the system of describing points, lines, planes and shapes in a multidimensional framework using numerical co-ordinates.

### • 1D - Space -

A single line where points are represented by one coordinate,  $x$

- equation:  $x = c$ , where  $x$  and  $c$  constant

2D space:

A flat surface where each point is represented by an ordered pair of coordinates  $(x, y)$ .

- equation:  $ax + by + c = 0$

3D Space:

A three dimensional region where each point is represented by a triplet of coordinates,  $(x, y, z)$

- equation:  $ax + by + cz + d = 0$

Topic 04:

Evaluate:  $x^2 + y^2 + 3xy - 6x + 8y - 12 = 0$

- a) Intersection point
- b) Angle between the lines
- c) Bisectors equation
- d) Test the equation represents pair of straight line or not.

Sol<sup>n</sup>: To evaluate those points we need a second degree equation first.

$\therefore$  Given equation:  $x^2 + y^2 + 3xy - 6x + 8y - 12 = 0$

Comparing with general equation.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$a = 1, b = 1, c = -12, h = \frac{3}{2}, g = -3, f = 4$$

a) Intersection point:

- Intersection points represents as  $(\alpha, \beta)$

We know,

$$\begin{aligned} (\alpha, \beta) &= \left( \frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right) \\ &= \left( \frac{\frac{3}{2} \cdot 4 - 1 \cdot (-3)}{1 \cdot 1 - (\frac{3}{2})^2}, \frac{(-3) \cdot \frac{3}{2} - 1 \cdot 4}{1 \cdot 1 - (\frac{3}{2})^2} \right) \\ &= \left( -\frac{36}{5}, \frac{34}{5} \right) \end{aligned}$$

b) Angle between the lines.

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\Rightarrow \tan \theta = \frac{2\sqrt{\left(\frac{3}{2}\right)^2 - 1 \cdot 1}}{1+1}$$

$$\Rightarrow \tan \theta = \frac{\sqrt{5}}{2}$$

$$\therefore \theta = 48.189^\circ$$

$\left  \begin{array}{l} \text{here,} \\ h = \frac{3}{2} \\ a = 1 \\ b = 1 \end{array} \right.$
---

c) Bisector equation:

$$\text{From (a), } (\alpha, \beta) = \left(-\frac{36}{5}, \frac{34}{5}\right)$$

$\therefore$  bisector is,

$$\frac{(x-\alpha)^2 - (y-\beta)^2}{a-b} = \frac{(x-\alpha)(y-\beta)}{h}$$

$$\Rightarrow h \left\{ (x-\alpha)^2 - (y-\beta)^2 \right\} = (a-b)(x-\alpha)(y-\beta)$$

$$\Rightarrow \frac{3}{2} \left\{ \left(x + \frac{36}{5}\right)^2 - \left(y - \frac{34}{5}\right)^2 \right\} = (1-1) \left(x + \frac{36}{5}\right) \left(y - \frac{34}{5}\right)$$

$$\Rightarrow x^2 + 2 \cdot x \cdot \frac{36}{5} + \frac{1296}{25} - y^2 + 2 \cdot y \cdot \frac{34}{5} - \frac{1156}{25} = 0 \times \frac{2}{3}$$

$$\Rightarrow x^2 - y^2 + \frac{72}{5}x + \frac{68}{5}y + \frac{28}{5} = 0$$

$$\Rightarrow 5x^2 - 5y^2 + 72x + 68y + 28 = 0$$

d) Test the equation represents pair of straight line or not.

If  $\Delta = 0$ , represents a second degree equation a pair of straight line.

$$\therefore \Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 1 \cdot 1 \cdot (-12) + 2 \cdot 4 \cdot (-3) \cdot (3/2) - 1 \cdot 4^2 - 1 \cdot (-3)^2$$
$$- (-12) \cdot (3/2)^2$$

$$= -46$$

$\therefore x^2 + y^2 + 3xy - 6x + 8y - 12 = 0$  equation represents a conics.