CHAPTER
04 Elasticity

- Price Elasticity of Demand
- Elastic and Inelastic Demand
- Point Elasticity
- Slope and Elasticity
- Arc Elasticity
- Own Price and Cross Price Elasticity of Demand
- Income Elasticity
- AR, MR and Elasticity
- Profit Maximization

The second chapter of the book focuses the determinants of demand and supply. Among many other factors, price substantially influences demand and supply. A certain percent change in price may cause a comparatively large change in quantities of demand or quantities of supply of some goods than others depending on the sensitivity of quantities of demand or supply to price. Elasticity measures the degree of responsiveness of quantities of demand or supply to price, income and other variables concerned.

4.1 Price Elasticity of Demand

The ratio between percentage change in demand and percentage change in price is called price elasticity of demand.

i.e., $\text{Price Elasticity of Demand} = \dfrac{\%\,\text{change in quantity of demand}}{\%\,\text{change in price}}$. According to this formula, price elasticity of demand is the percentage change in quantity demanded of a good due to 1% change in its price.

From the above definition five types of elasticities can be distinguished.

1. **Unitarily Elastic Demand**: If percentage change in quantity of demand is equal to percentage change in price, demand is unitarily elastic.
   Suppose a 10% increase in price results in a 10% fall in demand. $\text{Price Elasticity of Demand} = \dfrac{10\%}{10\%} = 1$. A unitarily elastic demand curve is neither very flat nor very steep. Figure 4.1 displays a unitarily elastic demand curve.

Unitarily Elastic Demand

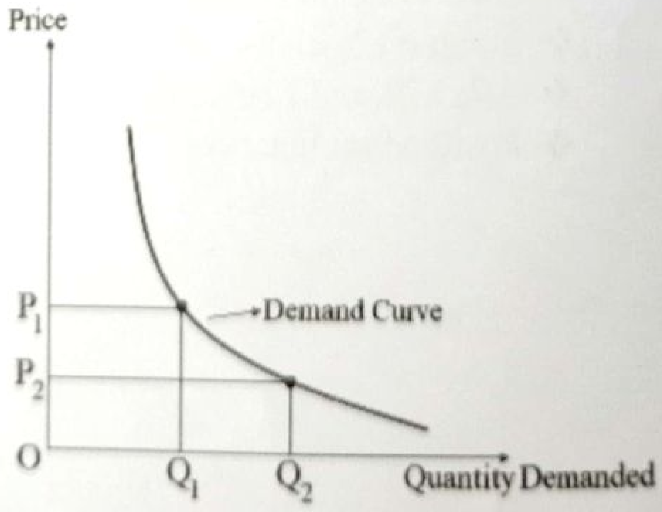

Figure- 4.1

2. **Elastic Demand**: If percentage change in quantity demanded is larger than percentage change in price, demand would be elastic. Suppose a 5% change in price results in a 10% change in quantity of demand.

Price Elasticity of Demand $= \frac{10\%}{5\%} = 2$. In the case of elastic demand, elasticity coefficient (e) would be greater than 1. In figure 4.2 movement from point A to B along demand curve represents a fall in price from 6 to 4 and rise in quantity demanded from 5 to 14.

Percentage change in price $= \frac{\Delta P}{P} \times 100 = \frac{6-4}{6} \times 100 = 33.33\%$

Percentage change in quantity demanded $= \frac{\Delta Q}{Q} \times 100 = \frac{14-5}{5} \times 100 = 180\%$

(ignoring the direction of change).

Thus, Price Elasticity of Demand (e) $= \frac{180\%}{33.33\%} = 5.40 > 1$. Here demand is elastic.

A comparatively flat demand curve represents higher elasticity of demand.

Elastic Demand

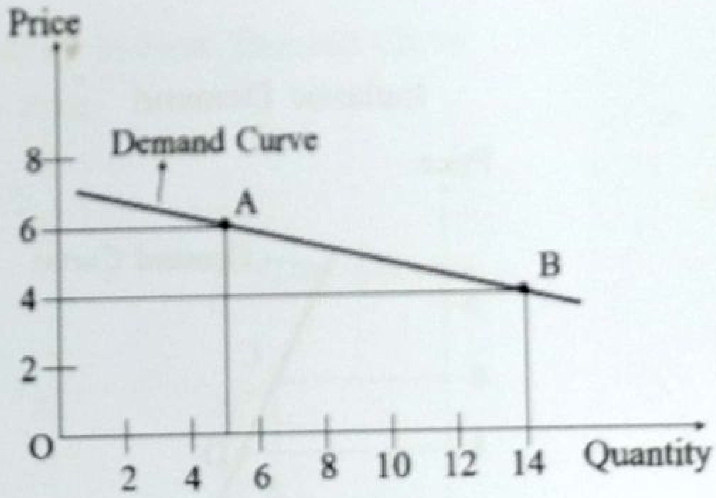

Figure- 4.2

3. **Perfectly Elastic Demand**: If quantity of demand changes without any change in price, demand is perfectly elastic. In this special case, demand curve would be horizontal which is a distinguishing feature of perfectly competitive market structure where price remains fixed but the consumers can buy any amount they desire. Elasticity coefficient (e) turns out to be infinity (∞), because something divided by zero equals infinity $(e = \frac{k\%}{0\%} = \infty)$.

Perfectly Elastic Demand Curve

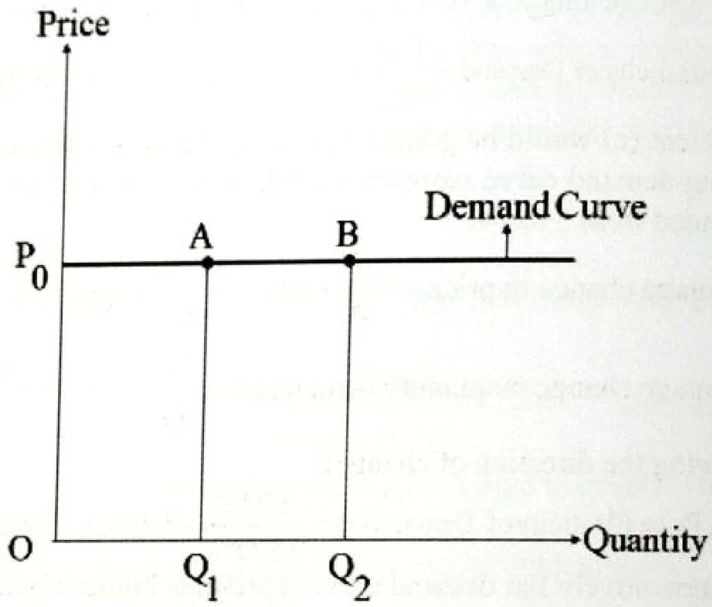

Figure- 4.3

Inelastic Demand

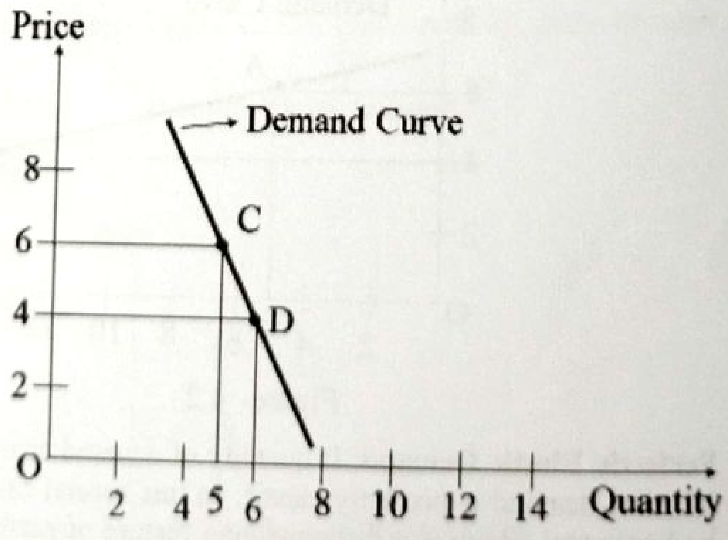

Figure- 4.4

Figure 4.3 is the graphical presentation of perfectly elastic demand curve. Price remaining fixed at P_0, quantity varies from Q_1 to Q_2.

4. **Inelastic Demand**: If percentage change in quantity of demand is smaller than percentage change in price, demand is inelastic.

   In case of inelastic demand, elasticity coefficient (e) is smaller than one. Suppose a 10% change in price causes 5% change in demand. $\text{Elasticity, e} = \frac{5\%}{10\%} = \frac{1}{2} < 1$. In figure 4.4, movement from point C to D along the demand curve associates a fall in price from 6 to 4 and a rise in quantity demanded from 5 to 6. Percentage change in price is 33.33% and percentage change in quantity demanded is 20%.

   Thus, $\text{Price Elasticity of Demand (e)} = \frac{20\%}{33.33\%} = 0.60 < 1$.

   Here demand is inelastic.

5. **Perfectly Inelastic Demand**: When demand remains completely unresponsive to price, i.e., no change in demand occurs following changes in price, demand is said to be perfectly inelastic. Perfectly inelastic demand curve takes vertical shape as Figure 4.5.

Perfectly Inelastic Demand Curve

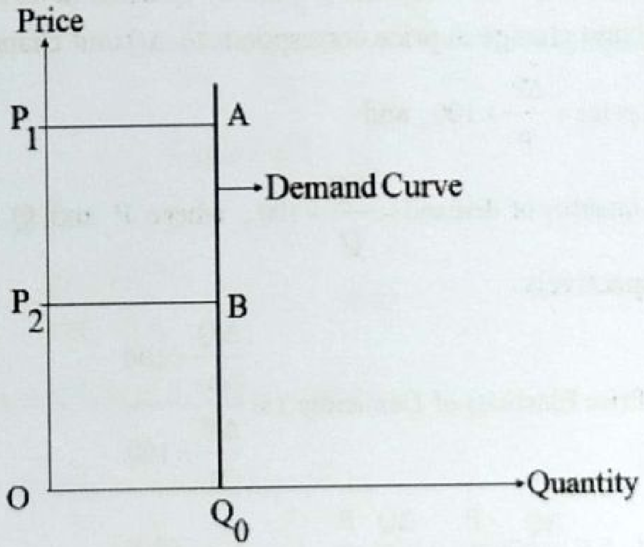

Figure- 4.5

In the occasion of perfectly inelastic demand, elasticity coefficient is zero. Suppose there is k% change in price which causes no change in demand. $\text{Elasticity (e)} = \frac{0\%}{k\%} = 0$.

Figure 4.6 exhibits the demand curve of a product where demand at any price is 40. In this case, price elasticity of demand is zero, and the good is viewed as price-neutral.

Price-neutral Demand

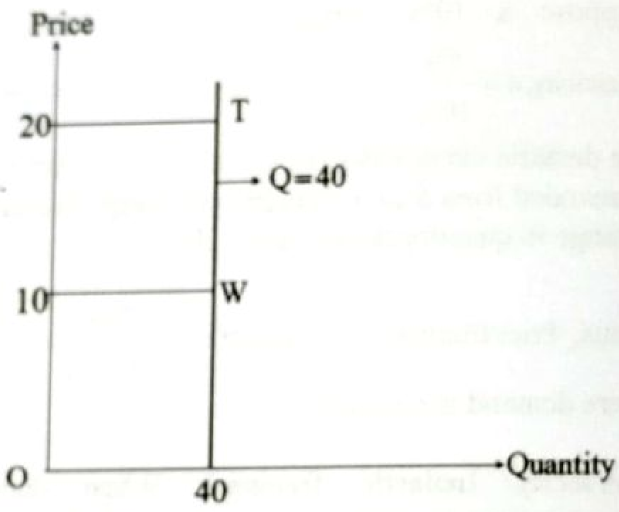

Figure- 4.6

4.2 Point Elasticity of Demand

Elasticity at a certain point of demand curve is called point elasticity. Assume the demand function $Q = f(P)$ revealing quantity of demand as a function of price. Let us suppose ΔP unit change in price corresponds to ΔQ unit change in quantity of demand.

$\%\text{ change in price} = \frac{\Delta P}{P} \times 100$, and

$\%\text{ change in quantity of demand} = \frac{\Delta Q}{Q} \times 100$, where P and Q denote initial price and quantity respectively.

$$\text{Therefore, Price Elasticity of Demand}(\varepsilon) = \frac{\frac{\Delta Q}{Q} \times 100}{\frac{\Delta P}{P} \times 100}$$

$$\text{or, } \varepsilon = \frac{\Delta Q}{Q} \times \frac{P}{\Delta P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} \quad \ldots\ldots \quad \ldots\ldots \quad (4.1)$$

DH is a straight-line demand function in figure 4.7. We are interested to measure elasticity at point A where price and quantity are OP_1 and OQ_1 respectively. In order to measure elasticity at point A, we consider neighbouring point B where price and quantity are OP_2 and OQ_2 respectively.

Change in price, $\Delta P = P_1P_2 = AM$

Change in quantity, $\Delta Q = Q_1Q_2 = MB$

Initial price, $P = OP_1 = Q_1A$

Initial quantity, $Q = OQ_1 = P_1A$

Point Elasticity Measurement

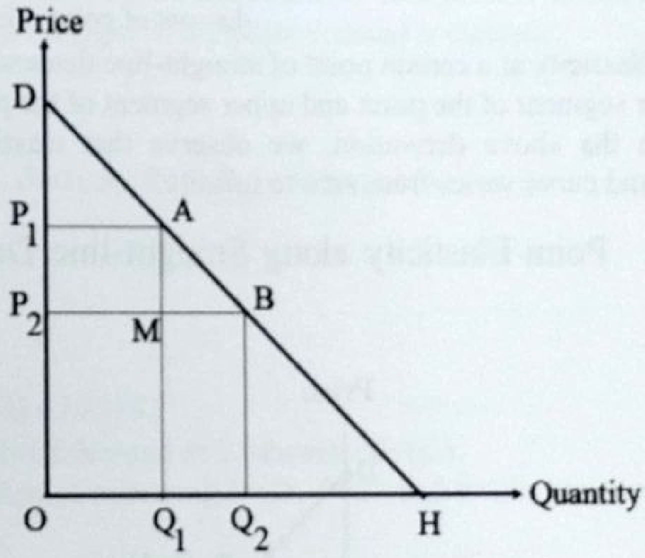

Figure- 4.7

Put the above values into (4.1)

$$\text{Elasticity}(\varepsilon) = \frac{\Delta Q}{\Delta P}.\frac{P}{Q} = \frac{MB}{AM} \times \frac{Q_1A}{P_1A} \quad ..... \quad ..... \quad (4.2)$$

Right angled triangles AMB and AQ_1H are similar.

Thus, $\frac{MB}{AM} = \frac{Q_1H}{Q_1A}$. Use this information in (4.2).

$$\text{Elasticity}(\varepsilon) = \frac{MB}{AM} \times \frac{Q_1A}{P_1A} = \frac{Q_1H}{Q_1A} \times \frac{Q_1A}{P_1A}$$

$$\varepsilon = \frac{Q_1H}{P_1A} \quad ..... \quad ..... \quad (4.3)$$

Again two right-angled triangles AQ_1H and DP_1A are similar.

Therefore, $\frac{Q_1H}{HA} = \frac{P_1A}{AD}$ $\Rightarrow \frac{Q_1H}{P_1A} = \frac{HA}{AD}$

Use the above information in (4.3)

$$\text{Elasticity at point A} = \frac{Q_1H}{P_1A} = \frac{HA}{AD} = \frac{\text{lower segment of point A}}{\text{upper segment of point A}}$$

$$= \frac{\text{distance of point A from quantity axis}}{\text{distance of point A from price axis}}$$

i.e., elasticity at a certain point of straight-line demand curve is the ratio between lower segment of the point and upper segment of the point of demand curve. From the above derivation, we observe that elasticity along a straight-line demand curve varies from zero to infinity.

Point Elasticity along Straight-line Demand Function

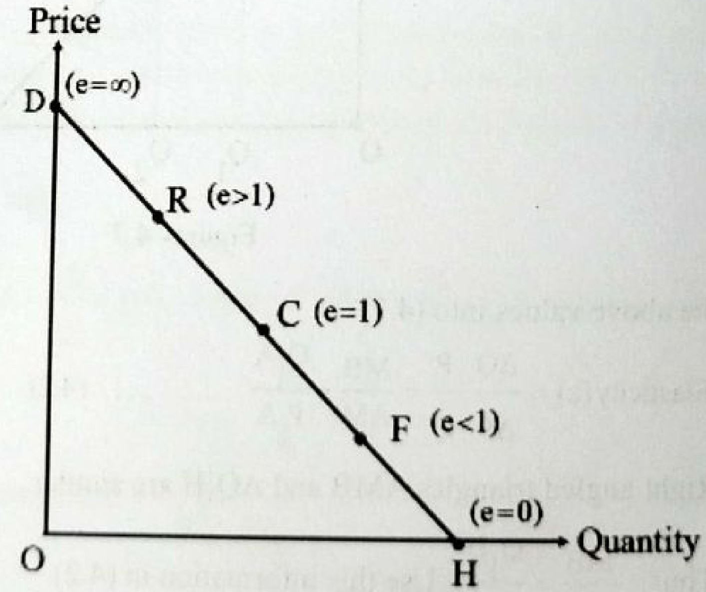

Figure- 4.8

DH is a straight-line demand curve in figure 4.8. Suppose midpoint of this demand curve is C.

$$\text{Elasticity at point C} = \frac{\text{lower segment of point C}}{\text{upper segment of point C}} = \frac{CH}{DC} = 1; \quad \because CH = DC$$

$$\text{Elasticity at point S} = \frac{\text{lower segment of point S}}{\text{upper segment of point S}} = \frac{SH}{DS} > 1; \quad \because SH > DS$$

$$\text{Elasticity at point D} = \frac{\text{lower segment of point D}}{\text{upper segment of point D}} = \frac{DH}{0} = \infty$$

$$\text{Elasticity at point F} = \frac{\text{lower segment of point F}}{\text{upper segment of point F}} = \frac{FH}{DF} < 1; \quad \because FH < DF$$

$$\text{Elasticity at point H} = \frac{\text{lower segment of point H}}{\text{upper segment of point H}} = \frac{0}{HD} = 0$$

It is evident that on the left of the midpoint of a straight-line demand curve, demand is elastic; and on the right of midpoint demand is inelastic.

Use of Calculus

Replacing $\frac{\Delta Q}{\Delta P}$ by $\frac{dQ}{dP}$, elasticity formula (4.1) becomes

$$\varepsilon = \frac{dQ}{dP}.\frac{P}{Q}$$

Example 4.1

Given the demand function $Q = 100 - 2P$

i. Express elasticity of demand as a function of price.

ii. Determine elasticities assuming P=25, P=30 and P=20. Make comment on your results.

Solution

i. Given $Q = 100 - 2P$; $\frac{dQ}{dP} = -2$

We know elasticity, $\varepsilon = \frac{dQ}{dP}.\frac{P}{Q} = -2.\frac{P}{100-2P}$

i.e., $\varepsilon = -\frac{2P}{100-2P}$ which is function of price.

ii. Setting P=25; $\varepsilon = -\frac{2P}{100-2P} = -\frac{50}{50} = -1$

P=30; $\varepsilon = -\frac{2P}{100-2P} = -\frac{60}{40} = -\frac{3}{2}$ and

P=20; $\varepsilon = -\frac{2P}{100-2P} = -\frac{40}{60} = -\frac{2}{3}$

We observe negative value of elasticity in each case. This comes from the inverse relationship between price and quantity demanded. While commenting on elasticity values, it is recommended to take account of the absolute elasticities. This practice leads to following results:

At P=25, elasticity coefficient $e = 1$. That means, demand is unitarily elastic at this price.

At P=30; $e = \frac{3}{2} > 1$; i.e., demand is elastic, and when P=20; $e = \frac{2}{3} < 1$; i.e., demand is inelastic.

4.3 Slope and Elasticity

Sometimes it is confused that slope and elasticity are identical. In fact, demand functions of different slope may adhere to same elasticity or the opposite. Several cases are depicted below.

4.3.1. Different Slopes but Equal Elasticity

In figure 4.9, demand cure DH is flatter than DF. Elasticity of demand curve DF at point A is $\frac{AF}{DA} = \frac{OP}{PD}$.

Equal Elasticity with Different Slopes

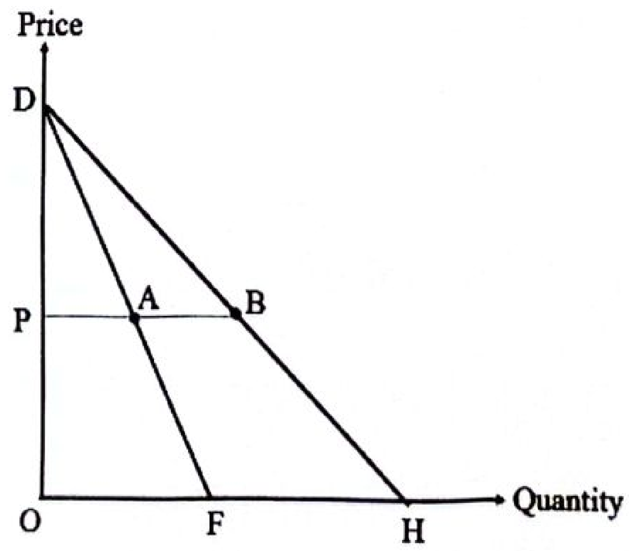

Figure- 4.9

Elasticity of demand curve DH at point B is $\frac{BH}{DB} = \frac{OP}{PD}$.

This proves that demand curves with different slopes result in same elasticity. Note that in this case price is assumed fixed and both the demand curves have identical vertical intercept.

4.3.2. Different Slopes and Different Elasticity

At a certain price the elasticity of a flatter demand curve would be higher than that of a steeper demand curve if their intercepts are unequal. Figure 4.10 illustrates this feature.

Elasticity of Flatter Demand Curve

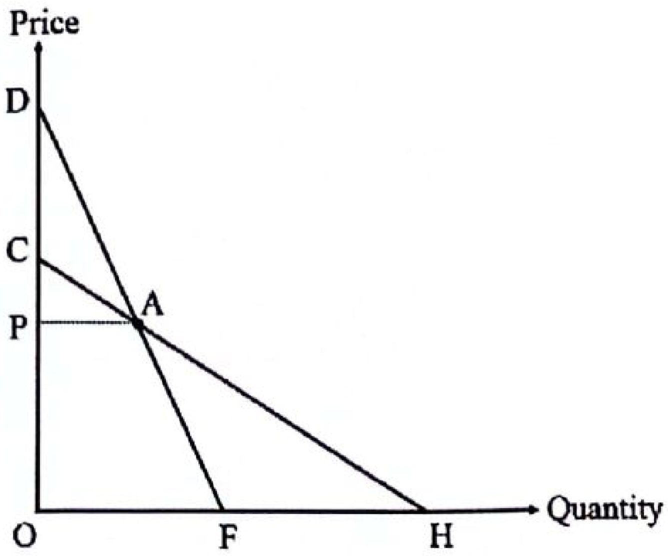

Figure- 4.10

In figure, demand curve CH is flatter than DF.

Elasticity of CH at point A $= \dfrac{AH}{CA} = \dfrac{OP}{PC}$ and

elasticity of DF $= \dfrac{AF}{DA} = \dfrac{OP}{PD}$.

Since $\dfrac{OP}{PC} > \dfrac{OP}{PD}$, it suggests that the flatter demand entails higher elasticity.

4.3.3. Equal Slope but Unequal Elasticity

Price remaining fixed, elasticity of a demand curve staying on the right is attributable to lower price elasticity though their slopes are same.

Elasticity Comparison of Equal Sloped Demand Curves at Constant Price

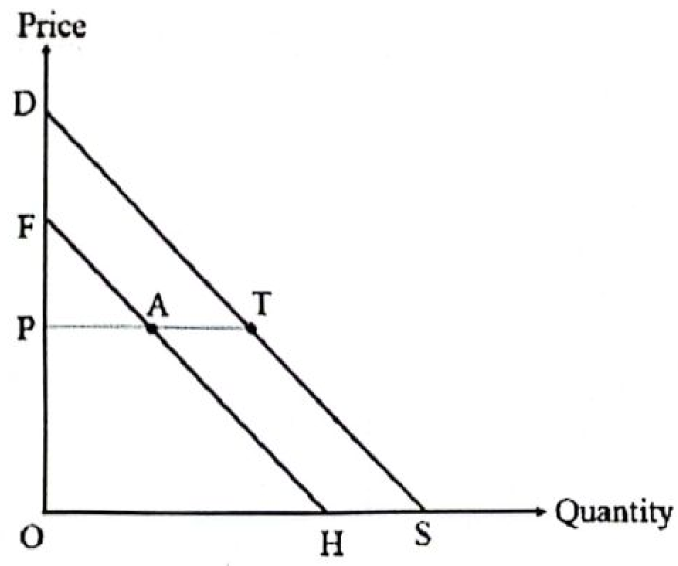

Figure- 4.11

In figure 4.11, demand curves FH and DS are parallel. Elasticity of FH at point A $= \frac{AH}{FA} = \frac{OP}{PF}$ and elasticity of DS at point T $= \frac{TS}{DT} = \frac{OP}{PD}$.

Here, $\frac{OP}{PF} > \frac{OP}{PD}$. This means, at a fixed price elasticity along DS is smaller than elasticity along FH. The opposite occurs if quantity is kept constant.

In figure 4.12 demand curves FH and VW are parallel. Quantity is assumed fixed at OQ.

Elasticity of demand curve FH at point Z $= \frac{ZH}{FZ} = \frac{QH}{OQ}$ and elasticity of demand curve VW at point Y $= \frac{YW}{VY} = \frac{QW}{OQ}$.

The demand function Q = 100 – 2P is drawn as a straight-line in figure 4.13.

Point Elasticity versus Arc Elasticity

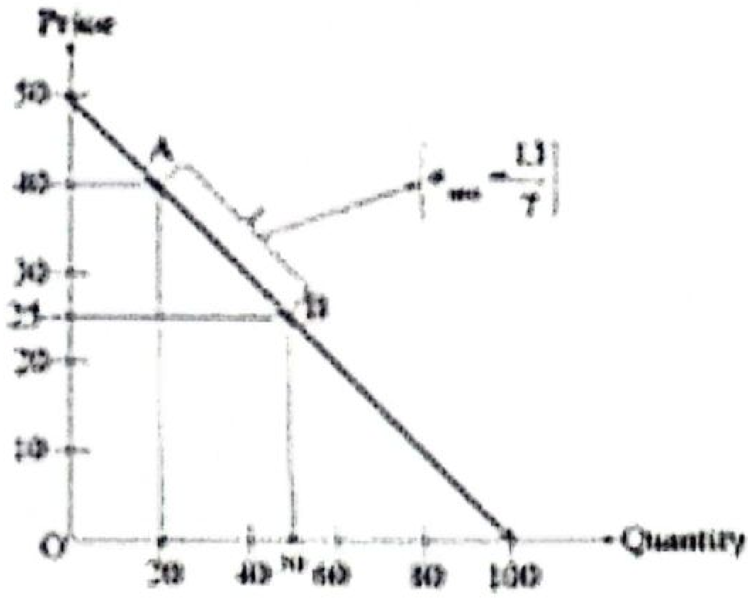

Figure- 4.13

At point A, $P_1 = 40$, $Q_1 = 20$

At point B, $P_2 = 25$, $Q_2 = 50$

$\Delta Q = 20 - 50 = -30$; $\Delta P = 40 - 25 = 15$

Arc elasticity between A and B

$$\varepsilon_{arc} = \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2} = \frac{-30}{15} \cdot \frac{40 + 25}{20 + 50} = -\frac{13}{7}$$

$$e_{arc} = \frac{13}{7}$$

4.5 Constant Elasticity Demand Function

Along a constant elasticity demand function, elasticity remains equal at each point. Rectangular hyperbolic demand functions fall in this class. Price elasticity of a rectangular hyperbolic demand function $Q = \frac{100}{P}$ is computed below.

$$\frac{dQ}{dP} = -\frac{100}{P^2}$$

$$\varepsilon = \frac{dQ}{dP}.\frac{P}{Q} = -\frac{100}{P^2}.\frac{P}{100/P}$$

$$= -\frac{100}{P^2}.\frac{P^2}{100} = -1$$

$e = 1$, which is a constant.

Example 4.2

Consider the demand function $Q = \frac{k}{P^n}$ (k and n are positive constants)

Compute elasticity of demand. Does elasticity depend on price? When will demand be elastic?

Solution

$$Q = \frac{k}{P^n} = kP^{-n}; \qquad \therefore \frac{dQ}{dP} = -nkP^{-n-1}$$

$$\text{Elasticity, } \varepsilon = \frac{dQ}{dP}.\frac{P}{Q} = -nkP^{-n-1} \times \frac{P}{kP^{-n}}$$

$$= -nP^{-n-1+1+n}$$

$$= -nP^{0}$$

$$= -n$$

Elasticity does not depend on price. The function given seems to be constant elasticity demand function.

$|\varepsilon| = e = n$, implying that demand would be elastic if $n > 1$.

4.6 Cross Elasticity of Demand

The ratio between percentage change in quantity of demand for one good and percentage change in price of another good is termed as cross price elasticity of demand.

For illustration, assume the demand function: $Q_X = f(P_X, P_y, P_Z, M)$

Q_X : demand for X

P_X : price of X

P_y : price of Y

P_Z : price of Z

M : income

Demand for X depends on its own price, price of Y, Z and income (M). Two cross price elasticities can be defined as below.

$$\text{Elasticity of demand for X with respect to price of Y } \left(\varepsilon_{xy}\right) = \frac{\%\text{ change in demand for X}}{\%\text{ change in price of Y}}$$

$$= \frac{\Delta Q_x}{\Delta P_y} \frac{P_y}{Q_x}$$

$$\approx \frac{\partial Q_x}{\partial P_y} \frac{P_y}{Q_x}$$

Analogously, elasticity with respect to price of Z $\left(\varepsilon_{XZ}\right) = \dfrac{\partial Q_x}{\partial P_z} \dfrac{P_z}{Q_x}$

Sign of cross elasticity determines the relation between goods. Positive cross elasticity indicates substitute goods and negative cross elasticity complementary goods.

4.7 Income Elasticity

Income Elasticity is defined as the ratio between percentage change in quantity of demand and percentage change in income.

$$\text{Income Elasticity of Demand} = \frac{\%\text{ change in demand}}{\%\text{ change in income}}$$

$$\text{Income elasticity } (\eta) = \frac{\partial Q_x}{\partial M} . \frac{M}{Q_x}$$

Positive income elasticity refers to a normal good and negative income elasticity refers to inferior good. Normal goods are further categorized as necessary and luxury. If percentage change in quantity demanded is less than or equal to percentage change in income, the good is necessary. On the other hand, if percentage change in quantity demanded is greater than percentage change in income, the good is a luxury. Income elasticity coefficient below or equal to unity signifies necessary good and above unity luxury good.

Example 4.3

Given the demand function

$q_x = 60 - 2P_x + 3P_y + \sqrt{M}$; $(P_x = 20, P_y = 15, M = 100)$

i. Calculate demand for X at the given price and income set.
ii. Determine own price, cross price and income elasticity of demand.
iii. Make comments on elasticity coefficients.

Solution

i. Demand for X, $q_x = 60 - 2P_x + 3P_y + \sqrt{M}$

$$= 60 - (2 \times 20) + (3 \times 15) + \sqrt{100}$$

$$= 75$$

ii. Given $q_x = 60 - 2P_x + 3P_y + \sqrt{M}$

$$\frac{\partial q_x}{\partial P_x} = -2; \quad \frac{\partial q_x}{\partial P_y} = 3 \quad \text{and} \quad \frac{\partial q_x}{\partial M} = \frac{1}{2\sqrt{M}}$$

Own price elasticity, $\varepsilon_{xx} = \frac{\partial q_x}{\partial P_x}\frac{P_x}{q_x} = -2 \times \frac{20}{75} = -\frac{8}{15}$

Cross price elasticity, $\varepsilon_{xy} = \frac{\partial q_x}{\partial P_y}\frac{P_y}{q_x} = 3 \times \frac{15}{75} = \frac{3}{5}$

Income Elasticity, $\eta = \frac{\partial q_x}{\partial M}\frac{M}{q_x} = \frac{1}{2\sqrt{M}}\frac{M}{q_x} = \frac{\sqrt{M}}{2q_x} = \frac{10}{2 \times 75} = \frac{1}{15}$

iii. Own price elasticity coefficient $\left|\varepsilon_{xx}\right| = e = \frac{8}{15} < 1$. Demand is inelastic.

Cross elasticity coefficient, $\varepsilon_{xy} = \frac{3}{5} > 0$. Since cross elasticity is positive, X and Y are substitutes.

Income elasticity $\eta = \frac{1}{15} > 0$. Positive income elasticity refers to a normal good. Moreover, income elasticity coefficient is below unity and thus the good is necessary.

4.8 Total Revenue Approach to Measure Elasticity

Total revenue is the product of price and quantity. At 5 dollar price per unit if 20 kg is sold then total revenue generated equals $(5 \times 20 =)100$ dollars. A rise in price leads to a fall in quantity demanded but this does not necessarily suggest a fall in total revenue. If demand is extremely responsive to price then a rise in price would be accompanied by a sharp fall in quantity of demand and hence total revenue. But in the event of a less responsive demand, total revenue will increase with an increase in price because of a little drop in demand relative to increase in price. Total revenue may even remain unaffected with a change in price. Three varieties of elasticities are defined on the basis of the direction of total revenue with price.

First, if total revenue remains unaltered with any change in price, demand is unitarily elastic. Table shows unitarily elastic demand.

Table 4.1
Unitarily Elastic Demand

Price	Quantity	Total Revenue
20	30	600
25	24	600

Second, if total revenue falls as price rises or vice versa, demand is elastic. We observe more responsive demand in the table provided where total revenue falls with an increase in price due to a sharp fall in quantity of demand.

Table 4.2
Elastic Demand

Price	Quantity	Total Revenue
20	30	600
25	20	500

Third, if total revenue falls as price falls or total revenue rises as price rises, demand is inelastic. In the table we watch, quantity of demand did not fall that much although price increased, which rather resulted an increase in total revenue.

Table 4.3
Inelastic Demand

Price	Quantity	Total Revenue
20	30	600
25	28	700

4.9 Average Revenue (AR), Marginal Revenue (MR) and Elasticity (e)

General form of the demand function expresses the quantity of demand as a function of price,

$$\text{i.e., } Q = f(P)$$

Price elasticity of demand, $\varepsilon = \frac{dQ}{dP}.\frac{P}{Q}$

Elasticity coefficient (absolute value of elasticity), $e = -\frac{dQ}{dP}.\frac{P}{Q}$

Total revenue, $R = P \times Q$

Average revenue, $AR = \frac{R}{Q} = \frac{P \times Q}{Q} = P$

Marginal revenue, $MR = \frac{dR}{dQ} = \frac{d}{dQ}(P \times Q)$

$$\Rightarrow MR = P + Q\frac{dP}{dQ}$$

$$\Rightarrow MR = P(1 + \frac{Q}{P}\frac{dP}{dQ})$$

$$\Rightarrow MR = P\left(1 + \frac{1}{\frac{P}{Q}\frac{dQ}{dP}}\right)$$

$$\Rightarrow MR = P\left(1 - \frac{1}{e}\right) \quad ..... \quad ..... (4.4) \ (\because \quad e = -\frac{dQ}{dP}.\frac{P}{Q})$$

$$\Rightarrow MR = AR\left(1 - \frac{1}{e}\right) \quad (\because P \equiv AR)$$

$$\Rightarrow \frac{MR}{AR} = 1 - \frac{1}{e}$$

$$\Rightarrow \frac{1}{e} = 1 - \frac{MR}{AR} = \frac{AR - MR}{AR},$$

this follows $e = \frac{AR}{AR - MR}$, which signifies the relationship among AR, MR and elasticity (e).

Equation (4.4) reveals an important relation between marginal revenue and elasticity.

Three points are worth mentioning with regard to $MR = P\left(1-\frac{1}{e}\right)$:

1. If demand is unitarily elastic (e=1), MR=0
2. Elastic demand (e>1) yields positive MR.

   Suppose, e=4; $MR = P\left(1-\frac{1}{4}\right) = \frac{3}{4}P$ which is positive.

3. If demand is inelastic (e < 1), MR becomes negative.

Assuming $e = \frac{1}{4}$, $MR = P\left(1-\frac{1}{1/4}\right) = P(1-4) = -3P$ which is negative. A producer refuses to produce in the inelastic portion of demand curve because of negative MR. Profit maximization, as shown below, requires $MR = MC$. Marginal cost (MC) of production is always positive hence MR should be positive as well. Positive marginal revenue would only be accrued if demand is elastic (e>1).

4.10 Profit Maximization

Profit (π) is defined as the difference between total revenue (TR) and total cost (TC), where both are functions of quantity (Q).

$$\pi = R(Q) - C(Q)$$

Figure 4.14 plots profit function as inverse 'U'-shaped. At Q_0 quantity profit is maximum, characterized by zero slope of profit function at point B.

Slope of profit function is $\frac{d\pi}{dQ}$.

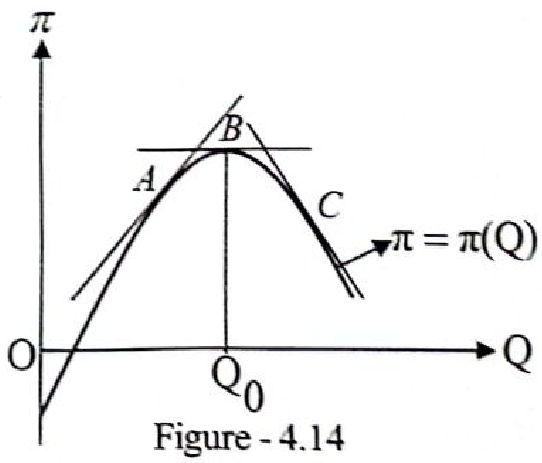

Figure - 4.14

Profit function $\pi = R(Q) - C(Q)$

$$\frac{d\pi}{dQ} = \frac{d}{dQ}R(Q) - \frac{d}{dQ}C(Q)$$

$$\Rightarrow \frac{d\pi}{dQ} = MR - MC$$

Setting slope $\frac{d\pi}{dQ} = 0$, we obtain $MR - MC = 0$; or, $MR = MC$

Thus $MR = MC$ is termed as the first order condition of profit maximization. To examine the second order condition, take a look at figure 4.14. Movements from point A to B to C suggest gradually smaller slope of profit function. This implies a fall in $\frac{d\pi}{dQ}$ with an increase in quantity (Q).

Algebraically, $\frac{d}{dQ}\left(\frac{d\pi}{dQ}\right) < 0$

$$\Rightarrow \frac{d}{dQ}(MR - MC) < 0$$

$$\Rightarrow \frac{d}{dQ}(MR) - \frac{d}{dQ}(MC) < 0$$

$\Rightarrow$ slope of MR $-$ slope of MC < 0

$\Rightarrow$ slope of MR $<$ slople of MC - which is second order condition of profit maximization.

Example 4.4

Consider the demand function $Q = 100 - 2P$

i. Find total revenue, average revenue and marginal revenue as functions of quantity.

ii. Determine elasticity coefficient assuming $Q = 80$.

Solution

i. Given, $Q = 100 - 2P$

$$\Rightarrow 2P = 100 - Q$$

$$\Rightarrow P = 50 - 0.5Q$$

Total revenue, $R = P \times Q = (50 - 0.5Q)Q = 50Q - 0.5Q^2$

Average revenue, $AR = \frac{R}{Q} = \frac{P \times Q}{Q} = P = 50 - 0.5Q$

Marginal revenue, $MR = \frac{dR}{dQ} = \frac{d}{dQ}(50Q - 0.5Q^2) = 50 - Q$

Elasticity coefficient, $e = \frac{AR}{AR - MR} = \frac{50 - 0.5Q}{50 - 0.5Q - 50 + Q}$

$$\text{or, } e = \frac{50 - 0.5Q}{0.5Q}$$

$$\text{Setting } Q = 80, \quad e = \frac{50 - 0.5 \times 80}{0.5 \times 80}$$

$$\text{or, } e = \frac{1}{4}$$

Example 4.5

Prove that MR function is two times steeper than AR function assuming $AR = 90 - 3Q$.

Solution

Given $AR = 90 - 3Q$

$$\text{slope of AR} = \frac{d}{dQ}(AR) = \frac{d}{dQ}(90 - 3Q) = -3$$

$$\text{Total revenue, } R = AR \times Q = (90 - 3Q)Q = 90Q - 3Q^2$$

$$\text{Marginal revenue, } MR = \frac{dR}{dQ} = \frac{d}{dQ}(90Q - 3Q^2) = 90 - 6Q$$

$$\text{slope of MR} = \frac{d}{dQ}(MR) = \frac{d}{dQ}(90 - 6Q) = -6 = 2 \times (-3) = 2 \times \text{slope of AR}$$

We find the slope of MR is two times the slope of AR. Thus MR would be two times steeper than AR. This axiom always holds true provided that AR function appears linear.

4.11 Graphical Representation of AR, MR and Elasticity

In Figure 4.15, MR function is drawn as two times steeper than AR function. Corresponding AR and MR functions are $AR = 90 - 3Q$ and $MR = 90 - 6Q$.

$$e = \frac{AR}{AR - MR}$$

$$\text{or, } e = \frac{90 - 3Q}{90 - 3Q - 90 + 6Q}$$

$$= \frac{90 - 3Q}{3Q}$$

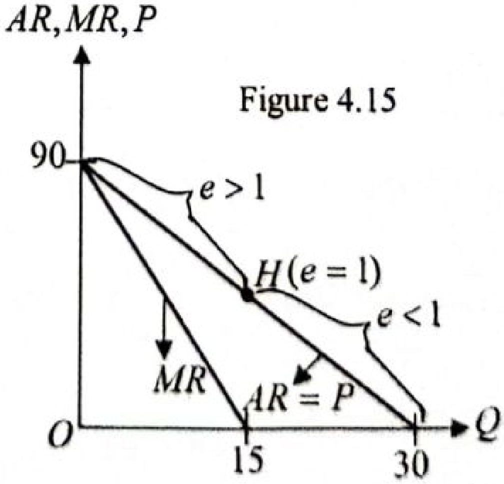

Figure 4.15

At Q=15, $e = \dfrac{90-3Q}{3Q} = \dfrac{90-(3\times 15)}{3\times 15} = 1$,

corresponding point on demand curve in figure 4.15 is H where elasticity is 1. Marginal revenue at this output level is zero. Any quantity below 15 would account elasticity greater than one. For example, if Q is assumed 10,

$e = \dfrac{90-3Q}{3Q} = \dfrac{90-(3\times 10)}{3\times 10} = 2 > 1$. Q < 15 defines elastic range of demand function where marginal revenue remains positive and Q > 15 defines inelastic range of demand and negative marginal revenue.

Example 4.6

Suppose the demand function $P = 100 - 2Q$. At what range of output is demand elastic?

Solution

Given $P = AR = 100 - 2Q$

Total revenue, $R = (P \times Q) = (100 - 2Q)Q = 100Q - 2Q^2$

Marginal revenue, $MR = \dfrac{dR}{dQ} = 100 - 4Q$

Elasticity, $e = \dfrac{AR}{AR - MR} = \dfrac{100-2Q}{100-2Q-100+4Q} = \dfrac{100-2Q}{2Q}$

At elastic range of demand curve, $e > 1$

$$\frac{100-2Q}{2Q} > 1$$

$$\Rightarrow 100 - 2Q > 2Q$$

$$\Rightarrow 100 > 4Q$$

$$\Rightarrow Q < 25$$

Demand would be elastic at any output level below 25.

Example 4.7

Given the demand function $Q = 80 - 2P$. At what range of price is demand elastic?

Solution

$$Q = 80 - 2P$$

$$\Rightarrow \frac{dQ}{dP} = -2$$

Elasticity of demand, $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -2 \times \frac{P}{80 - 2P} = -\frac{2P}{80 - 2P}$

Elasticity coefficient $e = \frac{2P}{80 - 2P}$

Elastic demand is featured as $e > 1$

$$\Rightarrow \frac{2P}{80 - 2P} > 1$$

$$\Rightarrow 2P > 80 - 2P$$

$$\Rightarrow 4P > 80$$

$$\therefore \quad P > 20$$

Demand would be elastic over the range $P > 20$

Example 4.8

Consider two demand functions of two individuals: $Q_1 = 16 - 4P$ & $Q_2 = 20 - 2P$. Calculate point price elasticity at a price of 2 dollars for each individual and for the market.

Solution

Individual I's demand function $Q_1 = 16 - 4P$;

$$\frac{dQ_1}{dP} = -4; \quad Q_1 = 16 - (4 \times 2) = 8$$

Elasticity $\varepsilon_1 = \frac{dQ_1}{dP} \cdot \frac{P}{Q_1} = -4 \times \frac{2}{8} = -1$

Individual II's demand function $Q_2 = 20 - 2P$;

$$\frac{dQ_2}{dP} = -2; \quad Q_2 = 20 - (2 \times 2) = 16$$

Elasticity $\varepsilon_2 = \frac{dQ_2}{dP} \cdot \frac{P}{Q_2} = -2 \times \frac{2}{16} = -\frac{1}{4}$

Market demand function: $Q = Q_1 + Q_2 = 16 - 4P + 20 - 2P$

$$\text{or, } Q = 36 - 6P$$

$$\therefore \frac{dQ}{dP} = -6; \quad Q = 36 - 6 \times 2 = 24$$

$$\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -6 \times \frac{2}{24} = -\frac{1}{2}$$

Example 4.9

i. Determine revenue maximizing price and quantity by assuming the demand function $P = 10000 - 4Q$.

ii. Make a comment on elasticity assuming two different prices, 6000 and 7000 for the demand function in (i).

Solution

i. Total revenue $R = P \times Q = (10000 - 4Q)Q = 10000Q - 4Q^2$

First order condition of revenue maximization $\frac{dR}{dQ} = 0$

$$\text{Here, } \frac{dR}{dQ} = 10000 - 8Q$$

$$\text{Setting } \frac{dR}{dQ} = 0; \quad 10000 - 8Q = 0 \quad \therefore Q = 1250$$

Plug $Q = 1250$ into demand function. $P = 10000 - (4 \times 1250) = 5000$

Thus revenue maximizing price and quantity are 1250 and 5000 respectively.

ii. Plug P = 6000 into the demand equation:

$6000 = 10000 - 4Q; \Rightarrow 4Q = 4000$

$\therefore Q = 1000$

Total revenue $R = P \times Q = 6000 \times 1000 = 6000000$

Then plug P = 7000 into demand equation:

$7000 = 10000 - 4Q; \Rightarrow 4Q = 3000$

$\therefore Q = 750$

Total revenue $R = P \times Q = 7000 \times 750 = 5250000$

An increase in price results in a decrease in total revenue. Therefore, demand is elastic.

Example 4.10

Work out arc elasticity of demand assuming two different prices 50 and 80 for the demand function $P = 100 - \frac{Q}{50}$

Solution

Given $P = 100 - \frac{Q}{50}$; this follows $\frac{Q}{50} = 100 - P$

$$\Rightarrow Q = 5000 - 50P$$

$$\Rightarrow \frac{dQ}{dP} = -50$$

At initial price $P_1 = 50$, demand, $Q_1 = 5000 - 50 \times 50 = 2500$

At a different price $P_2 = 80$, demand, $Q_2 = 5000 - 50 \times 80 = 1000$

Arc elasticity, $\varepsilon_{arc} = \frac{dQ}{dP} \cdot \frac{P_1 + P_2}{Q_1 + Q_2} = -50 \times (\frac{50 + 80}{2500 + 1000}) = -50 \times \frac{130}{3500} = -\frac{13}{7}$

Example 4.11

Estimated demand function of a newly launched product is $Q = 10000 - 200P$. Price is set at 10 dollars per unit. Do you recommend a price reduction to increase sales revenue?

Solution

Given $Q = 10000 - 200P$; for $P = 10, Q = 8000$

$$\frac{dQ}{dP} = -200$$

Elasticity, $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -200 \times \frac{10}{8000} = -\frac{2000}{8000} = -\frac{1}{4}$

Elasticity coefficient $e = \frac{1}{4} < 1$, i.e., demand is inelastic. Thus a reduction in price will reduce revenue. Hence price reduction is not recommended.

Example 4.12

Price and income elasticities of good X are -0.4 and 0.8 respectively. Demand for the good is 5 quintal at a price of 20 dollars and income 20000 dollars.

i. Explain the nature of good on the basis of income elasticity.
ii. If income rises to 25000 dollars, what will be the quantity demanded of good X?
iii. If the price of X rises to 22 dollars per kg at the initial income level, what will be the quantity demanded?

Solution

i. Since income elasticity is positive, the good is normal. Moreover, income elasticity falls below unity therefore the good is necessary.
ii. Initial income = 20000, new income = 25000. Change in income $\Delta M = 5000$. Initial demand = 5 quintal. We have to determine demand. This can be done using income elasticity information.

We know income elasticity, $\eta = \frac{\Delta Q}{\Delta M} . \frac{M}{Q}$

$$\text{or, } 0.8 = \frac{\Delta Q}{5000} \times \frac{20000}{5}$$

$$\therefore \Delta Q = 1$$

This means demand will increase by 1 quintal following the given change in income. Amount of demand for the product would be 5+1=6 quintal.

iii. Initial price = 20 dollars, Changed price = 22 dollars. Change in price, $\Delta P = 2$. We have to compute demand for the good.

Price elasticity, $\varepsilon = \frac{\Delta Q}{\Delta P} . \frac{P}{Q} = -0.4$

$$\text{or, } \frac{\Delta Q}{2} . \frac{20}{5} = -0.4$$

$$\therefore \Delta Q = -0.2$$

Demand will fall by 0.2 quintals following an increase in price from 20 to 22 dollars. New demand for the good would be (5-0.2) = 4.8 quintals.

4.12 Determinants of Price Elasticity

Quantity of demand is said to be more elastic if a certain amount change in price causes a comparatively larger change in quantity of demand. Several factors are responsible for quantity of demand to be more or less responsive to price. Few of them are discussed below.

1. **Nature of Goods**: Change in the price of necessary goods does not cause a considerable change in quantity of demand for the goods because people have to consume the goods unavoidably. Therefore, demand for necessary goods is less elastic. But quantity of demand for luxury items changes considerably following change in price, thus luxury items' demand is more elastic.

2. **Availability of Substitutes:** If substitutes are available then increase in price of a good will cause a sharp reduction in its quantity of demand because people will buy the substitutes at a relatively low price. Thus, in the presence of substitutes, demand is more elastic.

3. **Complementarity Relation:** If one good is complementarily used with another good then change in price does not cause a substantial change in quantity of demand. For example, lubricating oil is complementarily used with fuel. Increase in price of lubricating oil will not cause a considerable change in its quantity of demand as long as use of car remains unchanged. Therefore, demand for lubricating oil is less elastic.

4. **Share in Total Budget:** There are some goods in the consumption bundle of the consumer whose share in total budget is very small. Firebox is used in the kitchen to start up the gas stove. Total spending on firebox per month is very small. Change in the price of such products does not cause substantial changes in their quantity of demand. Hence demand is less elastic.

5. **Postponement Possibility:** Some goods have multiple uses. If some of the uses can be curtailed then increase in price of the good will lead to a sizeable drop in its quantity of demand. For example, milk is used to make a variety of delicious foods. If the consumer can postpone some of the uses of milk for a short period, increase in price of milk will cause a sharp decrease in its quantity of demand. Hence demand is more elastic. But use of some goods cannot be postponed, thereby demand for those goods is inelastic.

6. **Time Span:** There are some goods whose demand is less elastic in the short run but highly elastic in the long run. If price of petroleum increases then car users cannot avoid its use in the short run. But in the long run, they will convert their vehicles into gas-driven ones. As a consequence, long run demand would be more elastic although short run demand is inelastic.

7. **Habit and Addiction:** The individuals who are intrinsically habituated in using some special goods cannot give up the consumption of those goods instantly. For example, the smokers cannot avoid smoking at once even though price of cigarette is doubled. People who use to have special drugs cannot avoid them so quickly. Demand for such good is less elastic.

Exercise 4

1. Consider the demand function $Q = 500 - 4P$. Compute price elasticity of demand assuming $i)$ $P = 100$, $ii)$ $P = 50$ and $ii)$ $P = 20$. At what range of price is demand elastic?

2. Compute elasticity as a function of price from the demand function $P = 500 - 0.5Q$. Find the price at which demand is unitarily elastic.

3. Assume the demand function $Q = \dfrac{1000}{P^3}$. Does elasticity depend on price? What type of demand function is this? Make comment on elaticity coefficient.

4. Sumaiya Publication offers a price reduction from Tk. 400 to Tk. 300 which results in an increase in its volume of sale of a book from 500 to 1000. Is demand for the book elastic?

5. Given the demand function

   $X = 60 - 2P_x + 3P_y + \sqrt{B}$; $\quad (P_x = 10, P_y = 15, B = 625)$

   i) Determine own price, cross price and income elasticity of demand.
   ii) Make comment on elasticity coefficients.

6. Compute own price, cross price and income elasticity of demand from the demand function below.

   $$Q_1 = \frac{1000P_2\sqrt[3]{M}}{P_1P_3} \quad (P_1 = 10,\ P_2 = 20,\ P_3 = 5,\ M = 250000).$$

   Here Q_1 stands for demand for commodity 1. P_1, P_2 and P_3 stand for price of commodity 1, 2 and 3 respectively. M refers to income. Examine the relationship between commodity 1 and 3.

7. Price and income elasticities of good X are -0.4 and 0.8 respectively. Demand for the good is 5 quintal at a price of 20 dollars and income 20000 dollars.
   i. Explain the nature of good on the basis of income elasticity.
   ii. If income rises to 25000 dollars, what will be the quantity demanded of good X?
   iii. If the price of X rises to 22 dollars per kg at the initial income level, what will be the quantity demanded?