CHAPTER 06

Theory of Cost

- Cost Function
- Short Run vs Long Run Cost
- Average Cost and Marginal Cost
- Economies and Diseconomies
- Envelope Curve
- Accounting cost and Economic Cost

6.1 Cost Function

Cost function shows the dependence of total cost (C) on output (Q). Many other factors like the technology, the prices of factors and the government policy may also influence the cost of production. These factors are viewed as the shifting factors.

General form of the cost function is $C = f(Q)$. Total fixed cost (TFC) does not vary with output but total variable cost (TVC) increases with output. At zero output there will have only fixed cost. If output increases, total variable cost initially increases at a decreasing rate then at an increasing rate. Total cost is the sum of total fixed cost and total variable cost, i.e., TC=TFC+TVC.

TFC, TVC and TC

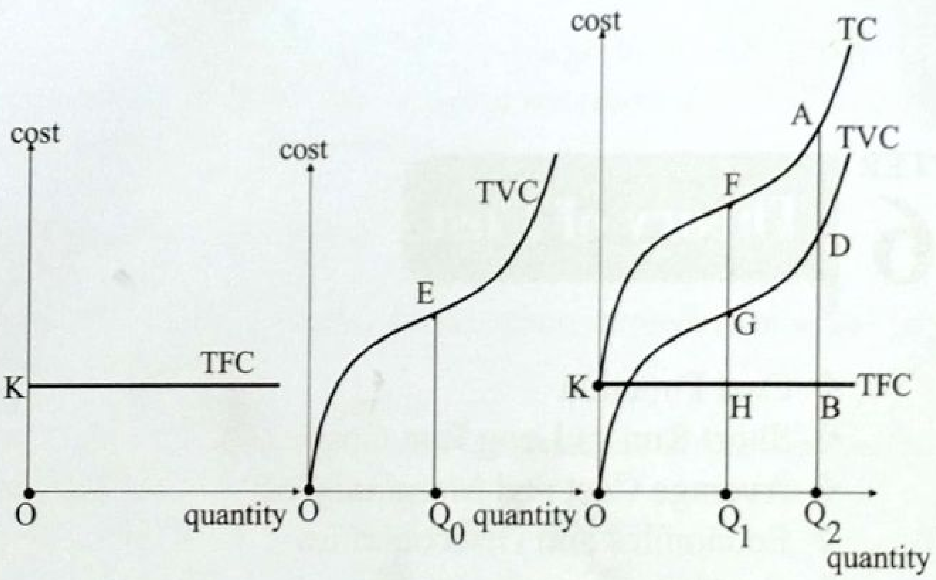


Figure- 6.1

Figure 6.1 describes the shapes of fixed cost and variable cost. In the left panel TFC is drawn as a horizontal line because of the constancy of the fixed cost. In the middle panel the total variable cost starts from the point of origin because at zero output variable cost is also zero. Total variable cost increases at a decreasing rate in the output range from zero to Q_0. When output exceeds Q_0 the variable cost increases at an increasing rate. Total cost is obtained by summing the fixed cost and the variable cost.

At zero output, TFC = OK, TVC = 0; TC = OK+0 = OK ; K is the first point of total TC in the right panel of Figure 6.1

At OQ_1 output, TFC = Q_1H, TVC = Q_1G;
TC = $Q_1H+Q_1G = Q_1F$; (Here, $Q_1H=GF$)

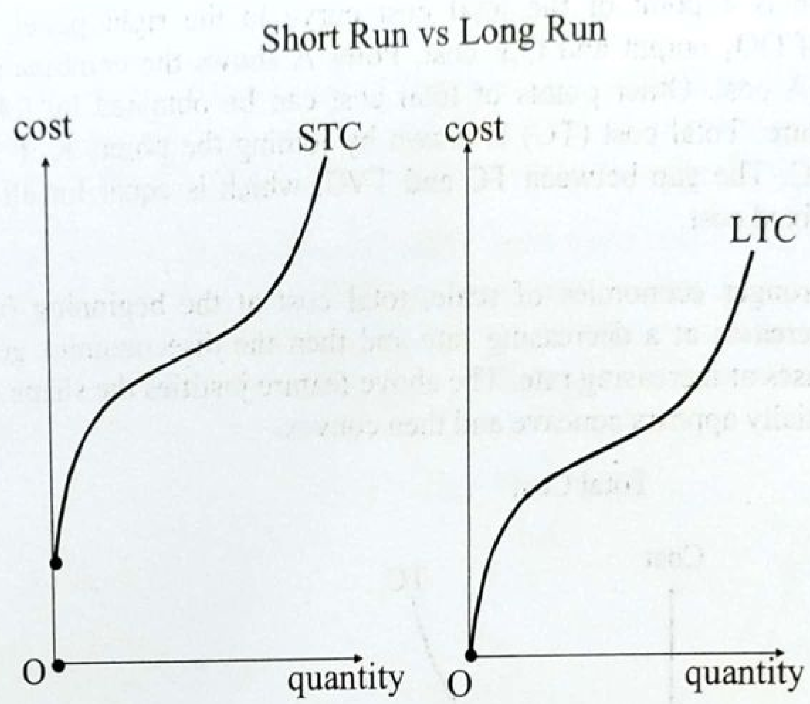


Figure - 6.3

Figure 6.3 clarifies the difference between short run total cost (STC) and long run total cost (LTC). Intercept of STC in the left panel reflects the fixed cost. LTC, in the right panel, has no fixed cost and thus emanates from the point of origin.

6.3 Average Cost (AC)

Total cost per unit of output is called average cost. If, for example, total cost of producing 5 units of output is 80 dollars, average cost is 80/5=16 dollars.

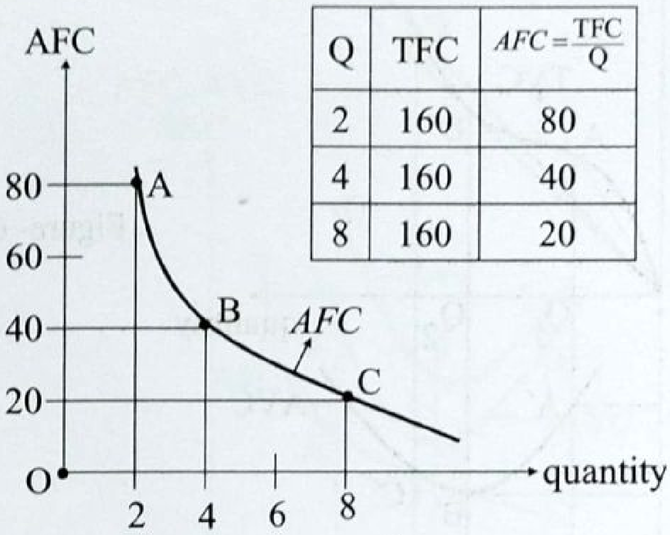
$$\text{Average cost, } AC = \frac{TC}{Q}$$

$$\text{Short run average cost, } SAC = \frac{STC}{Q} = \frac{TFC + TVC}{Q} = \frac{TFC}{Q} + \frac{TVC}{Q} = AFC + AVC$$

$$\text{i.e, } SAC = AFC + AVC$$

The fixed cost per unit of output measures average fixed cost. Suppose total fixed cost (TFC) of production is 160 thousand Pounds. Average fixed cost (AFC) of producing 10 units of output would be 16 thousand Pounds. If output is increased to 20 units, AFC would be 8 thousand Pounds. AFC will keep decreasing with the increase in output.

Average Fixed Cost



Q	TFC	$AFC = \frac{TFC}{Q}$
2	160	80
4	160	40
8	160	20

Figure- 6.4

AFC is drawn in Figure 6.4 by assuming total fixed cost equal to 160. Average fixed costs for 2, 4 and 8 units of output are 80, 40 and 20 respectively. Corresponding points are A, B and C. By joining these points AFC is drawn. At all points of AFC, the rectangles have equal area. This follows from the fact that each rectangle's length measures AFC and width quantity. AFC multiplied by quantity is TFC, which is throughout constant. That means, each rectangle has identical area. Thus, AFC is rectangular hyperbolic.

The variable cost per unit of output is called average variable cost. Average variable cost (AVC) primarily falls with output and beyond a certain level of output AVC rises, eventually AVC is 'U'-shaped.

Since $AVC = \frac{TVC}{Q}$, AVC would be the slopes of the rays emanating from the point of origin and meeting TVC at corresponding output levels. We observe successive rays at the beginning get flatter and flatter but subsequently they become steeper, referring to decreasing nature of AVC at the early stage and increasing nature afterwards. Figure 6.5 illustrates the approach of obtaining AVC from TVC.

Average Variable Cost

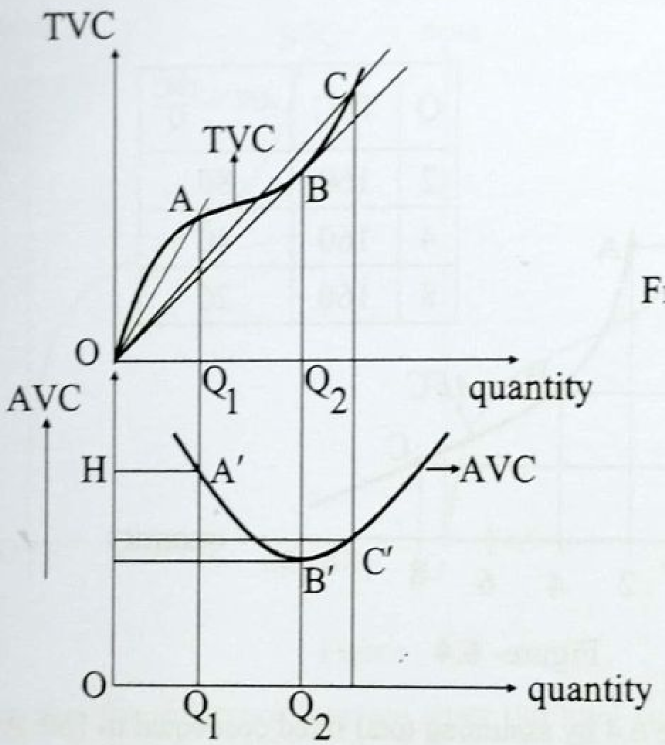


Figure- 6.5

At OQ_1 output, $TVC=Q_1A$, $AVC = \frac{TVC}{Q} = \frac{Q_1A}{OQ_1}$ = slope of OA. The slope of OA is measured by the vertical distance OH in the lower panel of Figure 6.5. Point A′ shows the combination of OQ_1 output and OH amount of average variable cost. Other points of AVC, namely B′ and C′ are obtained by applying the same technique. By joining all possible points, AVC is found 'U'-shaped.

The vertical summation of AFC and AVC yields short run average cost (SAC). Figure 6.6 provides the method of finding SAC from AFC and AVC.

Suppose, output = OQ_1

$AFC = Q_1A$

$AVC = Q_1B$

$SAC = Q_1A+Q_1B = Q_1C$; $(Q_1A=BC)$

C is one point of SAC.

When output = OQ_2; $AFC=Q_2D$; $AVC=Q_2E$;

$SAC=Q_2D+Q_2E=Q_2F$; $(Q_2D=EF)$

Short Run Average Cost

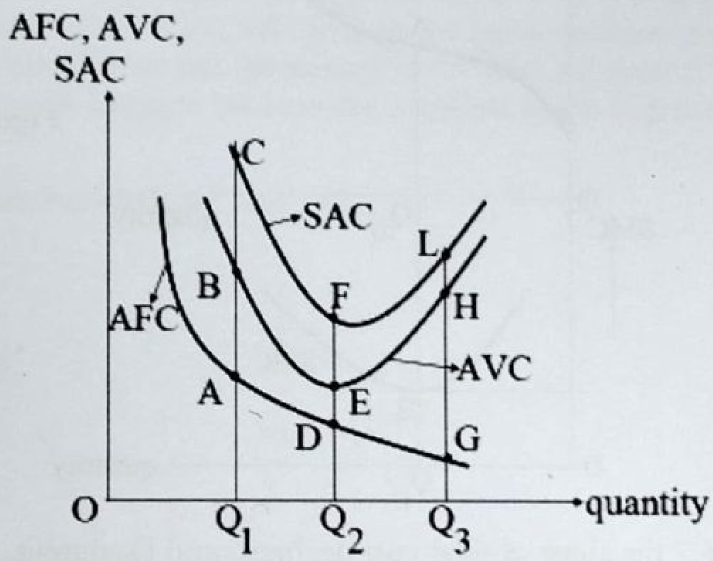


Figure- 6.6

F is another point of SAC. Similarly, L is the point of SAC. By joining points like C, F and L, the short run average cost (SAC) is found. SAC is also 'U'-shaped as the AVC. Difference between SAC and AVC stands for AFC which is ever decreasing. As a consequence, SAC and AVC seem to get closer, but they never meet because AFC never vanishes.

6.4 Marginal Cost

Marginal cost is the change in total cost due to one unit change in output. Given the total cost function $C = f(Q)$, marginal cost $MC = \frac{dC}{dQ}$. The slope of total cost geometrically measures marginal cost. Along the concave segment of the total cost curve, slope decreases and so does marginal cost. Afterwards slope of total cost, hence amount of marginal cost, increases.

Short Run Marginal Cost

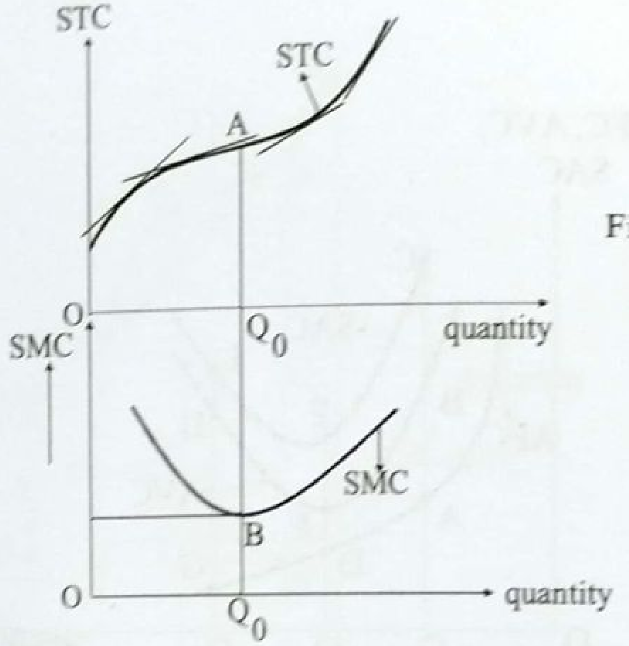


Figure- 6.7

In Figure 6.7 the slope of total cost declines until Q_0 output, due to which marginal cost (MC) curve slopes downward. When output exceeds Q_0, slope of total cost increases thus marginal cost increases and MC curve slopes upward. Marginal cost (MC) curve is also 'U'-shaped as the AC curve. AC and MC, however, maintain a specified relationship.

6.5 Relation between AC and MC

Three points become worth mentioning while describing the relationship between AC and MC.

First, if the amount of marginal cost remains smaller than average cost then average cost falls. This can alternatively be stated as when AC falls, MC stays below AC. Note that no matter whether marginal cost rises or falls, AC will certainly fall if MC is smaller than AC.
Second, at the minimum of average cost, it is equal to the marginal cost, and finally average cost increases if the amount of marginal cost is higher than average cost.

Figure 6.8 explains the relationship between AC and MC. Until Q_0 output MC stays below AC thus AC falls. At Q_0 output AC reaches minimum where AC=MC. When output exceeds Q_0, MC lies above AC, therefore AC rises.

In fact, the direction of the average is determined by the marginal. Suppose a student's average grade (CGPA) after the completion of 2nd semester is 3.8. If in third semester he secures GPA 3.5, which is marginal score, then his average

(CGPA) will fall. Fall in average is caused by lower marginal grade than the average grade. Suppose, after 3rd semester his average (CGPA) is 3.7. If in 4th semester he scores 3.6, again his average grade will fall though marginal increased. This is because his marginal score is still below average score. Suppose the student's average grade (CGPA) after the completion of 4th semester is 3.68. If in 5th semester he scores 3.68 then his average GPA will remain the same because marginal and average are the same. If in 6th semester the student earns GPA 3.9 then, of course, his average grade (CGPA) will increase because the marginal grade is greater than the average grade.

Relation between AC and MC

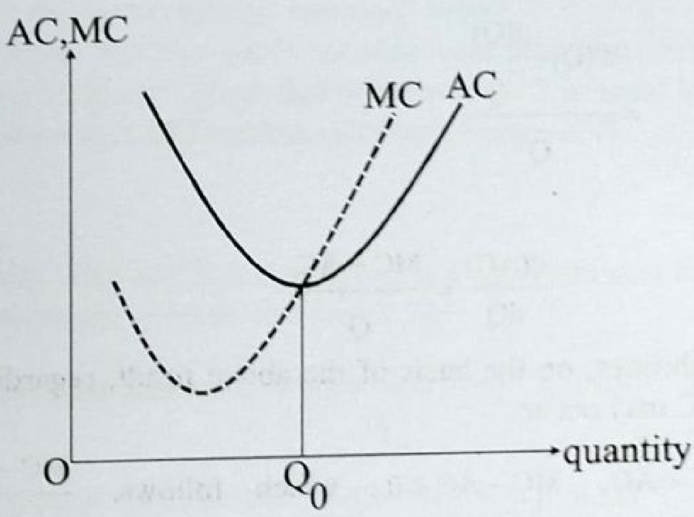


Figure- 6.8

The above example clears how the marginal value influences the average value to change. All real life average and marginal functions maintain the same nature of relation. The relation between average cost and marginal cost has been further verified below by means of calculus.

Assume total cost function: $C = f(Q)$

$$\text{Marginal cost: } MC = \frac{dC}{dQ} = f'(Q)$$

$$\text{Average cost: } AC = \frac{C}{Q} = \frac{f(Q)}{Q}$$

$$\text{Slope of average cost: } \frac{d(AC)}{dQ} = \frac{d}{dQ}\frac{f(Q)}{Q}$$

$$= \frac{Qf'(Q) - f(Q)}{Q^2}$$

$$= \frac{Q\left[f'(Q) - \dfrac{f(Q)}{Q}\right]}{Q^2}$$

$$= \frac{f'(Q) - \dfrac{f(Q)}{Q}}{Q}$$

$$\therefore \frac{d(AC)}{dQ} = \frac{MC - AC}{Q}$$

Three distinct possibilities, on the basis of the above result, regarding the relation between AC and MC may occur.

i. If $MC < AC$, $MC - AC < 0$, which follows, $\frac{MC - AC}{Q} < 0$, thus $\frac{d(AC)}{dQ} < 0$, i.e., slope of AC is negative, i.e., AC declines. This proves that when MC is smaller than AC, AC falls.

ii. At minimum point of AC, $\frac{d(AC)}{dQ} = 0$

$$\text{or, } \frac{MC - AC}{Q} = 0$$

$$\text{or, } MC - AC = 0$$

$$\text{or, } MC = AC$$

This refers, when AC reaches minimum, AC=MC.

iii. If $MC > AC$, $MC - AC > 0$; or, $\frac{MC - AC}{Q} > 0$, thus $\frac{d(AC)}{dQ} > 0$, i.e., slope of AC is positive, i.e., AC rises. This proves that when MC is greater than AC, AC rises.

Example 6.1

Consider the cost function

$$C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$$

i. Is this short run cost function? Why?
ii. Write out the total variable cost function. Find minimum average variable cost. Show that minimum AVC is equal to MC.
iii. Show that MC reaches minimum before AVC.

Solution

i. Yes. The cost function given is a short run cost function because of the existence of fixed cost worth 50.

ii. Total variable cost, $TVC = \frac{1}{3}Q^3 - 7Q^2 + 111Q$

Average variable cost $AVC = \frac{1}{3}Q^2 - 7Q + 111$

$$\therefore \frac{d(AVC)}{dQ} = \frac{2}{3}Q - 7$$

First order condition of minimum AVC is: $\frac{d(AVC)}{dQ} = 0$

$$\text{or, } \frac{2}{3}Q - 7 = 0$$

$$\text{or, } \frac{2}{3}Q = 7$$

$$\therefore Q = 7 \times \frac{3}{2} = 10.5$$

$\frac{d^2(AVC)}{dQ^2} = \frac{2}{3} > 0$, which is second order condition of AVC minimization.

Therefore, at $Q = 10.5$ AVC is minimum.

Plug $Q = 10.5$ into AVC function.

$$\text{Minimum AVC} = \frac{1}{3} \times (10.5)^2 - (7 \times 10.5) + 111$$
$$= 36.75 - 73.5 + 111 = 74.25$$

Cost function: $C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$

Marginal cost: $MC = \frac{dC}{dQ} = Q^2 - 14Q + 111$

Plug $Q = 10.5$ into the MC function.

$$MC = (10.5)^2 - (14 \times 10.5) + 111 = 74.25$$

It proves that minimum AVC is equal to MC.

iii. We got, $MC = Q^2 - 14Q + 111$; $\therefore \frac{d(MC)}{dQ} = 2Q - 14$

First order condition of minimum MC is: $\frac{d(MC)}{dQ} = 0$

$$or,\ 2Q - 14 = 0$$
$$\therefore Q = 7$$

$$\frac{d^2(MC)}{dQ^2} = 2 > 0$$

$\therefore$ At $Q = 7$ MC is minimum.

We observe that MC reaches minimum when $Q = 7$ and AVC reaches minimum when $Q = 10.5$. This suggests, MC reaches minimum before AC.

Example 6.2

Long run cost function $C(Q) = Q^3 - 50Q^2 + 1000Q$. Show that MC and AC are equal at minimum AC.

Solution

Average cost $AC = \frac{C(Q)}{Q} = \frac{Q^3 - 50Q^2 + 1000Q}{Q} = Q^2 - 50Q + 1000$

$$\therefore \frac{d(AC)}{dQ} = 2Q - 50$$

First order condition of minimum AC is: $\frac{d(AC)}{dQ} = 0$

$$\text{or,}\ \ 2Q - 50 = 0$$

$$\therefore Q = 25$$

$\frac{d^2(AC)}{dQ^2} = 2 > 0$. Thus, at $Q = 25$, AC is minimum.

Minimum $AC = (25)^2 - (50 \times 25) + 1000 = 625 - 1250 + 1000 = 375$

Given $C(Q) = Q^3 - 50Q^2 + 1000Q$; $MC = \frac{dC}{dQ} = 3Q^2 - 100Q + 1000$

Plug $Q = 25$ into MC function: $MC = (3 \times 25^2) - (100 \times 25) + 1000 = 375$
Thus, minimum AC is equal to MC.

6.6 Long-Run Average Cost (LAC)

Long-run average cost is derived from the short-run average cost (SAC). Short run average cost curves are called plant curve. In long run planning, firms get sufficient time to change plant size if they want to expand output.
Firm should not change plant size at a low rate of output because this rather increases unit cost. According to Fgure 6.9 if, for example, firm produces Q_1 output by using SAC_2 plant then per unit cost would be Q_1E whereas use of SAC_1 plant will involve only Q_1D cost. Therefore, best decision for the firm is to use SAC_1 plant until Q_2 output. If the firm wants to produce output above Q_2 then it should switch from SAC_1 to SAC_2 plant, which will lower average cost of production.

Long Run Average Cost

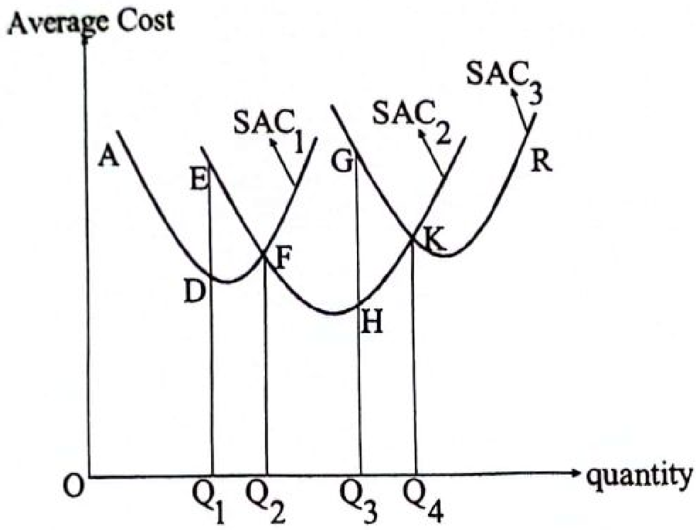


Figure- 6.9

Efficient firm should not move onto SAC_3 unless output exceeds Q_4. Careful look at Figure 6.9 clears the fact that the segment of SAC_2 on the left of point F and the segment of SAC_1 on the right of point F are irrelevant. For similar reasoning, segment of SAC_3 on the left of point K would be of no use. If one thinks of the shape of long-run average cost curve then the points A, D, F of SAC_1; F, H, K of SAC_2 and K, R of SAC_3 should be the relevant points.

Long Run Average Cost

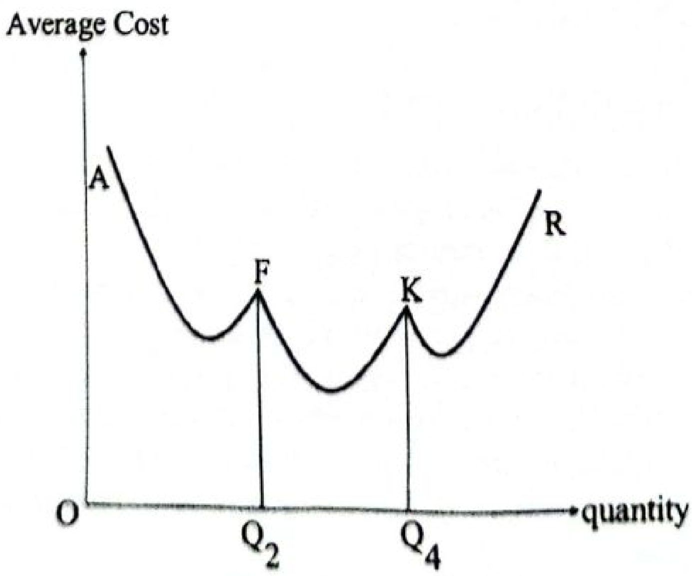


Figure- 6.10

Figure 6.10 is the primary shape of long run average cost which is drawn by taking the usable portions of SACs. Consideration of only three SACs yields an LAC that looks like a wave. But there may be hundreds of plants available to the firm. Figure 6.11 exhibits a pool of plants. If all possible plants are taken into account then LAC would appear as a smooth U-shaped curve as Figure 6.12. Note that every point of LAC corresponds one SAC. Virtually the LAC envelopes the SACs, for which LAC is called an envelope curve. As stated earlier, SAC is known as plant curve. Of all possible plants, the one that stays at the minimum point of LAC is called optimum plant. SACs that remain on the left of minimum point of LAC are less than optimum plants and on the right more than optimum plants.

Collection of Plants

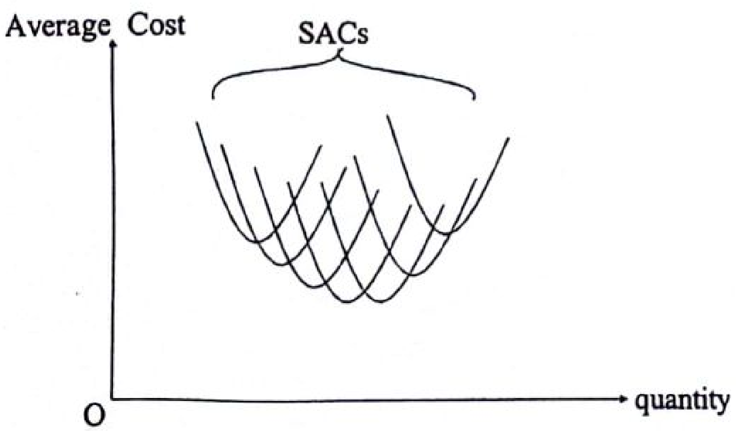


Figure- 6.11

Long Run Average Cost

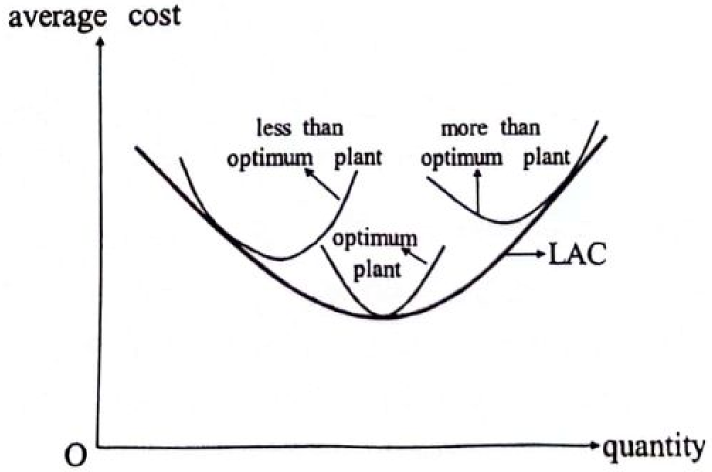
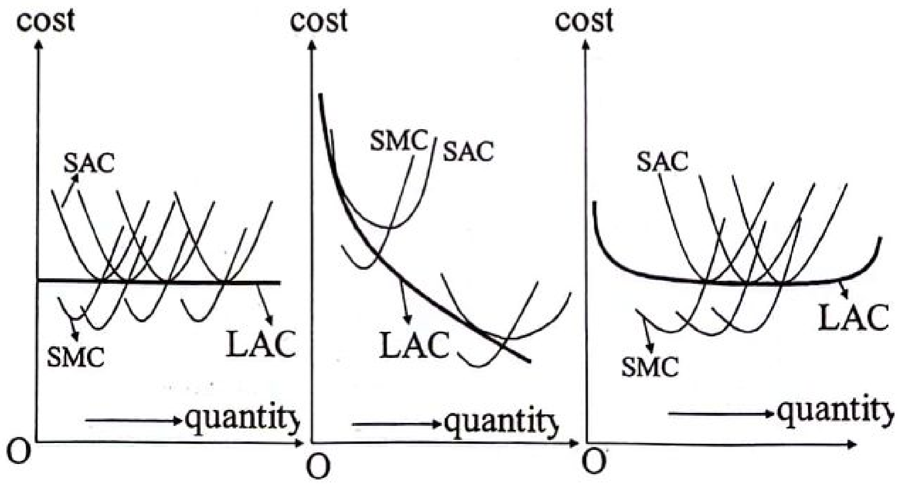


Figure- 6.12

Three types of plants are sketched in Figure 6.12. Firm's long run planning is made on the basis of LAC, therefore LAC is called planning curve. Note that U-shape of LAC is attributable to the fundamental microeconomic intuition, which justifies that the producer enjoys economies of scale at the beginning of production and thus average cost of production falls. Gradually the economies of scale are absorbed and the diseconomies become stronger due to which average cost of production increases. If the economies are equally counterbalanced by the diseconomies then the LAC would be horizontal.

Possible Shapes of LAC



Horizontal LAC Downward LAC Saucer shaped LAC

Figure- 6.13

Left panel of figure 6.13 demonstrates horizontal LAC. In this situation all plants are optimum. Unlike SAC, LAC can be throughout downward or saucer shaped.

6.7 Accounting Cost and Economic Cost

Accounting cost refers to the visible costs or explicit costs of production. Wages, rent, interest, advertisement, transportation, utility charges are some examples of explicit cost because these costs are visible and directly computable. Economic cost is derived by adding the implicit cost to the explicit cost. Implicit cost mainly refers to the opportunity cost. Suppose an entrepreneur uses his own house for running business. In lieu of using the house for business purposes he could rent out the house and gain money. The money that is foregone through the use of the house for business purposes than renting out is the opportunity cost of using the house for business. By adding this cost to other explicit costs, economic cost is computed. Implicit costs may take many other forms. As a separate example, consider an economics graduate who rejects a job offer to set a commercial plant. The set-up requires various costs that are visible and thus they constitute accounting cost. However, the income that is lost because of the rejection of the job offer is the implicit cost of the business. Adding this cost to other visible costs gives rise to the economic cost.

Example 6.3

An individual rejects a job offer of Tk. 20,000 per month to run his business. The person uses his building that could have earned Tk. 1,20,000 as rent per year. Other annual costs of production are as below.

Transportation	50,000 Taka
Advertisement	12,000 Taka
Wage bills	1,50,000 Taka
Utility charges	25,000 Taka

Compute accounting cost and economic cost.

Solution

Accounting cost = Explicit cost

= Transportation + Advertisement + Wage + Utility

= 50,000 + 12,000 + 1,50,000 + 25,000

= 2,37,000

Implicit cost = salary forgone + rent lost

= (20,000 X12) + 1,20,000

= 2,40,000 + 1,20,000

= 3,60,000

Economic Cost = Accounting cost + Implicit cost

= 2,37,000 + 3,60,000

= 5,97,000

Exercise 6

1. Assume the total cost function $C = 100 + 5Q^2$.
   i) Figure out fixed cost and variable cost.
   ii) Is it a short-run cost function? Why?
   iii) Draw the average fixed cost (AFC) and average variable cost (AVC) curves corresponding to the above cost function.
   iv) Show that the marginal cost curve is linear and upward.

2. Given the cost function $C = Q^3 - 2Q^2 + 3Q + 500$
   i) Determine TVC, AVC and MC.
   ii) What is the minimum amount of AVC?
   iii) Show that at the minimum of AVC, AVC = MC.

3. Given the long run total cost function $C = Q^3 - 61Q^2 + 1528Q$. Prove that at the minimum point of average cost, LAC=LMC.

4. Suppose total cost function: $C = \frac{1}{3}Q^3 - 5Q^2 + 100Q$. Find the rate of output at which marginal cost reaches minimum. Is this rate of output larger than the average cost minimizing rate of output?

5. Suppose a recent graduate rejects a job offer of Tk. 50,000 per month and invests a sum of Tk. 100,000 that could earn 12% interest each year. Other expenses are:

Advertising	Tk.12,000
Depreciation	Tk.10,000
Utilities	Tk.6,000
Salaries	Tk.60,000
Taxes and other expenses	Tk.10,000

Compute accounting cost and economic cost.