Electrical Circuit Analysis

Terms of Electricity

Voltage: Voltage is the electrical pressure, a potential force or difference in the electrical charge between two points. Voltage is measured in volts(V).

Current: Current is the electrical flow moving through a wire. Current flow in a wire pushed by voltage. The following equation relates charge to current.

$$I = \frac{Q}{t}$$

Where, I= current, Q= charge, t= time

Resistance: Resistance opposes current flow. It is like electrical "friction". This resistance slows the current flow. Every electrical component or element has resistance. And this resistance changes electrical energy into another form of energy-heat, light, motion. Resistance is measured in ohms (Ω).

The resistance of an element is determined from the resistivity (ρ), length(L), and cross-sectional area (A).

$$R = \frac{\rho L}{A}$$

Electrical Circuit Elements (Components)

Electronic elements that make up a circuit are connected together by conductors to form a complete circuit. If these connecting conductors are ideal conductors (i.e. they have no resistance) then all parts of the circuit can be classified into two main categories depending on whether they deliver or absorb energy from the circuit:

- Active components
- Passive components

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Active Component

An active component is an electronic component which supplies energy to a circuit. Active elements have the ability to electrically control electron flow (i.e. the flow of charge). All electronic circuits must contain at least one active component.

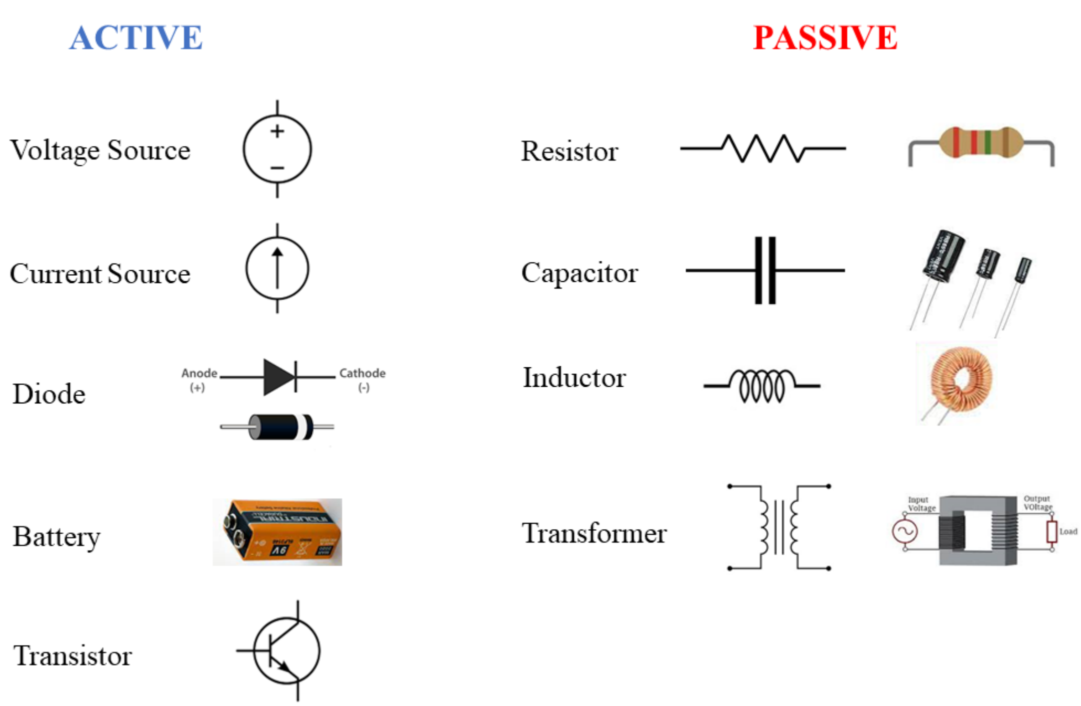
Common examples of active components include:

- Voltage sources
- Current sources
- Generators
- Transistors
- Diodes

Question: Why transistor is an active element?

Ans: Although not as obvious as a current or voltage source – transistors are also an active circuit component. This is because transistors are able to amplify the power of a signal.

As this amplification is essentially controlling the flow of charge – transistors are hence classified as an active component.



Passive Components

A passive component is an electronic component which can only receive energy, which it can either dissipate, absorb or store it in an electric field or a magnetic field. Passive elements do not need any form of electrical power to operate.

As the name 'passive' suggests – passive devices do not provide gain or amplification. Passive components cannot amplify, oscillate, or generate an electrical signal.

Common examples of passive components include:

- Resistors
- Inductors
- Capacitors
- Transformers

Question: Why transformer is a passive element?

Ans: A transformer is a passive electronic component. Although this can seem surprising since transformers are often used to raise voltage levels – remember that power is kept constant.

When transformers step up (or step down) voltage, power and energy remain the same on the primary and secondary side. As energy is not actually being amplified – a transformer is classified as a passive element.

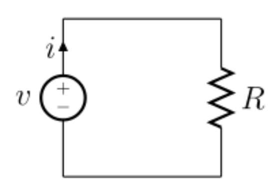
Question: Explain active and passive elements in electrical circuit with appropriate examples.

Electrical Network

Any interconnection of electric circuit elements or components is called as networks. There are different types of networks like lumped network, distributed network, linear network, active and passive network.

Active network

A network consisting of active elements such as op-amps, transistors, along with other elements is called an active network. An active element is an electronic component that supplies energy to a circuit.



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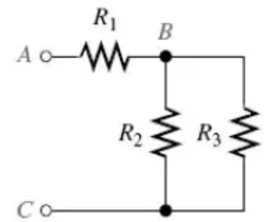
Passive network

A network consisting of passive elements only such as resistors, capacitors and inductors in known as passive network. A passive element is an electrical component that does not generate power, instead dissipates or stores it.



Question: Explain Active and passive network with appropriate circuit diagram.

Ohm's Law

Ohm's law states that at constant temperature the current through a conductor between two points is directly proportional to the voltage across the two points.

Mathematically, Ohm's law can be expressed as,

$$I \alpha V$$

Introducing the constant of proportionality, the conductivity G in the above equation, we get,

$$I = GV$$

$$\text{Or, } I = \frac{1}{R}V$$

$$\text{Or, } V = IR$$

Where,

- R is the resistance of the conductor in Ohm (Ω),
- G is the conductivity of the conductor in Siemens (S),
- I is the current through the conductor in Amperes (A),
- V is the voltage or potential difference measured across the conductor in Volts (V).

Ohm's law is applicable to both DC and AC.

Conductance: Conductance is the ability of an element to conduct electric current; it is measured in siemens (S).

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Plotting Ohm's Law

Ohm's Law tells us that if a conductor is at a constant temperature, the current flowing through the conductor is directly proportional to the voltage across it. This means that if we plot voltage on the x-axis of a graph and current on the y-axis of the graph, we will get a straight-line.

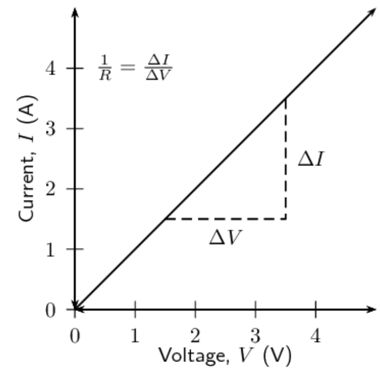

Figure 1: The I-V characteristics graph of a linear resistor.

Where, $slope = \frac{1}{R}$

Question: Explain Ohm's law with appropriate graph.

Dissipated Power

The power at the terminals of a resistor is the product of the terminal voltage and current.

$$P = VI$$

Power in a resistor in terms of current

$$P = I^2R$$

Power in a resistor in terms of voltage

$$P = \frac{V^2}{R}$$

Problem 1: The 560 Ω resistor is connected to a circuit which causes a current of 42.4 mA to flow through it. Calculate the voltage across the resistor and the power it is dissipating.

Solution: The voltage across the resistor is given by Ohm's law

$$V = IR = 0.0424 * 560 = 23.7\, V$$

The dissipated power can be calculated in several different ways. For instance,

$$P = VI = 23.7 * 0.0424 = 1.005\, W$$

Alternatively,

$$P = \frac{V^2}{R} = \frac{23.7^2}{560} = 1.003\, W$$

Or,

$$P = I^2R = 0.0424^2 * 560 = 1.003\, W$$

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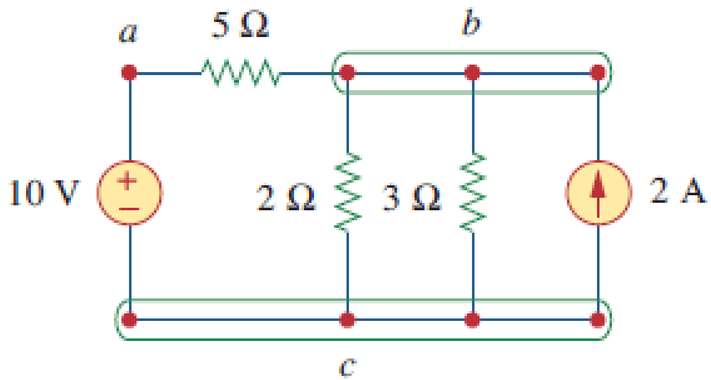
Nodes, Branches, Loops

Branch: A branch represents a single element such as a voltage source or a resistor.

In other words, a branch represents any two-terminal element. The circuit in Fig. 2 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.



Node: A node is the point of connection between two or more branches.

A node is usually indicated by a dot in a circuit. If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node. The circuit in Fig. 2. has three nodes a, b, and *c*.

Loop: A loop is any closed path in a circuit.

Kirchhoff's Law

The conservation of energy and conservation of charge when applied to electrical circuits are known as Kirchhoff's law. These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

Kirchhoff's First Law: Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

The total current or charge entering into a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node.

In other words,

The algebraic sum of all the currents at any node in a circuit equals zero.

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This law represents a mathematical statement of the fact that charge cannot accumulate at a node. *A node is not a circuit element*, and it certainly cannot store, destroy, or generate charge.

Mathematically, KCL implies that

$$\sum I_{entering} = \sum I_{leaving}$$

$$\text{Or, } \sum I_{total} = 0$$

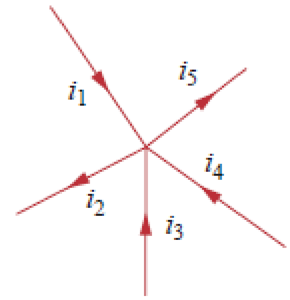

Figure 2: Currents at a node illustrating KCL.

Consider the node in Fig. 2. Applying KCL gives

$$i_1 + i_3 + i_4 = i_2 + i_5$$

Or,

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

Kirchhoff's Voltage Law: Kirchhoff's voltage law is based on the principle of conservation of energy. This requires that the total work done in taking a unit positive charge around a closed path and ending up at the original point is zero.

The algebraic sum of the voltages around any closed path is zero.

In other words,

In any closed loop network, the total voltage around the loop is equal to the sum of all voltage drops within the same loop.

Expressed mathematically, KVL states that

$$\sum V = 0$$

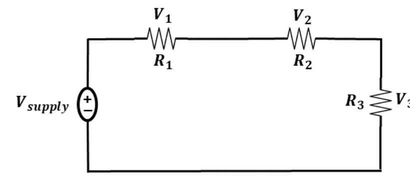

Figure 3: A single loop circuit illustrating KVL.

Or, it may be interpreted as

$$Sum\ of\ voltage\ drops = Sum\ of\ voltage\ rises$$

Applying KVL in Fig. 3,

$$V_{supply} = V_1 + V_2 + V_3$$

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Question: Explain Kirchhoff's current law and voltage law with proper figure.

Current Source: A current source is a simple circuit, which will provide a current which remains constant regardless of the load placed at its output.

Or,

A current source is a source that maintains the current at a particular value almost independent of the load conditions.

$$i = \frac{V}{R+r}, R \gg r$$

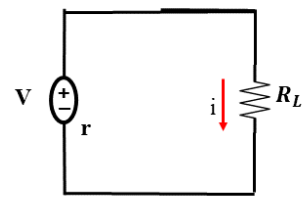

Figure 4: Voltage source

In voltage source, internal resistance remains almost zero.

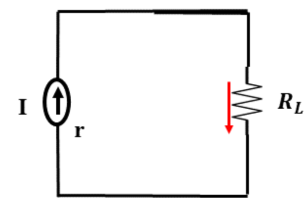

Figure 5: Current Source.

When, r >>R; R+r = r;

$$I = \frac{V}{r}$$

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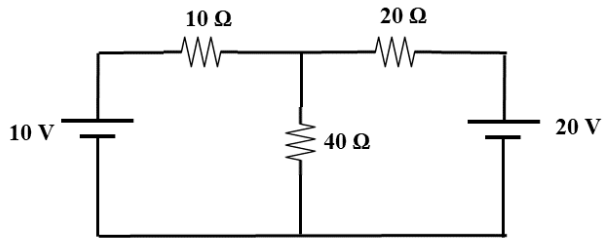
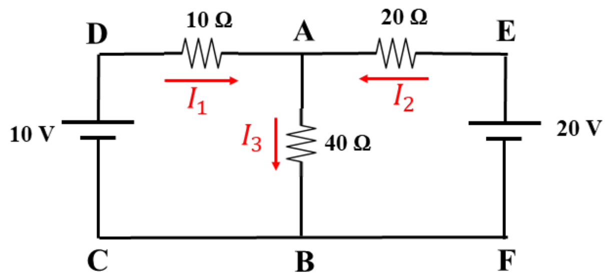
Problem 2: Find the current flowing in the 40Ω resistor in the following figure shown in below.



Solution:



Applying KCL at node A : $I_1 + I_2 = I_3$ (1)

Applying KVL at loop DABCD: $10I_1 + 40I_3 - 10 = 0$ (2)

Applying KVL at loop EABFE: $20I_2 + 40I_3 - 20 = 0$ (3)

Using equ^n (1) in equ^n (2), we get

$$10I_1 + 40(I_1 + I_2) - 10 = 0$$

$$Or,\ 50I_1 + 40I_2 - 10 = 0 \quad (4)$$

Using equ^n (1) in equ^n (3), we get

$$20I_2 + 40(I_1 + I_2) - 20 = 0$$

$$Or, \quad 40I_1 + 60I_2 - 20 = 0 \quad (5)$$

Now, Applying 3 X (4) – 2 X (5), we get

$$70I_1 + 10 = 0$$

$$Or, I_1 = -\frac{1}{7}\ A$$

Putting the value of I_1 into equation (4)

$$40\left(-\frac{1}{7}\right) + 40I_2 - 10 = 0$$

$$I_2 = 0.429\ A$$

From equation (1)

$$I_3 = -0.143 + 0.429 = 0.286\ A$$

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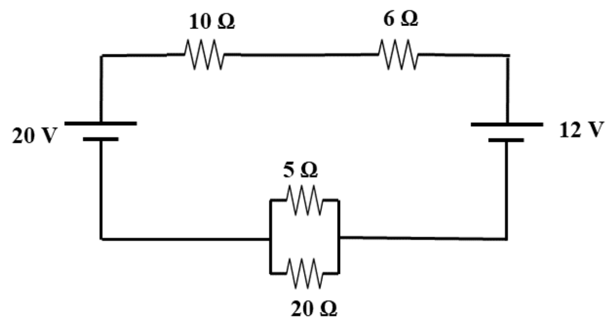
Problem 3: Determine the electric current that flows in circuit as shown in Figure below.

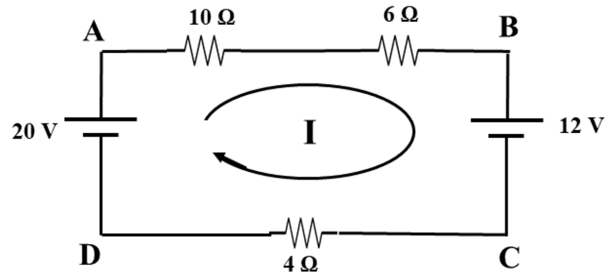


Solution:

5 Ω and 20 Ω are connected in parallel. The equivalent resistance

$$\frac{1}{R_p} = \frac{1}{5} + \frac{1}{20}$$

Or, $R_p = 4\ \Omega$



Applying KVL at loop ABCDA

$$10I + 6I + 12 + 4I - 20 = 0$$

Or, $20I = 8$

Or, $I = 0.4$ A

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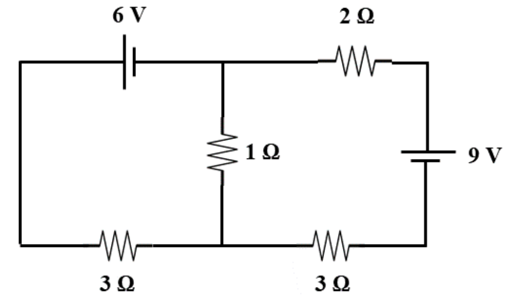
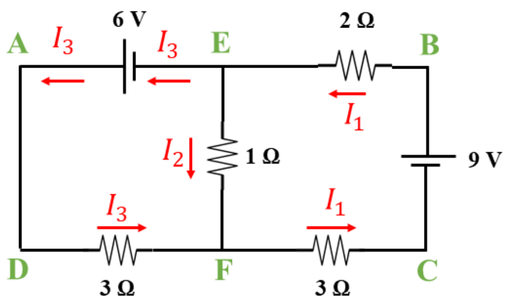
Problem 4: Calculate the current that flows in the 1Ω resistor in the following circuit.



Solution:



We can denote the current that flows from 9V battery in I_1 and it splits into I_2 and I_3 in the junction.

Applying KCL at point E,

$$I_1 = I_2 + I_3 \qquad (1)$$

Now, consider the loop EFCBE and apply KVL,

$$I_2 + 3I_1 - 9 + 2I_1 = 0$$

$$Or, 5\, I_1 + I_2 = 9 \qquad (2)$$

Applying KVL at the loop EADFE,

$$-6 + 3\, I_3 - I_2 = 0$$

$$Or, \quad -6 + 3\,(I_1 - I_2) - I_2 = 0 \quad [From\ eq. (1)]$$

$$Or, -6 + 3I_1 - 3I_2 - I_2 = 0$$

$$Or, 3I_1 - 4I_2 = 6 \qquad (3)$$

Solving equation (2) and (3), we get

$$I_1 = 1.83\ A$$

$$I_2 = -0.13\ A$$

It implies that the current in the 1Ω resistor flows from F to E.

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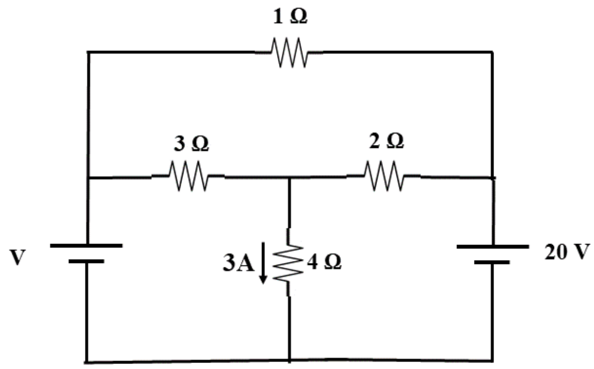
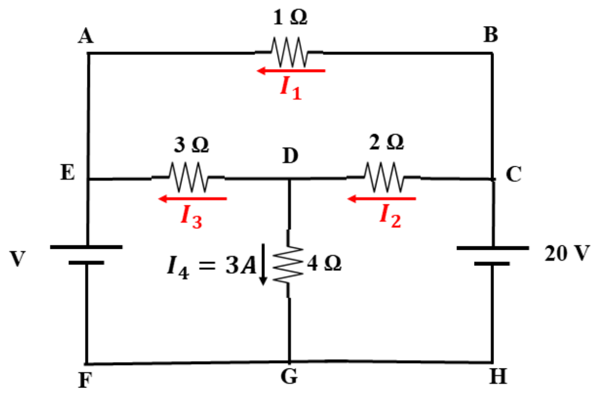
Problem 5: Given the circuit below with 3A of current running through the 4Ω resistor as indicated in the diagram. Determine

a) The current through each of the other resistors.
b) The voltage of the battery on the left.



Solution:



a) Apply KVL in the loop CDGHC

$$2I_2 + 4I_4 - 20 = 0$$

$$Or, \quad 2I_2 + 4 \times 3 - 20 = 0$$

$$Or, \quad 2I_2 = 8$$

$$Or, \quad I_2 = 4A$$

Apply KCL at point D

$$I_2 = I_3 + I_4$$

$$Or, \quad I_3 = I_2 - I_4 = 4 - 3 = 1A$$

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Apply KVL at the loop BAEDCB

$$I_1 - 3I_3 - 2I_2 = 0$$

$$Or, \quad I_1 - 3 - 8 = 0$$

$$Or, \quad I_1 = 11\ A$$

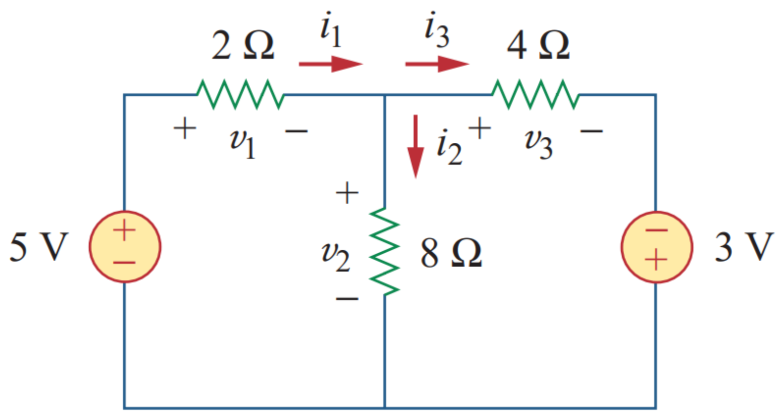
b) Apply KVL at EDGFE

$$-3I_3 + 4I_4 - V = 0$$

$$Or, \quad -3 \times 1 + 4 \times 3 - V = 0$$

$$Or, \quad V = 9V$$

Problem 6: Find the currents and voltages in the circuit shown in the Figure below.



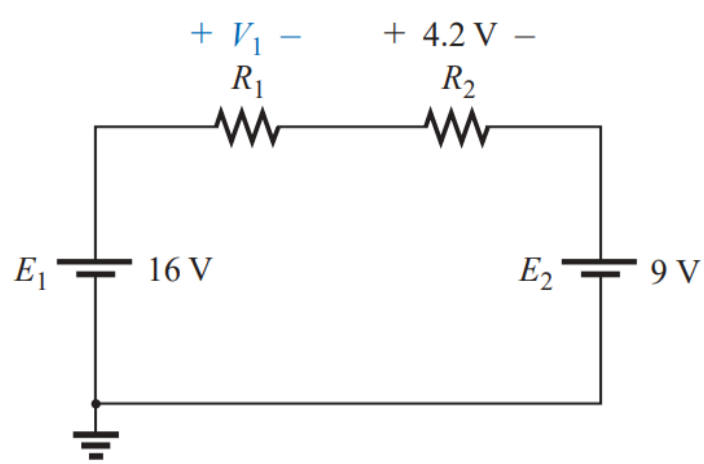
Solution: Try it yourself.

Answer: $V_1 = 3V$, $V_2 = 2V$, $V_3 = 5V$, $V_1 = 3V$, $I_1 = 1.5$ A, $I_2 = 0.25A$, $I_3 = 1.25A$.

Problem 7: Determine the unknown voltages for the networks shown in Figure below.

Solution: Try it yourself.

Answer: $V_1 = 2.8$ V



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Resistors in Series

A circuit consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow.

Two elements are in series if

1. They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).

2. The common point between the two elements is not connected to another current-carrying element.

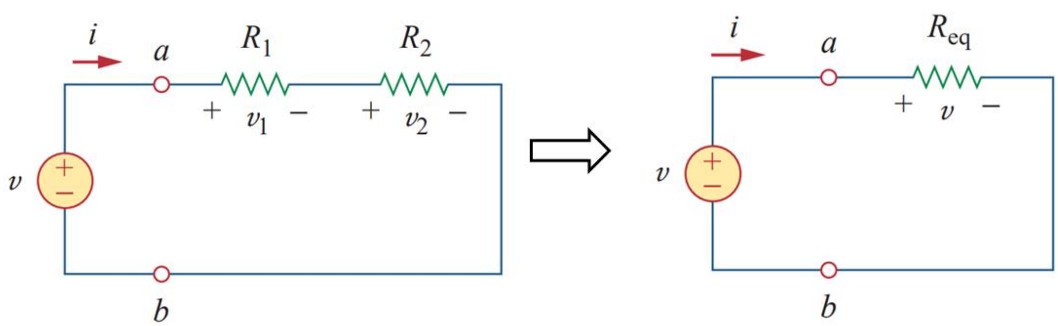

Figure 6: Resistors in Series (left), Equivalent Circuit.

The two resistors with resistance R_1 and R_2 in Figure 6. Are connected in series to a source of emf V. By current conservation, the same current, I, is in each resistor.

The current is the same through series elements.

The total voltage drop from a to b across both elements is the sum of the voltage drops across the individual resistors.

$$V = IR_1 + IR_2 = I(R_1 + R_2)$$

The two resistors in series can be replaced by one equivalent resistor (Figure 6) with the identical voltage drop $V = IR_{eq}$ that implies that

$$R_{eq} = R_1 + R_2$$

The above arguments can be extended to N resistors placed in series. The total resistance of a series circuit is the sum of the resistance levels.

$$R_{eq} = R_1 + R_2 + \ldots\ldots\ldots\ldots\ldots\ldots = \sum_{i=1}^{N} R_i$$

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Resistors in Parallel

Let consider two resistors R_1 and R_2 that are connected across a source of emf , V.

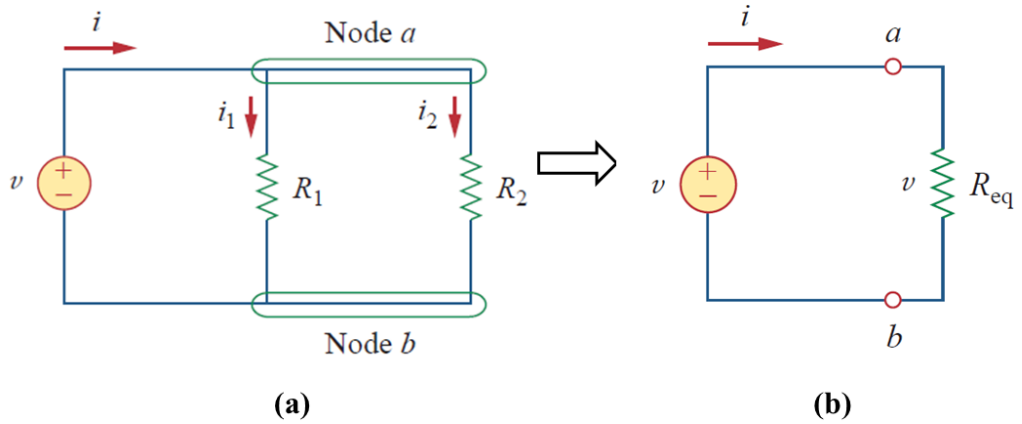

Figure 7: (a) Two resistors in parallel, (b) Equivalent circuit.

Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

By current conservation, the current I that passes through the source of emf must divide into a current I_1 that passes through resistor R_1 and a current I_2 that passes through resistor R_2. Each resistor individually satisfies Ohm's law. $V_1 = I_1R_1$ and $V_2 = I_2R_2$. However, the potential across the resistors are the same, $V = V_1 = V_2$. Current conservation that implies

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

The two resistors in parallel can be replaced by one equivalent resistor R_{eq} with $V = IR_{eq}$. Comparing these results, the equivalent resistance for the two resistors that are connected in parallel is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\text{Or}, R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

This result easily generalizes to N resistors connected in parallel.

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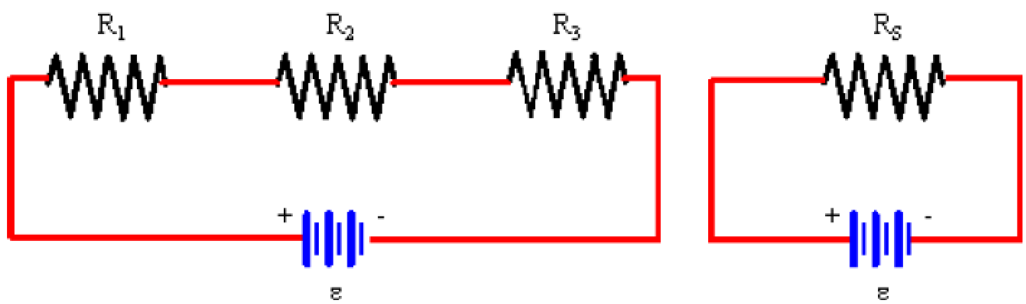
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \ldots .. = \sum_{i=1}^{N} \frac{1}{R_i}$$

Problem 8: In the diagram below, $R_1 = 5\Omega$, $R_2 = 10\Omega$, $R_3 = 15\Omega$. The battery supplies an emf of $\mathcal{E} = 0.30V$.

(i). What is the equivalent resistance, R_s?

(ii). What is the current through each resistor?

(iii). What is the voltage drop across each resistor?



Solution:

i) Since there are no nodes or branches between the three resistors, the three resistors are in series. For series resistors, the equivalent resistance is

$$R_S = R_1 + R_2 + R_3 = 5\ \Omega + 10\ \Omega + 15\ \Omega = 30\ \Omega\ .$$

ii) Resistors in series each carry the same current as their equivalent resistance, RS. The definition of equivalent means that the three resistors could be replaced by RS without affecting any other aspect of the circuit.

Using Ohm's Law the current through R_S is

$$I = \frac{\varepsilon}{R_S} = \frac{0.3}{30} = 0.01\ A$$

Thus R_1, R_2, and R_3 each carry 0.01 A.

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iii) Again using Ohm's Law, the voltage across each resistor is given by V = IR. The results are given below.

$$V_1 = IR_1 = 0.01 \times 5 = 0.05 \text{ V}$$

$$V_2 = IR_2 = 0.01 \times 10 = 0.1 \text{ V}$$

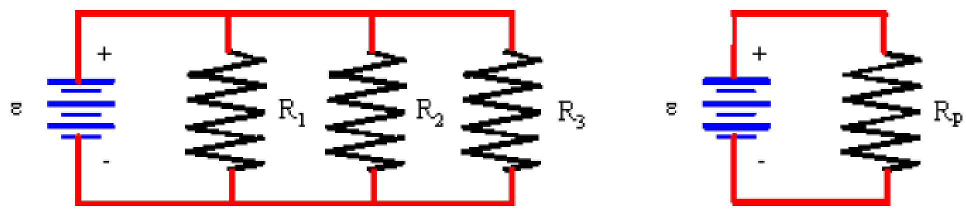
$$V_3 = IR_3 = 0.01 \times 15 = 0.15 \text{ V}$$

Problem 9: In the diagram below, $R_1 = 5\Omega$, $R_2 = 10\Omega$, $R_3 = 15\Omega$. The battery supplies an emf of $\mathcal{E} = 0.30$V.

(i). What is the equivalent resistance, R_p?

(ii). What is the voltage drop across each resistor?

(iii). What is the current through each resistor?



Solution:

i) Since the three resistors share the two common points or nodes, the three resistors are in parallel. For parallel resistors, the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{5} + \frac{1}{10} + \frac{1}{15} = \frac{11}{30}$$

$$\text{Or}, R_{eq} = \frac{30}{11} = 2.727\ \Omega$$

ii) Resistors in parallel each have the same voltage drop as their equivalent resistance, R_p. The definition of equivalent means that the three resistors could be replaced by R_p without affecting any other aspect of the circuit. So, the voltage drops across, R_p, and thus across R_1, R_2, R_3 is $\mathcal{E} = 0.30$V.

iii) Using Ohm's law, the current through each resistor is given by $I = \frac{V}{R}$.

$$I_1 = \frac{\varepsilon}{R_1} = \frac{0.30}{5} = 0.06\ A$$

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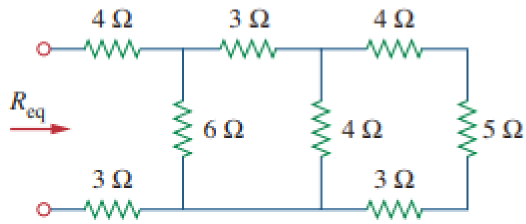
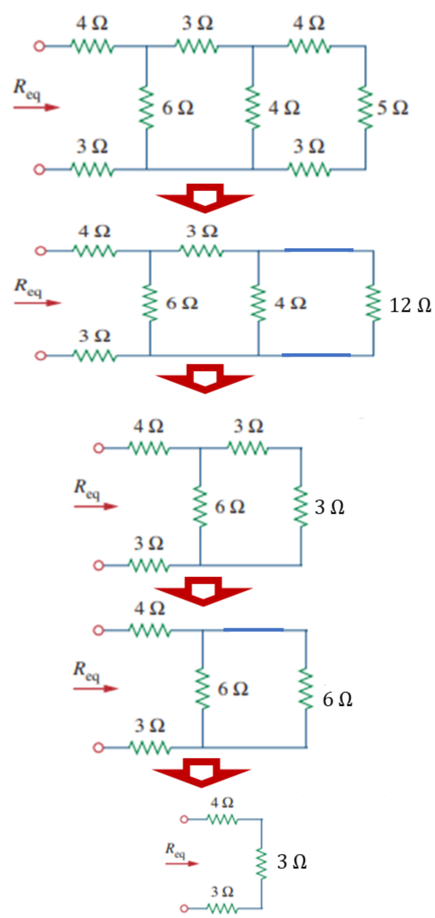
$$I_2 = \frac{\varepsilon}{R_2} = \frac{0.30}{10} = 0.03\ A$$

$$I_3 = \frac{\varepsilon}{R_3} = \frac{0.30}{5} = 0.06\ A$$

Problem 10: Find the total equivalent resistance for the circuit shown in Figure below.



Solution:



4 Ω, 5 Ω, and 3 Ω are connected in series. So, their equivalent resistance is

$$4\,\Omega + 5\,\Omega + 3\,\Omega = 12\,\Omega$$

12 Ω and 4 Ω are connected in parallel. Their equivalent resistance is

$$12\,\Omega \text{ II } 4\,\Omega = \left(\frac{1}{12} + \frac{1}{4}\right)^{-1} = 3\,\Omega$$

3 Ω and 3 Ω are connected in series and their equivalent resistance is

$$3\,\Omega + 3\,\Omega = 6\,\Omega$$

6 Ω and 6 Ω are connected in parallel and their equivalent resistance is

$$6\,\Omega \text{ II } 6\,\Omega = \left(\frac{1}{6} + \frac{1}{6}\right)^{-1} = 3\,\Omega$$

4 Ω, 3 Ω, and 3 Ω are connected in series. So, their equivalent resistance is

$$R_{eq} = 4\,\Omega + 3\,\Omega + 3\,\Omega = 10\,\Omega$$

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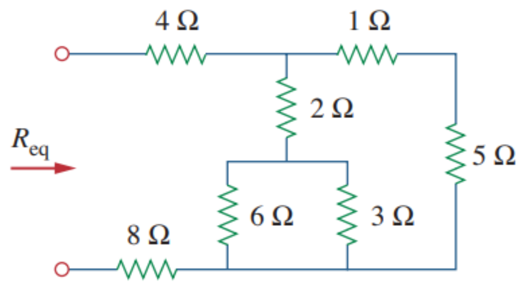
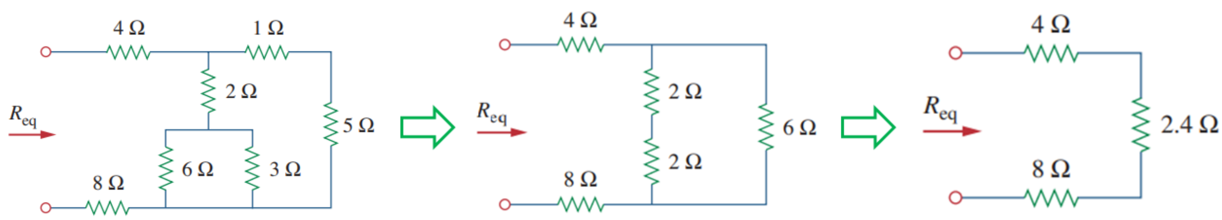
Problem 11: Find R_{eq} for the circuit shown in Figure below.



Solution:



To get R_{eq}, we combine resistors in series and in parallel.

The 6 Ω and 3 Ω resistors are in parallel, so their equivalent resistance is

$$6\,\Omega \text{ II } 3\,\Omega = \left(\frac{1}{6}+\frac{1}{3}\right)^{-1} = 2\,\Omega$$

Also, the 1 Ω and 5 Ω resistors are in series; hence their equivalent resistance is

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

Then, the two 2 Ω resistors are in series, so the equivalent resistance is

$$2\,\Omega + 2\,\Omega = 4\,\Omega$$

Then, This 4 Ω resistor is now in parallel with the 6 Ω resistor their equivalent resistance is

$$4\,\Omega \text{ II } 6\,\Omega = \left(\frac{1}{4}+\frac{1}{6}\right)^{-1} = 2.4\,\Omega$$

Finally, the three resistors 4 Ω, 2.4 Ω, and 8 Ω are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\,\Omega + 2.4\,\Omega + 8\,\Omega = 14.4\,\Omega$$

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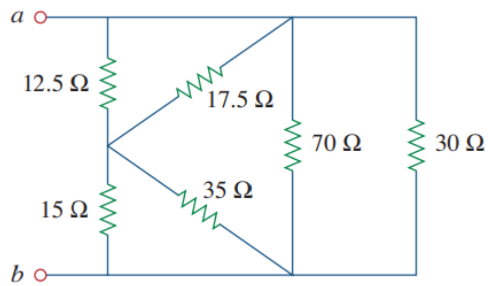
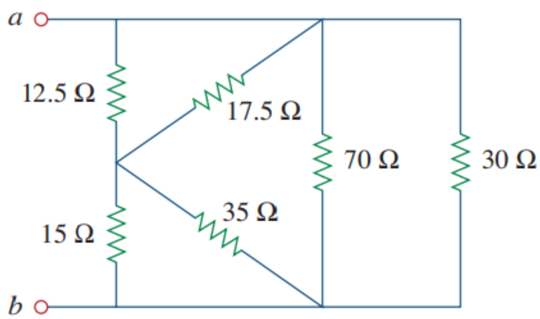
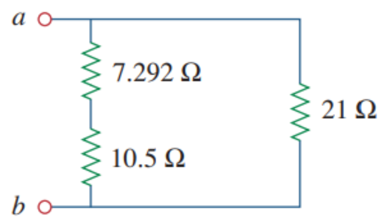
Problem 12: Find the equivalent resistance at terminal a-b in the Figure shown in below.



Solution:





70 Ω and 30 Ω resistors are in parallel, their equivalent resistance is

$$70\ \Omega \text{ II } 30\ \Omega = \left(\frac{1}{70} + \frac{1}{30}\right)^{-1} = 21\ \Omega$$

12.5 Ω and 17.5 Ω resistors are in parallel, their equivalent resistance is

$$12.5\ \Omega \text{ II } 17.5\ \Omega = \left(\frac{1}{17.5} + \frac{1}{12.5}\right)^{-1} = 7.292\ \Omega$$

15 Ω and 35 Ω resistors are in parallel, their equivalent resistance is

$$15\ \Omega \text{ II } 35\ \Omega = \left(\frac{1}{15} + \frac{1}{35}\right)^{-1} = 10.5\ \Omega$$

Then, the 7.292 Ω and 10.5 Ω resistors are in series, so the equivalent resistance is

$$7.292\ \Omega + 10.5\ \Omega = 17.792\ \Omega$$

Finally, 17.792 Ω and 21 Ω resistors are in parallel, their equivalent resistance is

$$17.792\ \Omega \text{ II } 21\ \Omega = \left(\frac{1}{17.792} + \frac{1}{21}\right)^{-1} = 9.632\ \Omega$$

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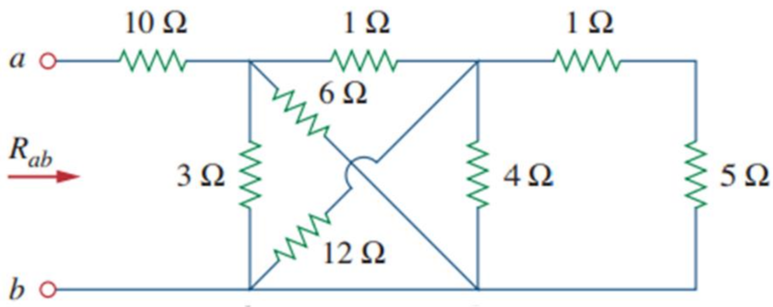
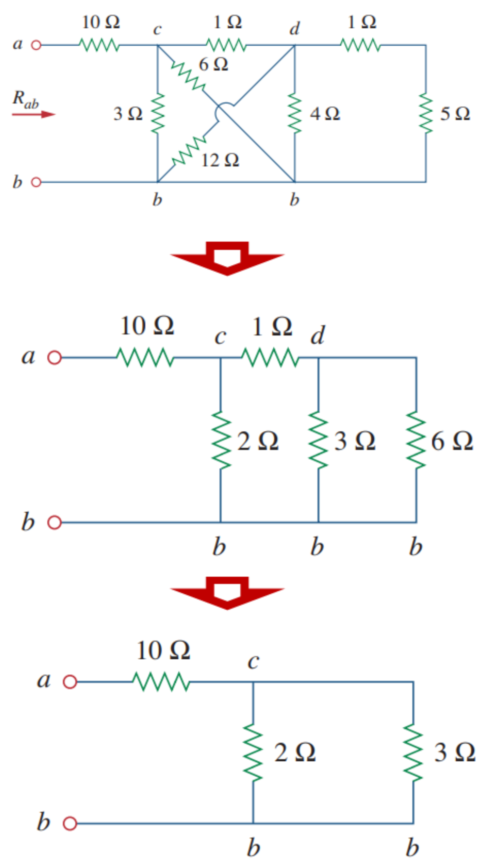
Problem 13: Find the equivalent resistance at terminal a-b in the Figure shown in below.



Solution:

The 3 Ω and 6 Ω resistors are in parallel because they are connected to the same two nodes c and b. Their combined resistance is

$$3\,\Omega \text{ II } 6\,\Omega = \left(\frac{1}{3}+\frac{1}{6}\right)^{-1} = 2\,\Omega$$



Similarly, the 12 Ω and 4 Ω resistors are in parallel because they are connected to the same two nodes d and b. Their combined resistance is

$$12\,\Omega \text{ II } 4\,\Omega = \left(\frac{1}{12}+\frac{1}{4}\right)^{-1} = 3\,\Omega$$

Also, the 1 Ω and 5 Ω resistors are in series; hence, their equivalent resistance is

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

Then, the 6 Ω and 3 Ω resistors are in parallel, hence equivalent resistance is 2 Ω. The equivalent 2 Ω resistance are connected in series with 1 Ω resistance and the combined resistance is

$$1\,\Omega + 2\,\Omega = 3\,\Omega$$

And we combine the 2 Ω and 3 Ω resistors in parallel to get

$$2\,\Omega \text{ II } 3\,\Omega = \left(\frac{1}{2}+\frac{1}{3}\right)^{-1} = 1.2\,\Omega$$

Finally, 1.2 Ω and 10 Ω are connected in series, so that

$$R_{ab} = 1.2\,\Omega + 10\,\Omega = 11.2\,\Omega$$

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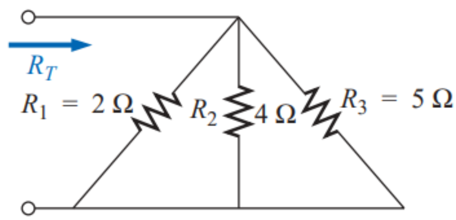
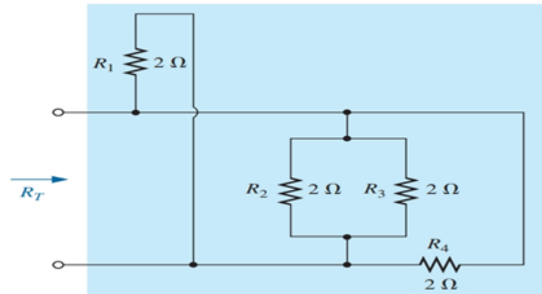
Problem 14: Find the total resistance for the network configuration shown in Figure below.

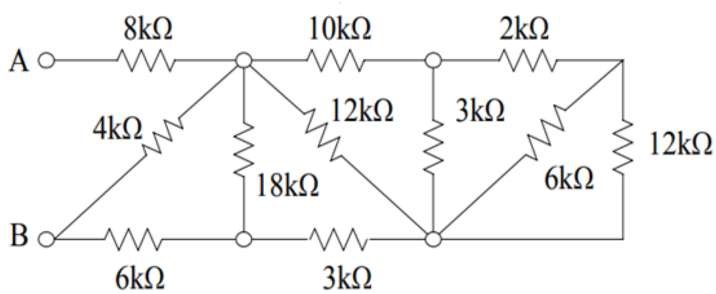
(a)



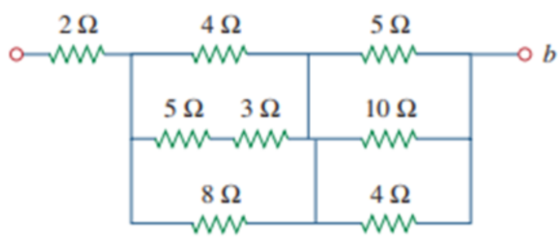
(b)



(c)



(d)



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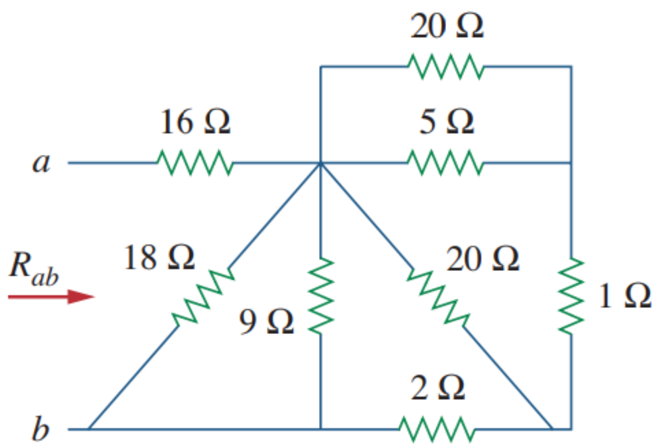
Problem 15: Find R_{ab} for the circuit shown in Figure below.



Solution: Try it yourself

Answer: $R_{ab} = 19\ \Omega$

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Voltage Divider Rule

In a series circuit,

the voltage across the resistive elements will divide as the magnitude of the resistance levels.

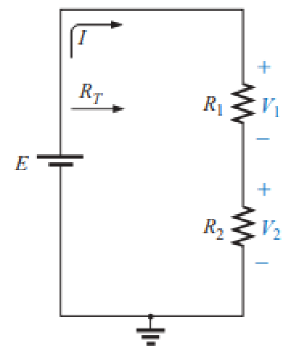

Figure 8: Developing voltage divider rule.

A method referred to as the voltage divider rule (VDR) that permits determining the voltage levels without finding the current. The rule can be derived by analyzing the network of Fig. 8.

$$R_T = R_1 + R_2$$

$$\text{and, } I = \frac{E}{R_T}$$

Applying Ohm's law,

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right) R_1 = \frac{R_1}{R_T} E$$

With

$$V_2 = IR_2 = \left(\frac{E}{R_T}\right) R_2 = \frac{R_2}{R_T} E$$

Note that the format for V_1 and V_2 is

$$V_x = \frac{R_x}{R_T} E$$

Where V_x is the voltage across R_x, E is the impressed voltage across the series elements, and R_T is the total resistance of the series circuit.

In words, the voltage divider rule states that the voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

Question: Explain Voltage Divider Rule with appropriate figure and equation.

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Problem 16: Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit of Fig. 9.

Solution:

$$V_1 = \frac{R_1}{R_T}E = \frac{2}{2+5+8} \times 45 = 6\text{ V}$$

$$V_3 = \frac{R_3}{R_T}E = \frac{8}{2+5+8} \times 45 = 24\text{ V}$$

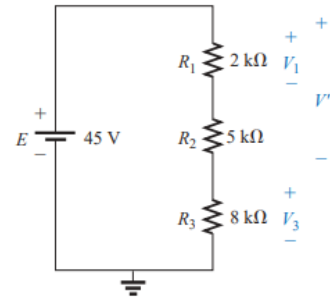

Figure 9

Problem 17: Design the voltage divider of Fig. 10 such that $V_{R_1} = 4\ V_{R_2}$.

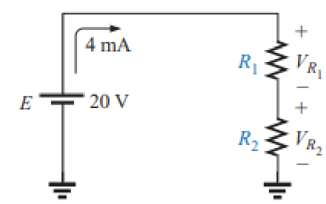

Figure 10

Solution:

The total resistance is defined by

$$R_T = \frac{E}{I} = \frac{20\text{ V}}{4\text{ mA}} = 5\text{ k}\Omega$$

Since $V_{R_1} = 4\ V_{R_2}$, $R_1 = 4\ R_2$

Thus, $R_T = R_1 + R_2 = 4R_2 + R_2 = 5R_2$

And, $5\ R_2 = 5\text{ k}\Omega$

Or, $R_2 = 1\text{ k}\Omega$

and, $R_1 = 4\text{ k}\Omega$

Problem 18: Referring to Fig. 11.

a) Determine V_2 by simply noting that $R_2 = 3R_1$.
b) Calculate V_3.
c) Noting the magnitude of V_3 compared to V_2 or V_1, determine R_3 by inspection.
d) Calculate the source current I.
e) Calculate the resistance R_3 using Ohm's law, and compare it to the result of part (c).

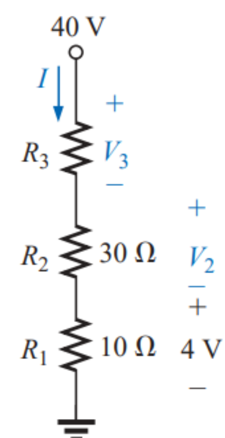

Figure 11

Solution: Try it yourself.

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Current Divider Rule

As the name suggests, the current divider rule (CDR) will determine how the current entering a set of parallel branches will split between the elements.

For two parallel elements of equal value, the current will divide equally.

For parallel elements with different values, the smaller the resistance, the greater the share of input current.

For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

The current-divider circuit shown in Fig. 12 consists of two resistors connected in parallel across a current source. The current divider is designed to divide the current between R_1 and R_2. We find the relationship between the current and the current in each resistor (that is, I_1 and I_2) by directly applying Ohm's law and Kirchhoff's current law.

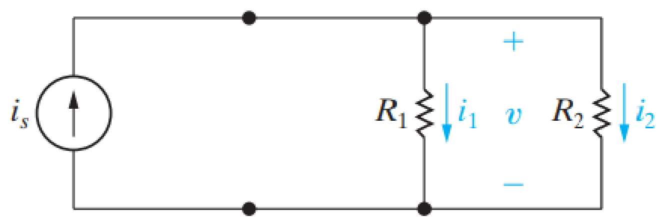

Figure 12: Current Divider Circuit.

The input current I_s equals $\frac{V}{R_T}$, where R_T is the total resistance of the parallel branches. Substituting $V = I_x R_x$ into the above equation, where I_x refers to the current through a parallel branch of resistance R_x, we have

$$I_s = \frac{V}{R_T} = \frac{I_x R_x}{R_T}$$

$$\text{And, } I_x = \frac{R_T}{R_x} I_s$$

which is the general form for the current divider rule. In words, the current through any parallel branch is equal to the product of the total resistance of the parallel branches and the input current divided by the resistance of the branch through which the current is to be determined.

For the current I_1,

$$I_1 = \frac{R_T}{R_1} I_s$$

And for the current I_2,

$$I_2 = \frac{R_T}{R_2} I_s$$

Question: Explain Current Divider Rule with appropriate figure and equation.

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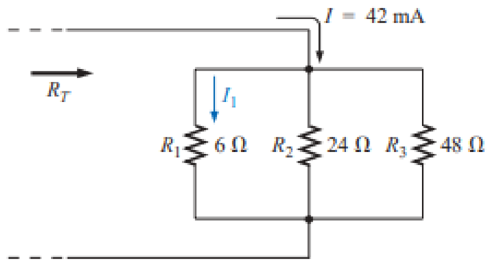
Problem 19: Using current divider rule, find the current I_1 for the network of figure shown in below.



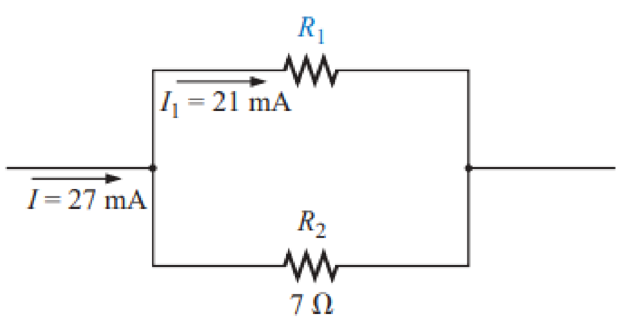
Solution:

6 Ω, 24 Ω and 48 Ω resistors are in parallel, their total resistance is

$$R_T = (\frac{1}{6} + \frac{1}{24} + \frac{1}{48})^{-1} = 4.363\ \Omega$$

With $I_1 = \frac{R_T}{R_1} I_s = \frac{4.363}{6} \times 42\ \text{mA} = 30.54\ \text{mA}$

Problem 20: Determine the resistance R1 to effect the division of current in figure shown in below.



Solution:

Applying the current divider rule,

$$I_1 = \frac{R_T}{R_1} I$$

$$\text{Where, } R_T = (\frac{1}{R_1} + \frac{1}{R_2})^{-1} = (\frac{R_1+R_2}{R_1R_2})^{-1} = \frac{R_1R_2}{R_1+R_2}$$

$$\text{So, } I_1 = \frac{R_T}{R_1} I = \frac{R_2}{R_1+R_2} I$$

$$\text{Or, } (R_1 + R_2)I_1 = R_2I$$

$$\text{Or, } R_1I_1 + R_2I_1 = R_2I$$

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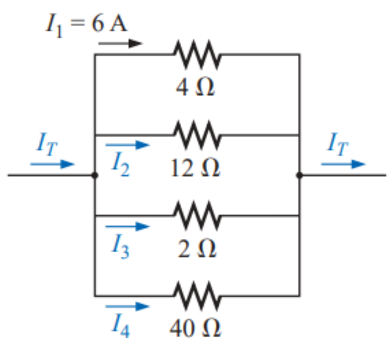
$$\text{Or, } R_1 I_1 = R_2 I - R_2 I_1$$

$$\text{Or, } R_1 = \frac{R_2(I - I_1)}{I_1}$$

Substituting values,

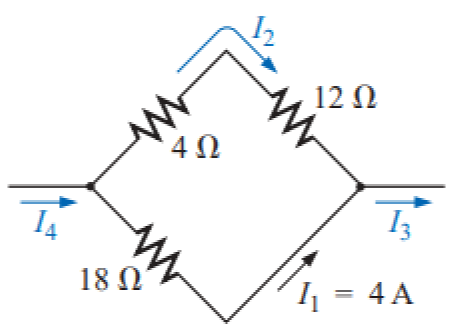
$$R_1 = \frac{7\Omega(27mA - 21mA)}{21mA} = 2\Omega$$

Problem 21: Using the current divider rule, find the unknown currents for the networks of Figure shown in below.



Solution: Try it yourself.

Problem 22: Using the current divider rule, find the unknown currents for the networks of Figure shown in below.



Solution: Try it yourself.

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Open and Short Circuits

An open circuit is simply two isolated terminals not connected by an element of any kind, as shown in Fig. 13(a). Since a path for conduction does not exist, the current associated with an open circuit must always be zero. The voltage across the open circuit, however, can be any value, as determined by the system it is connected to. In summary, therefore,

an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

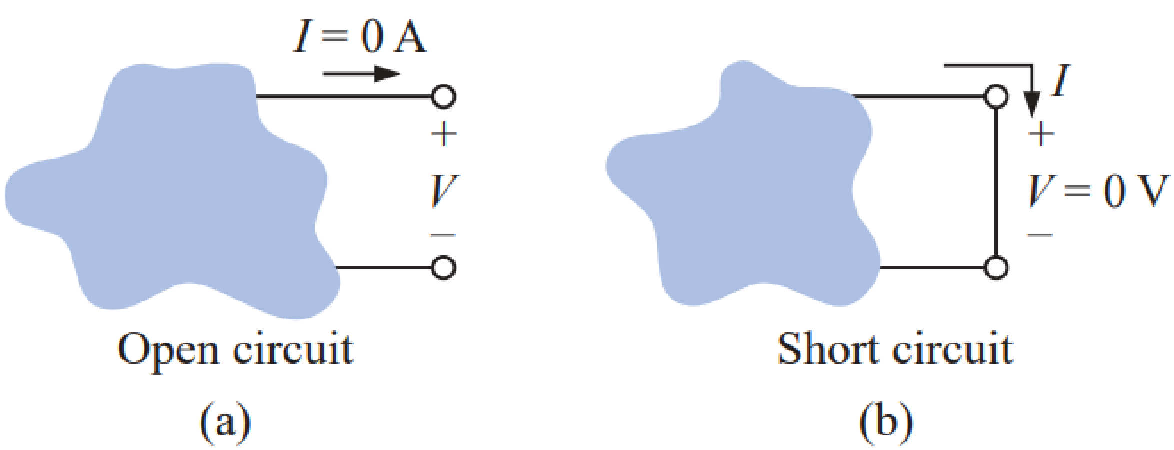

Figure 13: Two special network configurations.

A short circuit is a very low resistance, direct connection between two terminals of a network, as shown in Fig. 13(b). The current through the short circuit can be any value, as determined by the system it is connected to, but the voltage across the short circuit will always be zero volts because the resistance of the short circuit is assumed to be essentially zero ohms and $V = IR = I(0\Omega) = 0V$. In summary, therefore,

a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

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Problem 23: Determine the unknown voltage and current for each network of Fig. 14.

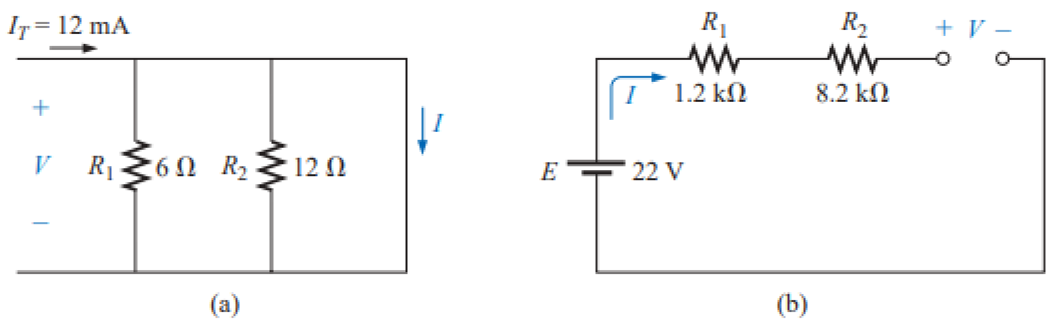

Figure 14.

Solution:

For the network of Fig. 14 (a), the current I_T will take the path of least resistance, and, since the short-circuit condition at the end of the network is the least-resistance path, all the current will pass through the short circuit. The voltage across the network is the same as that across the short circuit and will be zero volts, as shown in Fig. 15(a).

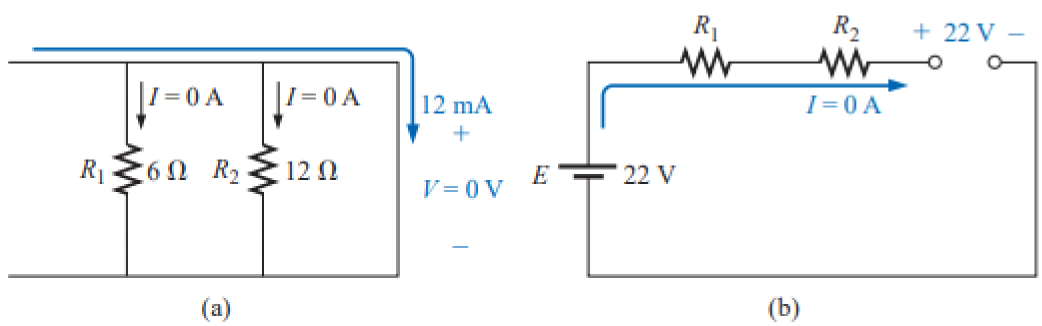

Figure 15: Solution to Problem 23.

For the network of Fig. 14(b), the open-circuit condition requires that the current be zero amperes. The voltage drops across the resistors must therefore be zero volts, as determined by Ohm's law [V_R = IR= (0)R= 0V], with the resistors simply acting as a connection from the supply to the open circuit. The result is that the open-circuit voltage will be E = 22 V, as shown in Fig. 15(b).

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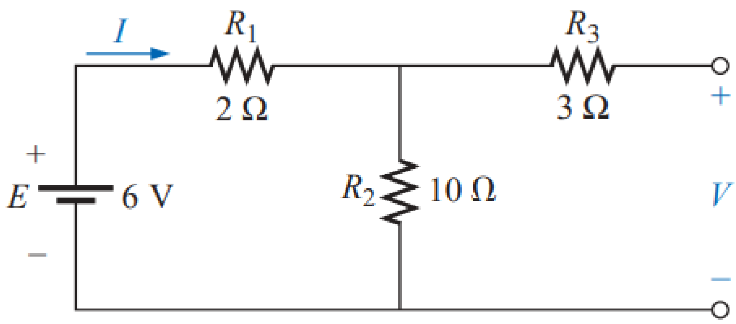
Problem 24: Determine V and I for the network shown in below if the resistor R_2 is shorted out.



Solution:

The redrawn network appears in Fig. 16.

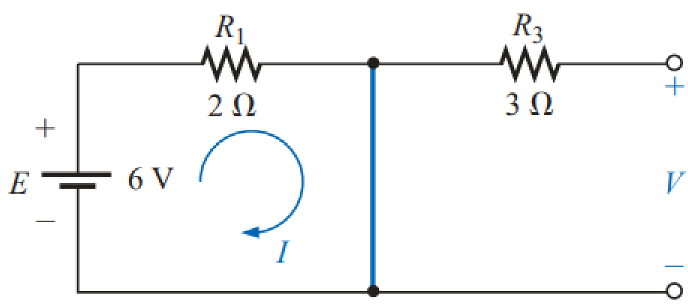


Figure 16: Help to solve Problem 24.

The current through the 3 Ω resistor is zero due to the open circuit, causing all the current I to pass through the jumper. Since $V_{3\Omega} = IR = (0)R = 0V$, the voltage V is directly across the short, and V= 0 V

With $I = \frac{E}{R_1} = \frac{6\,V}{2\Omega} = 3A$

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Problem 25: For the network of Fig. 17.

a) Determine I_s and V_L.

b) Determine I_s if R_L is shorted out.

c) Determine V_L if R_L is replaced by an open circuit.

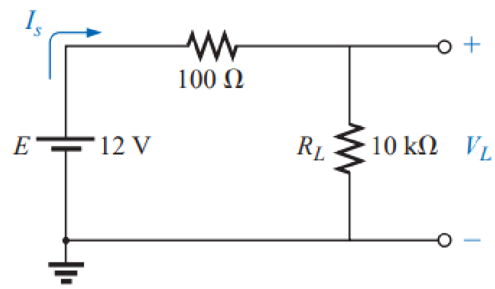

Figure 17.

Solution: Try it yourself

Problem 26: For the network of Fig. 18:

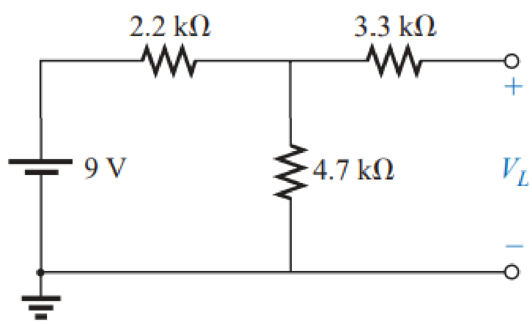

Figure 18.

a) Determine the open-circuit voltage V_L.

b) If the 2.2-kΩ resistor is short circuited, what is the new value of V_L?

c) Determine V_L if the 4.7-kΩ resistor is replaced by an open circuit.

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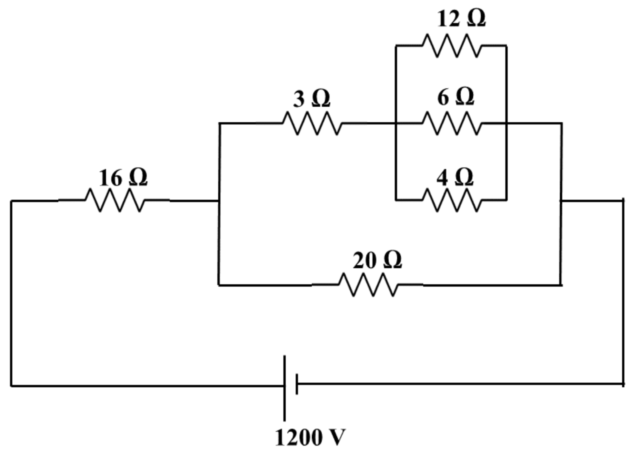
Resistors in Series and Parallel

Problem 27: Find the of the circuit below. Calculate the voltage drop over, and current through each resistor. Put your results in a table.



Solution:

The 12 Ω, 6 Ω and 4 Ω resistors are in parallel. So, their combined resistance is

$$12\ \Omega\ \text{II}\ 6\ \Omega\ \text{II}\ 4\ \Omega = (\frac{1}{12} + \frac{1}{6} + \frac{1}{4})^{-1} = 2\ \Omega$$

The 2 Ω and 3 Ω resistors are in series and their combined resistance is

$$2\ \Omega + 3\ \Omega = 5\ \Omega$$

And the 5 Ω and 20 Ω resistors in parallel to get

$$5\ \Omega\ \text{II}\ 20\ \Omega = (\frac{1}{5} + \frac{1}{20})^{-1} = 4\ \Omega$$

The 4 Ω and 16 Ω resistors are in series and the total resistance is

$$R_T = 4\ \Omega + 16\ \Omega = 20\ \Omega$$

Now, the current flow through 20 Ω is

$$I = \frac{V}{R_T} = \frac{1200\ V}{20\ \Omega} = 60\ A$$

So, the current flow through 4 Ω and 16 Ω resistors will be 60 A.

The voltage drop across 4 Ω resistor is = 60 A× 16 Ω = 960 V

The voltage drop across 4 Ω resistor is = 60 A× 4 Ω = 240 V

Therefore, the voltage drop across 5 Ω and 20 Ω resistors is 240 V.

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So, current flow through 5 Ω resistor is 240/5= 48 A

and current flow through 20 Ω resistor is 240/20= 12 A.

And we can say current flow through 2 Ω and 3 Ω resistors is 48 A.

Voltage drop across 3Ω resistor = 3 × 48 = 144 V

Voltage drop across 2Ω resistor = 2 × 48 = 96 V

So, voltage drop across the 12 Ω, 6 Ω and 4 Ω is 96 V.

current flow through 12 Ω resistor = 96/12= 8 A

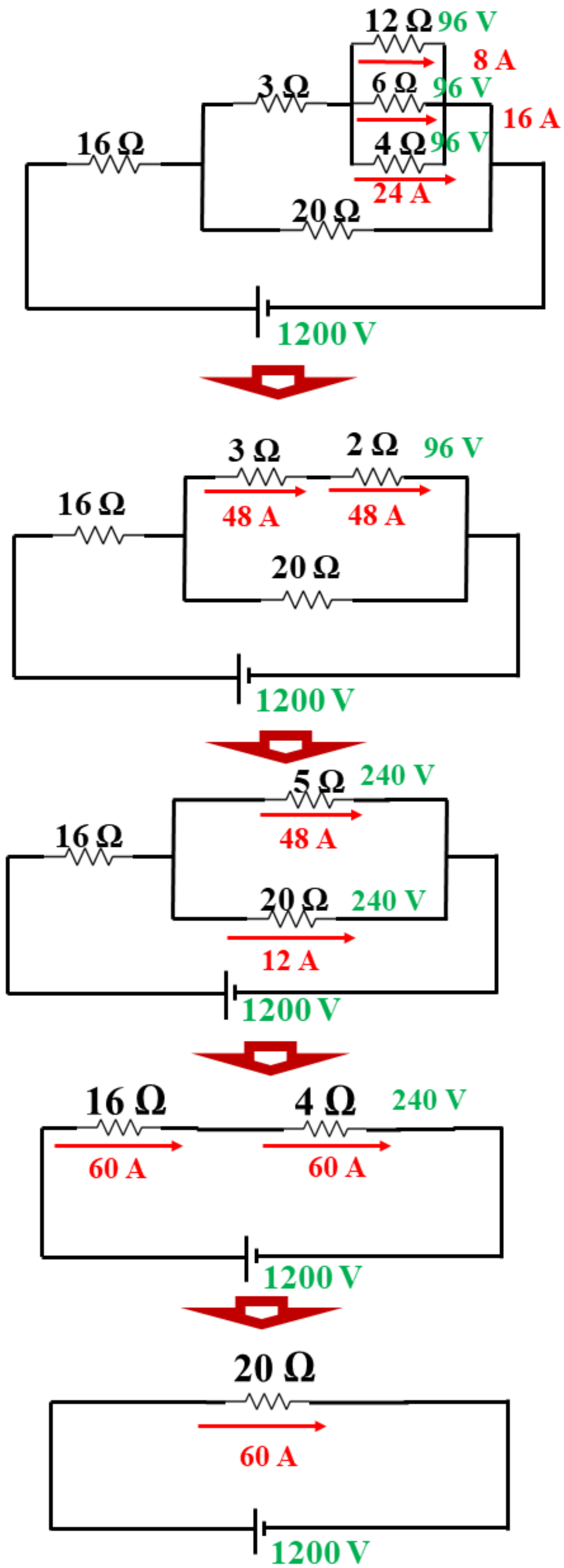
current flow through 6 Ω resistor = 96/6= 16 A

current flow through 4 Ω resistor = 96/12= 8 A

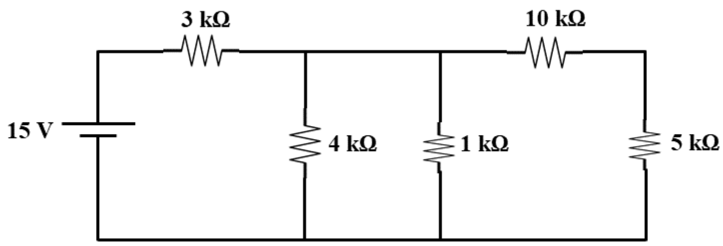
We can summarize all the findings in a table.

Resistor, R (Ω)	Voltage, V (V)	Current flow, I (A)
16	960	60
20	240	12
3	144	48
12	96	8
6	96	16
4	96	24

N.B.: The circuit flow chart of this problem is shown in the next page.


12 Ω
96 V
8 A
3 Ω
6 Ω
96 V
16 A
16 Ω
4 Ω
96 V
24 A
20 Ω
1200 V
3 Ω
2 Ω
96 V
16 Ω
48 A
48 A
20 Ω
1200 V
5 Ω
240 V
16 Ω
48 A
20 Ω
240 V
12 A
1200 V
16 Ω
4 Ω
240 V
60 A
60 A
1200 V
20 Ω
60 A
1200 V

Problem 28: Answer the following questions for the circuit shown in below.



i) What is the total resistance in the circuit?

ii) What is the current supplied by the battery?

iii) Determine the voltage drop over, and current flow through each resistor.

Solution:

The 10 kΩ and 5 kΩ resistors are in series and their combined resistance is

$$10\text{ k}\Omega + 5\text{ k}\Omega = 15\text{ k}\Omega$$

The 15 kΩ, 1 kΩ and 4 kΩ resistors are in parallel. So, their combined resistance is

$$15\text{ k}\Omega \text{ II } 1\text{ k}\Omega \text{ II } 4\text{ k}\Omega = (\frac{1}{15} + \frac{1}{1} + \frac{1}{4})^{-1} = 0.76\text{ k}\Omega$$

The 0.76 kΩ and 3 kΩ resistors are in series and the total resistance is

$$R_T = 0.76\text{ k}\Omega + 3\text{ k}\Omega = 3.76\text{ k}\Omega$$

Now, the current flow through 3.76 kΩ is

$$I = \frac{V}{R_T} = \frac{15\,V}{3.76\,k\Omega} = 3.99\,mA$$

So, the current flow through 0.76 kΩ and 3 kΩ resistors will be 3.99 mA.

The voltage drop across 3 kΩ resistor is = 3.99 mA× 3 kΩ = 12 V

The voltage drop across 0.76 kΩ resistor is = 3.99 mA× 0.76 kΩ = 3 V

Therefore, the voltage drop across the 15 kΩ, 1 kΩ and 4 kΩ resistors is 3 V.

So, current flow through 15 kΩ resistor is 3V / 15 kΩ = 0.2 mA

Current flow through 1 kΩ resistor is 3 V / 1 kΩ = 3 mA.

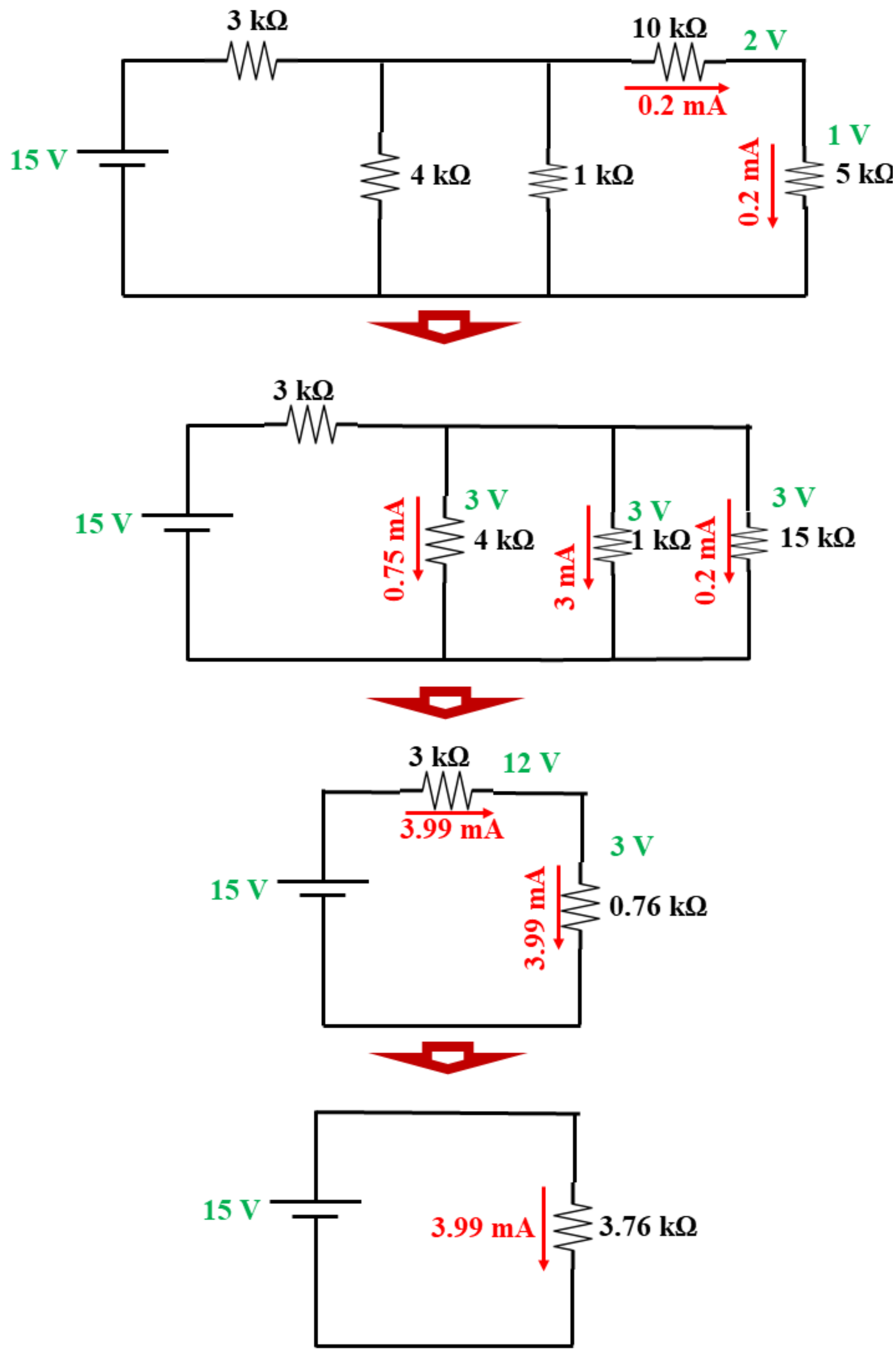
and current flow through 4 kΩ resistor is 3 V / 4 kΩ = 0.75 mA.

And we can say current flow through 10 kΩ and 5 kΩ resistors is 0.2 mA.

Voltage drop across 10 kΩ resistor = 0.2 mA × 10 kΩ = 2 V

Voltage drop across 5 kΩ resistor = 0.2 mA × 5 kΩ = 1 V

N.B.: The circuit flow chart of this problem is shown in the next page.


3 kΩ
10 kΩ
2 V
0.2 mA
15 V
4 kΩ
1 kΩ
1 V
0.2 mA
5 kΩ
3 kΩ
15 V
0.75 mA
3 V
4 kΩ
3 V
3 mA
1 kΩ
0.2 mA
3 V
15 kΩ
3 kΩ
12 V
3.99 mA
15 V
3 V
3.99 mA
0.76 kΩ
15 V
3.99 mA
3.76 kΩ

Problem 29: Find all the currents and voltages in the network in Fig. 19.

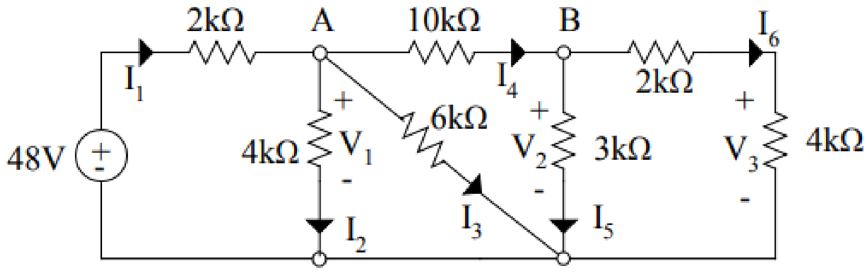


Figure 19.

Solution: Try it yourself.

Answer: $I_1 = 12\text{ mA},\ I_2 = 6\text{ mA}, I_3 = 4\text{ mA}, I_4 = 2\text{ mA}, I_5 = \frac{4}{3}\text{ mA}, I_6 = \frac{2}{3}\text{ mA}; V_1 = 24\text{ V}, V_2 = 4\text{ V}, V_3 = \frac{8}{3}\text{ V}$

Problem 30: Find R_{eq} and i_o in the circuit of Fig. 20.

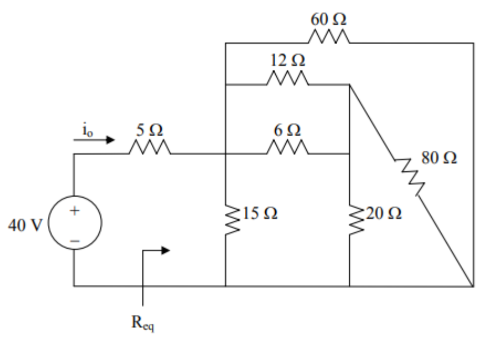


Figure 20.

Solution: Try it yourself.

Answer: $R_{eq} = 12.5\ \Omega$, $i_0 = 3.2$ A.

Problem 31: Find i and V_0 in the circuit of Fig. 21.

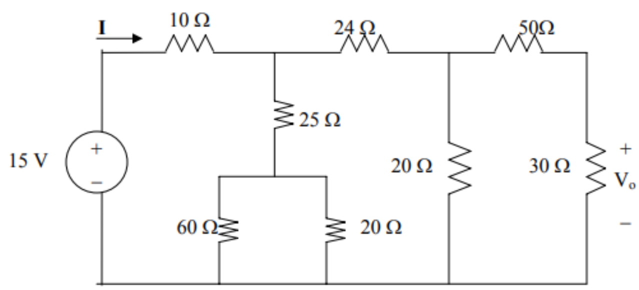


Figure 21.

Solution: Try it yourself.

Answer: $i = 0.5$ A, $V_0 = 1.5$ V.

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Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 22. This type of circuits can be simplified by using three-terminal equivalent networks. These are the wye (Y) or tee (T) network shown in Fig. 23 and the delta(Δ) or pi(π) network shown in Fig. 24. These networks occur by themselves or as part of a larger network.

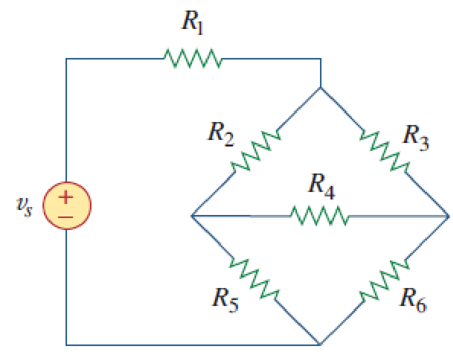

Figure 22: The Bridge Network.

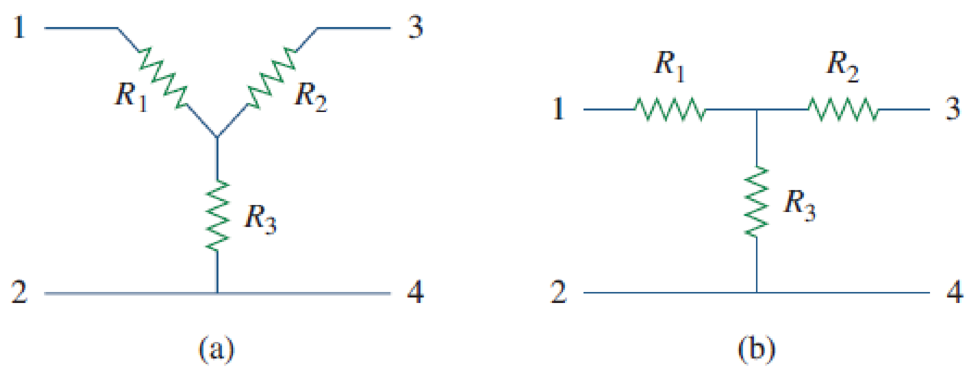

Figure 23: Two forms of the same network: (a) Y, (b) T.

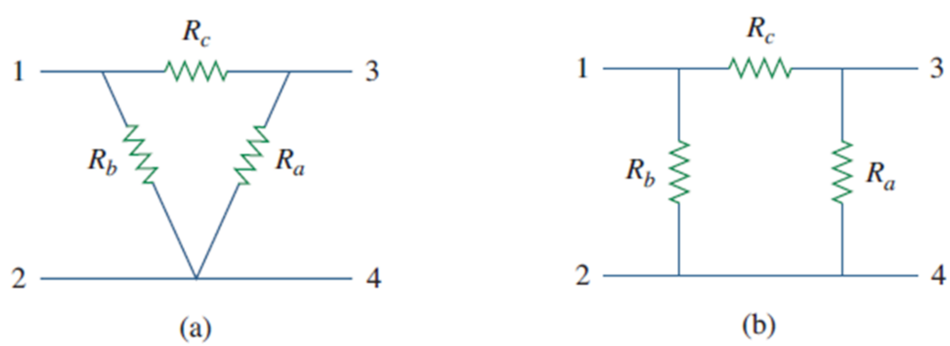

Figure 24: Two forms of the same network: (a) Δ, (b) π.

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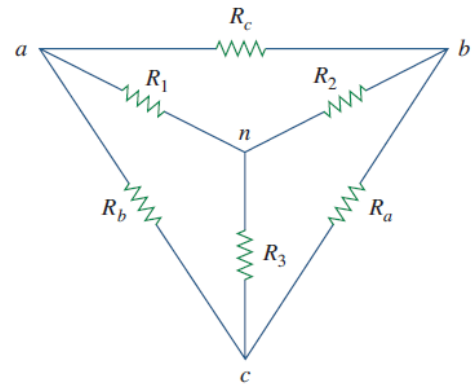
Delta to Wye Conversion

To obtain the conversion formulas for transforming a delta network to an equivalent wye network, we note



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note the following equations:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be balanced when

$$R_1 = R_2 = R_3 = R_Y,$$

$$R_a = R_b = R_c = R_\Delta$$

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \text{ or } R_\Delta = 3\,R_Y$$

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Example: Convert the Δ network in Fig. 25(a) to an equivalent Y network.

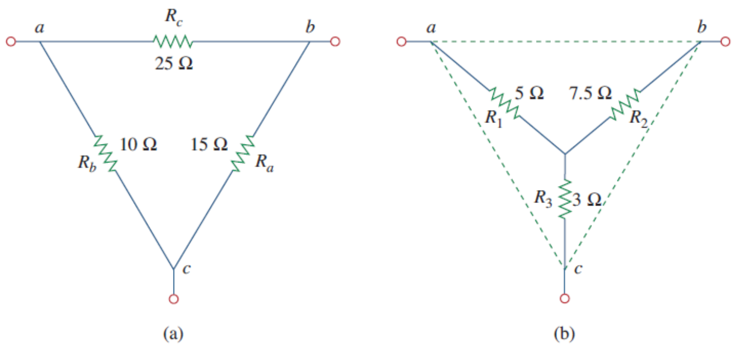

Figure 25: (a) Original Delta Network, (b) Y equivalent network.

Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{25 \times 10}{25+15+10} = 5\Omega$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{10 \times 15}{50} = 3\Omega$$

The equivalent Y network is shown in Fig. 25(b).

Problem 32: Transform the wye network in Fig. 26 to a delta network.

Solution: Try it yourself.

Answer: $R_a = 140\ \Omega, R_b = 70\ \Omega, R_c = 35\ \Omega$

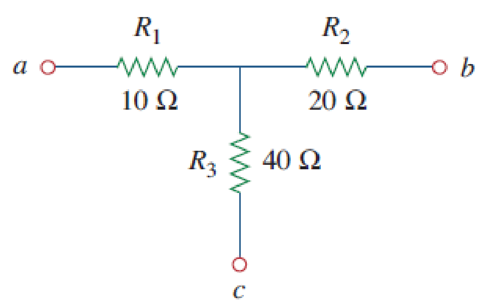

Figure 26.

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Problem 33: Obtain the equivalent resistance R_{ab} for the circuit in Fig. 27. And use it to find current i.

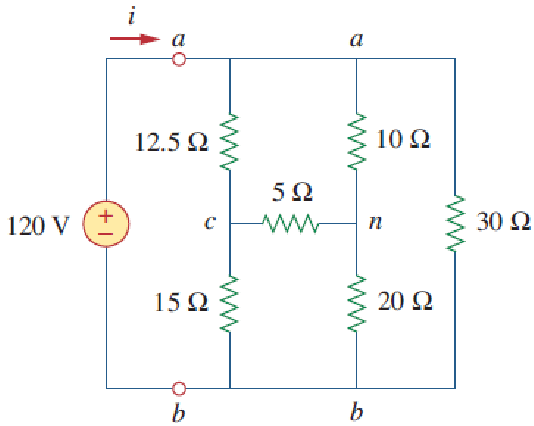

Figure 27.

Solution:

If we convert the Y network comprising the 5 Ω, 10 Ω, 20 Ω resistors, we may select

$R_1 = 10\ \Omega, R_2 = 20\ \Omega, R_3 = 5\ \Omega$

From the transformation equations

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10} = \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} == \frac{350}{5} = 70\ \Omega$$

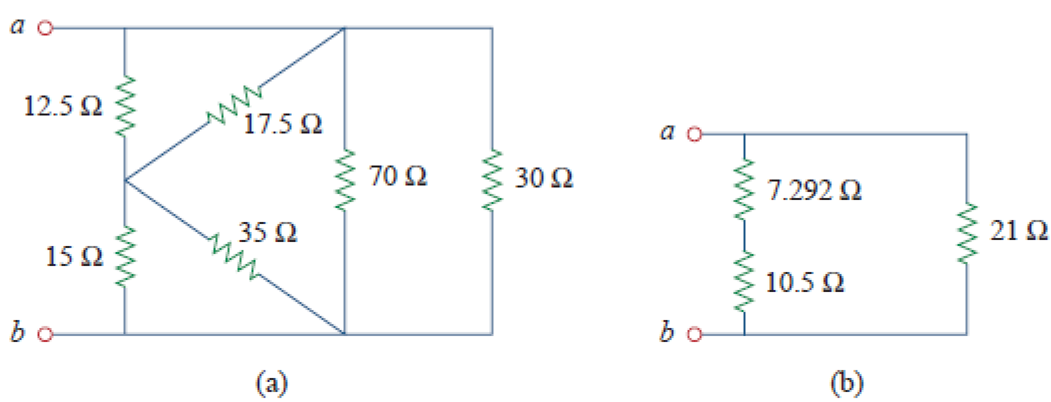

Figure 28: Equivalent circuits to Fig. 27, with voltage source removed.

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Follow the solutions of Problem 12 to find R_{ab}.

We find

$$R_{ab} = 9.632\ \Omega$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458\ A$$

Problem 34: For the bridge network in Fig.29, find R_{ab} and i.

Solution: Try it yourself.

Answer: 40 Ω, 2.5 A.

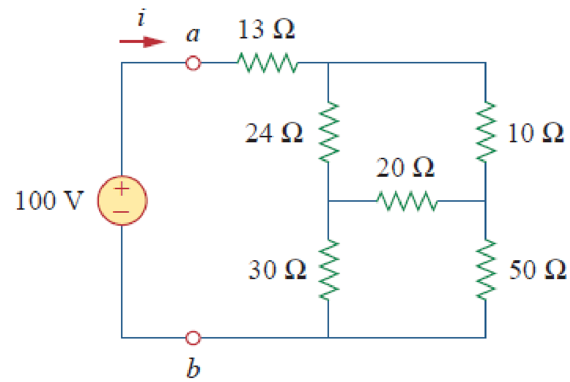

Figure 29.

Problem 35: Obtain the equivalent resistance R_{ab} in each of the circuits of Fig. 30. In (b), all resistors have a value of 30Ω.

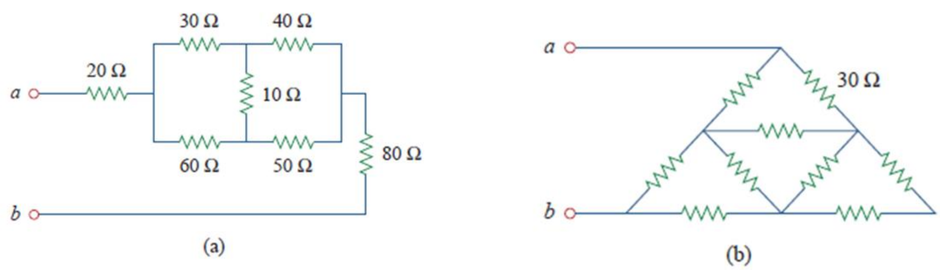

Figure 30.

Solution: Try it yourself.

Answer: (a) 142.32 Ω, (a) 33.33 Ω.

Problem 36: Determine V in the circuit of Figure 31.

Solution: Try it yourself.

Answer: 42.18 V.

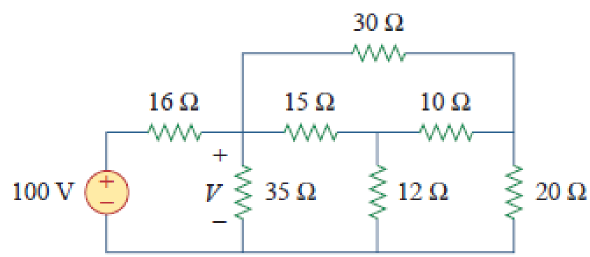

Figure 31.

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Problem 37: Find R_{eq} and I in the circuit of Fig. 32.

Solution: Try it yourself.

Answer: 12.21 Ω; 1.64 A.

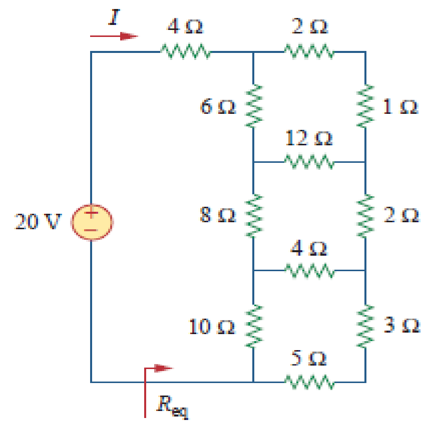

Figure 32.

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