

CHAPTER

**08**

## **Monopoly**

- ❖ Definition
- ❖ Monopoly Demand
- ❖ Short Run Equilibrium
- ❖ Monopoly Supply
- ❖ Long Run Equilibrium
- ❖ Perfect Competition vs Monopoly
- ❖ Price Discrimination

### 8.1 Definition

Monopoly is a market structure in which single seller sells a completely differentiated product. Monopolist's product has no substitute; therefore, the seller possesses full discretion with regard to the pricing decision. Since there is only one firm in monopoly, the firm itself is the industry.

Monopoly may be created due to various reasons attributable to the individual producer or government policies. Production of certain commodity may require some special types of raw materials that may only be owned by an individual firm. If this is so, the firm will emerge as the sole producer of that good. It thus becomes the monopolist. Sometimes market size allows only one firm to run business, thereby making the firm the monopolist. Apart from the exclusive ownership of raw materials, a producer may have patent right on the production technique of a particular product. Such patent right gives rise to monopoly. Under special circumstances, monopoly is created because of the government policy. If for certain reasons the government issues licence to a single producer then he or she becomes a monopolist.

### 8.2 Monopoly Demand

If the monopolist plans to increase volume of sale, it fixes a low price. In contrast, if a higher price is set then the amount of sale would be low. This suggests the inverse relation between price and quantity. Thus the demand function of the monopolist slopes downward.

Mechanical form of the linear demand function of a monopolist is

$$P = a - bQ \quad (a, b > 0)$$

$$\text{Total revenue: } R = P \times Q = (a - bQ)Q = aQ - bQ^2$$

$$\text{Average revenue: } AR = \frac{R}{Q} = \frac{P \times Q}{Q} = P = a - bQ$$

$$\text{Intercept of AR} = a; \text{ slope of AR} = \frac{d(AR)}{dQ} = -b$$

$$\text{Marginal revenue: } MR = \frac{dR}{dQ} = \frac{d}{dQ}(aQ - bQ^2) = a - 2bQ$$

$$\text{Intercept of MR} = a$$

$$\text{Slope of MR} = \frac{d(MR)}{dQ} = -2b = 2 \times (-b) = 2 \times \text{slope of AR}$$

AR and MR have identical intercept but the slope of MR is two times than that of AR. Thus, if the demand function is assumed linear then the MR would be two times steeper than AR.



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In Figure 8.1, the MR function is drawn as two times steeper than the AR function.

Corresponding AR and MR functions are:  $AR = a - bQ$  and  $MR = a - 2bQ$ .

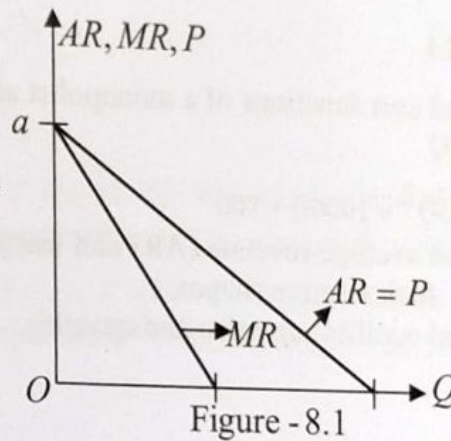


Figure - 8.1

### 8.3 Short Run Equilibrium

The monopolist attains equilibrium by maximizing profit. Two conditions of profit maximization are  $MR = MC$  and slope of  $MR < \text{slope of } MC$ .

In short run equilibrium, monopolist can make economic profit or loss depending on the volume of cost relative to earning. Figure 8.2 demonstrates monopolist's equilibrium with economic profit. Equilibrium price and quantity are OG and  $Oq_0$  respectively. Unit cost OF is smaller than price OG. Monopolist makes FG amount of profit per unit. Figure 8.3 illustrates monopolist's equilibrium without any profit or loss since price OK is equal to average cost. Monopolist can even make loss to a limited extent in the short run which is displayed in Figure 8.4. Unit price OW is smaller than unit cost OZ, resulting a loss equal to WZ per unit.

#### Short Run Equilibrium of Monopolist

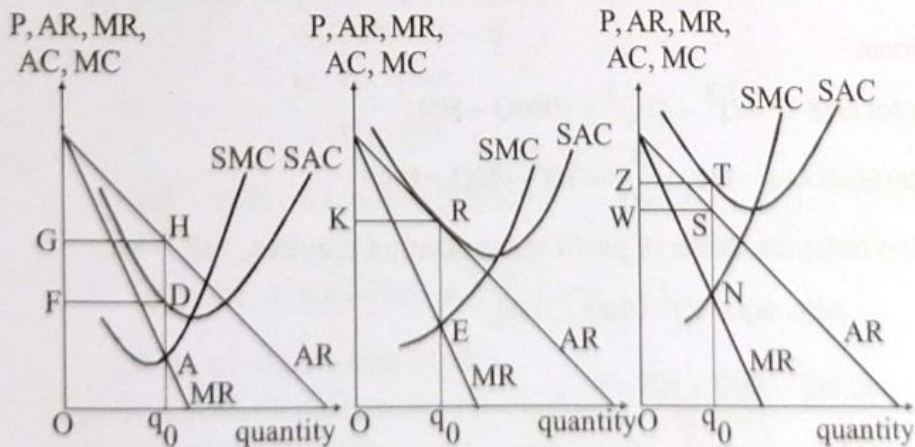


Figure- 8.2

Figure- 8.3

Figure- 8.4

**Example 8.1**

Demand and cost functions of a monopolist are as follows.

$$P = 500 - 2Q$$

$$C = Q^3 - 45Q^2 + 1000Q + 800$$

- i. Find average revenue (AR) and marginal revenue (MR). Make comment on their relative shapes.
- ii. Find equilibrium price and quantity.

**Solution**

i. Total revenue,  $R = P \times Q = (500 - 2Q)Q = 500Q - 2Q^2$

$$\text{Average revenue, } AR = \frac{R}{Q} = \frac{500Q - 2Q^2}{Q} = 500 - 2Q$$

$$\text{Marginal revenue, } MR = \frac{dR}{dQ} = 500 - 4Q$$

$$\text{Having, } AR = 500 - 2Q$$

$$\text{Intercept of } AR = 500; \text{ slope of } AR = \frac{d(AR)}{dQ} = -2$$

$$MR = 500 - 4Q$$

$$\text{Intercept of } MR = 500;$$

$$\text{slope of } MR = \frac{d(MR)}{dQ} = -4 = 2 \times (-2) = 2 \times \text{slope of } AR$$

AR and MR have equal intercept but the slope of MR is two times that of AR.

- ii. Given

$$\text{Total cost } C = Q^3 - 45Q^2 + 1000Q + 800;$$

$$\text{Marginal cost } MC = \frac{dC}{dQ} = 3Q^2 - 90Q + 1000$$

First order condition of profit maximization requires,  $MR = MC$

$$500 - 4Q = 3Q^2 - 90Q + 1000$$

$$\text{or, } 3Q^2 - 86Q + 500 = 0$$

$$\therefore Q = \frac{-(-86) \pm \sqrt{(-86)^2 - 4 \times 3 \times 500}}{2 \times 3}$$

$$= \frac{86 \pm \sqrt{7396 - 6000}}{6} = \frac{86 \pm 37.36}{6} = 20.56 \text{ \& } 8.11$$



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$$MC = 3Q^2 - 90Q + 1000$$

$$\text{Slope of MC} = \frac{d(MC)}{dQ} = 6Q - 90$$

When  $Q = 20.56$ , Slope of MC =  $(6 \times 20.56) - 90 = 33.36$ ; slope of MR = -4

Slope of MR < slope of MC. Thus profit would be maximum at  $Q = 20.56$ .

Set  $Q = 20.56$  into  $P = 500 - 2Q$ ;  $\therefore P = 500 - 2 \times 20.56 = 458.88$

Profit maximizing price and output are  $P^* = 458.88$  &  $Q^* = 20.56$ .

### Example 8.2

Assume demand function  $P = 1000 - 2Q$  & total cost  $TC = 50 + 20Q$

- Find monopoly price, output and profit;
- Compare competitive solution with monopoly solution;
- Determine the amount of deadweight loss

#### Solution

i. Given,  $P = 1000 - 2Q$

$$\text{Total revenue, } R = P \times Q = (1000 - 2Q)Q = 1000Q - 2Q^2$$

$$\text{Marginal revenue, } MR = \frac{dR}{dQ} = 1000 - 4Q$$

$$TC = 50 + 20Q$$

$$\text{Marginal cost, } MC = \frac{dC}{dQ} = 20$$

Monopolist's equilibrium condition,  $MR = MC$

$$\text{or, } 1000 - 4Q = 20$$

$$\text{or, } 4Q = 980$$

$$\therefore Q = 245$$

$$MR = 1000 - 4Q; \text{ slope of MR} = \frac{d(MR)}{dQ} = -4$$

$$MC = 20; \text{ slope of MC} = \frac{d(MC)}{dQ} = 0$$

Slope of MR < slope of MC

Thus monopolist's profit would be maximum if  $Q = 245$ .

Set  $Q = 245$  into  $P = 1000 - 2Q$

$$\text{or, } P = 1000 - (2 \times 245) = 1000 - 490 = 510$$

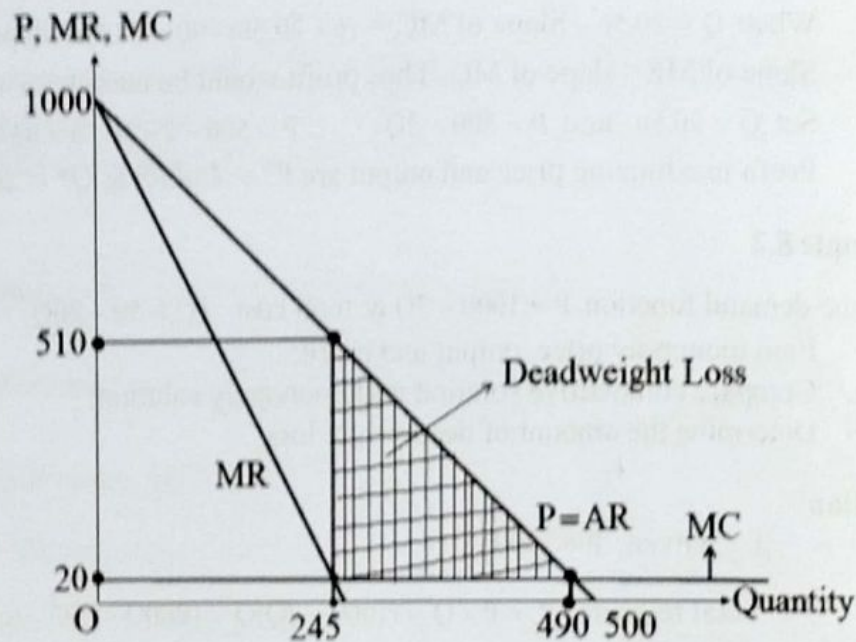
$$\text{Total revenue, } R = P \times Q = 510 \times 245 = 124950$$

$$\text{Total cost, } TC = 50 + (20 \times 245) = 50 + 4900 = 4950$$

## MICROECONOMICS with simple mathematics

$$\begin{aligned}\text{Profit gained by the monopolist} &= \text{total revenue} - \text{total cost} \\ &= 124950 - 4950 = 120000\end{aligned}$$

### Deadweight Loss



ii. Under perfect competition,  $P = MR = 1000 - 2Q$

Equilibrium under perfect competition requires  $P = MR = MC$

$$1000 - 2Q = 20$$

$$\text{or, } 2Q = 980$$

$$\therefore Q = 490$$

Set  $Q = 490$  into  $P = 1000 - 2Q$

$$\Rightarrow P = 1000 - (2 \times 490)$$

$$\Rightarrow P = 1000 - 980$$

$$\therefore P = 20$$

Monopoly price is much higher than competitive price and monopoly quantity is smaller than competitive quantity.

iii. Deadweight loss is demonstrated by the shaded triangle in the diagram. Area of this triangle measures the amount of deadweight loss.

$$\text{Deadweight loss} = \frac{1}{2} \times (490 - 245) \times (510 - 20) = 60025$$



## Monopoly

### 8.4 Monopolist's Supply Curve

Supply curve shows the one-to-one correspondence between price and quantity supplied. If different units are supplied at different prices, there holds one-to-one relationship between price and quantity of supply but monopolist supplies same quantity at different prices or different quantities at a same price. Thus it is argued that monopolist does not have any supply curve.

#### Monopoly Supply

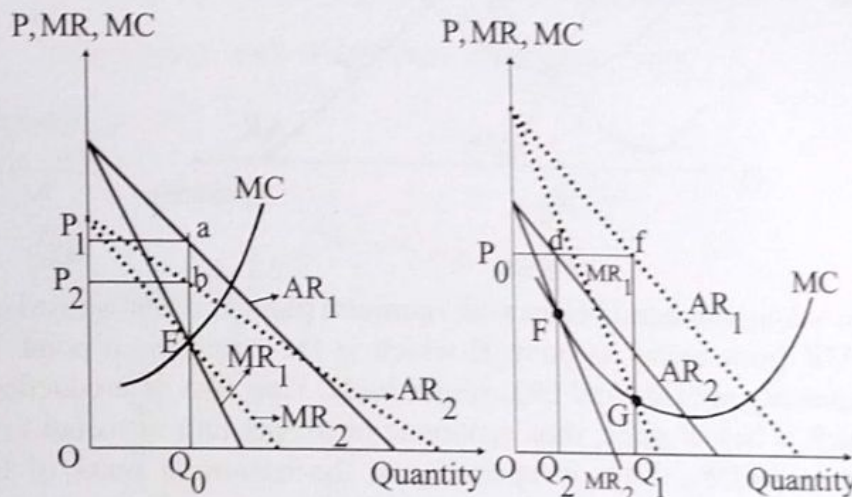


Figure- 8.5

Figure- 8.6

Figure 8.5 explains the situation of two different prices for the same quantity. MC intersects both MR<sub>1</sub> and MR<sub>2</sub> at point E. Prices corresponding to AR<sub>1</sub> and AR<sub>2</sub> are P<sub>1</sub> and P<sub>2</sub> respectively but quantity is fixed at OQ<sub>0</sub>. Since the monopolist is supplying the same quantity at different prices, it implies no one-to-one relation between price and quantity supplied. Figure 8.6 illustrates the opposite case of identical price for different quantities. MC cuts MR<sub>1</sub> at point G and MR<sub>2</sub> at point F. Price is fixed at P<sub>0</sub> but quantities are OQ<sub>1</sub> and OQ<sub>2</sub> respectively. There is no unique supply decision of the monopolist, which confirms the nonexistence of a supply curve.

### 8.5 Long Run Equilibrium

As opposed to the short-run equilibrium, monopolist earns supernormal profit in the long-run because monopolist is the single seller who finds ample opportunity of extracting benefits from the market. Thus the monopolist can exercise market power. Monopolist, however, can utilize optimum plant, less than optimum plant or more than optimum plant in the long-run equilibrium. Three different situations are demonstrated below.

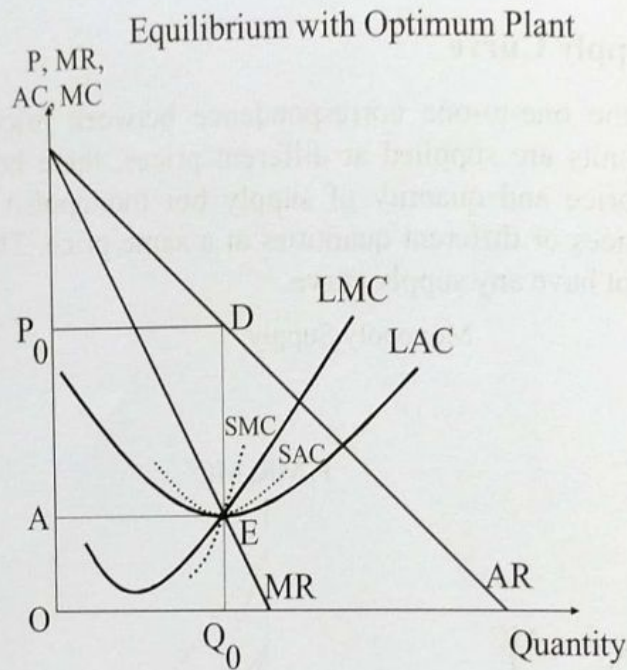


Figure- 8.7

Monopolist's long run equilibrium with optimum plant is demonstrated in figure 8.7. MC cuts MR from below at point E which is the equilibrium point. Equilibrium price and quantity are  $OP_0$  and  $OQ_0$  respectively. Unit cost of production appears to be  $OA$  which is below price, thus economic profit per unit turns out to be  $AP_0$  and total profit is  $AEDP_0$ . Firm is operating at the minimum point of LAC, which corresponds to the optimum plant denoted as SAC.

Equilibrium with Less Than Optimum Plant

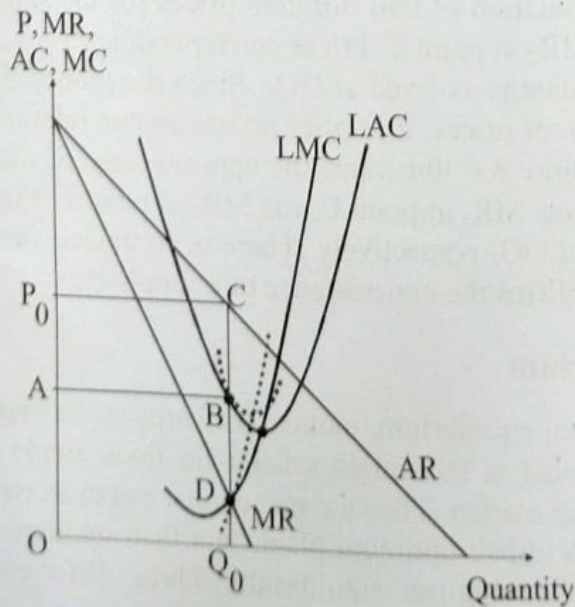


Figure- 8.8



Monopolist can make economic profit by using even less than optimum plant as in Figure 8.8. Equilibrium point is D where MC and MR intersect. Price and quantity are  $OP_0$  and  $OQ_0$  respectively. Average cost of production is OA and per unit profit is  $AP_0$  and total profit equals to  $ABCP_0$ . SAC and SMC are drawn as dashed curves. The concerned firm is operating at a point on the left of the minimum point of LAC which represents less than optimum plant. Less than optimum plant corresponds to the falling part of average cost curve. The reason behind falling average cost is the existence of stronger economies of scale. In production environment both economies and diseconomies may occur but at the beginning the economies remain stronger and thus per unit cost of production falls, making average cost curve downward. Falling average cost is favourable for the monopolist to make economic profit.

### Equilibrium with More Than Optimum Plant

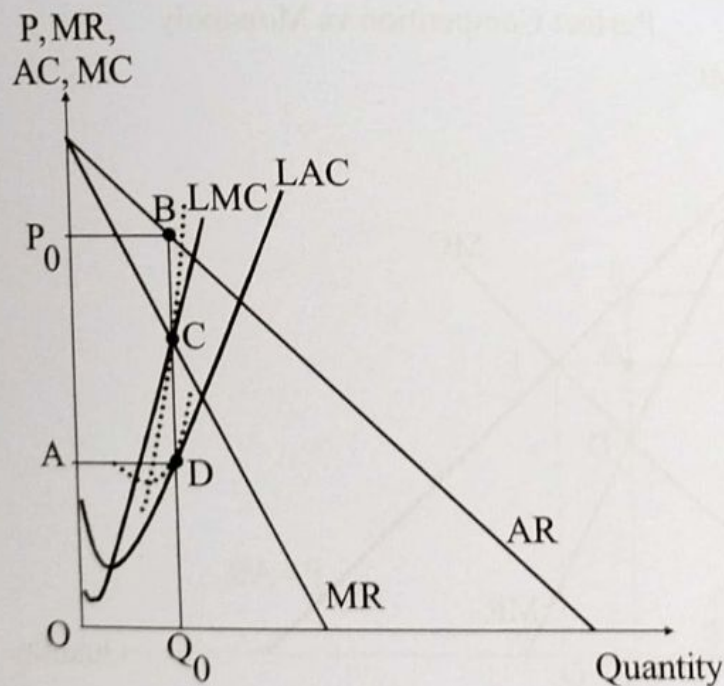


Figure- 8.9

Figure 8.9 describes the long run equilibrium with more than optimum plant.

Equilibrium point: C

Equilibrium price =  $OP_0$

Equilibrium quantity =  $OQ_0$

Total revenue = price  $\times$  quantity =  $OP_0 \times OQ_0$   
 $= OP_0BQ_0$

Average cost = OA

Total cost = average Cost  $\times$  quantity =  $OA \times OQ_0$   
 $= OADQ_0$

Profit = revenue - cost =  $OP_0BQ_0 - OADQ_0$   
 $= ADBP_0$

The monopolist operates at point D which is a point on the right of the minimum point of LAC. Short run average cost, which is drawn as a dashed curve, is tangent to LAC at a point which stays on the right of its minimum point. Thus it refers to more than optimum plant.

The above analysis suggests that a monopolist is able to earn economic profit in the long run regardless of the nature of the plant size.

## 8.6 Perfect Competition versus Monopoly: Dead-weight-loss

Compared to monopoly, perfectly competitive market structure yields higher output at a lower price. Amount of social welfare, measured as the sum of consumer surplus and producer surplus, squeezes under monopoly than perfect competition. Figure 8.10 below provides the explanation.

## Perfect Competition vs Monopoly

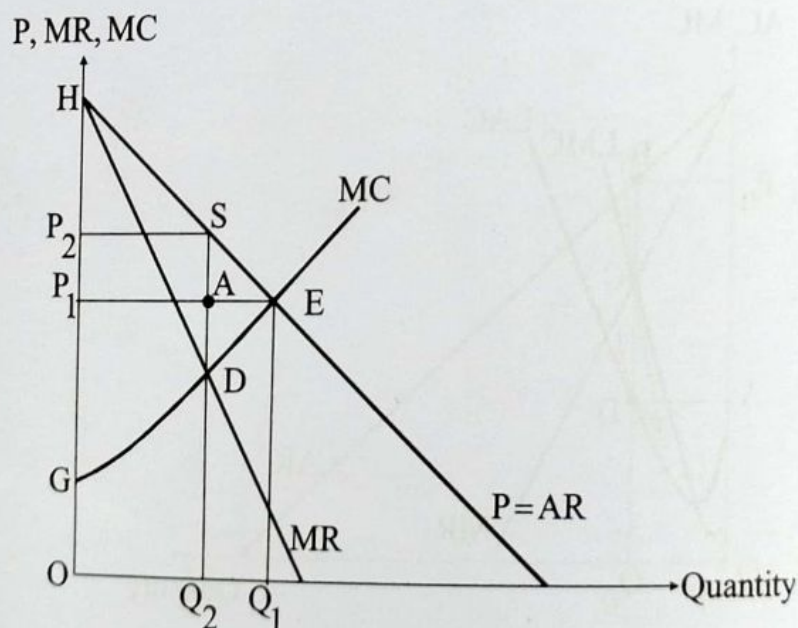


Figure- 8.10

Under perfect competition  $P = AR = MR$ . Equilibrium is determined at point E where  $P = MC$ . Competitive price and quantity are  $P_1$  and  $Q_1$  respectively. Total utility is measured as the area under the demand curve and total cost as the area under the supply curve over a range of quantity. Total expenditure of the consumer is equal to the price times quantity.



## Monopoly

### Under Perfect Competition

Total utility from the consumption of  $OQ_1 = OHEQ_1$

Total expenditure = price  $\times$  quantity =  $OP_1 \times OQ_1 = OP_1EQ_1$

Consumer surplus = Total utility – Total expenditure

$$= OHEQ_1 - OP_1EQ_1 = P_1HE$$

Total revenue of the producer = total expenditure of the consumer

$$= \text{price} \times \text{quantity} = OP_1 \times OQ_1 = OP_1EQ_1$$

Total cost of producing  $Q_1$  output =  $OGEQ_1$

Producer surplus = Total revenue – Total cost =  $OP_1EQ_1 - OGEQ_1 = P_1GE$

Thus social welfare under competition = Consumer surplus + Producer surplus

$$= P_1HE + P_1GE = GHE$$

### Under Monopoly

Since linear demand function is assumed, the marginal revenue function in monopoly would be two times steeper than the average revenue function. In figure 8.10 marginal revenue MR is drawn as two times steeper than average revenue AR. Monopolist's equilibrium is attained at point D where price and quantity are  $OP_2$  and  $OQ_2$  respectively.

Total utility of the consumer =  $OHSQ_2$

Total expenditure of the consumer =  $OP_2 \times OQ_2 = OP_2SQ_2$

Consumer surplus = Total utility – Total expenditure =  $OHSQ_2 - OP_2SQ_2 = HP_2S$

Total revenue of the producer = Total expenditure of the consumer =  $OP_2SQ_2$

Total cost of producing  $Q_2$  output =  $OGDQ_2$

Producer surplus = Total revenue – Total cost =  $OP_2SQ_2 - OGDQ_2 = GDSP_2$

Social welfare under monopoly = consumer surplus + producer surplus =  $HP_2S + GDSP_2 = GDSH$

Area GHE, the measure of social welfare under perfect competition, is clearly bigger than the area GDSH, implying a drop in social welfare under the monopoly than the perfect competition.

Drop in the social welfare =  $GHE - GDSH = DSE$ , which is the amount of welfare loss. Note that the amount of welfare loss, consisting of two areas ASE and ADE, is also called deadweight loss. The reason is discussed below.

Under perfect competition, consumer surplus =  $P_1HE$

Under monopoly, consumer surplus =  $HP_2S$

Loss in consumer surplus =  $P_1P_2SE = P_1P_2SA + ASE$

The amount  $P_1P_2SA$  is now going to the producer as revenue but the area  $ASE$  is not. Consumer loses this area but producer does not receive this, therefore it is termed as the deadweight loss.

Again under perfect competition, producer surplus is  $P_1GE$ , whereas under monopoly producer surplus is  $GDSP_2$

Producer surplus increases by  $P_1P_2SA$  and decreases by  $ADE$ . The increased amount, however, is a transfer from consumer to producer while the market is turned into monopoly from perfect competition but the area  $ADE$  lost by the producer is not received by anyone in the society. This accounts for second portion of deadweight loss. The amount of deadweight loss or efficiency loss altogether equals to  $ASE + ADE = DSE$ .

**Example 8.3** Find the amount of producer surplus and consumer surplus under both perfect competition and monopoly considering the following demand and supply functions. What is the amount of dead-weight-loss?

$$P_d = 10 - 2Q$$

$$P_s = 3Q$$

**Solution**

Equilibrium under perfect competition is achieved if  $P_d = P_s$

$$\Rightarrow 10 - 2Q = 3Q$$

$$\Rightarrow 5Q = 10$$

$$\therefore Q = 2$$

Equilibrium price under perfect competition can be obtained by plugging  $Q = 2$  into the demand or supply function. Here,  $P = 10 - 4 = 6$

Total utility of the consumer under perfect competition =

$$\int_0^2 (10 - 2Q)dQ = 10 \left[ Q \right]_0^2 - \left[ Q^2 \right]_0^2 = 20 - 4 = 16$$

Total expenditure = Price  $\times$  Quantity =  $6 \times 2 = 12$

Consumer surplus under perfect competition = Total utility - Total expenditure =  $16 - 12 = 4$

$$\text{Total cost of the producer for producing 2 units output} = \int_0^2 3QdQ = \frac{3}{2} \left[ Q^2 \right]_0^2 = 6$$

Producer surplus = Total revenue - Total cost =  $12 - 6 = 6$

Under perfect competition, consumer surplus + producer surplus =  $4 + 6 = 10$

In order to find equilibrium entities under monopoly, marginal revenue function has to be derived.



## Monopoly

Given the demand function  $P = 10 - 2Q$

Total revenue,  $R = PQ = (10 - 2Q)Q = 10Q - 2Q^2$

Marginal revenue,  $MR = \frac{dR}{dQ} = 10 - 4Q$

Equilibrium condition under monopoly is:  $MR = MC$

$$10 - 4Q = 3Q$$

$$\text{or, } 7Q = 10$$

$$\therefore Q = \frac{10}{7}$$

$$\therefore P = 10 - 2 \times \frac{10}{7} = \frac{50}{7}$$

Price and quantity under monopoly are  $\frac{50}{7}$  and  $\frac{10}{7}$  respectively.

Therefore, total expenditure of the consumer or total revenue of the producer =  $\frac{50}{7} \times \frac{10}{7} = \frac{500}{49}$

Consumer's utility obtained from the consumption of  $\frac{10}{7}$  units =  $\int_0^{10/7} (10 - 2Q)dQ$

$$= 10[Q]_0^{10/7} - [Q^2]_0^{10/7} = \frac{600}{49}$$

Consumer surplus under monopoly = Total utility - Total expenditure

$$= \frac{600}{49} - \frac{500}{49} = \frac{100}{49}$$

Total cost of the producer for producing  $\frac{10}{7}$  units output

$$= \int_0^{10/7} 3Q dQ = \frac{3}{2} [Q^2]_0^{10/7} = \frac{150}{49}$$

Producer surplus under monopoly = Revenue - Cost =  $\frac{500}{49} - \frac{150}{49} = \frac{350}{49}$

Under monopoly,

$$\text{consumer surplus} + \text{producer surplus} = \frac{100}{49} + \frac{350}{49} = \frac{450}{49} = 9.18$$

$$\text{Dead-weight-loss} = 10 - 9.18 = 0.82$$

## 8.7 Elasticity and Pricing Decision

Business firms set price in order to maximize profit. Pricing decision is influenced by the elasticity of demand. Profit maximization requires two conditions to be fulfilled:  $MR = MC$  and Slope of  $MR < \text{Slope of } MC$ . The first condition hinges upon elasticity of demand via the relationship between  $MR$  and elasticity.

General form of the demand function  $Q = f(P)$

Price elasticity of demand,  $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q}$

Elasticity coefficient (absolute value of elasticity),  $e = -\frac{dQ}{dP} \cdot \frac{P}{Q}$

Total revenue,  $R = P \times Q$

Average revenue,  $AR = \frac{R}{Q} = \frac{P \times Q}{Q} = P$

Marginal revenue,  $MR = \frac{dR}{dQ} = \frac{d}{dQ}(P \times Q)$

$$\Rightarrow MR = P + Q \frac{dP}{dQ}$$

$$\Rightarrow MR = P \left( 1 + \frac{Q}{P} \frac{dP}{dQ} \right)$$

$$\Rightarrow MR = P \left( 1 + \frac{1}{\frac{P}{Q} \frac{dQ}{dP}} \right)$$

$$\Rightarrow MR = P \left( 1 - \frac{1}{e} \right) \quad \dots \dots (8.1) \quad (\because e = -\frac{dQ}{dP} \cdot \frac{P}{Q})$$

(8.1) expresses the relationship between marginal revenue and elasticity. Since microeconomic theory suggests positive marginal cost ( $MC$ ) of production, marginal revenue should also be positive so that the first order condition of profit maximization  $MR = MC$  holds. But positive marginal revenue would only be accrued if elasticity coefficient ( $e$ ) lies above unity. This can be examined from

$$(8.1). MR = P \left( 1 - \frac{1}{e} \right)$$



## Monopoly

If demand gets inelastic, i.e.,  $e < 1$ , suppose  $e = \frac{1}{4}$ , this leads to

$$MR = P \left( 1 - \frac{1}{\frac{1}{4}} \right) = -3P \text{ which is negative. Negative MR can never be equal to}$$

positive MC. But if demand is assumed elastic ( $e > 1$ ), suppose  $e = 4$ , this follows

$$MR = P \left( 1 - \frac{1}{4} \right) = \frac{3}{4}P \text{ which is positive. In this case first order condition of profit maximization will properly be met.}$$

### AR, MR and Elasticity

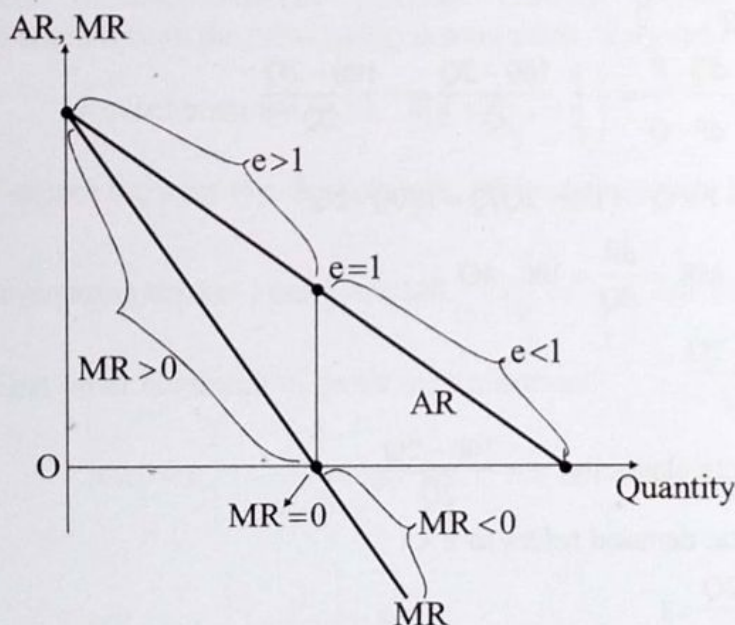


Figure- 8.11

Figure 8.11 shows the link between price elasticity and marginal revenue. Elasticity of demand is equal to 1 at the midpoint of straight-line demand curve, corresponding marginal revenue:  $MR = P \left( 1 - \frac{1}{e} \right) = P(1 - 1) = 0$ . Elasticity being greater than unity would result positive marginal revenue and less than unity negative marginal revenue. Thus profit maximizing pricing decision would be made in the elastic range of demand curve.

## MICROECONOMICS with simple mathematics

### Example 8.4

Given the demand function,  $P = 100 - 2Q$

- Find elasticity as a function of quantity
- Find marginal revenue
- Show that inelastic demand corresponds to negative marginal revenue

### Solution

i) Elasticity,  $\epsilon = \frac{dQ}{dP} \cdot \frac{P}{Q}$

ii) Given,  $P = 100 - 2Q$

$$\frac{dP}{dQ} = -2$$

$$\therefore \frac{dQ}{dP} = -\frac{1}{2}$$

$$\epsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -\frac{1}{2} \cdot \frac{100 - 2Q}{Q} = -\frac{100 - 2Q}{2Q}$$

Total revenue,  $R = P \times Q = (100 - 2Q)Q = 100Q - 2Q^2$

Marginal revenue,  $MR = \frac{dR}{dQ} = 100 - 4Q$

iii)  $\epsilon = -\frac{100 - 2Q}{2Q}$

Absolute elasticity,  $e = \frac{100 - 2Q}{2Q}$

Inelastic demand refers to  $e < 1$

$$\frac{100 - 2Q}{2Q} < 1$$

$$\Rightarrow 100 - 2Q < 2Q$$

$$\Rightarrow 100 < 4Q$$

$$\therefore Q > 25$$

Visit the MR function  $MR = 100 - 4Q$ . For any  $Q > 25$ , MR becomes negative.

For example,

$$\text{if } Q = 26, \quad MR = 100 - (4 \times 26) = 100 - 104 = -4 < 0$$



## 8.8 Price Discrimination

Monopolist has the chance to set discriminating price of its product because there is no substitute of monopolist's product. Price discrimination refers to the sale of identical product at different prices. This may occur in the same market or in different markets. Depending on the strength of discrimination, price discrimination is classified as

- Third degree price discrimination
- Second degree price discrimination and
- Perfect price discrimination or first degree price discrimination

### 8.8.1 Third Degree Price Discrimination

This form of discrimination holds in separate markets depending on the price elasticity. The market with low price elasticity gives the opportunity to charge high price and with high elasticity low price. Since different prices are charged at different markets, this form of price discrimination is the weakest form of discrimination. Trade-off between elasticity and price can be examined from the relationship among price, marginal revenue and elasticity.

$$\text{Revisit equation (8.1), } MR = P \left( 1 - \frac{1}{e} \right)$$

Suppose there are two markets with different elasticities  $e_1$  and  $e_2$ . Marginal

$$\text{revenues in market 1 and 2 are } MR_1 = P_1 \left( 1 - \frac{1}{e_1} \right) \text{ and } MR_2 = P_2 \left( 1 - \frac{1}{e_2} \right).$$

First order condition of profit maximization:

$$MR_1 = P_1 \left( 1 - \frac{1}{e_1} \right) = MC \text{ and}$$

$$MR_2 = P_2 \left( 1 - \frac{1}{e_2} \right) = MC$$

$$\text{Thus, } P_1 \left( 1 - \frac{1}{e_1} \right) = P_2 \left( 1 - \frac{1}{e_2} \right)$$

$$\text{or, } \frac{P_1}{P_2} = \frac{1 - \frac{1}{e_2}}{1 - \frac{1}{e_1}} \dots \dots (8.2)$$

## MICROECONOMICS with simple mathematics

Suppose elasticity in market 1 is higher than 2.

$$e_1 > e_2 \quad (\text{e.g., } 5 > 4)$$

$$\therefore \frac{1}{e_2} > \frac{1}{e_1} \quad \left(\frac{1}{4} > \frac{1}{5}\right)$$

$$\therefore \left(1 - \frac{1}{e_2}\right) < \left(1 - \frac{1}{e_1}\right) \quad (0.75 < 0.80)$$

$$\therefore \frac{1 - \frac{1}{e_2}}{1 - \frac{1}{e_1}} < 1 \quad \left(\frac{0.75}{0.80} < 1\right)$$

Using this expression in (8.2):  $\frac{P_1}{P_2} < 1$

$$\text{i.e., } P_1 < P_2$$

The assumption of higher elasticity leads to lower price in market 1.  
Figure 8.12 and 8.13 describe the fact that the higher elastic demand results in the lower price.

Elasticity Comparison of Two Demand Curves

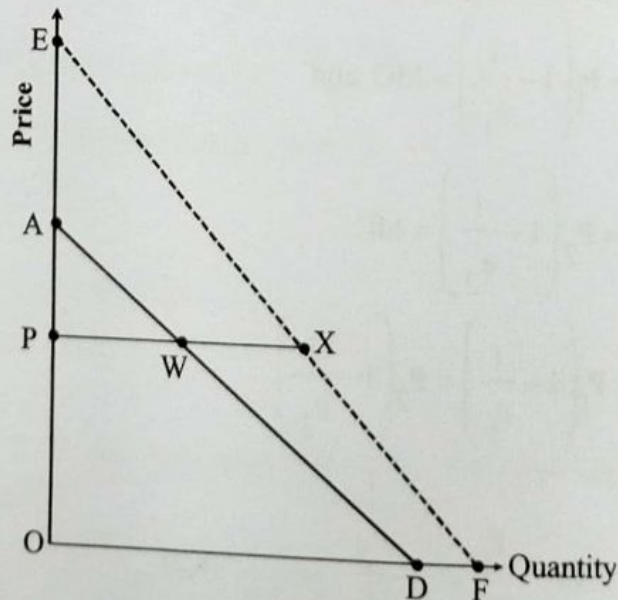


Figure- 8.12



## Monopoly

Two demand curves AD and EF have been drawn in figure 8.12. Let AD be the demand curve of market I and EF of market 2. Elasticities are different in two markets.

At a given price OP,

$$\text{Elasticity at point W of demand curve AD} = \frac{WD}{AW} = \frac{OP}{PA}$$

$$\text{Elasticity at point X of demand curve EF} = \frac{XF}{EX} = \frac{OP}{PE}$$

Since  $\frac{OP}{PA} > \frac{OP}{PE}$ , it suggests that at a given price demand curve AD has higher elasticity than EF. Figure 8.13 explains that the price corresponding to demand curve AD (market I) is lower.

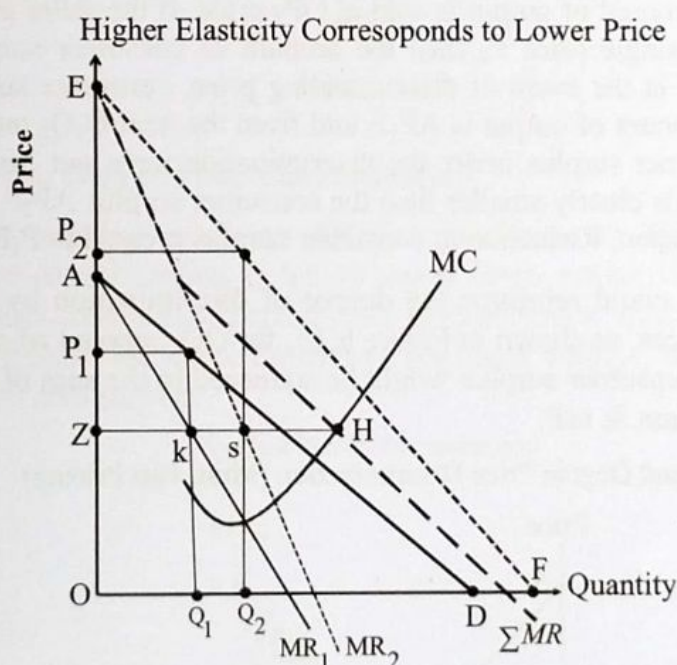


Figure- 8.13

Marginal revenue functions corresponding to AD and EF are  $MR_1$  and  $MR_2$  respectively.  $\Sigma MR$  is obtained by horizontally summing  $MR_1$  and  $MR_2$ . Marginal cost  $MC$  cuts  $\Sigma MR$  at point H. Horizontal line  $HZ$  cuts  $MR_1$  and  $MR_2$  at points  $k$  and  $s$  respectively. These two points are the profit maximizing points of two markets because  $MR_1 = MC$  at point  $k$  and  $MR_2 = MC$  at point  $s$ . Prices are  $OP_1$  and  $OP_2$  in market I and market 2 respectively. It is clear that a third degree price discrimination results in two different prices. Higher elastic demand curve AD results in a low price  $OP_1$  and less elastic demand curve involves a higher price  $OP_2$ .

Third degree price discrimination is most common in real life situation. Demand function faced by the seller of the good that is traded in the supermarkets is relatively less elastic as the rich customers visit such market who do not react much to changes in price. But the demand function for the same good sold in local markets has high price elasticity as the average customers will significantly react to a price change. Naturally, the price in the supermarkets remains relatively higher than in local markets.

### 8.8.2 Second Degree Price Discrimination

This form of price discrimination occurs in the same market. Identical product is sold at two or three different prices. Same customer purchases several units at a higher price, and the subsequent units at a lower price. This sort of price discrimination causes a reduction in the consumer surplus. Figure 8.14 illustrates two part pricing.  $OQ_1$  output is sold at  $OP_1$  price per unit and the next  $Q_1Q_2$  amount of output is sold at  $OP_2$  price. If the seller sells entire  $OQ_2$  output at a single price  $P_2$  then the amount of consumer surplus would be  $AP_2F$ . But in the event of discriminating price, consumer surplus from the first  $OQ_1$  amount of output is  $AP_1E$  and from the next  $Q_1Q_2$  output it is  $EGF$ . Total consumer surplus under the discrimination turns out to be  $AP_1E$  plus  $EGF$ , which is clearly smaller than the consumer surplus  $AP_2F$  in the absence of discrimination. Reduction in consumer surplus is equal to  $P_1P_2GE$ .

If the seller could reinforce the degree of discrimination by charging four different prices, as shown in Figure 8.15, for  $OQ_2$  amount of output then the amount of consumer surplus would be squeezed to the sum of little triangles  $Acb$ ,  $bhE$ ,  $Emn$  &  $nsF$ .

### Second Degree Price Discrimination (Multi Part Pricing)

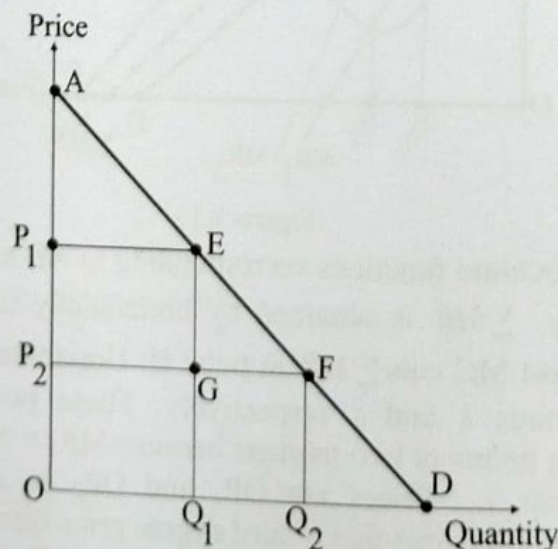


Figure- 8.14



## Monopoly

### Second Degree Price Discrimination (Multi Part Pricing)

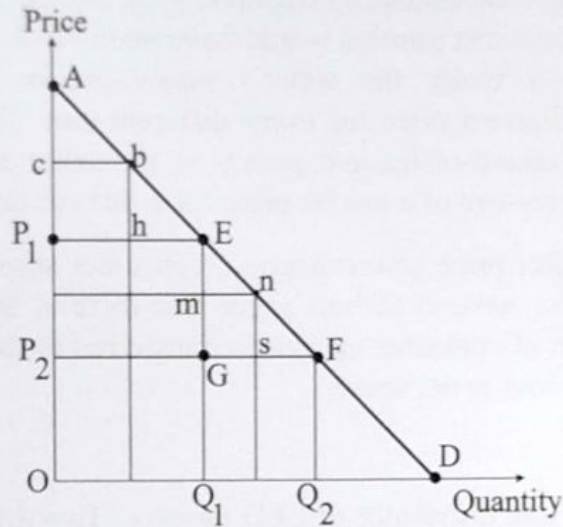


Figure- 8.15

### 8.8.3 First Degree Price Discrimination

If the monopolist can charge different price for every different unit, the form of price discrimination is the strongest form- which is called first degree price discrimination or perfect price discrimination. In this extreme case, the seller exploits the entire amount of consumer surplus, thus the consumer is left with zero surplus.

#### Perfect Price Discrimination

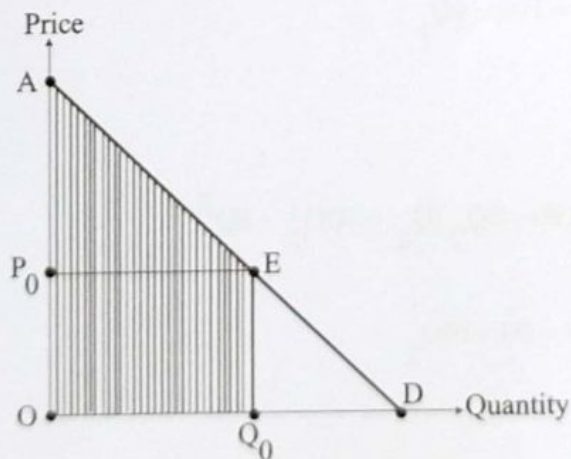


Figure- 8.16

Figure 8.16 displays the situation of perfect price discrimination. Had the consumer got the chance of consuming  $Q_0$  amount of the product at a single price  $P_0$ , consumer surplus would have been  $AP_0E$ . But the entire consumer surplus goes under the seller's possession having the opportunity of charging different price for every different unit. This pricing policy is also known as take-it-or-leave-it policy as the seller is able to put the buyers under the pressure of a unique price for a distinct unit.

Under perfect price discrimination, consumer surplus turns out to be zero. Nevertheless, several authors argue that there is no efficiency loss because full amount of consumer surplus is transferred to the producers, thus there is no welfare loss in net sense.

### Example 8.5

SAMSUNG is the sole distributor of LED monitor. Two different locations have the following demand functions for monitor.

Demand in location 1:  $P_1 = 100 - 2Q_1$

Demand in location 2:  $P_2 = 90 - 5Q_2$

Cost function:  $C = 100 + 2Q$  ( $Q = Q_1 + Q_2$ )

Show that the market in which elasticity is higher, price is lower.

Given,  $P_1 = 100 - 2Q_1$

$$R_1 = P_1 Q_1 = (100 - 2Q_1)Q_1 = 100Q_1 - 2Q_1^2$$

$$\therefore MR_1 = \frac{dR_1}{dQ_1} = 100 - 4Q_1$$

$$P_2 = 90 - 5Q_2$$

$$R_2 = P_2 Q_2 = (90 - 5Q_2)Q_2 = 90Q_2 - 5Q_2^2$$

$$\therefore MR_2 = \frac{dR_2}{dQ_2} = 90 - 10Q_2$$

$$C = 100 + 2Q$$

$$MC = \frac{dC}{dQ} = 2$$



## Monopoly

Equilibrium in location 1:

$$MR_1 = MC$$

$$\Rightarrow 100 - 4Q_1 = 2$$

$$\Rightarrow 4Q_1 = 98$$

$$\therefore Q_1 = 24.5$$

Set  $Q_1 = 24.5$  into  $P_1 = 100 - 2Q_1$

$$\Rightarrow P_1 = 100 - (2 \times 24.5) = 100 - 49 = 51$$

Equilibrium in location 2:

$$MR_2 = MC$$

$$\Rightarrow 90 - 10Q_2 = 2$$

$$\Rightarrow 10Q_2 = 88$$

$$\therefore Q_2 = 8.8$$

Set  $Q_2 = 8.8$  into  $P_2 = 90 - 5Q_2$

$$\Rightarrow P_2 = 90 - (5 \times 8.8) = 90 - 44 = 46$$

Given  $P_1 = 100 - 2Q_1$

$$\frac{dP_1}{dQ_1} = -2$$

$$\therefore \frac{dQ_1}{dP_1} = -\frac{1}{2}$$

Elasticity in market 1:

$$\epsilon_1 = \frac{dQ_1}{dP_1} \cdot \frac{P_1}{Q_1} = -\frac{1}{2} \cdot \frac{51}{24.5} = -1.04$$

$$\therefore e_1 = 1.04$$

$$\text{Given, } P_2 = 90 - 5Q_2$$

$$\frac{dP_2}{dQ_2} = -5$$

$$\therefore \frac{dQ_2}{dP_2} = -\frac{1}{5}$$

Elasticity in market 2:

$$\epsilon_2 = \frac{dQ_2}{dP_2} \cdot \frac{P_2}{Q_2} = -\frac{1}{5} \cdot \frac{46}{8.8} = -1.05$$

$$\therefore e_2 = 1.05$$

What observed is

$$e_1 = 1.04 \quad ; \quad P_1 = 51$$

$$e_2 = 1.05 \quad ; \quad P_2 = 46$$

Higher elasticity corresponds to lower price. (showed)



## Monopoly

### Exercise 8

1. Given the demand and cost function of a monopolist as follows  
 $Q = 500 - 0.5P$ ,  $C = \frac{1}{3}Q^3 - 7Q^2 + 100Q + 50$ 
  - i) Express price as a function of quantity
  - ii) Compare between AR and MR
  - iii) Find profit maximizing price and quantity
  - iv) Calculate maximum profit
2. Assume demand and cost functions as follows  
 $P = 500 - Q$   
 $C = 10 + Q^2$ 
  - i) Find monopoly price, output and profit
  - ii) Compare between monopoly solution and competitive solution
  - iii) What is the amount of deadweight loss?
3. Demand function faced by a monopolist is as below.  
 $Q = 1200 - 0.5P$ 
  - i) Find total revenue, and marginal revenue as function of quantity
  - ii) Compute price elasticity of demand
  - iii) Show that inelastic demand results in negative marginal revenue.
4. Using the following information regarding demand and cost conditions, show that higher elasticity of demand results in lower price.  
 $Q_1 = 100 - 2P_1$   
 $Q_2 = 200 - 10P_2$   
 $C = 10 + 5Q_1 + 5Q_2$
5. Demand function faced by a monopolist is  $P_d = 100 - Q$  and supply function is  $P_s = 4Q$ . Find the amount of producer surplus under monopoly.