Magnetics

Fields Due to Magnets

The magnetic properties of a magnet appear to originate at certain regions in the magnet which are referred to as the poles. In a bar magnet the poles are the ends.

Here are some experimental findings about magnets

1) Like poles are of two kinds
2) Like poles repel each other and unlike poles attract
3) Poles always seem to occur in equal and opposite pairs, and
4) When no other magnet is near, a freely suspended magnet sets so that the line joining its poles (i.e. its magnet axis) is approximately parallel to the earth's north – south axis.

The fourth finding suggests that the earth itself behaves like a large permanent magnet and it makes it appropriate to call the pole of a magnet which points (more or less towards the earth's geographical North Pole, the north pole of the magnet and the other the south pole.

Magnetic Field

The space surrounding a magnet where a magnetic force is experienced is called a magnetic field.

A magnetic field can be represented by magnetic field lines drawn so that:

(i) The line (or the tangent to it if is curved) gives the direction of the field at that point, and

(ii) The number of lines per unit cross-section area is an indication of the "strength" of the field.

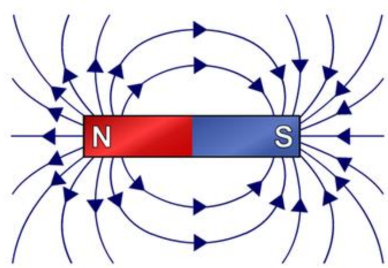


Figure 1: Magnetic Field Lines.

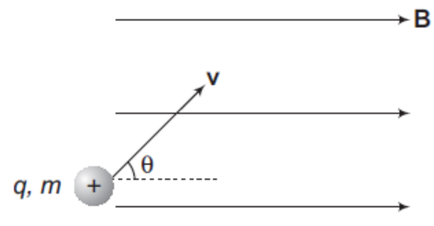
A magnetic field is established by a permanent magnet, by an electric current or by other moving charges. This magnetic field, in turn, exerts forces on other moving charges and current carrying conductors.

Magnetic Force

Magnetic Force can be defined as the attractive or repulsive force that is exerted between the poles of a magnet and electrically charged moving particles. Hence, it is a consequence of the electromagnetic forces. It does not require objects to be in physical contact.

Magnetic Force on a Moving Charge (F_m)

The magnetic force (**F**$\mathbf{F_m}$) on a charge q moving with velocity **v** in a magnetic field **B** is given, both in magnitude and direction, by



$$\overrightarrow{F_m} = q\left(\vec{v} \times \vec{B}\right) \qquad (1)$$

Following points are worthnoting regarding the above expression.

(i). The magnitude of $\overrightarrow{F_m}$ is

$$F_m = qvBsin\theta$$

where, θ is the angle between $\vec{v}$ and $\vec{B}$.

(ii). F_m is zero when,

i. B = 0, i.e. no magnetic field is present.

ii. q = 0, i.e. particle is neutral.

iii. v = 0, i.e. charged particle is at rest or

iv. θ = 0° or 180°, i.e. $\vec{v} \uparrow\uparrow \vec{B}$ or $\vec{v} \uparrow\downarrow \vec{B}$

(iii). F_m is maximum at θ = 90° and this maximum value is qvB.

(iv). The units of $\vec{B}$ must be the same as the units of $\frac{F}{qv}$. Therefore, the SI unit of B is equivalent to $\frac{N-s}{C-m}$. This unit is called the tesla (abbreviated as T), in honour of Nikola Tesla, the prominent Serbian-American scientist and inventor. Thus,

$$tesla = 1\,T = \frac{1\,N - s}{C - m} = \frac{1\,N}{A - m}$$

(v). In equation (1), q is to be substituted with sign. If q is positive, magnetic force is along $\left(\vec{v} \times \vec{B}\right)$ and if q is negative, magnetic force is in a direction opposite to $\left(\vec{v} \times \vec{B}\right)$. In other word, the direction of $\overrightarrow{Fm}$ will be perpendicular to both the direction of velocity $\vec{v}$ and the direction of magnetic field $\vec{B}$. Its exact direction is given by the law of vector product of two vectors.

Direction of $\mathbf{F_m}$

From the property of cross product we can infer that $\mathbf{F_m}$ is perpendicular to both **v** and **B** or it is perpendicular to the plane formed by **v** and **B**. The exact direction of $\mathbf{F_m}$ can be given by any of the following methods:

a) Direction of

$$F_m = (sign\ of\ q)(direction\ of\ \vec{v} \times \vec{B})$$

Or, as we stated earlier also,

$$\overrightarrow{F_m} \uparrow\uparrow \overrightarrow{(v} \times \vec{B})\ if\ q\ is\ positive\ and$$
$$\overrightarrow{F_m} \uparrow\downarrow \overrightarrow{(v} \times \vec{B})\ if\ q\ is\ positive\ and$$

b) **Fleming's left hand rule**

According to this rule, the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other. If the central finger shows the direction of velocity of positive charge ($\boldsymbol{v_{+q}}$) and forefinger shows the direction of magnetic field (**B**), then the thumb will give the direction of magnetic force ($\mathbf{F_m}$). If instead of positive charge we have the negative charge, then $\mathbf{F_m}$ is in opposite direction.

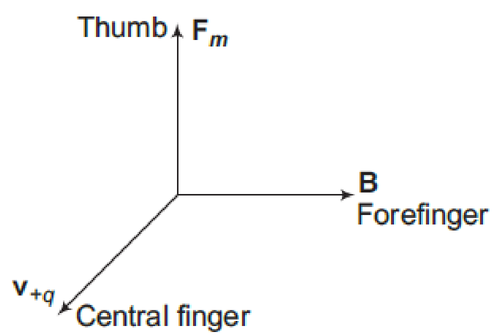


Figure 2: Fleming's left hand rule.

c) **Right hand rule**

Wrap the fingers of your right hand around the line perpendicular to the plane of **v** and **B** as shown in figure, so that they curl around with the sense of rotation from **v** to **B** through the smaller angle between them. Your thumb then points in the direction of the force $\mathbf{F_m}$ on a positive charge.

(Alternatively, the direction of the force $\mathbf{F_m}$ on a positive charge is the direction in which a right hand thread screw would advance if turned the same way).

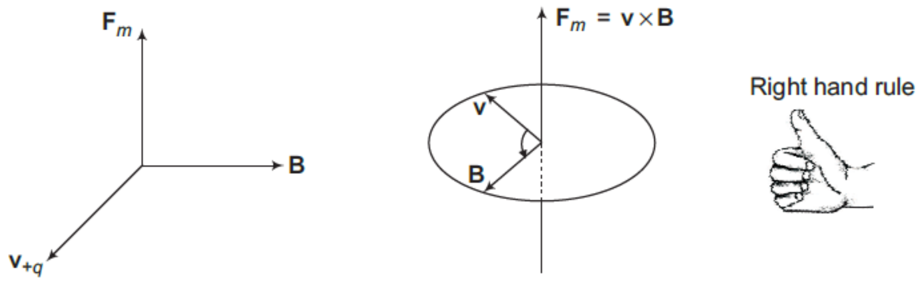


Figure 3: Right hand rule.

Magnetic Force on a Current Carrying Conductor

Figure 4 shows a straight segment of a conducting wire, with length *l* and cross-sectional area A; the current is from left to right. The wire is in a uniform magnetic field ***B***, perpendicular to the plane of the diagram and directed into the plane. Let us assume first that the moving charges are positive.

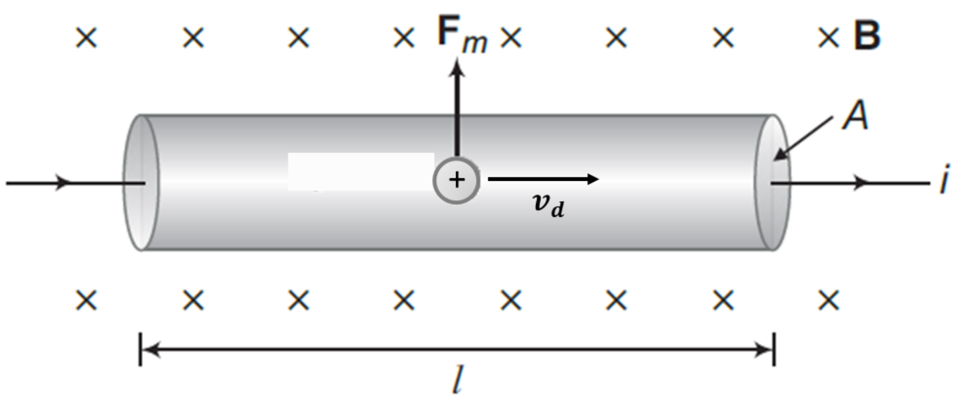


Figure 4: Help to understand the magnetic force on a current carrying conductor.

The drift velocity $\overrightarrow{v_d}$, perpendicular to $\vec{B}$. The average force on each charge is $\overrightarrow{F_m} = q\ (\overrightarrow{v_d} \times \vec{B})$, directed upward; since $\overrightarrow{v_d}$ and $\vec{B}$ are perpendicular, the magnitude of the force is $F_m = qv_dB$.

We can derive an expression for the total force on all the moving charges in a length *l* of a conductor with cross-sectional Area A. The number of charges per unit volume is *n;* a segment of conductor with length l has volume Al and contains a number of charges equal *to* nAl. The total force F_m on all the moving charges in this segment has magnitude

$$F_m = (nAl)(qv_dB) = (nqv_dA)(lB)$$

We have, total current $I = nAv_dq$, Or $v_d = \frac{I}{nAq}$

Therefore, we can rewrite,

$$F_m = IlB$$

If the field B is not perpendicular to the wire but makes an angle φ with it, we handle the situation the same way we did for a single charge. The magnetic force on the wire segment is

$$F_m = IlBsin\varphi$$

We can represent the segment of wire with a vector $\vec{l}$ along the wire in the direction of current; then the force $\overrightarrow{F_m}$ on this segment is

$$\overrightarrow{F_m} = I(\vec{l} \times \vec{B})$$

Lorentz Force

If a charged particle is moving in space where both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ are present, then the total force acting on the charged particle is called the Lorentz force.

The electric force acting on charged particle, $\overrightarrow{F_e} = q\vec{E}$ ……(1)

The magnetic force acting on the charged particle, $\overrightarrow{F_m} = q\left(\vec{v} \times \vec{B}\right)$

The total force acting on the charged particle,

$$\vec{F} = \overrightarrow{F_e} + \overrightarrow{F_m} = q\vec{E} + q\left(\vec{v} \times \vec{B}\right)$$

$$\text{Or, } \vec{F} = q\left[\vec{E} + \left(\vec{v} \times \vec{B}\right)\right]\text{……..(2)}$$

The force given by equation (2) is called the Lorentz force and the equation is known as Lorentz force equation.

Biot-Savart Law

Let us consider a conductor of an arbitrary shape carrying electric current i and P a point in vacuum at which the magnetic field is to be determined. Let us divide the conductor into infinitesimal current-elements. Let us consider a small current element, length *dl*.

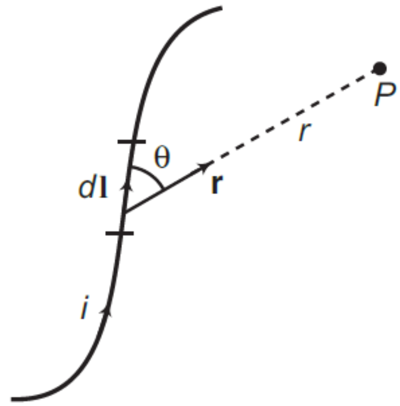


Figure 5: Help to understand Biot-Savart Law.

According to Biot-Savart law, the magnetic field dB produced due to this current element at
point P at a distance r from the element is-

(i) directly proportional to the current flowing in the element i.e. $dB \propto i$.

(ii) directly proportional to the length of element i.e. $dB \propto dl$.

(iii) directly proportional to sin of angle between current element and the line joining current element to point P i.e. $dB \propto sin\,\theta$.

(iv) inversely proportional to the square of the distance of the element from point P i.e.

$$dB\ \alpha\ \frac{1}{r^2}$$

Combining these, we get

$$dB\ \alpha\ \frac{idlsin\theta}{r^2}$$

$$Or, dB = \frac{\mu}{4\pi}\frac{idlsin\theta}{r^2} \ldots\ldots (1)$$

where, $\frac{\mu}{4\pi}$ is a dimensional constant of proportionality whose value depends upon the units used for the various quantities. It depends on the medium between the current element and point of observation (P). Here, μ is called the permeability of medium. Equation (1) is called Biot-Savart law. The product of current i and the length of element dl i.e. idl is called the current element. Current element is a vector quantity; its direction is along the direction of current.

If you place the conductor in vacuum or air, then μ is replaced by μ_0 and thus Biot-Savart law can be written as

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2} \ldots\ldots..(2)$$

μ_0 is called the permeability of free space or air. Its value in the SI system is assigned as-

$\mu_0 = 4\pi \times 10^{-7}$ weber/ampere-meter (WbA^{-1} m^{-1})

Thus, $\frac{\mu_0}{4\pi} = 10^{-7}WbA^{-1}\ m^{-1}$

μ_0 or , $\frac{\mu_0}{4\pi}$ may also be expressed in Newton/Ampere^2 (N/A^2).

The direction of magnetic field is perpendicular to the plane containing current element and the line joining point of observation to current element. Therefore, in vector form, Biot-Savart law can be expressed as-

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{l} \times \vec{r}}{r^3} \ldots\ldots\ldots.(3)$$

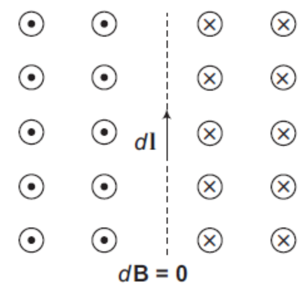


Figure 6: Direction of Magnetic field.

The resultant magnetic field at P due to the whole conductor can be found by integrating equation (3) over the entire length of the conductor. Thus

$$\vec{B} = \int d\vec{B}$$

Applications of Biot Savart Law

Let us consider few applications of Biot Savart law

i) Magnetic field surrounding a thin straight conductor:

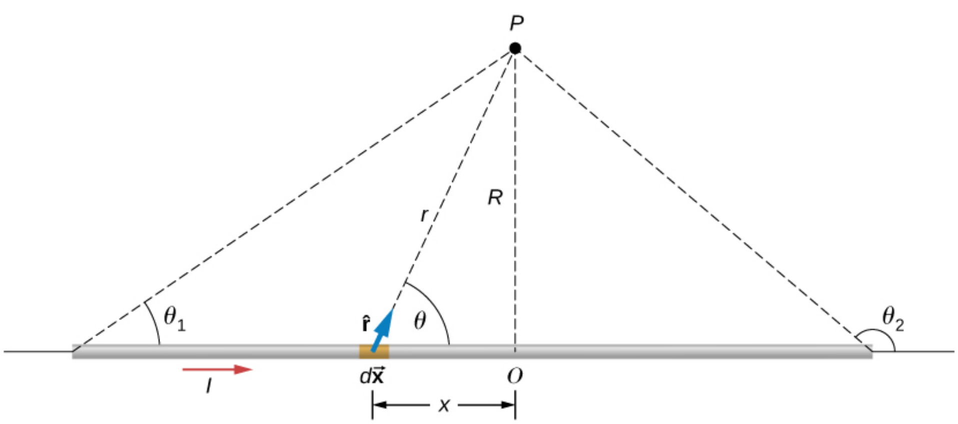


Figure 7: A section of a thin, straight current-carrying wire.

Figure 7 shows a section of an infinitely long, straight wire that carries a current **I**. The magnetic field at a point **P**, located a distance **R** from the wire is

$$\boldsymbol{B} = \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2R}}$$

The magnetic field lines of the infinite wire are circular and centered at the wire (Figure 8), and they are identical in every plane perpendicular to the wire. Since the field decreases with distance from the wire, the spacing of the field lines must increase correspondingly with distance. The direction of this magnetic field may be found with a second form of the **right-hand rule** (Figure 8). If you hold the wire with your right hand so that your thumb points along the current, then your fingers wrap around the wire in the same sense as $\vec{B}$.

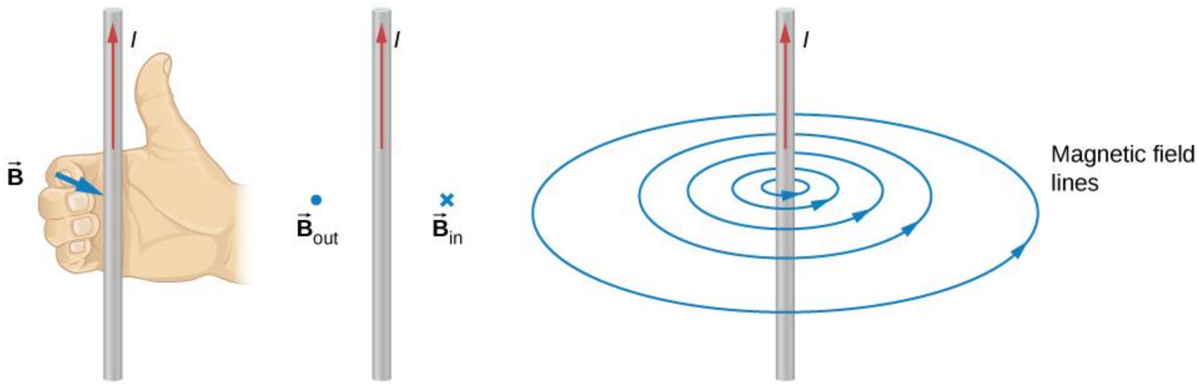


Figure 8: Some magnetic field lines of an infinite wire. The direction of $\vec{B}$ can be found with a form of the right-hand rule.

ii) Magnetic field on the centre of a circular loop

Suppose a current carrying circular loop has a radius *R*. Current in the loop is *i*. The magnetic field at point *O* on the centre of the loop is

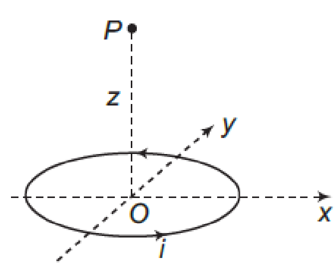


Figure 9.

$$\boldsymbol{B} = \frac{\boldsymbol{\mu_0 N i}}{\boldsymbol{2r}}$$

Where, N is the total loop number of a coil.

Direction of magnetic field on the axis of a circular loop can be obtained using the right hand thumb rule (Figure 10). If the fingers are curled along the current, the stretched thumb will point towards the magnetic field.

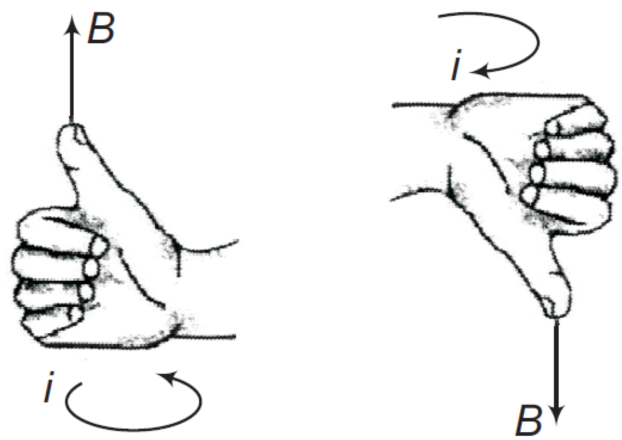


Figure 10.

Problem 1: An electron moving with velocity $5 \times 10^7\ ms^{-1}$ enters a magnetic field of $1\ Wbm^{-2}$at an angle of 90º to the magnetic field. Estimate the magnetic force acting on the electron.

Solution:

Given data,

$$v = 5 \times 10^7\ ms^{-1}$$

$$B = 1\ Wbm^{-2}$$

$$\theta = 90^{\circ}$$

$$q = 1.6 \times 10^{-19}\ C$$

$$F_m = ?$$

Now,

$$F_m = qvBsin\theta = (1.6 \times 10^{-19}\ C) \times (5 \times 10^7\ ms^{-1}) \times (1\ Wbm^{-2}) \times (sin90^{\circ})$$

$$= 8 \times 10^{-12}\ N$$

Problem 2: A charged particle is projected in a magnetic field $\boldsymbol{B} = (3\,\hat{\imath} + 4\,\hat{\jmath}) \times 10^{-2}\ T$. The acceleration of the particle is found to be $\boldsymbol{a} = (x\,\hat{\imath} + 2\,\hat{\jmath})\ ms^{-2}$. Find the value of x.

Solution:

As we read, $\boldsymbol{F_m} \perp \boldsymbol{B}$

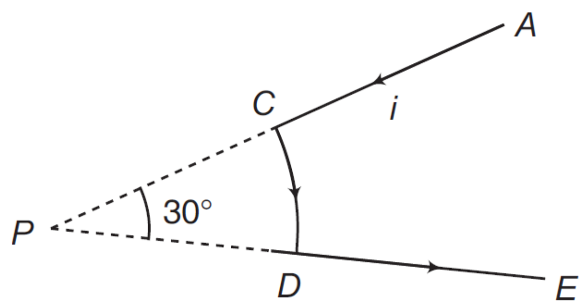
i.e. the acceleration $\boldsymbol{a} \perp \boldsymbol{B}$ **or, a . B** = 0

Or, $(x\,\hat{\imath} + 2\,\hat{\jmath}) \,.\, (3\,\hat{\imath} + 4\,\hat{\jmath}) \times 10^{-2} = 0$

Or, $(3x + 8) \times 10^{-2} = 0$

Or, $x = -\frac{8}{3}\ ms^{-2}$.

Problem 3: A current path shaped as shown in figure produces a magnetic field at P, the centre of the arc. If the arc subtends an angle of 30° and the radius of the arc is 0.6 m, what are the magnitude and direction of the field produced at P if the current is 3.0 A.



Solution:

The magnetic field at P due to the straight segments AC and DE is zero.

CD is arc of circle.

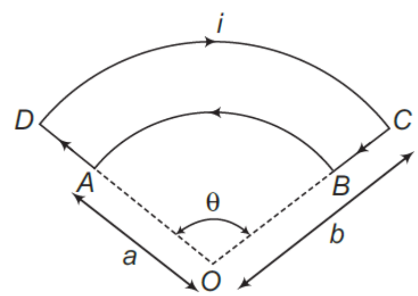
So, $B = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2R}\right)$

Or, $B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)\ \theta$

Or, $B = (10^{-7})\left(\frac{3.0}{0.6}\right)\ (\frac{\pi}{6})$

$= 2.62\ \times 10^{-7}\ T$, inward

Problem 4: Figure shows a current loop having two circular arcs joined by two radial lines. Find the magnetic field B at the centre O.



Solution:

Magnetic field at point O, due to wires CB and AD will be zero.

Magnetic field due to wire BA will be

$$B_1 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2a}\right)$$

Direction of field B_1 is coming out of the plane of the figure.

Similarly, field at O due to arc DC will be

$$B_2 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2b}\right)$$

Direction of field B_2 is going into the plane of the figure. The resultant field at O is

$$B = B_1 - B_2 = \frac{\mu_0 i\theta(b-a)}{4\pi ab}$$

Problem 5: An electron is moving vertically upward with a speed of $2 \times 10^{8}\ ms^{-1}$. Find out the magnitude and direction of the force on the electron exerted by a horizontal magnetic field of 0.50 $0.50\ Wbm^{-2}$ directed towards west? Also calculate the acceleration of the electron.

Solution:

Given data,

$$v = 2 \times 10^{8}\ ms^{-1}$$
$$B = 0.50\ Wbm^{-2}$$
$$\theta = 90^{\circ}$$
$$q = 1.6 \times 10^{-19}\ C$$
$$F_m = ?$$

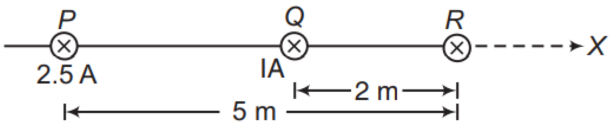
Now,

$$F_m = qvBsin\theta = (1.6 \times 10^{-19}\ C) \times (2 \times 10^{8}\ ms^{-1}) \times (0.50\ Wbm^{-2}) \times (sin90^{\circ})$$

$$= 1.6 \times 10^{-11}\ N, \text{towards north}$$

Using $F = ma$, Or, $a = \frac{F}{m} = \frac{1.6 \times 10^{-11}\ N}{9 \times 10^{-31}\ Kg} = 1.8 \times 10^{19}\ ms^{-2}$

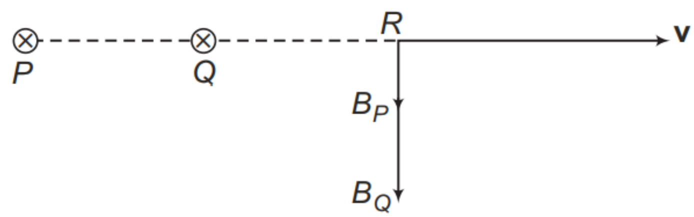
Problem 6: Two long parallel wires carrying currents 2.5 A and I (ampere) in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 m and 2 m respectively from a collinear point R (see figure).



a) An electron moving with a velocity of $4 \times 10^{5}\ ms^{-1}$ along the positive x-direction experiences a force of magnitude $3.2 \times 10^{-20}\ N$ at the point R. Find the value of I.
b) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 A may be placed, so that the magnetic induction at R is zero.

Solution:

a) Magnetic field at R due to both the wires P and Q will be downwards as shown in Figure.



Therefore, net field at R will be sum of these two.

$$B = B_P + B_Q$$
$$= \frac{\mu_0}{2\pi}\frac{I_P}{5} + \frac{\mu_0}{2\pi}\frac{I_Q}{2} = \frac{\mu_0}{2\pi}\left(\frac{2.5}{5} + \frac{I}{2}\right)$$
$$= \frac{\mu_0}{2\pi}(I+1) = 10^{-7}\,(I+1)$$

Net force on the electron will be

$$F_m = qvBsin\theta$$
$$\text{Or, } 3.2 \times 10^{-20} = (1.6 \times 10^{-19})(4 \times 10^5)\,(10^{-7})\,(I+1)$$
$$\text{Or, } I + 1 = 5$$
$$\text{Or, } I = 4\,A$$

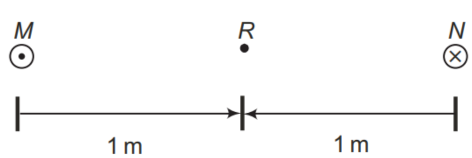
b) Net field at R due to wires P and Q is

$$B = 10^{-7}\,(I+1)\,T$$
$$B = 5 \times 10^{-7}\,T$$

Magnetic field due to third wire carrying a current of 2.5 A should be $5 \times 10^{-7}\,T$ in upward direction, so that net field at R becomes zero. Let distance of this wire from R be r. Then,

$$\frac{\mu_0}{2\pi}\frac{2.5}{r} = 5 \times 10^{-7}\,T$$
$$\text{Or, } \frac{(2 \times 10^{-7})(2.5)}{r} = 5 \times 10^{-7}$$
$$\text{Or, } r = 1\,m$$

So, the third wire can be put at M or N as shown in figure.



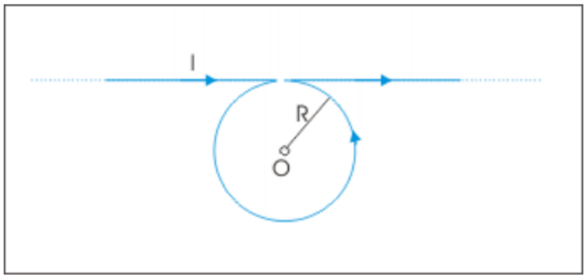
If it is placed at M, then current in it should be outwards and if placed at N, the current be inwards.

Problem 7: What is the force exerted on a straight wire of length 3.5 cm, carrying a current of 5 A, and situated at right angles to a magnetic field of flux density 0.2 T?

Solution: Try it yourself.

Answer: 0.035 N.

Problem 8: Calculate magnetic field at the center O for the current flowing through wire segment as shown in the figure. Here, current through wire is 10 A and radius of the circular part is 0.1 m.



Solution:

The magnitude of magnetic field due to circular wire is:

$$B_C = \frac{\mu_0 I}{2R} = \frac{4\pi \times 10^{-7} \times 10}{2 \times 0.1} = 6.28 \times 10^{-5}\ T, outward$$

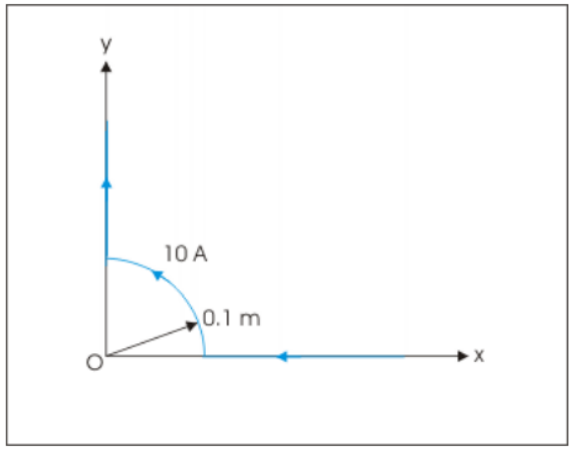
The magnitude of magnetic field due to straight wire is:

$$B_S = \frac{\mu_0 I}{2\pi R} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.1} = 20 \times 10^{-6}\ T, inward$$

Hence, magnitude of magnetic field at O is algebraic sum of two magnetic fields (we consider outward direction as positive) :

$$B = B_C - B_S = 6.28 \times 10^{-5} - 20 \times 10^{-6} = 4.28 \times 10^{-5}\ T, outward$$

Problem 9: Find the magnetic field at the corner O due to current in the wire as shown in the figure. Here, radius of curvature is 0.1 m for the quarter circle arc and current is 10 A.



Solution: Try it yourself.

Answer: 0.157 $\mu T, outward$

Problem 10: An electron experiences a magnetic force of magnitude $4.60 \times 10^{-15} N$, when moving at an angle of 60° with respect to a magnetic field of magnitude $3.50 \times 10^{-3} T$. Find the speed of the electron.

Solution: Try it yourself.

Answer: $9.46 \times 10^{6}\ ms^{-1}$.

Problem 11: A charged particle carrying $q = 1\mu C$ moves in uniform magnetic field with velocity $v_1 = 10^6\ ms^{-1}$ at angle 45° with x-axis in the x-y plane and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative z-axis. When the same particle moves with $v_2 = 10^6\ ms^{-1}$ along the z-axis it experiences a force F_2 in y-direction. Find

(a) magnitude and direction of the magnetic field,

(b) the magnitude of the force F_2 .

Solution:

(a) As magnetic force always acts perpendicular to magnetic field, magnetic field must be along x-axis.

$$F_1 = qv_1 B\ sin\theta_1$$

$$Or, B = \frac{F_1}{qv_1 sin\theta_1} = \frac{5\sqrt{2} \times 10^{-3}}{1 \times 10^{-6} \times 10^{6} \times \frac{1}{\sqrt{2}}}$$

$$B = 10^{-3}\ T$$

$$Or, \quad \vec{\boldsymbol{B}} = (10^{-3}\ T)\hat{\imath}$$

(b)

$F_2 = qv_2B\ sin\theta_2$

$= 1 \times 10^{-6} \times 10^{6} \times 10^{-3} \times \sin 90^{\circ}$

$= 10^{-3}$ N

$= 1\ mN$

Problem 12: Two long parallel transmission lines, 40.0 cm apart, carry 25.0 A and 75.0 A currents. Find all locations where the net magnetic field of the two wires is zero if these currents are in,

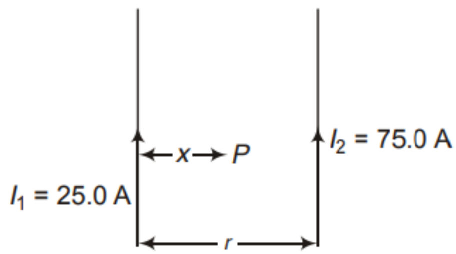
(a) The same direction,

(b) The opposite direction.

Solution:

(a) Consider a point P in between the conductors at a distance x from conductor carrying current I_1 $(25\ A)$.

Magnetic field at P



$B = \frac{\mu_0}{4\pi}\frac{I_1}{x} - \frac{\mu_0}{4\pi}\frac{I_2}{r-x} = 0$

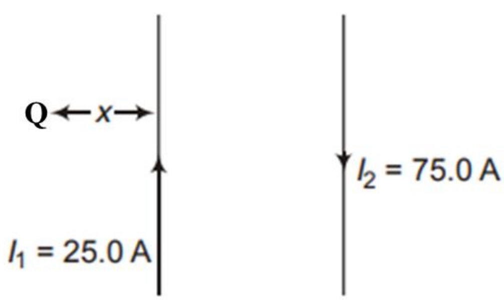
Or, $\frac{I_1}{x} = \frac{I_2}{r-x}$

Or, $\frac{r-x}{x} = \frac{I_2}{I_1}$

Or, $x = \frac{I_1}{I_1+I_2}r = \frac{25.0}{75.0 + 25.0} \times 40 = 10\ cm$

(b) Consider a point Q lying on the left of the conductor carrying current I_1 at a distance x from it.

Magnetic field at Q



$B = \frac{\mu_0}{4\pi}\frac{I_1}{x} - \frac{\mu_0}{4\pi}\frac{I_2}{r+x} = 0$

Or, $\frac{I_1}{x} = \frac{I_2}{r+x}$

Or, $\frac{r+x}{x} = \frac{I_2}{I_1}$

Or, $x = \frac{I_1}{I_2-I_1}r = \frac{25.0}{75.0 - 25.0} \times 40 = 20\ cm$

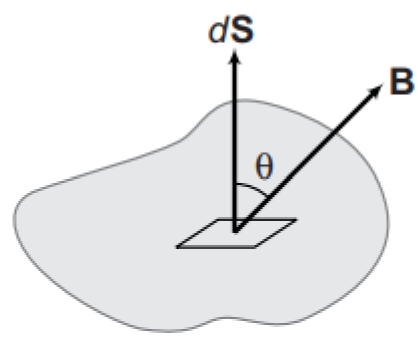
Electromagnetic Induction

In the chapter on current electricity, we learned that for a current to flow in a circuit, there must be an electromagnetic force (emf). But most of the electric devices used in the industry don't get their emf from batteries. Instead, it comes from a place where electricity is made. In these stations, energy from other sources is turned into electricity. For example, in a hydroelectric plant, the potential energy of gravity is turned into electric energy. In the same way, a nuclear plant turns nuclear energy into electric energy. But how is this change made? Or, what does this have to do with physics? All of these questions can be answered by the branch of physics called electromagnetic induction.

Magnetic Flux

Magnetic flux is the number of magnetic field lines passing through a closed surface. It measures the entire magnetic field strength that penetrates a specific region. In this case, there is no restriction on the area's size or orientation for the magnetic field.

Consider an element of area $\overrightarrow{dS}$ on an arbitrary shaped surface as shown in figure. If the magnetic field at this element is $\vec{B}$, the magnetic flux through the element is



$$\boldsymbol{d\varphi_B = \vec{B}.d\vec{S} = BdScos\theta}$$

Here, $\boldsymbol{d\vec{S}}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area d**S** and θ is the angle between **B** and d**S** at that element.

In general, $\boldsymbol{d\varphi_B}$ varies from element to element. The total magnetic flux through the surface is the sum of the contributions from the individual area elements.

$$\boldsymbol{\varphi_B == \int BdScos\theta = \int \vec{B}.d\vec{S}}$$

Note down the following points regarding the magnetic flux :

(i) Magnetic flux is a scalar quantity (dot product of two vector quantities is a scalar quantity)

(ii) The SI unit of magnetic flux is $tesla - metre^2$ $(1\,T - m^2)$. This unit is called weber (1Wb).

$$1\,Wb = 1\,Tm^2 = 1\,Nm/A$$

Thus, unit of magnetic field is also $weber/metre^2$ 2 (1 Wb/m^2).

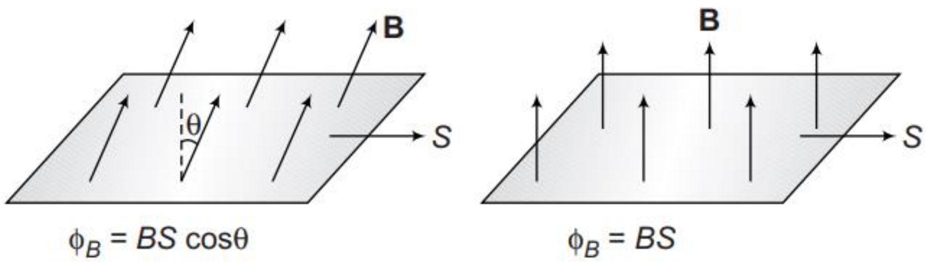
$$\text{Or, } 1\,T = 1\frac{Wb}{m^2}$$

(iii) In the special case in which B is uniform over a plane surface with total area S,

$$\varphi_B = BS\,cos\theta$$

If B is perpendicular to the surface, then cosθ =1 and

$$\varphi_B = BS$$



Faraday's Law

Faraday's law of electromagnetic induction (referred to as **Faraday's law**) is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF). This phenomenon is known as electromagnetic induction.

Faraday's First Law of Electromagnetic Induction

Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.

Or,

Whenever a conductor is placed in a varying magnetic field, an electromotive force is induced. If the conductor circuit is closed, a current is induced, which is called induced current.

Faraday's Second Law of Electromagnetic Induction

The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

$$\varepsilon = -\frac{d\varphi_B}{dt}$$

Where, ε is the electromotive force, φ_B is the magnetic flux.

If a circuit is a coil consisting of N loops all of the same area and if φ_B is the flux through one loop, an emf is induced in every loop, thus the total induced emf in the coil is given by the expression,

$$\varepsilon = -N\frac{d\varphi_B}{dt}$$

Where, N is the number of turns.

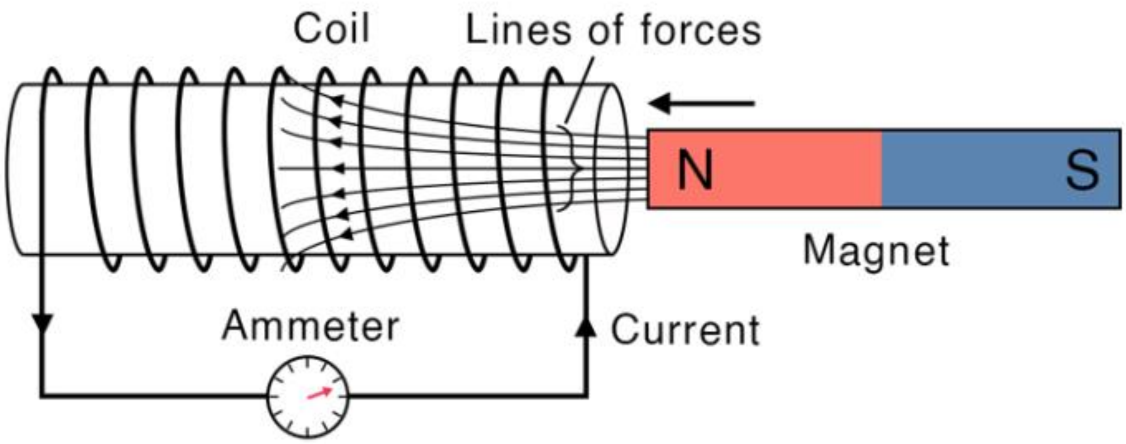


Figure 11: Help to understand Faraday's law.

Note down the following points regarding the Faraday's law:

(i) As we have seen, induced emf is produced only when there is a change in magnetic flux passing through a loop. The flux passing through the loop is given by

$$\boldsymbol{\varphi_B = BScos\theta}$$

This flux can be changed in several ways:

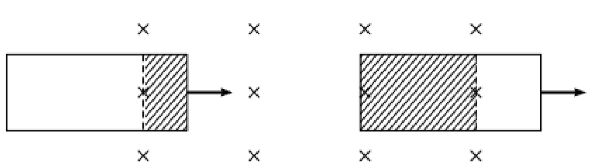
a) The magnitude of $\vec{B}$ can change with time. In the problems if magnetic field is given a function of time, it implies that the magnetic field is changing. Thus,

$$B = B(t)$$

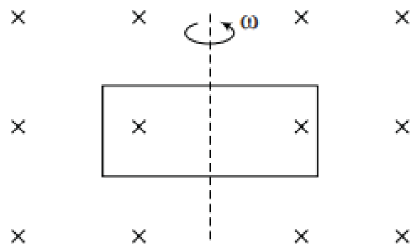
b) The current producing the magnetic field can change with time. For this, the current can be given as a function of time. Hence,

$$i = i(t)$$

c) The area of the loop inside the magnetic field can change with time.This can be done by pulling a loop inside (or outside) a magnetic field.



d) The angle θ between Band the normal to the loop (or s) can change with time.



This can be done by rotating a loop in a magnetic field.

e) Any combination of the above can occur.

(ii) When the magnetic flux passing through a loop is changed, an induced emf and hence, an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by

$$\boldsymbol{i = \frac{\varepsilon}{R} = \frac{1}{R}\left(-\frac{d\varphi_B}{dt}\right)}$$

Current starts flowing in the circuit, means flow of charge takes place. Charge flown in the circuit in time *dt* will be given by

$$\boldsymbol{dq = idt = \frac{1}{R}(-d\varphi_B)}$$

Lenz's Law

The negative sign in Faraday's equations of electromagnetic induction describes the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with the help of Lenz's law. This law states that:

"The direction of any magnetic induction effect is such as to oppose the cause of the effect."

Alternatively, L**enz's law of electromagnetic induction** states that the direction of the current induced in a conductor by a changing magnetic field (as per Faraday's law of electromagnetic induction) is such that the magnetic field created by the induced current ***opposes*** the initial changing magnetic field which produced it. The direction of this current flow is given by Fleming's right hand rule.

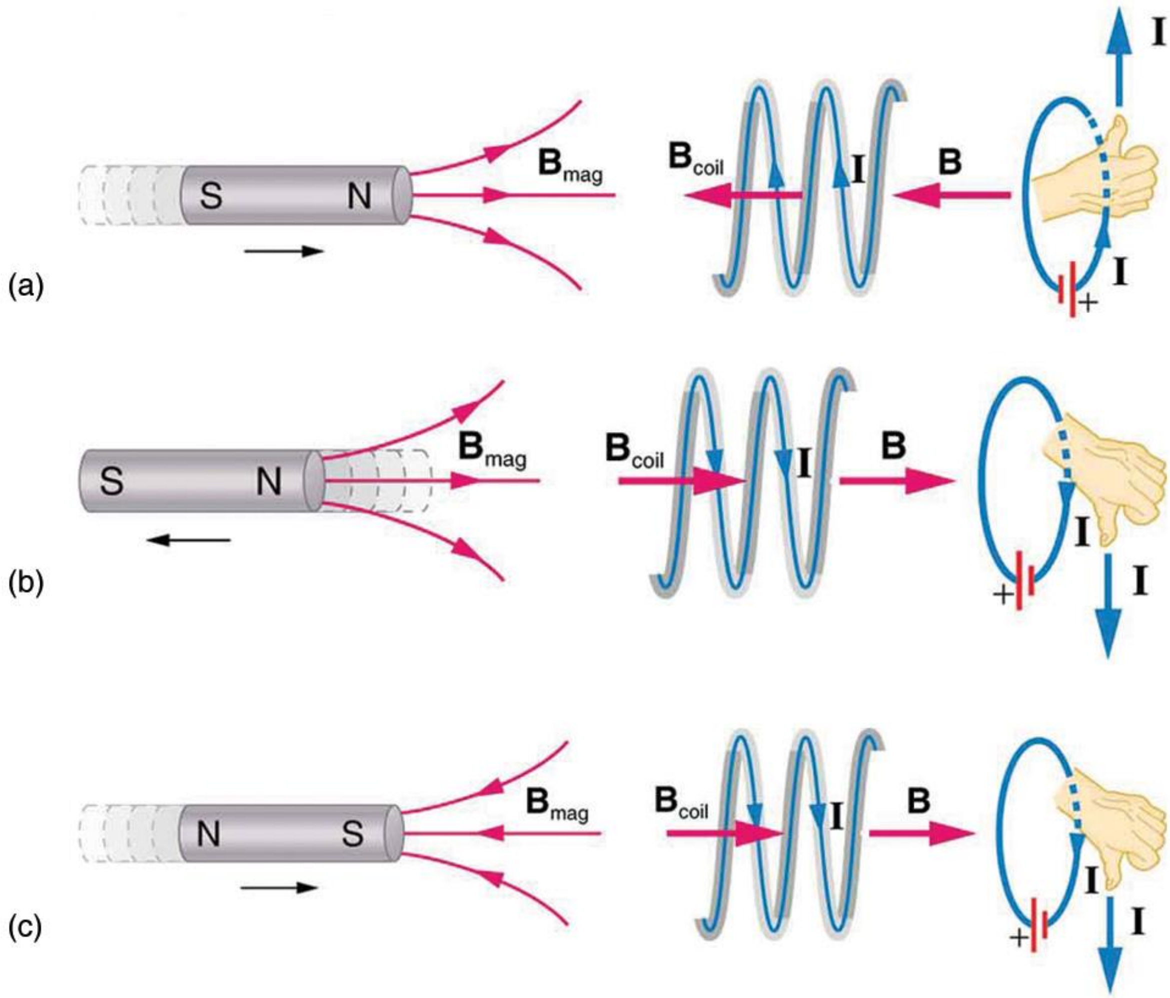
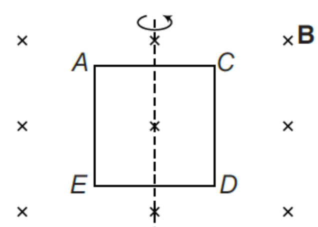


Figure 12: Help to understand Lenz's Law and direction of induced emf.

Problem 13: A square loop ACDE of area 20 cm^2 and resistance 5 W is rotated in a magnetic field **B** = 2T through 180°, (a) in 0.01 s and (b) in 0.02 s.

Find the magnitudes of average values of ε, i and dq in both the cases.



Solution:

Let us take the area vector **S** perpendicular to plane of loop inwards. So initially, **S**↑↑**B** and when it is rotated by 180°, **S**↑↓**B**.

Hence, initial flux passing through the loop,

$$\varphi_i = BScos\theta_i = (2)(20 \times 10^{-4})(\cos 0^{\circ}) = 4.0 \times 10^{-3}\, Wb$$

Flux passing through the loop when it is rotated by 180°,

$$\varphi_f = BScos\theta_f = (2)(20\times10^{-4})(\cos 180^{\circ}) = -4.0\times10^{-3}\,Wb$$

Therefore, change in flux,

$$d\varphi_B = \varphi_f - \varphi_i = -8.0\times10^{-3}\,Wb$$

a) Given, dt = 0.01s, R = 5 W

$$|\varepsilon| = \left|-\frac{d\varphi_B}{dt}\right| = \frac{8.0\times10^{-3}}{0.01} = 0.8\,V$$

$$i = \frac{|\varepsilon|}{R} = \frac{0.8}{5} = 0.16\,A$$

$$\text{And } dq = idt = 0.16\times0.01 = 1.6\times10^{-3}C$$

b) Given, dt = 0.02 s

$$|\varepsilon| = \left|-\frac{d\varphi_B}{dt}\right| = \frac{8.0\times10^{-3}}{0.02} = 0.4\,V$$

$$i = \frac{|\varepsilon|}{R} = \frac{0.4}{5} = 0.08\,A$$

$$\text{And } dq = idt = 0.08\times0.02 = 1.6\times10^{-3}C$$

Problem 14: A coil consists of 200 turns of wire having a total resistance of 2.0 **Ω**. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.5 T in 0.80 s, what is the magnitude of induced emf and current in the coil while the field is changing?

Solution:

From the Faraday's law,

Induced emf,

$$|\varepsilon| = \left|-\frac{Nd\varphi_B}{dt}\right| = N\frac{\Delta BS}{dt} = \frac{(200)\times(0.5-0)\times(18\times10^{-2})^2}{0.8} = 4.05V$$

Induced current,

$$i = \frac{|\varepsilon|}{R} = \frac{4.05}{5} = 0.16\,A$$

Problem 15: The magnetic flux passing through a metal ring varies with time t as : $\varphi_B = 3(at^3 - bt^2)\,Tm^2$ with a = 2.00 s^{-3} and b = 6.00 s^{-2} . The resistance of the ring is 3.0 Ω. Determine the maximum current induced in the ring during the interval from t = 0 to t = 2.0 s.

Solution:

Given, $\varphi_B = 3(at^3 - bt^2)$

Induced emf,

$$|\varepsilon| = \left|\frac{d\varphi_B}{dt}\right| = 9at^2 - 6bt$$

Induced current,

$$i = \frac{|\varepsilon|}{R} = \frac{9at^2 - 6bt}{3} = 3at^2 - 2bt$$

For current to be maximum,

$$\frac{di}{dt} = 0$$

$$\text{Or, } 6\,at - 2b = 0$$

$$\text{Or, } t = \frac{b}{3a}$$

i.e. at $t = \frac{b}{3a}$, current is maximum. This maximum current is

$$i_{max} = \left|3a(\frac{b}{3a})^2 - 2b(\frac{b}{3a})\right|$$

$$= \left|\frac{b^2}{3a} - \frac{2b^2}{3a}\right| = \frac{b^2}{3a}$$

Substituting the given values of a and b, we have

$$i_{max} = \frac{6^2}{3(2)} = 6.0\,A$$

Problem 16: A wire in the form of a circular loop of radius 10 cm lies in a plane normal to a magnetic field of 100 T. If this wire is pulled to take a square shape in the same plane in 0.1 s, find the average induced emf in the loop.

Solution: Try it yourself.

Answer: 6.4 V.

Problem 17: A closed coil consists of 500 turns has area 4 cm^2 and a resistance of 50 Ω. The coil is kept with its plane perpendicular to a uniform magnetic field of $0.2\, Wbm^{-2}$. Calculate the amount of charge flowing through the coil if it is rotated through 180°.

Solution: Try it yourself.

Answer: 1600μC.

Problem 18: A magnetic field in a certain region is given by $\vec{B} = \left(4.0\hat{\imath} - 1.8\,\hat{k}\right) \times 10^{-3} T$. How much flux passes through a 5.0 cm^2 area loop in this region if the loop lies flat on the x-y plane?

Solution: Try it yourself.

Answer: $9.0\ \times 10^{-7} Wb$

Inductance

Inductance is the magnetic analog of capacitance in electric phenomena. Like capacitance, inductance has to do with the geometry of a magnetic device and the magnetic properties of the materials making up the magnetic device. The capacitance C of an electric device is associated with the ability to store energy in the electric field of that device. The inductance L of a magnetic device is associated with the ability to store energy in the magnetic field of that device.

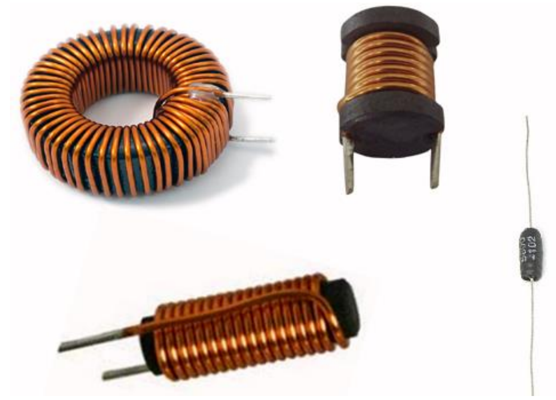


Figure 13: Practical Inductors.

Inductance is classified into two types as:

- Self-Inductance
- Mutual Inductance

Self-Inductance

Consider a single isolated circuit. When a current is present in the circuit, it sets up a magnetic field that causes a magnetic flux through the same circuit. This flux changes as the current in the circuit is changed. According to Faraday's law any change in flux in a circuit produces an induced emf in it. Such an emf is called a **self-induced emf**. The name is so called because the source of this induced emf is the change of current in the same circuit.

First Definition Consider a single conducting circuit includes a coil with N turns of wire. It carries a current i. This current generates a magnetic field $\mathbf{B}$ which gives rise to a magnetic flux φ_B linking the circuit. The total flux ($N\varphi_B$) linked with the coil is directly proportional to the current (i) in the coil, i.e.

$$N\varphi_B \propto i$$

Or,

$$N\varphi_B = Li$$

Where L is termed as the self-inductance of the coil or the coefficient of self-inductance, the self-inductance depends on the cross-sectional area, the permeability of the material, and the number of turns in the coil.

Thus,

$$L = \frac{N\varphi_B}{i}$$

We can define self-inductance (L) of any circuit as, the total flux per unit current. The SI unit of self-inductance is henry (H).

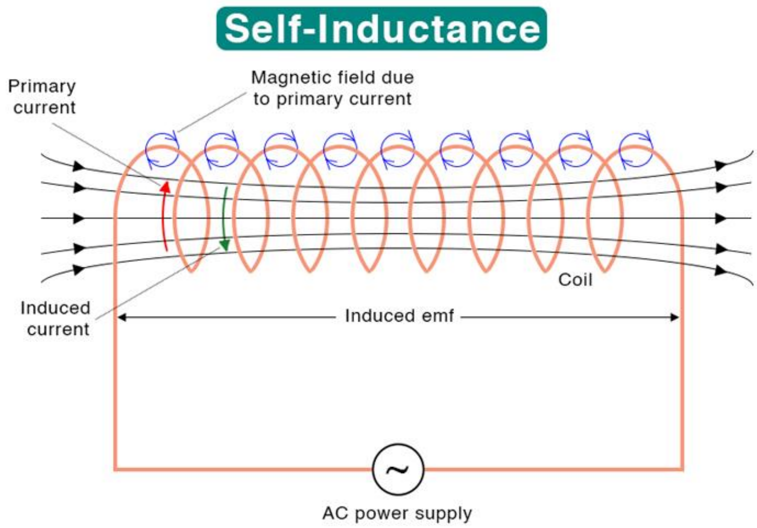


Figure 14: Self-Inductance

Second Definition If a current i is passed in a circuit and it is changed with a rate $\frac{di}{dt}$, the induced emf Ɛ produced in the circuit is directly proportional to the rate of change of current. Thus,

$$\varepsilon \propto \frac{di}{dt}$$

Or,

$$\varepsilon = -L\,\frac{di}{dt}$$

Where, L is the proportionality constant and the minus sign here is a reflection of Lenz's law. It says that the self-induced emf in a circuit opposes any change in the current in that circuit. From the above equation,

$$L = \left|\frac{-\varepsilon}{di/dt}\right|$$

This equation states that, the self-inductance of a circuit is the magnitude of self-inductance emf per unit rate of change of current.

Inductor: A circuit o part of a circuit, that is designed to have a particular inductance is called an inductor. The usual symbol for an inductor is

Thus, an inductor is a circuit element which opposes the change in current through it. It may be circular coil, solenoid etc.

Problem 19: What is the magnetic flux through one turn of a solenoid of self-inductance $8 \times 10^{-5}\ H$ when a current of 3.0 A flows through it? Assume that the solenoid has 1000 turns and is wound from wire of diameter 1.0 mm.

Solution: Given,

$$L = 8 \times 10^{-5}\ H, i = 3.0\ A\ and\ N = 1000\ turns$$

We know,

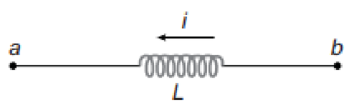
$$N\varphi_B = Li$$

The flux linked with one turn,

$$\varphi_B = \frac{Li}{N} = \frac{(8 \times 10^{-5})(3.0)}{1000}$$

$$= 2.4\ \times\ 10^{-7}\ Wb$$

Problem 20: The inductor shown in the figure has inductance 0.54 H and carries a current in the direction shown that is decreasing at a uniform rate, $\frac{di}{dt} = -0.030\ As^{-1}$:



(a) Find the self-induced emf.

(b) Which end of the inductor, a or b, is at a higher potential?

Solution:

(a) Self-induced emf,

$$\varepsilon = -L\frac{di}{dt} = -0.54 \times (-0.030) = 1.62 \times 10^{-2}\ V$$

(b) $V_{ba} = L\frac{di}{dt} = -1.62 \times 10^{-2}\ V$

Since, $V_{ba}(= V_b - V_a)$ is negative. It implies that $V_a > V_b$ or a is at higher potential.

Alternatively,

Current flowing from b to a is decreasing, Hence, a must be at higher potential.

Problem 21: The current through an inductor of 1 H is given by $i = 3t\ sint$. Find the voltage across the inductor.

Solution:

Potential difference across an inductor,

$$V = L\frac{di}{dt} = L\frac{d}{dt}(3t\ sint) = 3L[sint + tcost]$$

Problem 22: At the instant when the current in an inductor is increasing at a rate of 0.0640 As^{-1}, the magnitude of the self-induced emf is 0.0160 V.

(a) What is the inductance of the inductor?

(b) If the inductor is a solenoid with 400 turns, what is the average magnetic flux through each turn when the current is 0.720 A?

Solution:

(a)

$$|\varepsilon| = L\,\frac{di}{dt}$$

Or, $L = \frac{|\varepsilon|}{\frac{di}{dt}} = \frac{0.0160}{0.0640} = 0.250\ H$

(b) Flux per turn

$$\varphi_B = \frac{Li}{N} = \frac{0.250 \times 0.720}{400} = 4.5 \times 10^{-4}\ Wb$$

Problem 23: An induced emf of 20 mV is measured across a coil of 50 closely wound turns while the current through it increases uniformly from 0 A to 5.0 A in 0.10s.

(a) What is the self-inductance of the coil?

(b) With the current at 5.0 A, what is the flux through each turn of the coil?

Solution: Try it yourself.

Answer: (a) $4.0 \times 10^{-4}\ H$, (b) $4.0 \times 10^{-5}\ Wb$

Mutual Inductance

When there are two coils in the neighborhood of each other, a change of current, in one of them, would cause an induced e.m.f. in the other. This induced e.m.f. would last only as the current in the other coil is changing.

We can say:

The phenomenon of mutual induction implies the production of an induced emf in one coil, due to change of current in another neighboring coil. The induced emf would try to oppose the very cause to which it is due.

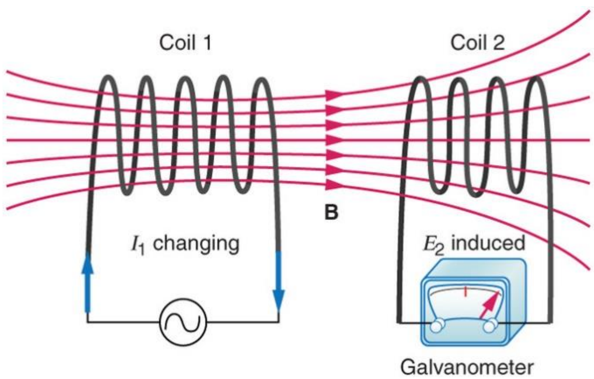


Figure 15: Mutual Inductance.

A current flowing in coil 1 produces magnetic field and hence, a magnetic flux through coil 2. If the current in coil 1 changes, the flux through coil 2 changes as well. According to Faraday's law this induces an emf in coil 2. In this way, a change in the current in one circuit can induce a current in a second circuit. This phenomenon is known as mutual induction. Like the self-inductance (L), two circuits have mutual inductance (M).

It also have two definitions as under:

First Definition: Suppose the circuit 1 has a current i_1 flowing in it. Then, total flux $N_2\varphi_{B_2}$ linked with circuit 2 is proportional to the current in 1. Thus,

$$N_2\varphi_{B_2} \propto i_1$$

$$Or, N_2\varphi_{B_2} = Mi_1$$

Here, the proportionality constant M is known as the mutual inductance M of the two circuits. Thus,

$$M = \frac{N_2\varphi_{B_2}}{i_1}$$

From this expression, *M* can be defined as the total flux $N_2\varphi_{B_2}$ linked with circuit 2 per unit current in circuit 1.

Second Definition: If we change the current in circuit 1 at a rate $\frac{di_1}{dt}$, an induced emf ε_2 is developed in circuit 2, which is proportional to the rate $\frac{di_1}{dt}$. Thus,

$$\varepsilon_2 \propto -\frac{di_1}{dt}$$

$$\text{Or, } \varepsilon_2 = -M\frac{di_1}{dt}$$

Here, the proportionality constant is again *M*. Minus sign indicates that ε_2 is in such a direction that it opposes any change in the current in circuit 1. From the above equation,

$$M = \left| -\frac{\varepsilon_2}{\frac{di_1}{dt}} \right|$$

This equation states that, **the mutual inductance of two circuits is the magnitude of induced emf, $\boldsymbol{\varepsilon_2}$ per unit rate of change of current, $\boldsymbol{\frac{di_1}{dt}}$.**

Note down the following points regarding the mutual inductance:

1. The SI unit of mutual inductance is henry (1 H).
2. *M* depends upon the distance between two coils, the orientations and sizes of both coils, the number of turns, the medium between two coils,etc.
3. **Reciprocity theorem:**

$$M_{12} = M_{12} = M$$

$$\varepsilon_2 = -M\frac{di_1}{dt}$$

$$\text{And } \varepsilon_1 = -M\frac{di_2}{dt}$$

$$M_{12} = \frac{N_2\varphi_{B_2}}{i_1}$$

$$\text{And, } M_{21} = \frac{N_1\varphi_{B_1}}{i_2}$$

The application of the concept of mutual inductance can be found in transformers, electric generators, and motors.

Problem 24: Two coils have mutual inductance of $3.25 \times 10^{-4}\ H$. The current in the first coil increases at a uniform rate of 830 As^{-1}.

a) What is the magnitude of induced emf in the 2nd coil? Is it constant?

b) Suppose that the current is instead in the 2nd coil, what is the magnitude of the induced emf in the 1st coil?

Solution:

a)

$$\varepsilon_2 = -M\,\frac{di_1}{dt}$$
$$= -(3.25 \times 10^{-4}\ H)(830\ As - 1) = -0.27\ V$$

As $\frac{di_1}{dt}$ is constant, induced emf is constant.

b) Coefficient of mutual induction remains same whether current flows in first coil or second. Hence,

$$\varepsilon_1 = -M\,\frac{di_2}{dt} = -0.27\ V$$

Problem 25: Two coaxial coils are very closer to each other and their mutual inductance is 3 mH. If a current *(50A)sin500t* is passed in one of the coils, then find the peak value of induced emf in the secondary coil.

Solution:

Here, $i = 50\sin 500t$

Induced emf of the mutual induction is given by

$$\varepsilon = -M\,\frac{di}{dt}$$
$$= -(5 \times 10^{-3})\frac{d}{dt}(50\sin 500t)$$
$$= -5 \times 10^{-3} \times 50\cos 500t\ \times 500$$
$$= -125\cos 500t$$

The peak value is 125 V.

Problem 26: Two solenoids A and B spaced close to each other and sharing the same cylindrical axis have 400 and 700 turns, respectively. A current of 3.50 A in coil A produced an average flux of 300 μTm^{-2} through each turn of A and a flux of 90 μTm^{-2} through each turn of B.

(a) Calculate the mutual inductance of the two solenoids.

(b) What is the self-inductance of A?

(c) What emf is induced in B when the current in A increases at the rate of 0.5 As^{-1}.

Solution:

(a)

$$M = \frac{N_B \varphi_B}{i_A}$$

$$= \frac{(700) \times (90 \times 10^{-6})}{3.5}$$

$$= 1.8 \times 10^{-2} H$$

(b)

$$L_A = \frac{N_A \varphi_A}{i_A}$$

$$= \frac{(400)(300 \times 10^{-6})}{3.5}$$

$$= 3.43 \times 10^{-2}\ H$$

(c)

$$\varepsilon_B = M\left(\frac{di_A}{dt}\right)$$

$$= (1.8 \times 10^{-2})(0.5)$$

$$= 9.0 \times 10^{-3}\ V$$

Problem 27: Calculate the mutual inductance between two coils when a current of 4 A changes to 12 A in 0.5 s in primary and induces an emf of 50 mV in the secondary. Also calculate the induced emf in secondary if current in the primary changes from 3 A to 9 A in 0.02 s.

Solution:

$$|\varepsilon| = M\frac{di}{dt} = M\frac{i_2 - i_1}{t}$$

Or, $50 \times 10^{-3} = M\frac{12-4}{0.5}$

Or, $M = \frac{50\times10^{-3}\times0.5}{8} = 3.125 \times 10^{-3}H = 3.125\ mH$

If current changes from 3 A to 9 A in 0.02 s.

$$|\varepsilon| = M\frac{di}{dt} = M\frac{i_2 - i_1}{t}$$

$$= 3.125 \times 10^{-3} \times \frac{9-3}{0.02}$$

$$= 0.9375\ V$$

Problem 28: A coil has 600 turns which produces $5 \times 10^{-3}\ Wb/turn$ of flux when 3 A current flows in the wire. This produced $6 \times 10^{-3}\ Wb/turn$ in 1000 turns secondary coil. When the switch is opened the current drops to zero in 0.2 s in primary. Find

(a) Mutual inductance

(b) The induced emf in the secondary

(c) The self inductance of the primary coil

Solution:

(a) Magnetic flux linked with secondary coil,

$$\varphi_{m_2} = Mi_1$$

Or, $M = \frac{\varphi_{m_2}}{i_1} = \frac{6.0\times10^{-3}\times1000}{3} = 2\ H$

(b) $\varepsilon = -M\frac{di_1}{dt} = -2 \times \frac{0-3}{0.2} = 30\ V$

(c) $L = \frac{\varphi_{m_1}}{i_1} = \frac{600\times5\times10^{-3}}{3} = 1\ H$

Problem 29: Two toroidal solenoids are wound around the same pipe so that the magnetic field of one passes through the turns of the other. Solenoid 1 has 700 turns and solenoid 2 has 400 turns. When the current in solenoid 1 is 6.52 A, the average flux through each turn of solenoid 2 is 0.0320 Wb.

(a) What is the mutual inductance of the pair of solenoids?

(b) When the current in solenoid 2 is 2.54 A, what is the average flux through each turn of solenoid 1?

Solution:

(a) Magnetic flux linked with the secondary coil,

$$N_2\varphi_2 = Mi_1$$

Or, $M = \frac{N_2\varphi_2}{i_1} = \frac{400 \times 0.0320}{6.52} = 1.96\ H$

(b)

$$\varphi_1 = Mi_2 = 1.96 \times 2.54 = 4.9784\ Wb$$

Flux per turn through primary coil $= \frac{\varphi_1}{N_1} = \frac{4.9784}{700} = 7.112\ \times 10^{-3}\ Wb/turn$