CHAPTER
03 Utility

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3.1 Cardinal vs. Ordinal Utility

The amount of happiness or pleasure created through the consumption of a good or a service is called utility. There are two different schools of thought regarding the measurability of utility. Some argue that utility can be cardinally measured and others disagree, who rather advocate ordinal utility. William Stanley Jevons, Léon Walras and Alfred Marshall were the principal proponents of cardinal theory of utility. Although outdated, cardinal measure of utility builds the basis of the law of demand. Downward sloping demand curve underlies the law of diminishing marginal utility- which is one of the prime provisos of cardinal utility theory. Among others, John Hicks and Roy Allen were the leading contributors in the field of ordinal theory of utility. Ordinalists believe that utility cannot be measured, can only be compared. For example, an individual can say that his utility from the consumption of an apple is greater than utility from an orange. Cardinalists, on the other hand, opine that individual knows for certain- how much utility is obtained from the consumption of an apple or an orange.

3.2 Total Utility and Marginal Utility

Amount of satisfaction obtained from the consumption of a certain quantity of a good or a service is called total utility (TU). Unit of measurement of total utility is util. The individual consumer can, however, measure utility in any unit. Change in total utility due to one unit change in consumption is called marginal utility. Table 3.1 describes the difference between total utility and marginal utility.

Table 3.1
Total Utility and Marginal Utility

Unit	Total Utility	Marginal Utility
1	10	10
2	18	8
3	24	6
4	28	4
5	30	2
6	30	0
7	28	-2

Total utility derived from 1 unit consumption is 10 util. Total utility from 2 unit consumption is 18 util. That means utility from second unit is 8 util, which is marginal utility. Since total utility remains unchanged at 30 util following an increase in consumption from 5 to 6 unit, marginal utility gained from 6^{th} unit is zero. Marginal utility from 7^{th} unit however is negative because increase in consumption from 6 unit to 7 unit lowers total utility from 30 to 28 util. Algebraically, slope of total utility measures marginal utility.

Suppose total utility, U = f (q) ; q stands for quantity of consumption

Marginal utility, MU = slope of total utility $= \frac{dU}{dq} = f'(q)$

Example 3.1

Assume the utility function $U = 100q - q^2$. Find marginal utility and draw the marginal utility function.

Solution

Marginal utility, $MU = \frac{dU}{dq} = 100 - 2q$

Table 3.2
Marginal Utility

Unit	Marginal Utility
0	100
10	80
20	60
30	40
40	20
50	0
60	-20

Marginal Utility

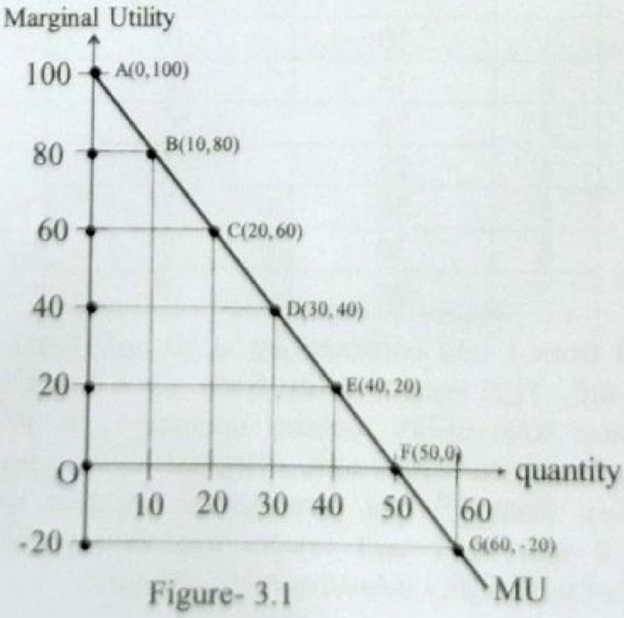

Figure- 3.1

Marginal utility line is drawn in figure 3.1 by measuring quantity of consumption along horizontal axis and marginal utility along vertical axis. Point A shows 100 units of marginal utility at zero quantity. Other points of marginal utility are B, C, D, E, F and G. Of these points, marginal utility is negative at point G where consumption is 60 unit.

3.3 Law of Diminishing Marginal Utility

Law of diminishing marginal utility states that marginal utility decreases with successive increase in consumption. Total utility, in the beginning, increases with consumption and once it falls but marginal utility always falls. Total utility curve is therefore inverse 'U'-shaped and marginal utility curve slopes downward. Figure 3.2 displays the shapes of total utility and marginal utility.

Law of Diminishing Marginal Utility

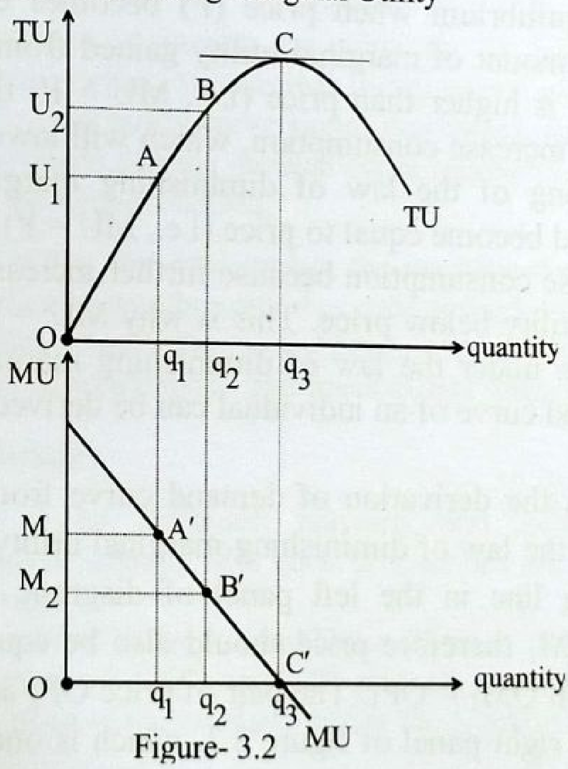

Figure- 3.2

Figure 3.2 describes law of diminishing marginal utility. In the upper panel of the diagram total utility is measured along vertical axis and consumption along horizontal axis. In the lower panel marginal utility is measured along vertical axis and consumption along horizontal axis. At Oq_1 consumption total utility is OU_1. Corresponding point of total utility is A. Marginal utility is the slope of total utility at point A, which is measured by the positive distance OM_1 along vertical axis of the lower panel of diagram. The pair of Oq_1 quantity and OM_1 marginal utility is designated by the point A' in the lower panel. A' is one point of marginal utility. At

Oq_2 quantity, total utility is OU_2, corresponding point is B in the upper panel. Slope of total utility at point B is smaller than slope of total utility at point A. Slope of total utility at point B measures marginal utility equal to OM_2 in the lower panel at Oq_2 quantity. The pair of Oq_2 quantity and OM_2 marginal utility is designated by the point B' in the lower panel. B' is another point of marginal utility. Total utility is maximum at point C in the upper panel of diagram. Lower panel representation of point C is C' where marginal utility is zero at Oq_3 quantity. If consumption goes beyond q_3, total utility decreases and marginal utility becomes negative, thereby marginal utility curve stays below quantity axis. By joining points A', B' and C', marginal utility curve MU is drawn which slopes downward- representing diminishing marginal utility.

Consumer's Equilibrium

Consumer attains equilibrium when price (P) becomes equal to marginal utility (MU). If the amount of marginal utility gained from the consumption of a good or service is higher than price (i.e., MU > P) then the consumer would be induced to increase consumption, which will lower marginal utility due to the functioning of the law of diminishing marginal utility. Once marginal utility would become equal to price (i.e., MU = P) and the consumer will no longer increase consumption because further increase in consumption will push marginal utility below price. This is why MU = P is termed as the equilibrium condition under the law of diminishing marginal utility. Using this condition, demand curve of an individual can be derived.

Figure 3.3 illustrates the derivation of demand curve from marginal utility analysis. Because of the law of diminishing marginal utility, MU is drawn as a downward sloping line in the left panel of diagram. At Oq_1 quantity, marginal utility is OM_1 therefore price should also be equal to OM_1. In the right panel of diagram $OM_1 = OP_1$. The pair of price OP_1 and quantity Oq_1 is denoted as D_1 in the right panel of figure 3.3, which is one point of demand curve. D_2 point shows the combination of price OP_2 and quantity Oq_2, where $OP_2 = OM_2$. Thus, D_2 is another point of demand curve. Right most point of demand curve is D_3 where price is zero and consumption is Oq_3. Note that the consumer will in no way increase consumption beyond Oq_3 because marginal utility would be negative but price cannot be negative.

Derivation of Demand Curve

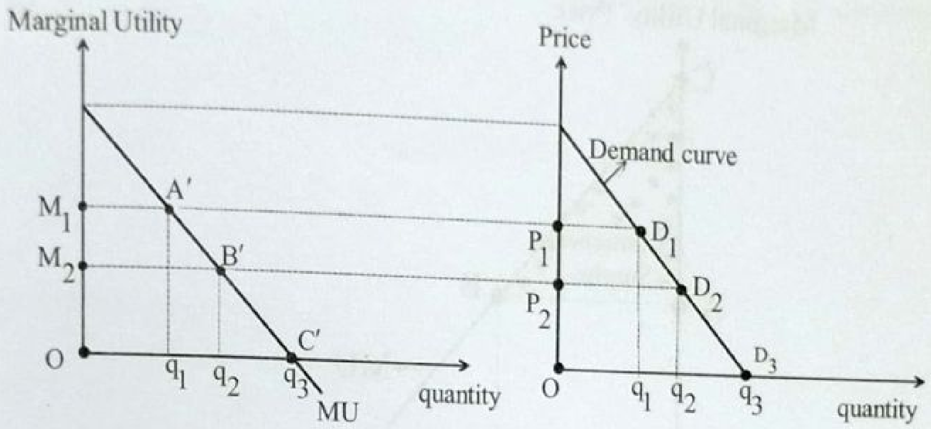

Figure- 3.3

Demand curve is drawn by joining the points D_1, D_2 and D_3. It is evident that demand curve is the portion of marginal utility curve which lies in the positive quadrant. Since marginal utility curve slopes downward, demand curve would also be downward. Downward sloping demand curve represents the inverse relationship between price and quantity demanded.

Cardinal theory of utility can only explain the negative slope of demand curve but individual's demand curve may be vertical or even upward if the good was respectively price neutral or Giffen. Ordinal theory of utility provides a complete theory of demand. Indifference curve is the fundamental instrument of analyzing ordinal theory of utility.

3.4 Consumer Surplus

Consumer surplus is the difference between total satisfaction obtained by a consumer from a given quantity of a good or a service and total expenditure made for the good or service.

Consumer surplus = Total Utility – Total Expenditure.

Suppose utilities from 1^{st} , 2^{nd} and 3^{rd} unit of a good are 10, 8 and 6 unit respectively. If the price of the good per unit is Tk. 5, total expenditure for 3 units of the good = 3X5=15.

Total utility from 3 units of the good in terms of Taka

= 10+8+6 = 24

Consumer Surplus = 24 – 15 = 9.

Geometrically, the area under marginal utility curve measures total utility. In figure 3.4 consumption of a typical commodity is shown OH. Price per unit is OA.

Total expenditure = price X quantity = OA X OH = OABH. Total utility obtained from the consumption of OH unit of consumption is OCBH.

Therefore, consumer surplus = total utility – total expenditure

= OCBH – OABH = CAB.

Consumer Surplus

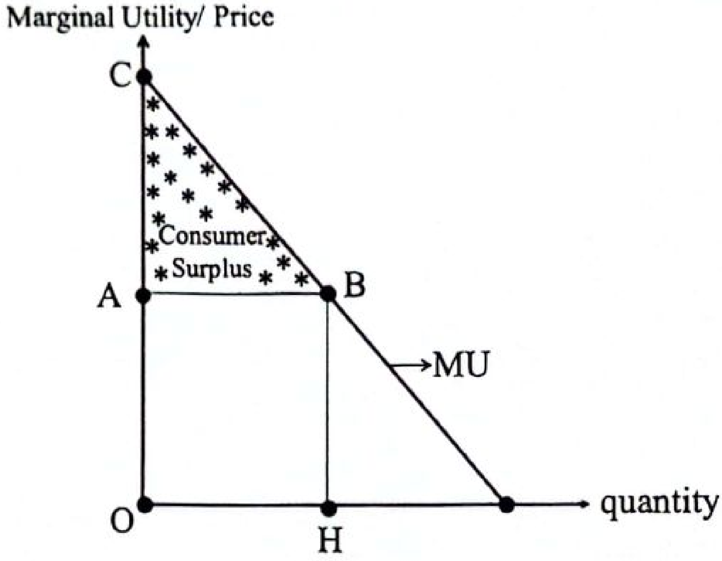

Figure- 3.4

Since the demand curve is a portion of marginal utility curve and integration of marginal utility yields total utility, total utility is computed by taking the integration of demand function within a certain limit.

Example 3.2

Assume the demand function $P = 100 - 2Q$. Compute the amount of consumer surplus from the consumption of 10 units of the commodity.

Solution

Here, amount of consumption (Q) = 10
Price per unit (P) = 100-2X10 = 100-20 = 80
Total expenditure = price X quantity = 10 X 80 = 800

Total utility gained from the consumption of 10 units of the commodity = $\int_0^{10} (100 - 2Q)dQ$

$$= 100 \int_0^{10} dQ - 2 \int_0^{10} QdQ$$

$$= 100[Q]_0^{10} - [Q^2]_0^{10}$$

$$= 100 \times 10 - 10^2$$

$$= 1000 - 100$$

$$= 900$$

Consumer surplus = total utility – total expenditure = 900–800 = 100

Example 3.3

Determine the amount of consumer surplus at equilibrium level of consumption assuming demand and supply functions below.

$Q_d = 18 - 2P$

$Q_s = -2 + 3P$

Solution

Equilibrium condition $Q_d = Q_s$

$$\Rightarrow 18 - 2P = -2 + 3P$$
$$\Rightarrow -5P = -20$$
$$\therefore \overline{P} = 4$$

$Q_d = Q_s = \overline{Q} = 18 - 2 \times 4 = 10$

Total expenditure = price X quantity = 4 X 10 = 40

Manipulate the demand function so that it becomes a function of quantity.

Given, $Q_d = 18 - 2P$

$$\Rightarrow 2P = 18 - Q$$

$$\Rightarrow P = 9 - \frac{1}{2}Q$$

Total utility from the consumption of 10 unit = $\int_0^{10} (9 - \frac{1}{2}Q)dQ$

$$= 9\int_0^{10} dQ - \frac{1}{2}\int_0^{10} QdQ$$

$$= 9[Q]_0^{10} - \frac{1}{4}[Q^2]_0^{10}$$

$$= (9 \times 10) - (\frac{1}{4} \times 10^2)$$

$$= 90 - 25$$

$$= 65$$

Consumer surplus = total utility – total expenditure = 65-40=25

3.5 Indifference Curve

An indifference curve shows the combinations of two goods that yield equal utility to the consumer. In figure 3.5 indifference curve II' is drawn by measuring consumption of X along horizontal axis and consumption of Y along vertical axis. At point A consumption of X and Y are Ox_1 and Oy_1 respectively. At point B consumption of X and Y are Ox_2 and Oy_2. If Ox_1 of X and Oy_1 of Y generate same utility as Ox_2 of X and Oy_2 of Y then consumer will remain indifferent between points A and B. Indifference curve is drawn by joining such indifferent points.

3.5.1 Marginal Rate of Substitution

Marginal rate of substitution (MRS) is defined as the amount of change in the consumption of one good for one unit change in consumption of another good so that total utility remains unaltered. More explicitly, marginal rate of substitution of X for Y (MRS_{xy}) is defined as the change in consumption of Y due to one unit change in consumption of X. In figure 3.5 movement from point A to B involves an increase in consumption of X by $\Delta x = x_1x_2$ unit and a decrease in consumption of Y by $\Delta y = y_1y_2$ unit.

Indifference Curve

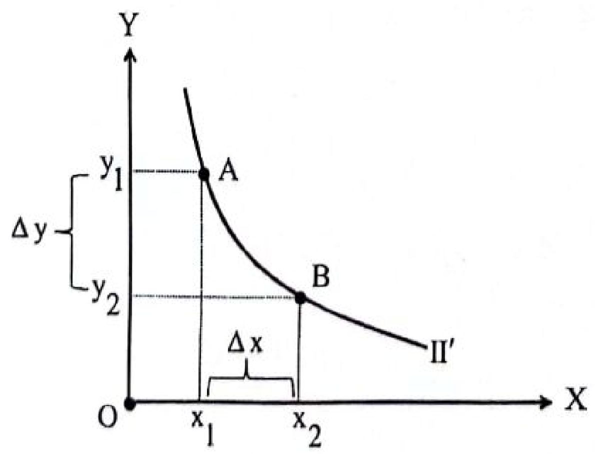

Figure- 3.5

x_1x_2 unit increase in consumption of X involves y_1y_2 unit decrease in consumption of Y.

$\therefore$ 1 unit increase in consumption of X involves $\dfrac{y_1y_2}{x_1x_2}$ unit decrease in consumption of Y.

By definition, therefore, $MRS_{xy} = \frac{y_1y_2}{x_1x_2} = \frac{\Delta y}{\Delta x}$ .... (3.1)

1 unit increase in consumption of X increases utility by MU_x unit

$\therefore \Delta x$ unit increase in consumption of X increases utility by $MU_x . \Delta x$ unit.

Analogously, 1 unit decrease in consumption of Y decreases utility by MU_y unit

$\therefore \Delta y$ unit decrease in consumption of Y decreases utility by $MU_y . \Delta y$ unit.

Since utility at both A and B are equal, therefore

Decrease in utility = increase in utility

$MU_y . \Delta y = MU_x . \Delta x$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{MU_x}{MU_y} \quad \text{..... .....} \quad (3.2)$$

Using (3.1) and (3.2), $MRS_{xy} = \frac{\Delta y}{\Delta x} = \frac{MU_x}{MU_y}$ .... (3.3)

Equation (3.3) expresses MRS as the ratio between marginal utilities.

Because of the law of diminishing marginal utility, increase in consumption of one good reduces its power of creating satisfaction. For that reason a consumer, who targets to keep utility constant, should reduce the consumption of Y successively by a smaller amount for every equal amount increase in consumption of X. This implies the diminishing nature of MRS.

3.5.2 Properties of an Indifference Curve

Indifference curve has the following four properties.

1. Indifference curve slopes downward to the right.
2. Indifference curve is convex to the origin.
3. Indifference curves do not intersect.
4. Higher indifference curve represents higher utility.

Slope of an Indifference Curve

If Indifference curve is drawn as an upward sloping curve like panel A in figure 3.6, movement from a to b will mean increases in consumption of both X and Y. Thus total utility at point b would be higher than total utility at point a, but along an indifference curve total utility cannot vary, therefore indifference curve will not be upward sloping.

Slope of Indifference Curve

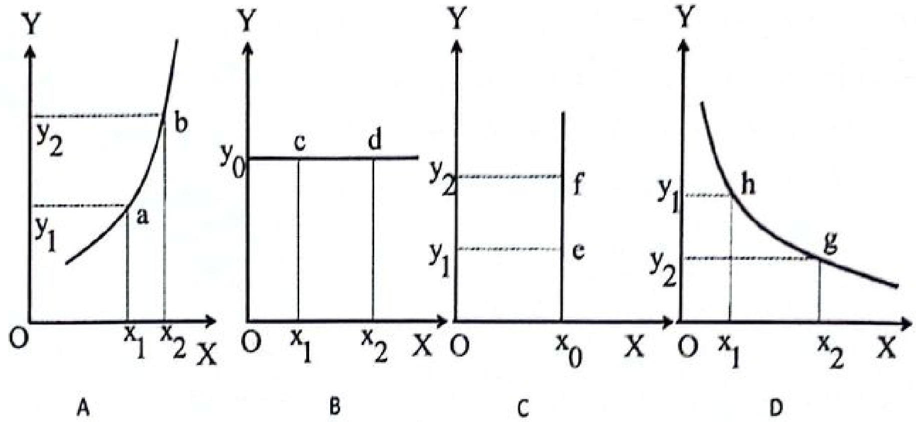

Figure- 3.6

If indifference curve is horizontal like panel B, movement from c to d will mean an increase in total utility because consumption of x is higher at point d than c although consumption of y is same. But utility is not allowed to vary along an indifference curve, therefore indifference curve will not be horizontal. For similar reasoning, indifference curve will not be vertical like panel C. An indifference curve will neither be upward, nor horizontal, nor vertical. It will only be downward as panel D. Movement from h to g displays an increase in consumption of X, accompanied by a decrease in consumption of Y, thereby total utility remains unchanged.

Algebraic Derivation of Slope of Indifference Curve

Equation of indifference curve: $U = U(x, y) = U_0$ (constant total utility).

$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy = 0$$

$$\Rightarrow MU_x dx + MU_y dy = 0$$

$$\Rightarrow MU_y dy = -MU_x dx$$

$\therefore \dfrac{dy}{dx} = -\dfrac{MU_x}{MU_y} < 0$, which is the slope of indifference curve.

Example 3.4

Determine the slope of the utility function: $U = x^{0.8} y^{0.3}$

Solution

Given, $U = x^{0.8} y^{0.3}$

$$MU_x = \frac{\partial U}{\partial x} = 0.8x^{-0.2} y^{0.3}$$

$$MU_y = \frac{\partial U}{\partial y} = 0.3x^{0.8} y^{-0.7}$$

Slope of the utility function: $\frac{dy}{dx} = -\frac{MU_x}{MU_y} = -\frac{0.8x^{-0.2} y^{0.3}}{0.3x^{0.8} y^{-0.7}} = -\frac{8}{3} \frac{y^{0.3+0.7}}{x^{0.8+0.2}} = -\frac{8}{3} \frac{y}{x}$

Example 3.5

Show that the slope of the utility function $U = x^{\alpha} y^{1-\alpha}$ is negative. $(0 < \alpha < 1)$

Solution

Given, $U = x^{\alpha} y^{1-\alpha}$

$$MU_x = \frac{\partial U}{\partial x} = \alpha x^{\alpha - 1} y^{1-\alpha}$$

$$MU_y = \frac{\partial U}{\partial y} = (1-\alpha) x^{\alpha} y^{1-\alpha-1} = (1-\alpha)\, x^{\alpha} y^{-\alpha}$$

Slope of the utility function:

$$\frac{dy}{dx} = -\frac{MU_x}{MU_y} = -\frac{\alpha x^{\alpha-1} y^{1-\alpha}}{(1-\alpha)\, x^{\alpha} y^{-\alpha}} = -\frac{\alpha}{1-\alpha} \frac{y^{1-\alpha+\alpha}}{x^{\alpha-\alpha+1}} = -\frac{\alpha}{1-\alpha} \frac{y}{x}, \text{ which is negative.}$$

Unusual Shapes of Indifference Curve

If indifference curve is drawn for a good and a bad together then it would be upward sloping because increase in consumption of good increases total utility and increase in consumption of bad decreases total utility. Consequently, total utility remains unchanged along an upward sloping indifference curve.

If the consumption of neuter is measured along horizontal axis then indifference curve will be horizontal because increase in the consumption of neuter neither increases nor decreases utility. For similar reasoning if the consumption of neuter is measured along vertical axis then indifference curve would be vertical.

Convexity of Indifference Curve

A downward sloping function may be straight-line, concave or convex to the origin. Figure 3.7 displays three different shapes of indifference curve. In panel A, indifference curve is drawn as a straight-line. Movement from a to b to c involves equal increment in consumption of X from Ox_1 to Ox_2 to Ox_3. Here, $x_1x_2 = x_2x_3$. Corresponding decreases in consumption of Y are y_1y_2 and y_2y_3 respectively. In figure, $y_1y_2 = y_2y_3$. That means, marginal rate of substitution between X and Y is constant. But because of the law of diminishing marginal utility MRS decreases. Needless to say that constancy of the MRS gives rise to a straight-line indifference curve. If indifference curve is drawn as a concave function as in panel B,

Convexity of an Indifference Curve

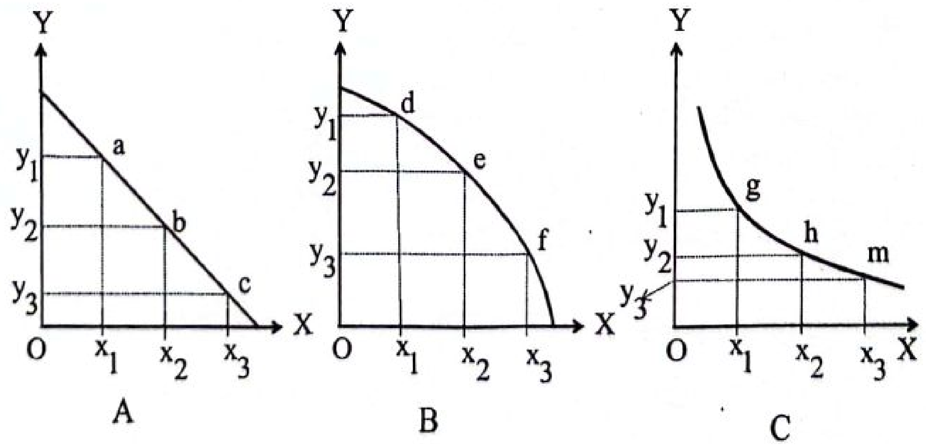

Figure- 3.7

equal increment in consumption of X results in successively larger amount of decreases in Y. In panel B, $x_2x_3 = x_1x_2$ but $y_2y_3 > y_1y_2$. In this case MRS increases, which causes an indifference curve to be concave. But increasing nature of MRS is a contradiction, hence indifference curve will not be concave. Convex indifference curve, as in panel C of figure 3.7, represents decreasing MRS. This becomes evident from panel C of figure 3.7, where $y_2y_3 < y_1y_2$ even though $x_2x_3 = x_1x_2$. For MRS to be diminishing, indifference curve should neither be straight-line nor concave, rather should be convex.

Intersection of Indifference Curves

Different indifference curves of an indifference map[1] cannot intersect each other. In figure 3.8, two indifference curves IC_1 and IC_2 intersect each other at point A. B and C are two other points of IC_1 and IC_2 respectively. Since A and B are two points of same indifference curve IC_1,

Total utility at point A = Total utility at point B .... (3.4)

Again, A and C are two points of same indifference curve IC_2, hence

Total utility at point A = Total utility at point C .... (3.5)

Using (3.4) and (3.5), one can infer that

Total utility at point B = Total utility at point C. But this conclusion is not valid because utility at point C would be obviously higher than utility at B as the consumption of Y is higher at point C than at B although consumption of X is same.

Intersection of Indifference Curves

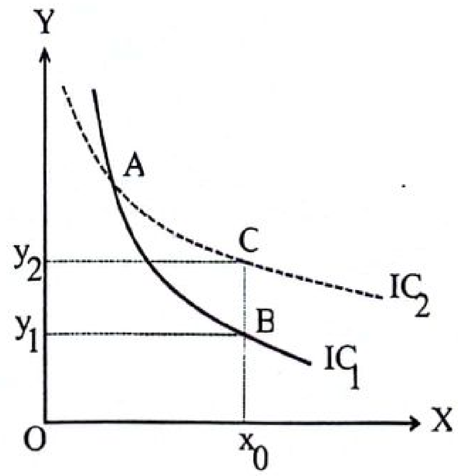

Figure- 3.8

Utility along Higher Indifference Curve

Two different indifference curves I_1 and I_2 are drawn in figure 3.9. I_1 is the lower indifference curve and I_2 is the higher indifference curve. D and E are two points of lower and higher indifference curves respectively.

[1] Indifference map contains all possible indifference curves of identical nature.

Consumption of X is constant at Ox_0 at both I_1 and I_2 but consumption of Y is higher at point E than D, thereby total utility at point E would be higher than total utility at point D. This proves that higher indifference curve represents higher total utility.

Utility at Higher Indifference Curve

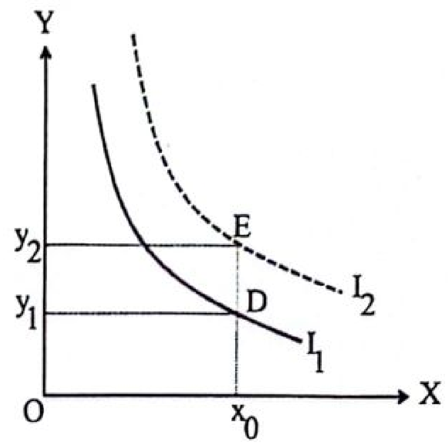

Figure- 3.9

3.6 Budget Line

Budget line shows the possible combinations of two goods that can be purchased by spending a given amount of money.

Suppose money income of a consumer, M = 100

Prices of X and Y are, $P_x = 5$ and $P_y = 4$ respectively.
If the consumer spends whole sum of money to buy X or Y then it can buy either 20 units of X or 25 units of Y. In figure 3.10 the corresponding budget line is AB, where $OA = \frac{M}{P_x} = \frac{100}{5} = 20$ and $OB = \frac{M}{P_y} = \frac{100}{4} = 25$. the budget line shows consumption possibilities at given market prices with a fixed money income of the consumer. The consumer is neither a saver nor a borrower. At each point of budget line, total budget is constant. Assume another point C where consumption possibilities of X and Y are 10 and 12.5 respectively, total budget is 100 as points A and B.

Budget Line

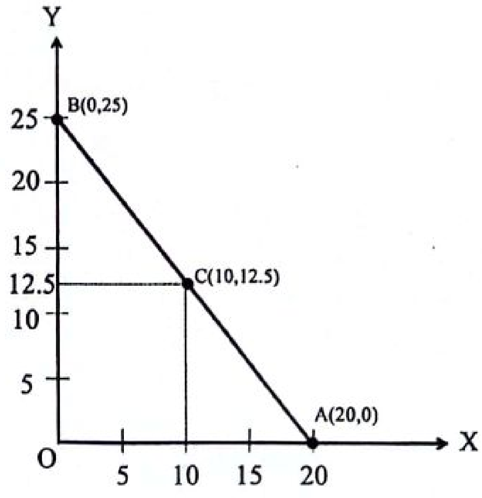

Figure- 3.10

Budget equation, $M = xP_x + yP_y$ .... (3.6)

xP_x is the total amount spent on good X and yP_y is the total amount spent on good Y.

x and y represent the quantities of goods X and Y respectively.

At point A consumption of Y = 0, setting y = 0 into (3.6)

$$M = xP_x$$

$$\Rightarrow x = \frac{M}{P_x} = OA$$

Analogously, at point B consumption of X = 0, setting x = 0 into (3.6)

$$M = yP_y$$

$$\Rightarrow y = \frac{M}{P_y} = OB$$

Slope of budget line AB $= -\frac{OB}{OA} = -\frac{M/P_y}{M/P_x} = -\frac{P_x}{P_y}$

The slope of a budget line can alternatively be derived algebraically.

Equation of the budget line $M = xP_x + yP_y$

$$\Rightarrow yP_y = M - xP_x$$

$$\Rightarrow y = \frac{M}{P_y} - x\frac{P_x}{P_y}$$

$$\therefore \frac{dy}{dx} = -\frac{P_x}{P_y} < 0 \quad ..... \quad ..... \quad \text{this is the slope of budget line.}$$

This suggests that slope of budget line is the negative of price ratio. If price varies then slope of budget line varies, but price remaining unchanged if income varies then budget line shifts parallelly without any variation in slope.
Figure 3.11 illustrates how budget line rotates from BA to BA′ following an increase in price of x from 5 to 10.

Impact of Price

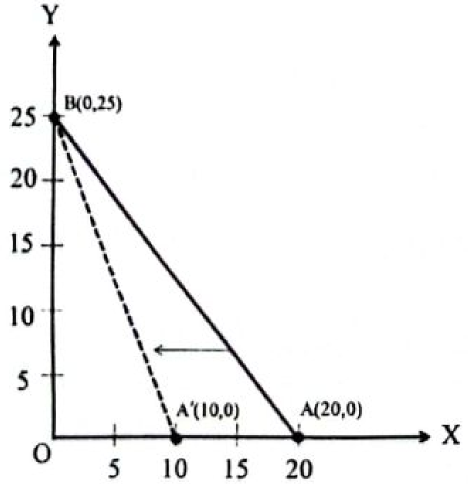

Figure- 3.11

If price of X decreases then point A will rotate rightwardly. Price of X remaining unchanged if price of Y increases then point B will move down, and if price of Y decreases then move up. In each case point A will remain unaltered.

Figure 3.12 demonstrates the impact of a fall in income.

If income decreases from 100 to 60 then the consumer can either buy 12 units of X (point E) or 15 units of Y (point F), budget line shifts left. In case of increase in income budget line shifts right.

Impact of Income

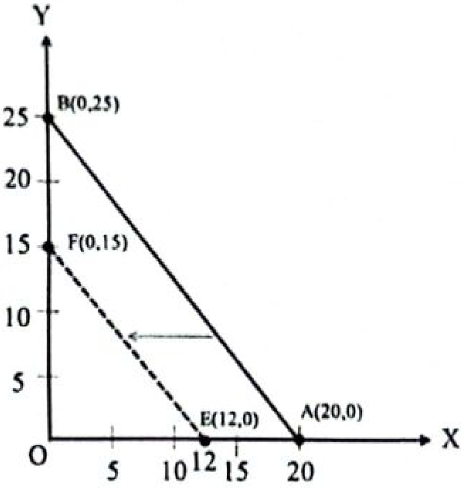

Figure- 3.12

3.7 Consumer's Equilibrium

A consumer achieves equilibrium if it can maximize utility by spending a given amount of money income. Limited income is represented by the budget line AB in figure 3.13, where $OA = \frac{M}{P_x}$ and $OB = \frac{M}{P_y}$. M, P_x and P_y stand for money income, price of X and price of Y respectively.

Consumer's Equilibrium

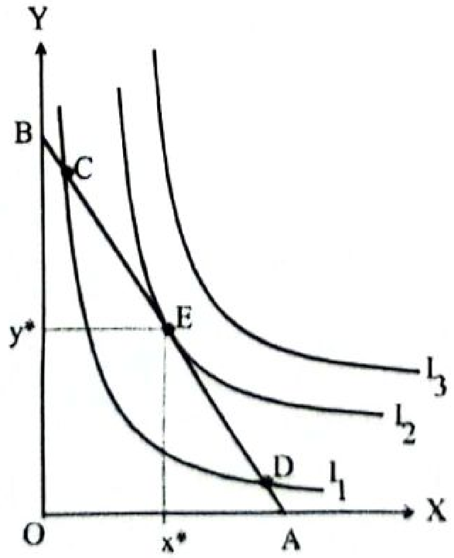

Figure- 3.13

Budget equation is $M = xP_x + yP_y$. Indifference map of the consumer contains numerous indifference curves. For the sake of simplicity, only three indifference curves I_1, I_2 and I_3 are drawn. Of these three indifference curves, I_3 represents highest utility but this level of utility is unattainable for the consumer because it is beyond the consumer's budget capacity. Points C, D and E are however within budget constraint. Among these points, consumer would naturally prefer point E because this point stands for higher utility than two other points although budget is the same at all three points. E is the equilibrium point because at this point the consumer can attain maximum utility by spending limited income. A comparative look at three points makes it clear that budget line is tangent to indifference curve at point E, i.e., slope of indifference curve is equal to slope of budget line- this is known as the first order condition or necessary condition of consumer's equilibrium.

Sufficient Condition

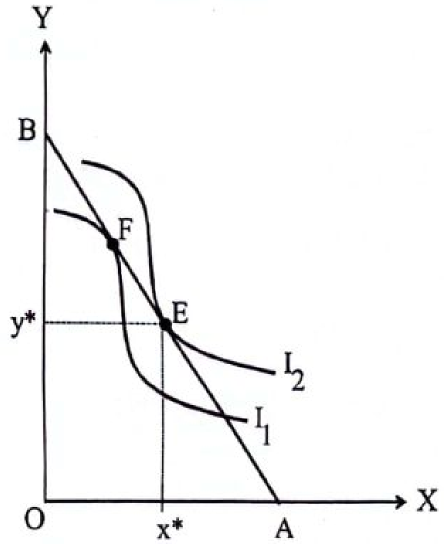

Figure-3.14

In order to examine the second order condition, look at figure 3.14 where both I_1 and I_2 indifference curves are tangent with budget line at points F and E respectively. Consumer's total utility is of course higher at point E than at F because E lies on higher indifference curve than F. Distinctive feature of point E is that indifference curve is convex to the origin at this point. Therefore, convexity of indifference curve is termed as the second order condition or sufficient condition of consumer's equilibrium.

Consumer's equilibrium under indifference curve theory requires two conditions to be fulfilled.

1. Budget line is tangent to indifference curve. This implies,
   slope of indifference curve = slope of budget line

$$\text{or, } -\frac{MU_x}{MU_y} = -\frac{P_x}{P_y}$$

$$\text{or, } \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

2. At the point of tangency, indifference curve should be convex to the origin. Convexity of indifference curve underlies diminishing MRS. Thus, diminishing MRS is termed as the sufficient condition of consumer's equilibrium.

In figure equilibrium consumption of X and Y are Ox* and Oy* respectively.

3.8 Income Effect

A change in equilibrium level of consumption due to a change in real income is called income effect. Suppose initial budget line A_1B_1 in figure 3.15 corresponds to an initial income. Equilibrium level of consumption of X and Y are Ox_1 and Oy_1 respectively. If income increases, budget line shifts to A_2B_2. New equilibrium point is F where consumption of X and Y are Ox_2 and Oy_2 respectively. Movement from E to F is called income effect.

Income effect on X = x_1x_2 and income effect on Y = y_1y_2 .
An increase in income leads to an increase in consumption of both X and Y. Both goods are normal in this case. Income consumption curve (ICC) is drawn by joining the equilibrium points generated via the changes in income. If both goods are assumed normal, ICC slopes upward.

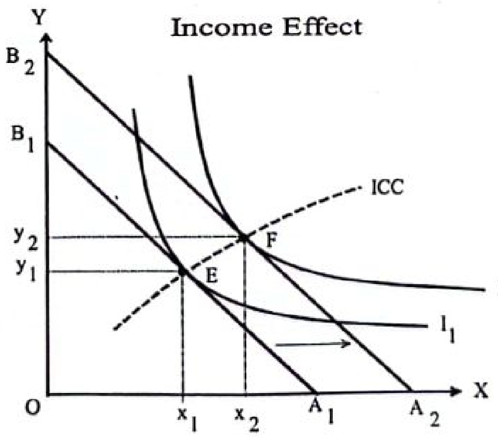

Figure- 3.15

In case of inferior goods, an increase in income leads to a decrease in consumption.

In figure 3.16, income effect on X = $-x_1x_2$ (an increase in income leads to a decrease in consumption of X).

Income effect on Y = y_1y_2 (increase in income leads to an increase in consumption of Y). In this case good X is inferior and Y is normal. ICC bends backward.

In figure 3.17, income effect on X is nil. Increase in income does not cause any change in consumption of X. Good X is income-neutral. Income effect on Y is positive, hence good Y is normal. ICC seems to be vertical between equilibrium points E and H.

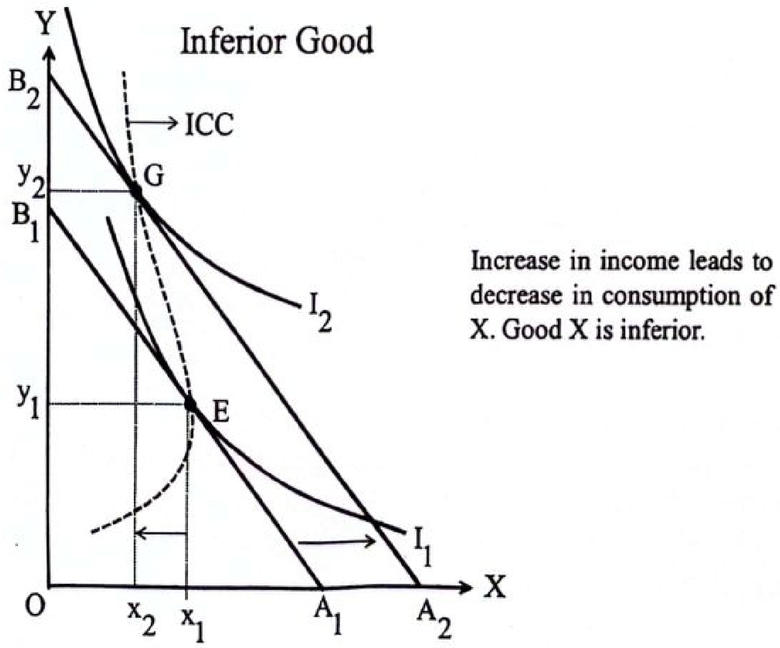

Figure- 3.16

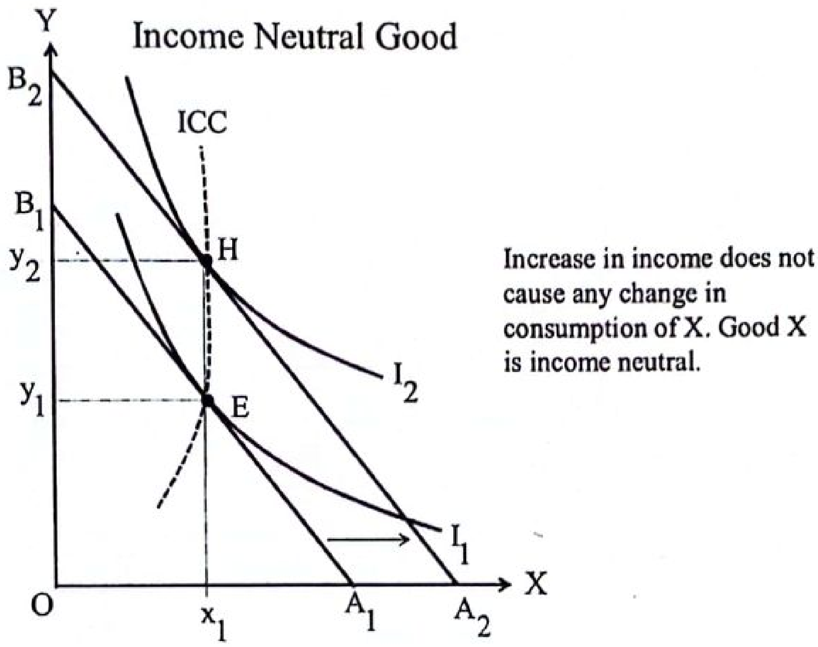

Figure- 3.17

3.9 Price Effect

Change in the level of equilibrium consumption due to a change in price is called price effect. Figure 3.18 describes the effect of a fall in price of X. Budget line rotates from AB_1 to AB_2 and equilibrium point moves from e_1 to e_2 following a decrease in price of X. Equilibrium consumption of X increases from Ox_1 to Ox_2. Price effect on X = x_1x_2.

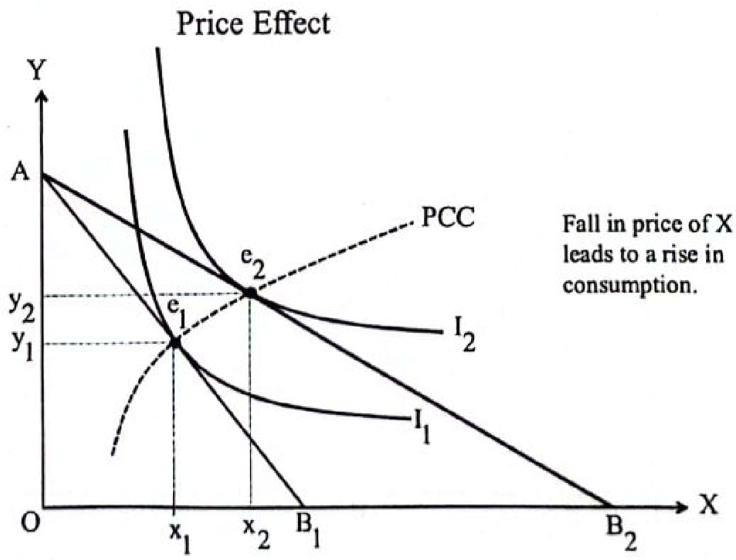

Figure- 3.18

Price Neutral Good

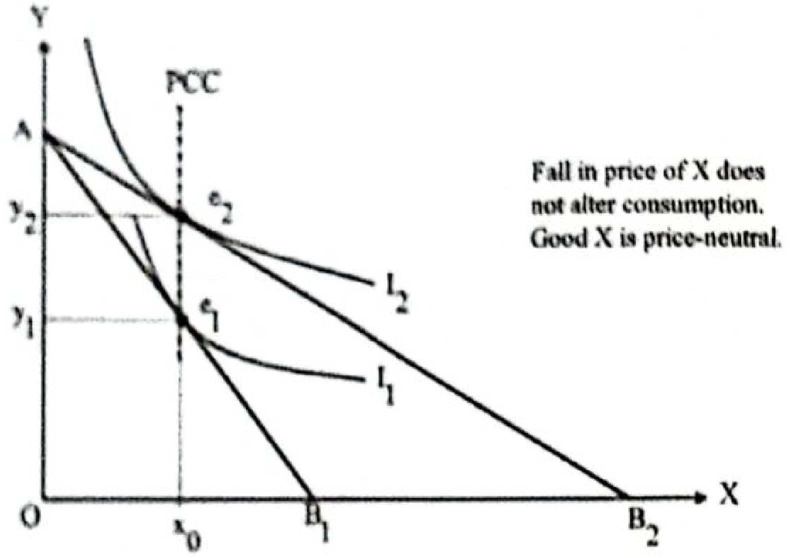

Figure- 3.19

A fall in price of X, in this case, leads to an increase in equilibrium level of consumption. Price consumption curve (PCC) is drawn by joining equilibrium points e_1 and e_2. PCC is defined as the locus of equilibrium points generated through the changes in price. If a change in price of X does not cause any change in consumption of X then the good X is said to be price-neutral. Price effect is nil in case of a price-neutral good. Figure 3.19 depicts X as a price neutral good. In this case equilibrium consumption of X remains unchanged at Ox_0 following a fall in price. Price effect on X equals to zero.

Giffen Good

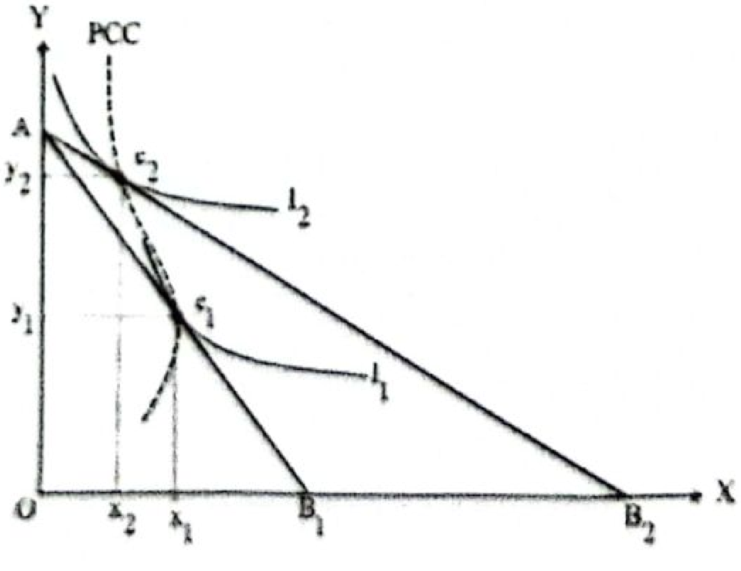

Figure- 3.20

Sometimes a fall in price may lead to a fall in equilibrium consumption or vice versa- this variant of good is called Giffen good after the name of Sir Robert Giffen. Figure 3.20 illustrates Giffen case. A fall in price of X reduces consumption from Ox_1 to Ox_2. Price effect on X = $-x_1x_2$. PCC is drawn by joining equilibrium points e_1 and e_2.

Price effect is the sum of substitution effect and income effect. Substitution effect refers to the change in equilibrium level of consumption due to change in price under the condition that real income remains unchanged. Real income is the purchasing power of nominal income. Suppose nominal income of a consumer is Tk. 400. If price of good X is Tk. 5 then purchasing power, hence real income, would be 400/5 = 80 unit X. If price of X falls to Tk. 4, real income increases to 400/4 = 100. In order to keep real income unchanged, an amount of nominal income has to be taken away from the consumer in case of a fall in price. In present case if 80 Taka is taken away from the consumer then her nominal income stands at 320 and real income remains fixed at 80 (=320/4) unit X. Note that in the event of increase in price, consumer should rather be given some money in order for real income to remain constant. The amount of nominal income that is to be taken away from the consumer in the event of a fall in price or given to the consumer in the event of a rise in price is called compensating variation (CV) in income.

Figure 3.21 decomposes price effect into substitution effect and income effect. Initial budget line AB_1 is tangent to indifference curve I_1 at point E. Equilibrium consumption of X is Ox_1. A fall in price of X yields new budget line AB_2 which is tangent to I_2 indifference curve at point F. Equilibrium consumption of X equals to Ox_2.

Price effect on X = x_1x_2. Fall in price causes an increase in consumption.

Because of a fall in price of X real income along budget line AB_2 is higher than AB_1. In order to explain substitution effect, real income has to be kept constant. Thus a certain amount of income is hypothetically taken away from the consumer, which causes a leftward shift in budget line from AB_2 to GK which is tangent to initial indifference curve I_1 at point H. Constancy of real income becomes evident from the fact that consumer attains same utility level using budget lines AB_1 and GK. Consumption of X at point H is equal to Ox'. Movement from E to H in figure 3.21 measures substitution effect[2].

---

[2] Here we follow the approach suggested by J. R. Hicks who proposed an adjustment in income so that consumer remains at same utility level. Slutsky, on the other hand, proposed an adjustment in income such that consumer is able to buy the previous commodity bundle under new situation.

Price Effect = Substitution Effect + Income Effect

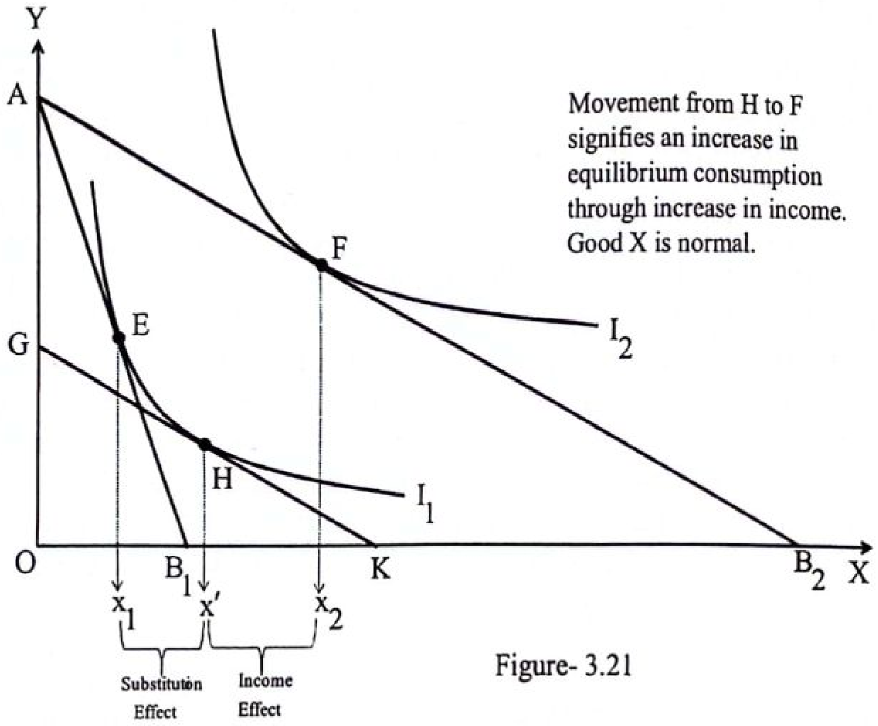

Figure- 3.21

Substitution effect on X = x_1x'.

If the income taken away is given back to the consumer, budget line shifts from GK to AB_2. Equilibrium point moves from H to F. equilibrium consumption of X increases from Ox' to Ox_2. Movement from H to F is called income effect.

Income effect on X = $x'x_2$.(increase in real income causes an increase in consumption, hence good X is normal)

Substitution effect + Income effect = $x_1x' + x'x_2 = x_1x_2$ = Price effect.

The above analysis proves that price effect is the sum of substitution effect and income effect. Here we considered normal good alone but this applies to income-neutral good, inferior good, price-neutral good and Giffen good as well.

Figure 3.22 demonstrates the case of income-neutral good. Initial equilibrium point is E where budget line AB_1 becomes tangent with indifference curve I_1. Equilibrium level of consumption of X equals to Ox_1. A fall in price of X causes a rotation in budget line from AB_1 to AB_2. Equilibrium point is R, where consumption of X is Ox_2.

Price effect on X = x_1x_2 . Fall in price causes an increase in consumption.

GK budget line is created through compensating variation in income. Budget lines AB_1 and GK exhibit equal amount of real income because both of these budget lines are tangent to the same indifference curve I_1, thereby utility attainable either by AB_1 or by GK is equal. Movement from E to H is substitution effect.

Substitution effect = x_1x_2.

If the income taken away is given back to the consumer, budget line shifts from GK to AB_2. Equilibrium point moves from H to R. Consumption of X remains unchanged at Ox_2.

Income effect on X = 0. (X is income-neutral)

Price Effect of an Income-Neutral Good

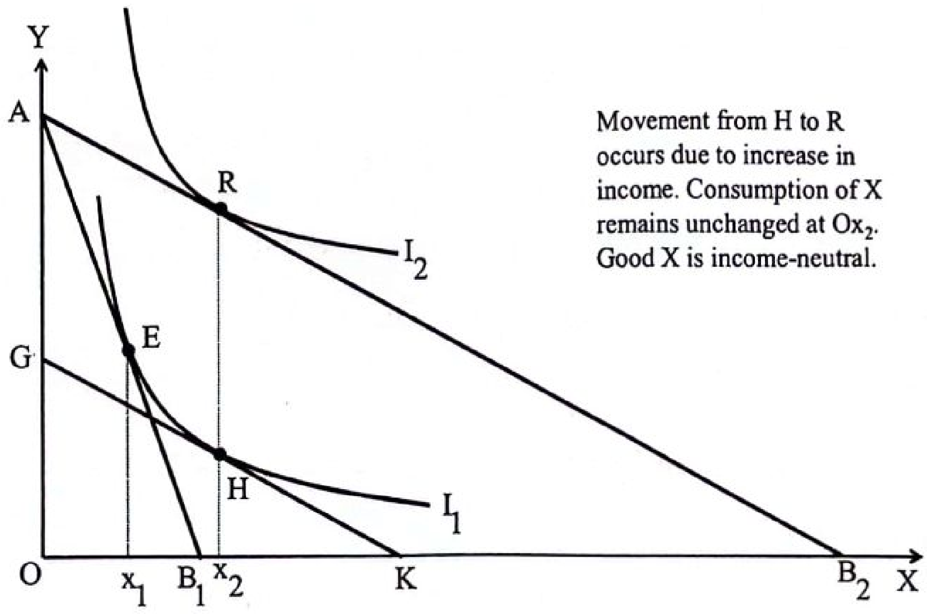

Figure- 3.22

Substitution effect + Income effect = $x_1x_2 + 0 = x_1x_2$ = Price effect.

That is, price effect of an income-neutral good is the sum of substitution effect and income effect.

Figure 3.23 displays the price effect of an inferior good. Fall in price of X moves equilibrium point from E to S.

Price Effect of an Inferior Good

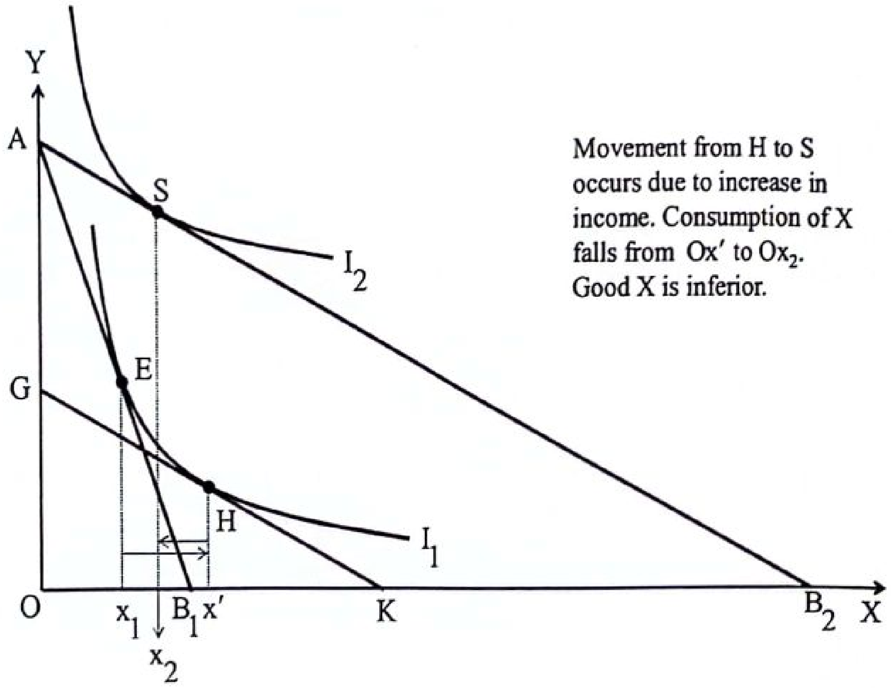

Figure- 3.23

Price effect on X = x_1x_2 (Consumption increases as price falls).

Substitution effect increases consumption from Ox_1 to Ox'.

Substitution effect = x_1x'.

Income effect reduces consumption from Ox' to Ox_2. Therefore, good X is inferior.

Income effect on X = $-x'x_2$.

Substitution effect + Income effect =

$$x_1x' + (-x'x_2) = x_1x' - x'x_2 = x_1x_2 = \text{Price effect.}$$

Although income effect appears negative the overall impact of a fall in price is an increase in consumption. This happens because negative income effect does not outweigh substitution effect. In this case good X may be viewed as weakly inferior. However, it may so happen that negative income effect is exactly equal to substitution effect or may outweigh substitution effect if the degree of inferiority gets larger.

Figure 3.24 demonstrates the case of a price-neutral good. Initial equilibrium point is E where consumption of X

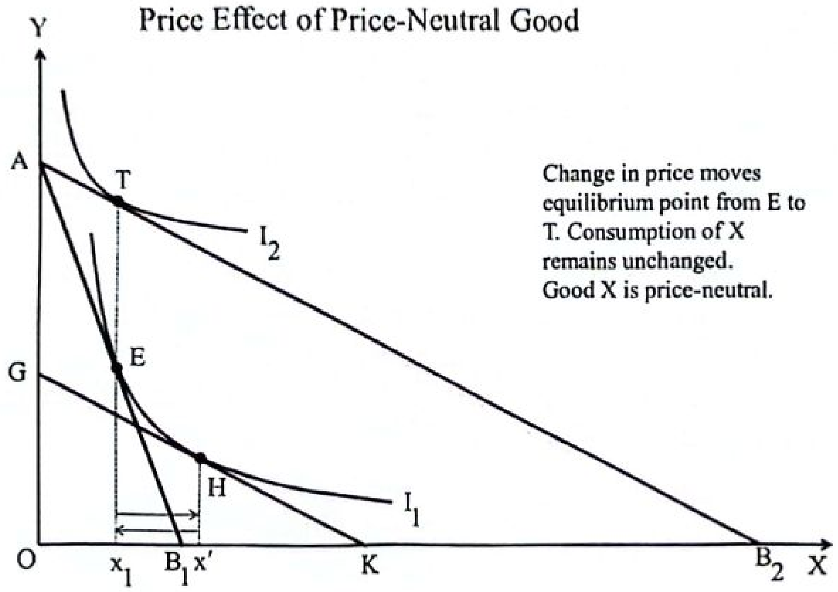

Figure- 3.24

equals to Ox_1. Fall in price of X generates new equilibrium point T on AB_2 budget line, where consumption of X is unchanged at Ox_1.

Price effect on X = 0; (Good X is price-neutral)

Movement from E to H characterizes substitution effect. Consumption of X increases from Ox_1 to Ox'.

Substitution effect $= x_1x'$

Movement from H to T indicates income effect.

Income effect = $-x'x_1$ (increase in income leads to decrease in consumption, thereby defining good X inferior)

Substitution effect + Income effect =

$$x_1x' + (-x'x_1) = x_1x' - x'x_1 = 0 = \text{Price effect.}$$

Here we showed the price effect of a price-neutral good as the sum of substitution effect and income effect. The analysis makes it clear that price-neutral goods are definitely inferior ones because price effect to be nil, income effect must be negative so that it offsets the increase in consumption via substitution effect in the situation of a fall in price. Good X can be viewed moderately inferior because decrease in consumption via income effect is equal to increase in consumption via substitution effect. If negative income effect becomes stronger than substitution effect then the good is ultra-inferior, which is known as Giffen good.

Figure 3.25 illustrates the decomposition of price effect of a Giffen good. Initial equilibrium occurs at point E where budget line AB_1 is tangent to the indifference curve I_1. Equilibrium level of consumption of X equals to Ox_1. Suppose price of X falls. New budget line is AB_2 which is tangent to the indifference curve I_2 at point K. Movement from E to K is price effect.

Price Effect of a Giffen Good

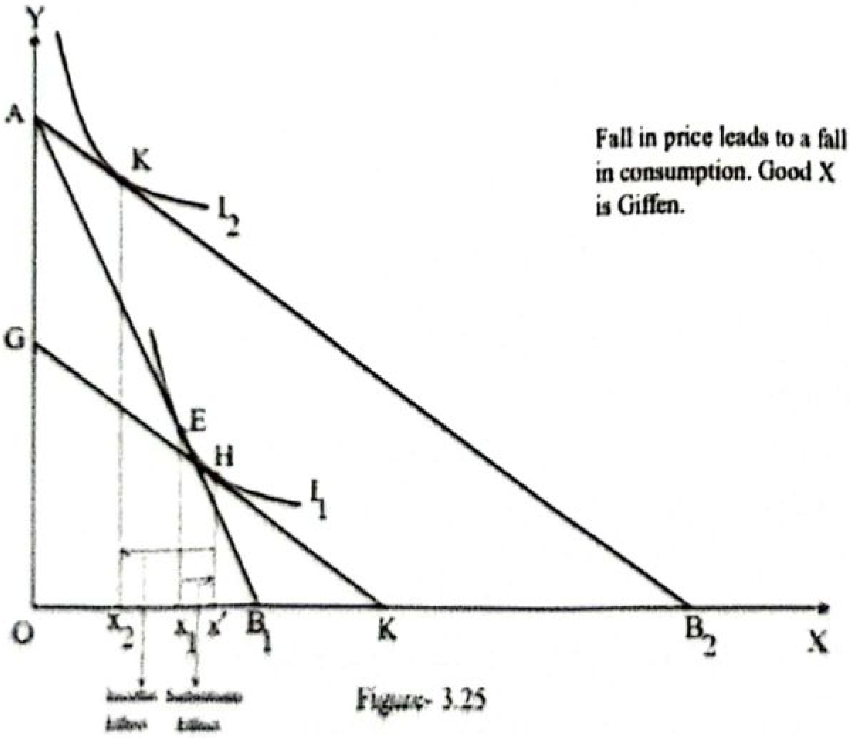

Figure- 3.25

Price effect on X = $-x_1x_2$; fall in price causes a fall in consumption of X, hence good X is Giffen.

Substitution effect is defined as the change in consumption due to change in price under the condition of constant real income. Real income along AB_2 budget line, however, is larger than AB_1. In order for keeping real income unchanged, an amount of nominal income has to be taken away from the consumer so that budget line shifts left and is tangential with initial indifference curve I_1. In figure such a budget line is GK which is tangent to I_1 indifference curve at point H. Movement from E to H is called substitution effect.

Substitution effect $= x_1x'$.

If the income taken away is given back to the consumer, budget line shifts from GK to AB_2. Equilibrium point moves from H to K. Consumption of X falls from Ox' to Ox_2.

Income effect $= -x'x_2$. Increase in income associates a decrease in consumption, hence good X is inferior. It may be noted here that decrease in consumption via income effect outweighs the increase in consumption via substitution effect. The net effect of a fall in price is therefore a fall in consumption, which is the feature of a Giffen good.

Substitution effect + Income effect $= x_1x' + (-x'x_2) = -(x'x_2 - x_1x') = -x_1x_2$ = Price effect

Finally, price effect of a Giffen good is found as the sum of substitution effect and income effect. The distinguishing feature of a Giffen good is that negative income effect outweighs the substitution effect. As a consequence, the final outcome of a fall in price is fall in consumption. Because of stronger negative income effect, Giffen goods are also named as ultra-inferior goods.

It may be observed that Giffenness of a good underlies inferiority. That means a commodity to be a Giffen one, its income effect must be negative. But negative income effect is attributable to inferior goods alone. Therefore, it is phrased that all Giffen goods are inferior. If the negative income effect does not outweigh substitution effect, then even being inferior the good will not be Giffen. That is why it is said that all inferior goods are not Giffen. We discussed two cases of inferior goods above. One was termed as weakly inferior and another moderately inferior. None of them is Giffen at all.

3.10 Slutsky Equation

In this section price effect is decomposed into substitution effect and income effect using calculus. Students taking the first course of microeconomics are advised to skip this section. The Slutsky equation, that expresses price effect as the sum of income and substitution effect, is derived by maximizing utility subject to budget constraint. Consider the following formation of a constrained optimization problem:

Maximize $U = U(x, y)$

Subject to $M = xP_x + yP_y$

Here, x and y are the quantities of two goods X and Y. P_x and P_y are their prices respectively.

Lagrangian function

$$L = U(x, y) + \lambda(M - xP_x - yP_y)$$

$$\frac{\partial L}{\partial x} = L_x = U_x - \lambda P_x$$

$$\frac{\partial L}{\partial y} = L_y = U_y - \lambda P_y$$

$$\frac{\partial L}{\partial \lambda} = L_\lambda = M - xP_x - yP_y$$

First order condition of utility maximization requires

$$L_x = U_x - \lambda P_x = 0 \qquad \text{or, } U_x = \lambda P_x \quad \ldots\ldots \quad \ldots\ldots \quad (1)$$

$$L_y = U_y - \lambda P_y = 0 \qquad \text{or, } U_y = \lambda P_y \quad \ldots\ldots \quad \ldots\ldots \quad (2)$$

$$L_\lambda = M - xP_x - yP_y = 0 \qquad \text{or, } M = xP_x + yP_y \quad \ldots\ldots \quad (3)$$

Dividing (1) by (2)

$$\frac{U_x}{U_y} = \frac{P_x}{P_y} \quad \ldots\ldots \quad \ldots\ldots \quad (4)$$

Take total differentials of first partial derivatives L_x, L_y and L_λ

$$dL_x = U_{xx}dx + U_{xy}dy - \lambda dP_x - P_x d\lambda = 0$$

$$\text{or,}\quad U_{xx}dx + U_{xy}dy - P_x d\lambda = \lambda dP_x \quad \text{.....} \quad \text{.....} \quad (5)$$

$$dL_y = U_{yx}dx + U_{yy}dy - \lambda dP_y - P_y d\lambda = 0$$

$$\text{or,}\quad U_{yx}dx + U_{yy}dy - P_y d\lambda = \lambda dP_y \quad \text{.....} \quad \text{.....} \quad (6)$$

$$dL_\lambda = dM - xdP_x - P_x dx - ydP_y - P_y dy = 0$$

$$\text{or,}\quad -P_x dx - P_y dy = -dM + xdP_x + ydP_y \quad \text{.....} \quad \text{.....}(7)$$

Arrange equations (5), (6) and (7) in matrix form

$$\begin{bmatrix} U_{xx} & U_{xy} & -P_x \\ U_{yx} & U_{yy} & -P_y \\ -P_x & -P_y & 0 \end{bmatrix} \begin{bmatrix} dx \\ dy \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda dP_x \\ \lambda dP_y \\ -dM + xdP_x + ydP_y \end{bmatrix} \quad \text{.....} \quad (8)$$

$$\text{Let,}\quad A = \begin{bmatrix} U_{xx} & U_{xy} & -P_x \\ U_{yx} & U_{yy} & -P_y \\ -P_x & -P_y & 0 \end{bmatrix}$$

$$|A| = 2P_x P_y U_{xy} - P_y^2 U_{xx} - P_x^2 U_{yy} \quad \text{....} \quad \text{....} \quad (9)$$

(note that $U_{xy} = U_{yx}$)

Using Cramer's rule, find the value of dx from the matrix system (8).

$$dx = \frac{\begin{vmatrix} \lambda dP_x & u_{xy} & -P_x \\ \lambda dP_y & u_{yy} & -P_y \\ -dM + xdP_x + ydP_y & -P_y & 0 \end{vmatrix}}{|A|}$$

$$\text{or, } dx = \frac{-\lambda P_y^2 dp_x + \lambda P_x P_y dP_y + (-dM + xdP_x + ydP_y)(-P_y U_{xy} + P_x U_{yy})}{|A|}$$

.... (10)

If we want to examine the impact of P_x on X then P_y and M would remain unchanged, thus $dP_y = dM = 0$. From equation (10)

$$\frac{\partial x}{\partial P_x} = \frac{-\lambda P_y^2}{|A|} + \frac{x(-P_y U_{xy} + P_x U_{yy})}{|A|} \quad \text{..... ..... (11)}$$

By definition $\frac{\partial x}{\partial P_x}$ measures price effect. In order to measure substitution effect, the utility of the consumer has to be kept constant and thus a change in money income is required. Change in income,

$$dM = xdP_x + P_x dx + ydP_y + P_y dy \quad \text{..... ..... (12)}$$

If utility remains unchanged, then $dU = 0$

Having $U = U(x, y)$

$dU = U_x dx + U_y dy = 0$

$$\frac{dy}{dx} = -\frac{U_x}{U_y} = -\frac{P_x}{P_y} \quad \text{(using equation (4))}$$

or, $P_x dx + P_y dy = 0$ .... (13)

Combining (12) and (13),

$$dM = xdP_x + ydP_y \quad \text{..... ..... (14)}$$

Plug $dP_y = 0$ and $dM = xdP_x + ydP_y$ in equation (10) to obtain substitution effect.

$$\left(\frac{\partial x}{\partial P_x}\right)_{\overline{U}} = -\frac{\lambda P_y^2}{|A|} \quad \ldots\ldots \quad \ldots\ldots \quad (15)$$, this is substitution effect.

Income effect $\frac{\partial x}{\partial M}$ can be derived from equation (10) by assuming $dP_x = dP_y = 0$

$$\frac{\partial x}{\partial M} = -\frac{(-P_y U_{xy} + P_x U_{yy})}{|A|} \quad \ldots\ldots \quad \ldots\ldots \quad (16)$$

Multiply both sides of equation (16) by –x and get weighted income effect.

$$-x\frac{\partial x}{\partial M} = \frac{x(-P_y U_{xy} + P_x U_{yy})}{|A|} \quad \ldots\ldots \quad \ldots\ldots \quad (17)$$

Using equations (11), (15) and (17)

$$\frac{\partial x}{\partial P_x} = \left(\frac{\partial x}{\partial P_x}\right)_{\overline{U}} - x\frac{\partial x}{\partial M} \quad \ldots\ldots \quad \ldots\ldots \quad (18)$$

Equation (18) is known as the Slutsky equation that expresses price effect as the sum of substitution effect and weighted income effect. The first part on the right hand side of the Slutsky equation $\left(\frac{\partial x}{\partial P_x}\right)_{\overline{U}}$ defines substitution effect. According to (15) $\left(\frac{\partial x}{\partial P_x}\right)_{\overline{U}} = -\frac{\lambda P_y^2}{|A|}$ which is always negative because the denominator $|A| = 2P_x P_y U_{xy} - P_y^2 U_{xx} - P_x^2 U_{yy}$ is positive, this comes from the fact that goods X and Y are substitutes thereby $U_{xy} > 0$, and because of the law of diminishing marginal utility $U_{xx} < 0$ and $U_{yy} < 0$.

Negative substitution effect implies a rise in consumption due to fall in relative price of a commodity and vice versa. The negative substitution has important bearing. If the utility function is convex or marginal rate of substitution is diminishing and the consumer observes a fall in price of a good then he or she would remain at the same level of utility by consuming more of that good. This holds for any type of good.

The second part of the Slutsky equation helps classifying goods. Income effect $\frac{\partial x}{\partial M}$ may be positive, negative or zero if the good is respectively normal, inferior or income neutral.

First, assume that the good is normal so that $\frac{\partial x}{\partial M} > 0$ and $-x\frac{\partial x}{\partial M} < 0$.

Price effect $\frac{\partial x}{\partial P_x}$ would then be large negative, implying a substantial rise in consumption due to fall in price and vice versa.

Secondly, if the good is assumed inferior then $\frac{\partial x}{\partial M} < 0$ and $-x\frac{\partial x}{\partial M} > 0$. If this positive value is smaller than the absolute value of substitution effect then again price effect would be negative. This suggests, if the good is weakly inferior then the price effect would be negative. If, on the other hand, the positive value of $-x\frac{\partial x}{\partial M}$ is equal to the absolute value of substitution effect then price effect would be zero, defining price neutral good. This suggests that a price neutral good is necessarily inferior good. Finally, in the extreme when the positive $-x\frac{\partial x}{\partial M}$ outweighs the absolute value of substitution effect then the price effect becomes positive, characterizing the Giffen good. This proves that the Giffen good is surely an inferior good.

3.11 Derivation of Demand Curve

A demand curve shows the combinations of price and consumption. Demand curve of an individual can be derived from equilibrium analysis under indifference curve theory. Figure 3.26 explains the derivation of the demand curve.

Upper panel of figure shows consumption of X and Y along two axes. Horizontal axis of lower panel measures consumption of X and vertical axis price of X. Initial budget line AB_1 is drawn assuming price of X equals to OP_1. Initial equilibrium point of the consumer is e_1 where consumption of X equals Ox_1. The pair of price OP_1 and consumption Ox_1 is designated by the point d_1 in the lower panel of diagram, which is one point of the demand curve. Suppose price of X falls from OP_1 to OP_2. New budget line is AB_2 and equilibrium point is e_2, where consumption of X equals Ox_2. The pair of price OP_2 and consumption Ox_2 is designated by point d_2 in the lower panel of diagram, which is another point of the demand curve. Other points of the demand curve can be derived by following the similar technique. Demand curve DD′ is drawn by joining the points like $d_1$and d_2.

Demand Curve Derivation

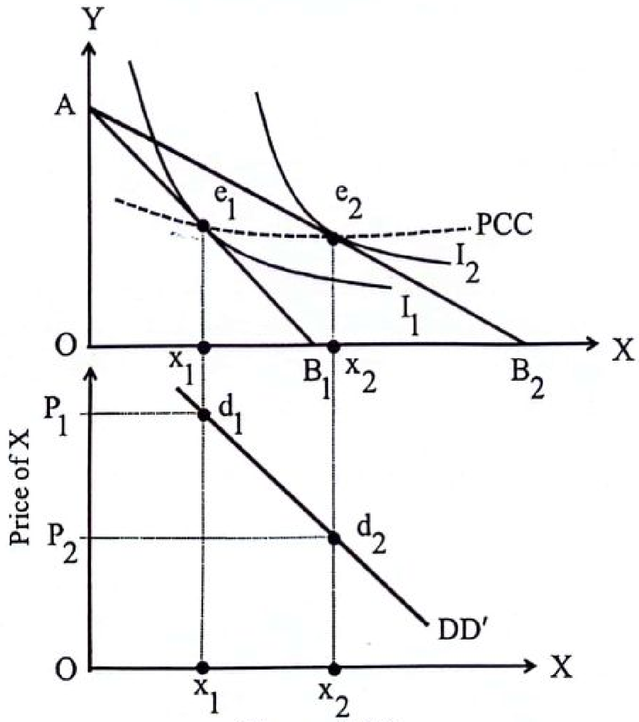

Figure- 3.26

Demand curve slopes downward suggesting an inverse relationship between price and quantity demanded. One may think that the good is normal. But demand curves of income neutral and inferior goods are also downward sloping.

Figure 3.27 displays the downward sloping demand curve of an income-neutral good. Movement from H to e_2 represents income effect. Change in income does not alter consumption of X, therefore income effect on X is zero.

Demand Curve of an Income-Neutral Good

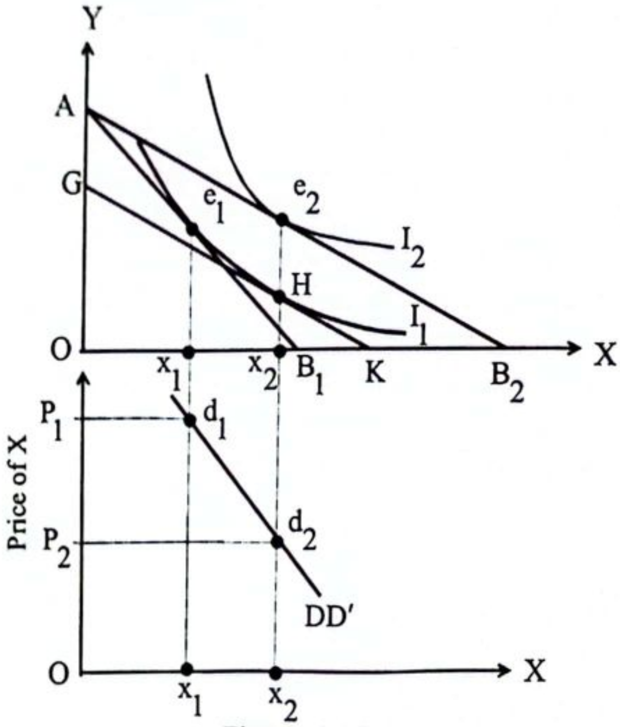

Figure- 3.27

Good X is income-neutral. However, substitution effect increases consumption from Ox_1 to Ox_2. d_1 point of lower panel shows the combination of price OP_1 and consumption Ox_1. One point of the demand curve is d_1. Another point of the demand curve is d_2 where price OP_2 is paired with consumption Ox_2.

Demand Curve of Inferior Good

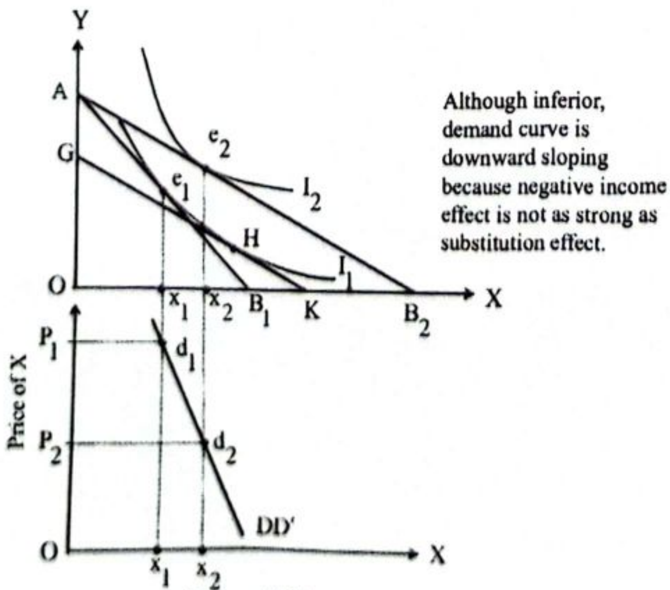

Figure- 3.28

It may be noted here that an increase in consumption from Ox_1 to Ox_2 in figure 3.27 is fully attributable to substitution effect. Income effect does not cause any change in level of consumption.

In case of a normal good, income effect also increases consumption in the event of a fall in price. Therefore, the increase in consumption of an income-neutral good is of course smaller than the increase that would have occurred if the good was normal. This is why, demand curve of an income-neutral good is steeper than that of a normal good.

Demand Curve of a Price-Neutral Good

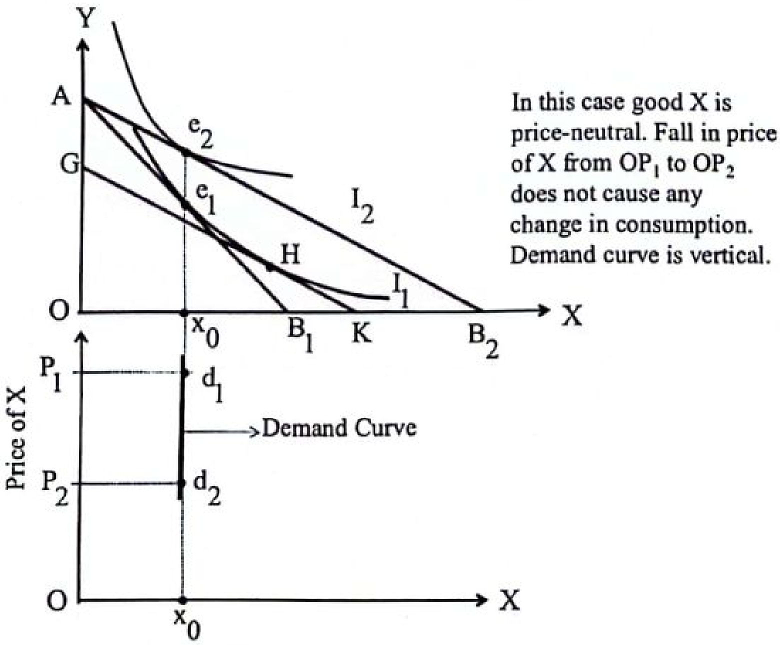

Figure- 3.29

Income effect of an inferior good turns out to be negative, thus increase in consumption by substitution effect is partially offset by negative income effect. Overall increase in consumption, due to decrease in price, becomes further smaller than what was for an income neutral good. Consequently, demand curve of an inferior good becomes steeper than that of an income-neutral good. Figure-3.28 portrays the derivation of inferior good's demand curve. If negative income effect fully offsets the substitution effect, fall in price does not cause any change in consumption. The good is price-neutral. Demand curve of a price-neutral good is vertical as in figure 3.29.

If negative income effect becomes stronger than substitution effect, a fall in price would be followed by a fall in quantity demanded, which is the case of a Giffen good. Because of a positive relationship between price and consumption, demand curve of a Giffen good slopes upward. Figure 3.30 illustrates the derivation of demand curve of a Giffen good. Budget line AB_1 corresponds to price of X equals OP_1. Equilibrium point in the upper panel is E_1 where consumption of X is Ox_1. Point d_1, in lower panel of diagram, exhibits price OP_1 and consumption Ox_1, which is one point of the demand curve. If price of X falls to OP_2, budget line rotates to AB_2, thereby generating a new equilibrium point E_2, where consumption of X falls to Ox_2. The pair of price OP_2 and consumption Ox_2 is denoted as d_2 in the lower panel of diagram, which is another point of the demand curve. Demand curve DD' is derived by joining the points d_1 and d_2. Because of a positive relationship between price and quantity demanded, the demand curve slopes upward.

Demand for a Giffen Good

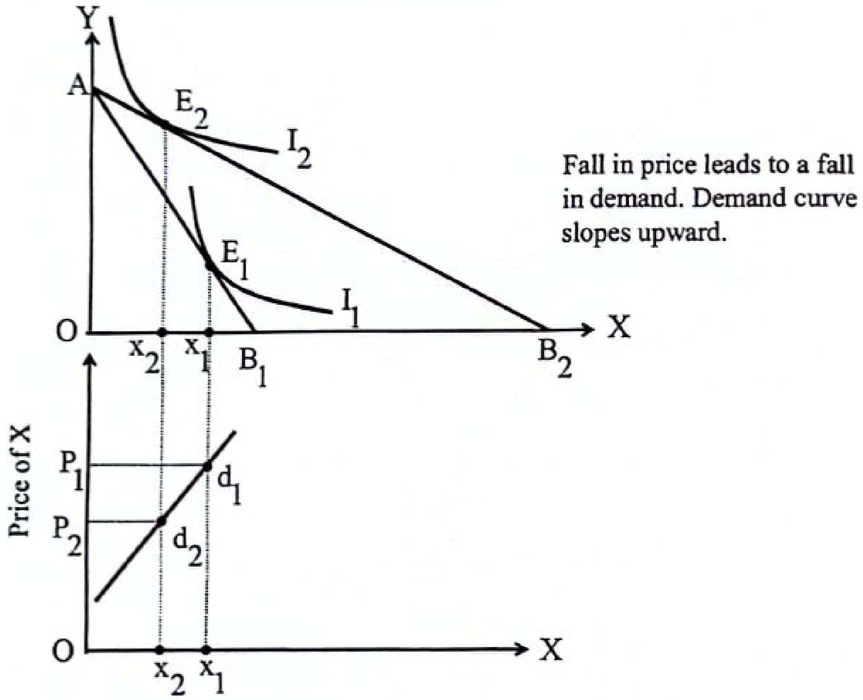

Figure- 3.30

Indifference curve approach can be viewed as a complete theory of demand because it can address all possibilities regarding the direction of change in quantity demanded following change in the price of a good. Cardinal theory of utility only provides the logic of inverse relationship between price and consumption. But in real life instances, other possibilities like price-consumption positive relationship or completely unresponsive consumption pattern may occur. Indifference curve approach can, in addition to usual law of demand, intensely address the latter aspects.

3.12 Revealed Preference Theory

Revealed preference theory deals with the preference revelation of individual consumer under different budget constraints. If, for example, there are two commodity bundles *A* and *B* that are affordable to the consumer and he decides to consume *A* then it is said that *A* is revealed to be preferred to *B*. The study of revealed preference theory opens the scope of testing the behavioral hypothesis that the consumer chooses the most preferred bundle from those available and this way the preference relation of the consumer can be discovered.

There are three assumptions of revealed preference theory.

1. The consumer's preference is of static nature. Preference does not change instantly. For example, if a consumer says that he prefers *A* to *B* then he would not say that he prefers *B* to *A*.
2. Diminishing marginal rate of substitution between goods holds and thus the choices are strictly convex.
3. The preference function is continuous and monotonic. There will have no discontinuity in the preference function.

Unique Commodity Bundle

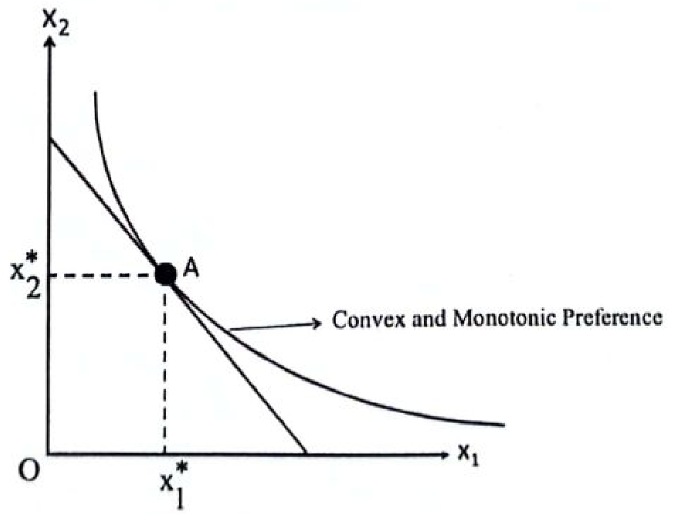

Figure-3.31

If the above assumptions hold then the consumer's most preferred affordable bundle would be unique. Figure- 3.31 shows the unique preference of the consumer under usual assumptions. Tangency of the budget line and convex preference function yields unique preference A, where consumption of x_1 and x_2 are x_1^* and x_2^* respectively.

3.12.1 Types of Preference Revelation

There are two types of preference revelation- Direct Preference Revelation and Indirect Preference Revelation. If the consumer can directly compare between commodity bundles then the revelation is direct. Suppose that the commodity bundle A is chosen when B is affordable then it is said that A is directly revealed to be preferred to B.

Direct Preference Revelation

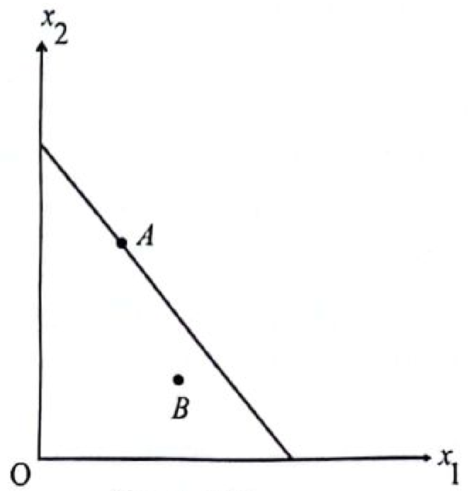

Figure-3.32

Figure 3.32 illustrates the feature of direct revelation. Two commodity bundles A and B are available to the consumer because they are within the budget constraint. If the consumer chooses A, it implies that A is directly revealed to be preferred to B because B was also available to him but he chose A.

Using notation it is written as $A \underset{D}{\succ} B$.

If the consumer cannot directly compare between commodity bundles rather indirectly can, then the revelation is indirect. Suppose A is directly preferred to B and B is directly preferred to C then, by transitivity, A is indirectly preferred to C, which is written as $A \underset{I}{\succ} C$.

This suggests, if $A \underset{D}{\succ} B$ and $B \underset{D}{\succ} C$ then $A \underset{I}{\succ} C$.

Indirect Preference Revelation

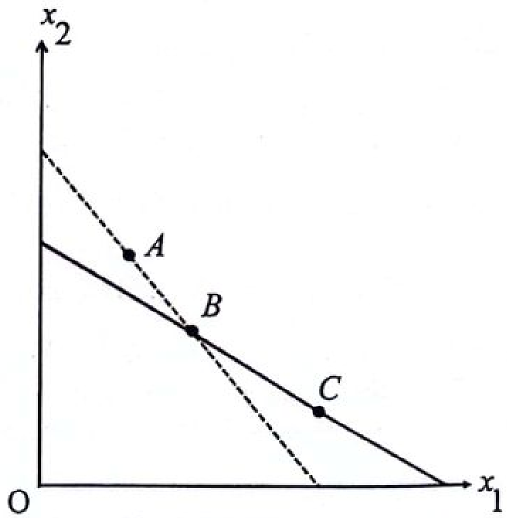

Figure-3.33

Indirect revelation is depicted in figure-3.33. When consumer compares between commodity bundles A and B then C was not available to him. Thus A and C are not directly comparable. If the consumer chooses A then it would mean that A is directly preferred to B. Again when B and C were available and consumer chose B then it would mean that B is directly preferred to C. Since A is directly preferred to B and B is directly preferred to C therefore A is indirectly preferred to C.

3.12.2.1 Weak and Strong Axioms of Revealed Preference

Revealed preference analysis would be operational if choices satisfy two criteria- the strong axiom and the weak axiom of revealed preference. The weak axiom of revealed preference (WARP) states that if commodity bundle A is directly revealed as to be preferred to B then it would never happen that B is directly revealed as to be preferred to A, i.e., if $A \underset{D}{\succ} B$ then not $\left(B \underset{D}{\succ} A\right)$.

The strong axiom of revealed preference (SARP) says that if the commodity bundle A is directly or indirectly revealed as to be preferred to B then it would never happen that B is directly or indirectly as to be preferred to A.

Using notation, if $A \underset{D}{\succ} B$ or $A \underset{I}{\succ} B$ then not $\left(B \underset{D}{\succ} A \text{ or } B \underset{I}{\succ} A\right)$.

Exercise 3

1. Find the marginal utility function assuming the total utility function: $U = 600q - q^2$
2. Derive demand curve from utility function given in (1).
3. Demand function of a consumer is $Q = 500 - 10P$. Compute the amount of consumer surplus from the consumption of 20 units of the commodity.
4. Determine the marginal rate of substitution from the utility function: $U = q_1q_2$
5. Find the slope of the indifference curve defined by the equation: $U = 25q_1^{0.5}q_2^{0.3}$
6. Suppose a consumer consumes two goods A & B whose prices are 5 Taka and 10 Taka respectively. Find the equation of budget line. Draw the budget line assuming M = 500. What will happen if price of A increases to 10 Taka per unit?
7. Draw the utility function U = xy, where x and y are the consumptions of X and Y respectively. Find equilibrium consumption of X and Y assuming M = 600, $P_x = 10$, $P_y = 20$.
8. Suppose income increases from 600 to 800. What is the measure of income effect on X if other variables remain unchanged as in (6) above?

APPENDIX B

DECOMPOSITION OF PRICE EFFECT CONSIDERING PRICE INCREASE

Price effects of five different types of goods have been decomposed into substitution effect and income effect by assuming an increase in price of X. Only graphical illustration is presented.

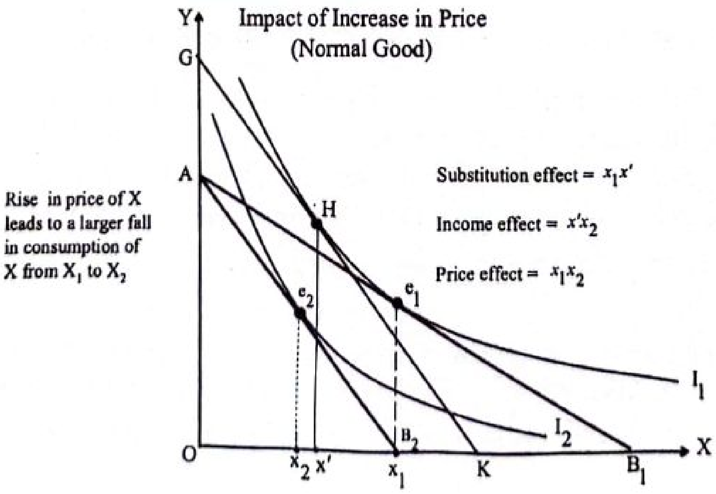

Figure- B.1