Static Electricity

Electric Charge

- It is an intrinsic property of particles (i.e. electrons and protons)
- Comes in both positive and negative amounts.
- Usually denote charge by letter “q”, unit of measure is the Coulomb, C, in SI units
    - Electron: $q_e = -e = -1.6022 \times 10^{-19}\, C$
    - Proton : $q_p = e = 1.6022 \times 10^{-19}\, C$
    - In fact, all charge is quantized in integer multiples of “e”.
- Most matter is electrically neural (balanced: equal amounts + and -)
- Can get a net imbalance of electric charge:
    - Silk on glass → excess + charge on glass
    - Fur on plastic → excess – charge on plastic
- Net charge is always conserved
- Like-sign charges repel
- Opposite-sign charges attract

Quantization of Charge

Charge is created by transfer of electrons; therefore, the net charge on an object is always an integral multiple of magnitude of charge on an electron/ elementary charge e. Mathematically, we can write

$$Q = ne$$

Where, n = 0, ± 1, ± 2, ± 3, ± 4, ± 5, ± 6……….. and $e = 1.6 \times 10^{-19}$ C

No charge is found in the fraction of e (as o.5 e or 0.7 e …). It means that electric charge can not be divided indefinitely. This property of charge is called ‘quantization’ or ‘atomicity of charge’.

Problem 1: Determine the number of excess electrons required to give an object a net charge of -3 C.

Solution:

$$Q = ne$$

$$Or, n = \frac{Q}{e} = \frac{-3\ C}{-1.6 \times 10^{-19}\ C} = 1.9 \times 10^{19}\ C$$

Problem 2: Two identical, conducting sphere have different charges. Sphere 1 has a net charge of $Q_1 = -9.6 \times 10^{-18}C$ and sphere 2 has 30 excess electrons. If the two spheres are brought into contact and then separated.

a) What charge will each have?
b) How many electrons will each have?
c) What charge is transferred?
d) How many electrons are transferred?

Solution:

a)

$$Q_1 = -9.6 \times 10^{-18}C$$

$$Q_2 = ne = 30 \times (-1.6 \times 10^{-19}C) = Q_1 = -4.8 \times 10^{-18}C$$

$$Q_{combined} = -9.6 \times 10^{-18}C + (-4.8 \times 10^{-18}C) = -1.44 \times 10^{-17}C$$

$$Q_{each} = \frac{-1.44 \times 10^{-17}C}{2} = -7.2 \times 10^{-18}C$$

b)

$$N_E = \frac{Q_{each}}{e} = \frac{-7.2 \times 10^{-18}C}{-1.6 \times 10^{-19}C} = 45$$

c)

$$Q_{tr} = -9.6 \times 10^{-18}C - (-7.2 \times 10^{-18}C) = -2.4 \times 10^{-18}C$$

d)

$$N_{tr} = \frac{Q_{tr}}{e} = \frac{-2.4 \times 10^{-18}C}{-1.6 \times 10^{-19}C} = 15$$

Coulomb’s Law

Coulomb's law or Coulomb's inverse-square law, is a law of physics describing the electrostatic interaction between electrically charged particles.

Colomb’s law states that the magnitude of the electrostatic force of attraction or repulsion between two electrically charged bodies is directly proportional to the product of the charge of the charged bodies and inversely proportional to the square of the distance between the center of the charged bodies.

If two point charges q_1 and q_2 are separated by a distance r then the magnitude of the force of repulsion or attraction between them is

$$F \propto \frac{q_1 q_2}{r^2}$$

Or,

$$F = K \frac{q_1 q_2}{r^2}$$

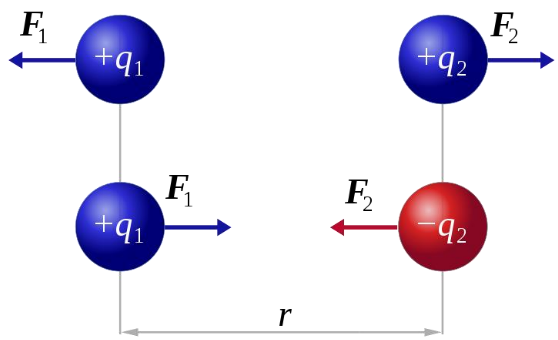


Figure 1: Coulomb interaction between two charges.

Where, K is the proportionality constant.

And $K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$ is the electrostatic constant.

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}m^{-2}$ is the electric permittivity of the free space.

When the charged particles are placed in the dielectric medium then the electrostatic constant will be

$$K = \frac{1}{4\pi\varepsilon_0 k} = \frac{1}{4\pi\varepsilon}$$

Where, $\varepsilon = \varepsilon_0 k$, k is the dielectric constant.

Therefore, This force (F) acting simultaneously on point charges (q_1) and (q_2), is given by

$$F = \frac{1}{4\pi\varepsilon_0 k}\frac{q_1 q_2}{r^2}$$

The dielectric constant of air or free space is equal to 1. So, the electrostatic force on free space

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

The direction of electrostatic force always acts along line joining centre of two charges.

Limitations of Coulomb's Law

1. Coulomb's law cannot be applied if the point charges are not at rest. The reason for this is that when the two charged particles are brought together, the distribution of charges on them changes.
2. Coulomb's law cannot be applied if the shape of the charges is arbitrary because, in irregular shapes, it becomes difficult to determine the distance between the particles.
3. Coulomb's Law can only be applied in those cases where the inverse square law is obeyed.
4. Coulomb's law cannot be used directly to calculate the charge on big planets. This is because of the arbitrary shapes of planets which makes determining the exact distance between them difficult.

Principle of superposition of Electric Force

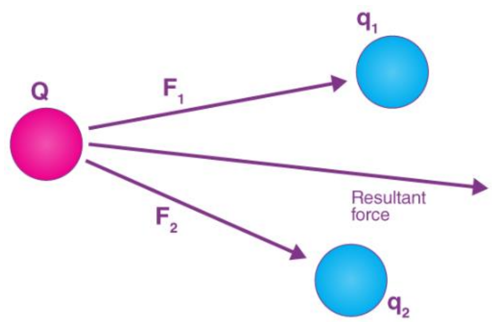
The principle of superposition states that every charge in space creates an electric field at point independent of the presence of other charges in that medium. The resultant electric field is a vector sum of the electric field due to individual charges.

The electrostatic force on charge Q is

$$\underset{F}{\rightarrow} = \underset{F}{\rightarrow}{}_{Qq_1} + \underset{F}{\rightarrow}{}_{Qq_2}$$

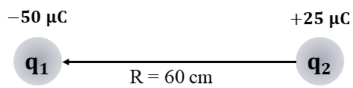
For the normalize causes, we can say

$$\underset{F}{\rightarrow} = \underset{F_1}{\rightarrow} + \underset{F_2}{\rightarrow} + \underset{F_3}{\rightarrow} + \cdots$$



Problem 3: A metal sphere is given a charge of -50 μC and a second sphere located 60 cm away is given a charge of +25 μC. Determine the magnitude and the direction of the force exerted on a positive charge.

Solution:



Given,

$q_1 = -50\ \mu C = -50 \times 10^{-6}\ C$

$q_2 = +25\ \mu C = 25 \times 10^{-6}\ C$

$R = 60\ cm = 0.6\ m$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}$$

$$= 9 \times 10^9 \times \frac{50 \times 10^{-6} \times 25 \times 10^{-6}}{(0.6)^2}$$

= 31.3 N; Towards q_1 or towards the negative x-axis.

Problem 4: Two point charges of 1.8×10^{-6} C and 2.4×10^{-6} C produce a force of 2.2×10^{-3} N on each other. How far apart are these two charges?

Solution:

$q_1 = 1.8 \times 10^{-6}\ C$

$q_2 = 2.4 \times 10^{-6}\ C$

$F = 2.2 \times 10^{-3}\ N$

$r = ?$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$Or, r^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{F}$$

$$Or, r = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{F}}$$

$$Or, r = \sqrt{9 \times 10^9 \times \frac{1.8 \times 10^{-6} \times 2.4 \times 10^{-6}}{2.2 \times 10^{-3}}} = 4.2\ m$$

Problem 5: Two charges of 9 μC and 4 μC, when placed in a dielectric medium at a distance of 3 m will exert the force $9 \times 10^{-3}\ N$. Calculate the dielectric constant of the medium.

Solution:

Given

$$q_1 = 9\ \mu C = 9 \times 10^{-6}\ C$$

$$q_2 = 4\ \mu C = 4 \times 10^{-6}\ C$$

$$F = 9 \times 10^{-3}\ N$$

$$r = 3\ m$$

$$k = ?$$

By formula

$$F = \frac{1}{4\pi\varepsilon_0 k}\frac{q_1 q_2}{r^2}$$

$$Or, k = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\frac{1}{F}$$

$$= 9 \times 10^9 \times \frac{9 \times 10^{-6} \times 4 \times 10^{-6}}{3^2}\frac{1}{9 \times 10^{-3}}$$

$$= 4$$

Problem 6: Two negative point charges of equal magnitude are 30 cm apart in a medium having dielectric constant 5. The repulsive force between them is 20 N. Determine the magnitude of each charge.

Solution: Try it yourself.

Answer: $3.16 \times 10^{-5}\ C$.

Problem 6: Find the ration of the magnitude of electrical force and the magnitude of gravitational force for two electrons placed at some distance apart in air. Where, charge , $e = -1.6 \times 10^{-19}\ C$, $m_e = 9.1 \times 10^{-31}\ kg$, $K = 9 \times 10^9\ Nm^2\ C^{-2}$, $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

Solution:

$$F_E = K\ \frac{q_1 q_2}{r^2}$$

And, $F_G = G\ \frac{m_1 m_2}{r^2}$

$$\frac{F_E}{F_G} = \frac{K.\ q_1 q_2}{r^2} \times \frac{r^2}{G.m_1 m_2}$$

$$= \frac{K.\ q_1 q_2}{G.m_1 m_2}$$

$$= \frac{9 \times 10^9 \times (-1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times (9.1 \times\ 10^{-31})^2}$$

$$\approx 10^{42}$$

Problem 7: Charges of magnitude 100 mC each are located in a vacuum at the corners A, B, and C of an equilateral triangle measuring 4 meters on each side. If the charge at A and C are positive and the charge B negative. What is the magnitude and direction of the total force on the charge at C?

Solution:

The situation is shown in Figure. Let us consider the forces acting on C due to A and B.

Now,

The force of repulsion on C due to A,

$F_{CA} = \frac{1}{4\pi\varepsilon_0} \frac{q \times q}{a^2}$, along AC

The force of attraction on C due to B,

$F_{CB} = \frac{1}{4\pi\varepsilon_0} \frac{q \times q}{a^2}$, along CB

Thus, the two forces are equal in magnitude. The angle between them is 120º. The resultant force F is given by

$$F = \sqrt{F_{CA}^2 + F_{CB}^2 + 2F_{CA} \times F_{CB} \cos 120°}$$

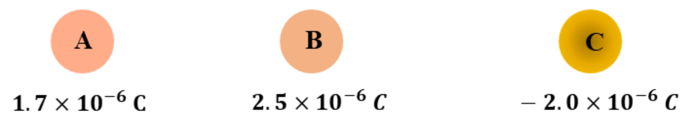
$$= \frac{q^2}{4\pi\varepsilon_0 a^2}$$

$$= \frac{9 \times 10^9 \times (100 \times 10^{-6})^2}{4^2}$$

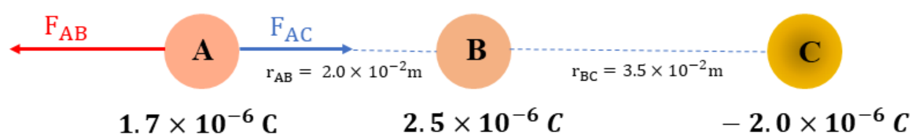
$$= 5.625\ N$$

This force is parallel to AB.

Problem 8: A charge of $1.7 \times 10^{-6}\ C$ is placed $2.0 \times 10^{-2} m$ from a charge of $2.5 \times 10^{-6}\ C$ and $3.5 \times 10^{-2} m$ from a charge of $-2.0 \times 10^{-6}\ C$ as shown in Figure. What is the net electric force on the $1.7 \times 10^{-6}\ C$ charge?



Solution:



$$F_{AB} = K\,\frac{q_A q_B}{r_{AB}^2}$$

$$= \frac{(9 \times 10^9)(1.7 \times 10^{-6})(2.5 \times 10^{-6})}{(2.0 \times 10^{-2})^2}$$

$$= 95.6\ N$$

$$F_{AC} = K\,\frac{q_A q_C}{r_{AC}^2}$$

$$= \frac{(9 \times 10^9)(1.7 \times 10^{-6})(2.0 \times 10^{-6})}{(3.5 \times 10^{-2})^2}$$

$$= 25.0\ N$$

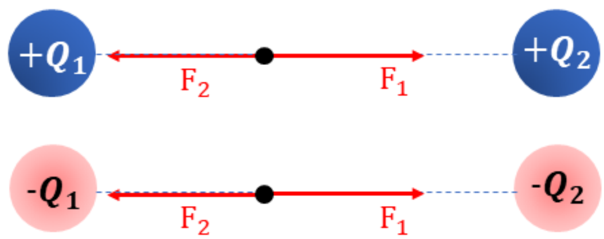
$$F_{net} = F_{AB} - F_{AC}$$

$$= (95.6 - 25.0) N$$

$$= 70.6\ N$$

Determine the position where net electric force is zero

Like Charges



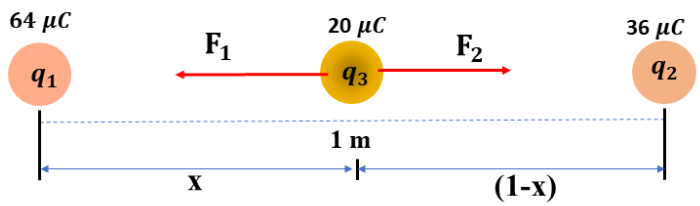
Position will be inside
between two charges.
$F_1 = F_2$

Unlike Charges

Position will be outside near to the
smaller charge.
$F_1 = F_2$

Problem 9: Two charges of 64 μC and 36 μC are placed 1m apart in a medium of dielectric constant 2.5. At what point between two, a charge of 20 μC experiences equal forces.

Solution:



Let, x be a distance from q_1 ,where q_3 experiences equal force.

The force between q_1 and q_3 is

$$F_1 = \frac{1}{4\pi\varepsilon_0 k}\frac{q_1 q_3}{r_{13}^2}$$

$$= 9 \times 10^9 \times \frac{64 \times 20}{2.5 \times x^2}$$

Similarly, the force between q_2 and q_3 is

$$F_2 = \frac{1}{4\pi\varepsilon_0 k}\frac{q_2 q_3}{r_{23}^2}$$

$$= 9 \times 10^9 \times \frac{36 \times 20}{2.5 \times (1-x)^2}$$

By condition,

$$F_1 = F_2$$

Or, $9 \times 10^9 \times \frac{64\times20}{2.5\times x^2} = 9 \times 10^9 \times \frac{36\times20}{2.5\times(1-x)^2}$

Or, $\frac{64}{x^2} = \frac{36}{(1-x)^2}$

Or, $\frac{8}{x} = \frac{6}{(1-x)}$

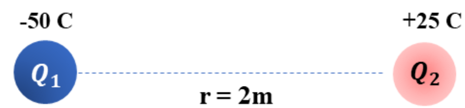
Or, $8 - 8x = 6x$

Or, $6x + 8x = 8$

Or, $14x = 8$

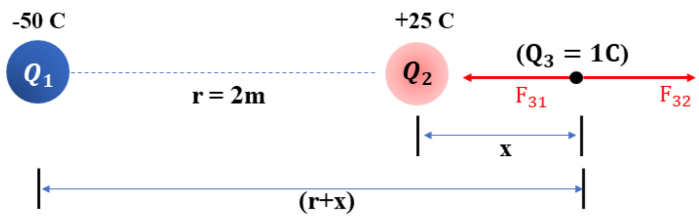
Or, $x = \frac{8}{14} = 0.57\ m$

Problem 10:



Given the two spheres as shown above, where should the third positively test charged sphere ($Q_3 = 1C$) be placed such that it experiences no net force.

Solution:



Let x be the distance from Q_2 where Q_3 experiences no net force, i.e. equal forces in opposite direction.

$$F_{31} = F_{32}$$

Or, $\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_3}{(r+x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{x^2}$

Or, $\frac{Q_1}{(r+x)^2} = \frac{Q_2}{x^2}$

Or, $\frac{50}{(r+x)^2} = \frac{25}{x^2}$

Or, $\frac{50}{25} = \frac{(r+x)^2}{x^2}$

Or, $2 = (\frac{r+x}{x})^2$

Or, $\sqrt{2} = \frac{r+x}{x}$

Or, $\sqrt{2} = \frac{r}{x} + 1$

Or, $\sqrt{2} - 1 = \frac{r}{x}$

Or, $x = \frac{r}{\sqrt{2}-1}$

Or, $x = \frac{2}{\sqrt{2}-1} = 0.4142\ m$