CHAPTER 3: SETS

SOLUTION

(a) $n(\xi) = n(P \cup Q) + 3 = 20 \Rightarrow\Rightarrow\Rightarrow n(P \cup Q) = 20 - 3 = 17$

$\Rightarrow\Rightarrow\Rightarrow n(P \cup Q) = x+y+z = 17$ .....(1)

$n(P) = x+y = 12$ $\quad n(Q) = y+z = 10$

therefore, $n(P)+n(Q) = 12+10 = x+y+y+z$ .....(2)

by way of replacing (1) into (2): $17+y = 22$

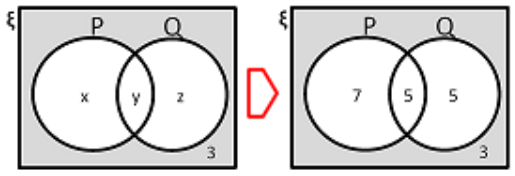
$y = \underline{5}$

by way of replacing y=5 into n(P): $x+5 = 12$

$x = \underline{7}$

by way of replacing y=5 into n(Q): $5+z = 10$

$z = \underline{5}$



(b) (i) $n(P \cup Q) = 7+5+5 = 17$

(ii) $n(P' \cup Q) = 5+5+3 = 13$

(iii) $n[P \cup (P \cup Q)'] = 7+5+3 = 15$

- **Combined operations** involve both the **intersection** and **union** of sets and **the complement** of sets
- When considering combined or mixed operations, it is advisable to follow the guidelines as per below:
  - ❑ Always perform operations from **left to right**
  - ❑ If there are brackets, perform operations **within the bracket first**, and to be **followed by operations left to right**

EXAMPLE 3.4

It is given $\xi = \{x: 1 \le x \le 10, x \text{ is an integer}\}$; P= {odd numbers} ; Q= {prime numbers} and R= {1,2,3,4,5}

List the elements of each of the following sets.

(a) $(P \cap Q) \cup R$

(b) $(R \cap P)' \cup Q$

(c) $Q' \cap (P \cup R)$

(d) $(P \cup Q \cup R)' \cap (P \cap R)$

SOLUTION

$\xi = \{1,2,3,4,5,6,7,8,9,10\}$; P= {1,3,5,7,9} ; Q= {2,3,5,7} ; R= {1,2,3,4,5}

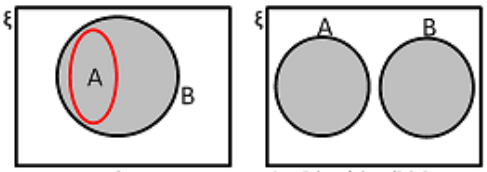
(a) $P \cap Q = \{2,3,5,7\}$; therefore $(P \cap Q) \cup R =$ **{1,2,3,4,5,7}**

(b) $R \cap P = \{1,3,5\}$; $(R \cap P)' = \{2,4,6,7,8,9,10\}$
therefore $(R \cap P)' \cup Q =$ **{2,3,4,5,6,7,8,9,10}**

(c) $Q' = \{1,4,6,8,9,10\}$; $P \cup R = \{1,2,3,4,5,7,9\}$;
therefore $Q' \cap (P \cup R) =$ **{1,4,9}**

(d) $P \cup Q \cup R = \{1,2,3,4,5,7,9\}$; $(P \cup Q \cup R)' = \{6,8,10\}$; $P \cap R = \{1,3,5\}$
therefore, $(P \cup Q \cup R)' \cap (P \cap R) = \{\ \}$

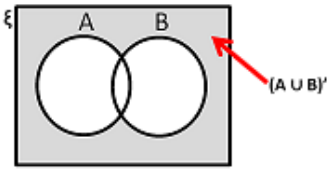
- $\cup$ denotes union of sets where **A ∪ B** is the set of elements that belong to A or B or both. When **A ⊂ B**, then **A ∪ B = B**.
- The diagram below illustrates both possibilities.



A ∪ B when **A ⊂ B** $\quad$ **A ∪ B** involving disjoint sets

- The following are true about the union of two sets, A and B
  - ❑ **A ⊂ (A ∪ B)**
  - ❑ **B ⊂ (A ∪ B)**
  - ❑ **(A ∩ B) ⊂ (A ∪ B)**

- **(A ∪ B)'** means the complement of the union of sets are all the elements that are not elements of A and B.



EXAMPLE 3.2

It is given that $\xi = \{x: 20 \le x \le 60, x \text{ are integers}\}$; M= {multiples of 10} and N= {multiples of 15}

(a) List the elements of (i) M (ii) N (iii) M ∪ N (iv) M ∩ N

(b) Find the value of (i) n(M) (ii) n(N) (iii) $n(\xi)$ (iv) $n(M \cup N)'$

(c) Draw a Venn diagram that illustrates the number of elements in each region.

SOLUTION

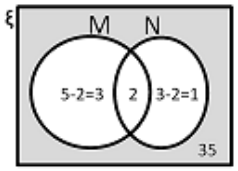
(a) (i) M= {20,30,40,50,60} (ii) N= {30,45,60}

(iii) M ∪ N= {20,30,40,45,50,60} (iv) M ∩ N= {30,60}

(b) (i) n(M)=5 (ii) n(N)=3 (iii) $n(\xi)$=60-20+1=41

(iv) $n(M \cup N)' = n(\xi) - n(M \cup N) = 41 - 6 = 35$

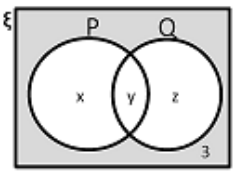
(c) Venn diagram



EXAMPLE 3.3

The Venn diagram shows the universal set **ξ**, set **P** and set **Q**. It is given that $n(\xi)=20$; $n(P)=12$ and $n(Q)=10$. Find the followings:

(i) $n(P \cup Q)$ (ii) $n(P' \cup Q)$ (iii) $n[P \cup (P \cup Q)']$



EXAMPLE 2.2

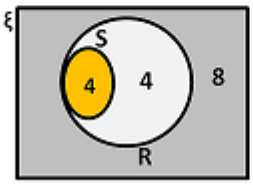
It is given that
$\xi = \{x : 15 \le x \le 30, x \text{ is an integer}\}$,
$R = \{x : x \text{ are odd numbers}\}$ and
$S = \{x : x \text{ are prime numbers}\}$

a. Find the value of (i) n(R) (ii) n(R')
b. List the elements of **S**
c. Find n(**S'**)
d. State the relationship between **R, S** and **ξ**
e. Draw a Venn diagram illustrating the number of elements in **R,S** and **ξ**

SOLUTION

$\xi = \{15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30\}$,
$R = \{15,17,19,21,23,25,27,29\}$ $S = \{17,19,23,29\}$

a. (i) n(**R**) = 8 (ii) n(**R'**) =8
b. S = {17,19,23,29}
c. n(**S'**)=n(**ξ**) – n(**S**) = 16-4=12
d. S ⊂ R ⊂ ξ
e.



NOTE:

When x is an integer :

- **a ≤ x ≤ b;** number of elements = **b – a + 1**
- **a ≤ x < b;** number of elements = **b – a**
- **a < x < b;** number of elements = **b – a – 1**

3. Intersection of Sets, Union of Sets and combined Operations on Sets

- ∩ denotes an intersection of two or more sets where its elements are common between one set to another. When **A ⊂ B,** then **A ∩ B = A.** When the two or more sets are disjoint, then **A ∩ B = Ø**

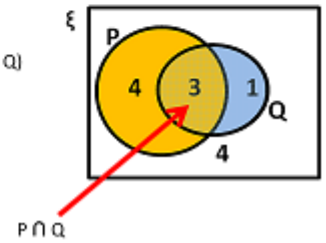
EXAMPLE 3.1

Given that P= {A,B,G,Z,U,L,N} and Q= {B,U,L,K}
(a) List the elements of P ∩ Q
(b) State the value of n(P ∩ Q)
(c) If the universal set, **ξ** = {A,B,G,Z,U,L,K,R,I,N,T,E}, list the elements of (P ∩ Q)'
(d) Draw a Venn diagram to represent the number of elements in sets P,Q and **ξ.** Then shade the area that represents (P ∩ Q)

SOLUTION

(a) P ∩ Q is also an intersection of {A,**B**,G,Z,**U**,**L**} and {**B**,**U**,**L**,K}; as such P ∩ Q = {B,U,L}
(b) n(P ∩ Q) = 3
(c) (P ∩ Q)' = {A,G,Z,K,R,I,N,T,E}
(d)

n(**ξ**)=12
n((P ∩ Q)' = n(**ξ**) - n(P ∩ Q)
= 12-3 = 9



1. Elements, Number of Elements, Empty Sets & Equal Sets

- ∈ means '**is an element of**' or '**belongs to**'
- ∉ means '**not an element of**' or '**doesn't belong to**'
- n(**A**) denotes **the number of elements in set A**
- **{ }** or **Ø** denotes **an empty set** or **null set**
- = denotes an equality between 2 sets where **both sets have the same elements in them**

EXAMPLE 1.1

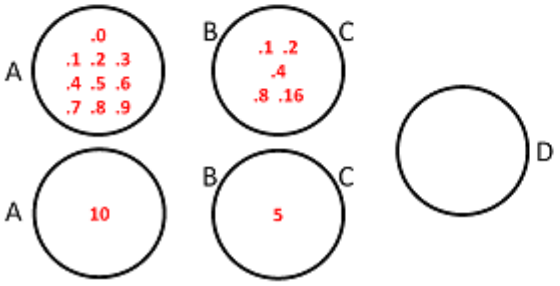
Given that **A** = { 0,1,2,3,4,5,6,7,8,9 } **B** = { 1,2,4,8,16 }
C = { $2^0, 2^1, 2^2, 2^3, 2^4$ }
D = { prime numbers between 7 and 11 }

5 ∈ A 5 ∉ B 8 ∈ C 9 ∉ C

n(A)=10 n(B)=5 n(C)=5 n(D) = 0

Since there is no element in D, D = { } or Ø

B = C is a correct statement because B and C has the equal elements
B = A is a false statement because not every element in A is in B



2. Subsets, Universal Sets and Complement of a Set

- ⊂ denotes **subsets** that indicates **every elements of one set is also an element of another set**
- **ξ** denotes **universal set** that indicates **the set contains all the elements under consideration in a given discussion**
- ' denotes the **complement of a set** that indicates **a non elements of certain set but still belongs to the universal set**

EXAMPLE 2.1

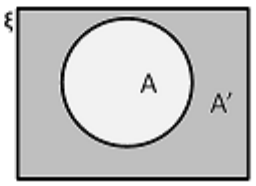
Given that **ξ** = { 0,1,2,3,4,5,6,7,8,9}
A = { 1,3,5 } and **B** = { 0,1,2,3,4,5 }

From the given information above, we can conclude the followings:

Both **A** and **B** ⊂ **ξ** **A ⊂ B**
(take note that the smaller set is always on the left of the bigger set and an empty set is part of any set)

n(A)=3 n(B)=6 number of subsets=2^n
The number of subsets A =2^3 = 8
= { },{1},{3},{5},{1,3},{1,5}, {3,5}, {1,3,5},

Since **A** = { 1,3,5 }, therefore **A'** = { 0,2,4,6,7,8,9 }
Since **B** = { 0,1,2,3,4,5 } , therefore **B'** = { 6,7,8,9 }



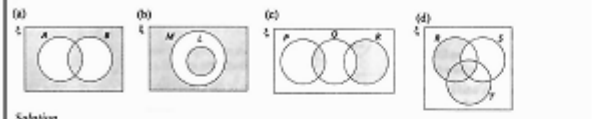
n(**A'**) = 7 and n(**B'**) = 4

alternatively, we can also apply the following steps:
n(**A'**) = n(**ξ**) – n(**A**) = 10-3=7
n(**B'**) = n(**ξ**) – n(**B**) = 10-6=4

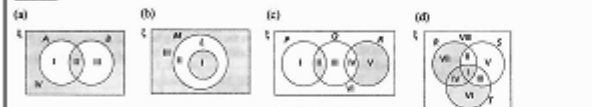
n(**ξ**) = n(**A**) + n(**A'**)

EXAMPLE 3.5

Use set notation to represent each of the following shaded regions.



Solution



Region IV: $(A \cup B)'$
Region II: $A \cap B$
$\therefore (A \cup B)' \cup (A \cap B)$

Region I: L
Region III: M'
$\therefore L \cup M'$

Region II: $P \cap Q$
Region V: $Q' \cap R$
$\therefore (P \cap Q) \cup (Q' \cap R)$

Region I: $R \cap S \cap T$
Regions IV, VI, VII: $S' \cap (R \cup T)$
$\therefore (R \cap S \cap T) \cup [S' \cap (R \cup T)]$

EXAMPLE 3.6

The Venn diagram below show sets L, M and N such that the universal set ξ = L∪M∪N. On the diagrams, shade:

(a) The set M∩N (b) the set (M'∩N)∪L

(b) L: regions I, II
M': region IV
N: regions II, III, IV
$(M' \cap N)$: region IV
$(M' \cap N) \cup L$: regions I, II, IV

Solution

(a) M: regions I, II, III, V
N: regions II, III, IV
$M \cap N$: regions II, III

$(M' \cap N) \cup L$

EXAMPLE 3.7

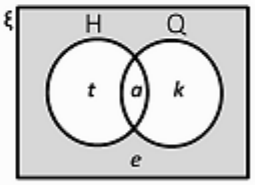
A certain school has 320 students who stay in the school hostel. 400 students, including some who stay in the hostel, participate in a written quiz. The symbols below illustrate the various groups of students in the school:
ξ = {all students in school}; **H**= {students who stay in school hostel} ; **Q**= {students who participate in the quiz}

(a) Draw a Venn diagram to illustrate the relationship between ξ, **H** and **Q**.

(b) It is given that the number of students who stay in the hostel but do not participate in the quiz is t. This number represents 1/5 of the students who participate in the quiz but do not stay in hostel. Calculate the value of t.

SOLUTION

(a) n(H)=t+a n(Q)=a+k n(H∩Q)=a n(ξ)=t+a+k+e



(b) t+a=320.....(1)
a+k=400.....(2)
t=(1/5) k.....(3)

sub. (3) into (1):
(1/5)k+a=320.....(1.1)

subtract (1.1) from (2):
a+k-(1/5)k-a = 400-320
(4/5)k=80 and k=80 x 5/4 =**100**

sub. k=100 into (3) in order to find t:
t=(1/5) x 100 = **20**