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Applications of tensegrity structures in civil engineering

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Abstract

The objective of the present paper is to describe the applications of tensegrity structures in civil engineering (roofs, domes, stadiums, etc.). The term of tensegrity was introduced by Fuller in the middle 50th of XX century. There are several definitions of this concept. For the purpose of this paper the tensegrity is defined a pin-joined system with a particular configuration of cables and struts that form a statically indeterminate structure in a stable equilibrium. Infinitesimal mechanism should exist in a tensegrity with equivalent self-stress state. Major advantages of tensegrity are: large stiffness-to-mass ratio, deployability, reliability and controllability.

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Keywords: Tensegrity; Pin-joned structures; Domes

1. Introduction

The word tensegrity, is a contraction of tensile integrity. This term has been proposed to name the structural rules, involving the creation of complex systems elements which are only compression or tension. It was coined by R. B. Fuller in his patent from 1962 [1]. The meaning of the word is vague and different interpretations are possible. Fuller [1] describes a tensegrity structure as “an assemblage of tension and compression components arranged in a discontinuous compression system…”. Referring to the work by Fuller, Pugh [9] defines a tensegrity system as: “A tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space”. Hanaor [4] describes tensegrity structures as

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"internally prestressed, free-standing pin-jointed networks, in which the cables or tendons are tensioned against a system of bars or struts". A broader interpretation by Miura and Pellegrino [7] is "that a tensegrity structure is any structure realized from cables and struts, to which a state of prestress is imposed that imparts tension to all cables." A narrower interpretation, also by Miura and Pellegrino, adds to the above definition the notion that "as well as imparting tension to all cables, the state of prestress serves the purpose of stabilising the structure, thus providing first-order stiffness to its infinitesimal mechanisms." Nowadays tensegrity systems are defined as: "systems in stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components" [8].

A large amount of literature on the geometry, art form, and architectural appeal of tensegrity structures exists, but there is little on the dynamics and mechanics of these structures. Form finding results for simple symmetric structures appear and show an array of stable tensegrity units is connected to yield a large stable system, which can be deployable. Several reasons are given why tensegrity structures should receive new attention from mathematicians and engineers: tension stabilizes; tensegrity structures are efficient, deployable, easily tunable.

2. Examples of tensegrity like structures in civil engineering

The tensegrity concept has found applications within architecture and civil engineering, such as towers, large dome structures, stadium roofs, temporarily structures and tents.

Towers which composed of interconnected tensegrity modules are the best known tensegrity structures. The two towers, designed by Kenneth Snelson: Needle Tower and the Needle Tower II are an example.

The well-known Munich Olympic Stadium of Frei Otto for the 1972 Summer Olympics, and the Millennium Dome of Richard Rogers for celebrating the beginning of the third millennium are both tensile structures, close to the tensegrity concept. The Seoul Olympic Gymnastics Hall, for the 1988 Summer Olympics, and the Georgia Dome, for the 1996 Summer Olympics, are examples of tensegrity concepts in large structures. A pair of tensegrity skeletons, supporting a membrane roof, has been constructed at Chiba,Japan in 2001.

An important example of Tensegrity being employed in roof structures is the stadium at La Plata (Argentina), based on a prize winning concept developed by architect Roberto Ferreira. The design adapts the patented Tenstar Tensegrity roof concept to the twin peak contour and the plan configuration, and consequently, it is more similar to a cable-dome structure than to a conventional roof structure. The first studies for the design of tensegrity grids were carried out by Snelson, but its applications were limited. For the past few years, the main focus has been in the development of double-layer Tensegrity grids and foldable Tensegrity systems. This kind of grid has its most feasible possibilities in the field of walls, roofs and covering structures [3].

3. Description of analyzed structures

Warnow Tower (Fig.1), measuring 49.2 meters tall and, with the addition of a 12.5 meter "needle", totalling 62.3 meters in height and 5 meters in diameter, was the tallest tensegrity tower ever built. The structure consists of six modules Simplex 8.3 meters in height each The tower was engineered by MERO Structures, Incorporated and erected at the 2003 Gardening Fair in Rostock, Germany. A prism was composed of three steel-tube compression members, three heavy-duty diagonal cables and three thin horizontal cables. Each stacked prism in turn was rotated by 30 degrees. To enable the tower to achieve an even greater height, the architects added a stainless steel needle, hung by ropes from the top prism, adding an additional 12.50 m to the tower. The tower was founded on a concrete base and foundation piles with a diameter of 8 meters. The base also featured floor lamps that illuminated the structure at night.

White Rhino (Fig.2), a building covered with membrane roofs supported by two tensegrity skeletons, has been constructed at Chiba in Japan in June, 2001. The building is constructed in the University of Tokyo's experimental centre and houses different laboratories of the university. The name, White Rhino, comes from the exterior appearance of the roofs, the white colour and two "horns", where the membrane roofs are pushed up from inside by two isolated posts supported by tensegrity skeletons. These isolated struts absorb large deformation of membrane roof and transmits the force from membrane roof to the tensegrity frame. One of the two tensegrity frames is about ten meters high while the other is seven meters high [5].

Fig. 1. Warnow Tower, a front view.

Fig. 2. White Rhino, a view inside.

4. Evaluation

The concept of tensegrity concerns specific trusses which consist of compression and tensile components which stabilize each other despite the fact that there are mechanisms in the structures. Tensegrity systems are characterized by the number of infinitesimal mechanisms and self-stress states. According to a qualitative analysis of trusses we can identify whether the structure is infinitesimally geometrically variable and whether there are self-stress states.

This study focuses on tensegrity structures. The analysis was made using the decomposition matrix method which describes the elongations in a truss according to singular values (Singular Value Decomposition). This method allows checking if the structure is tensegrity type. In the SVD decomposition a given matrix is presented in the form of the product of the unitary square matrix, the rectangular diagonal matrix with non-negative real coefficients and the Hermitian conjugation of unitary square matrix. Coefficients of the diagonal matrix are called singular values of the analyzed matrix. When the given matrix has real coefficients the unitary matrices become orthogonal matrices and the Hermitian conjugation becomes a transposition [6].

The subject of the analysis is N-membered, supported truss with following characteristics: material constants E_e, cross-sectional areas A_e and bar lengths L_e. Its mechanical properties are described by three linearized equations: compatibility, material properties and equilibrium with boundary conditions included

$$\mathbf{\Delta} = \mathbf{Bq}\,,\ \ \mathbf{S} = \mathbf{E\Delta}\,,\ \ \mathbf{B}^T\mathbf{S} = \mathbf{P} \tag{1}$$

where $\mathbf{q}$ is displacement vector of length M, $\mathbf{\Delta}$ is extension vector, $\mathbf{S}$ is normal force vector, $\mathbf{E}$ is elasticity matrix, $\mathbf{P}$ is load vector and $\mathbf{B}$ is compatibility matrix which can be determined directly or using the formalism of the finite element method [2]. The singular value decomposition of an $N\times M$ real matrix $\mathbf{B}$ is a factorization of the form:

$$\mathbf{B} = \mathbf{YNX}^T \tag{2}$$

where $\mathbf{Y}$ is an $N\times N$ real orthogonal matrix, $\mathbf{X}$ is an $M\times M$ real orthogonal matrix and $\mathbf{N}$ is an $N\times M$ rectangular diagonal matrix. Let us consider two eigen problems

$$\left(\mathbf{BB}^T - \mu\,\mathbf{I}\right)\mathbf{y} = 0 \ \text{ and } \left(\mathbf{B}^T\mathbf{B} - \lambda\,\mathbf{I}\right)\mathbf{x} = 0 \tag{3}$$

with the solutions in the form of eigenvalues and eigenvectors (normalized)

$$\mu_1, \mathbf{y}_1;\ \mu_2, \mathbf{y}_2;\ \ldots;\ \mu_N, \mathbf{y}_N \ \text{ and } \lambda_1, \mathbf{x}_1;\ \lambda_2, \mathbf{x}_2;\ \ldots;\ \lambda_N, \mathbf{x}_N \tag{4}$$

Full solutions of the above eigen problems can be expressesd in the condensed forms

$$\mathbf{BB}^T = \mathbf{YMY}^T \text{ and } \mathbf{B}^T\mathbf{B} = \mathbf{XLX}^T \tag{5}$$

$$\mathbf{M} = diag\{\mu_1 \quad \mu_1 \quad \dots \quad \mu_N\},\ \mathbf{L} = diag\{\lambda_1 \quad \lambda_1 \quad \dots \quad \lambda_M\},\ \mathbf{Y} = [\mathbf{y}_1\ \mathbf{y}_2\ \dots\ \mathbf{y}_N],\ \mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2\ \dots \mathbf{x}_M] \tag{6}$$

One can notice that the product $\mathbf{BB}^\mathbf{T}$ can be considered as a matrix of symmetrised equations of equilibrium with non-negative eigenvalues. Zero eigenvalues (if any) are related to the non-zero solution of homogeneaous equations (**P**=**0**) named self-stress. The self-stress can be considered as an eigenvector related to zero eigenvalue).

In a similar way the product $\mathbf{B}^\mathbf{T}\mathbf{B}$ can be considered as a particular form of linear stiffness matrix with unit elasticity matrix. The eigenvalues are non-negative. Zero eigenvalues (if any) are related to the finite or infinitesimal mechanisms, but in general the information from the null-space analysis alone does not suffice to establish the difference. The mechanism can be considered as an eigenvector related to zero eigenvalue). To establish if the mechanism is infinitesimal it is necessary to apply the nonlinear analysis with the use of geometric stiffness matrix, which is possible if the self-stress exist. Lack of self- stress means that the mechanism is finite.

Based on the above two eigen problems it is easy to proof the singular value decomposition of the matrix **B**

$$\mathbf{BB}^T = \mathbf{YNX}^T\mathbf{XN}^T\mathbf{Y}^T = \mathbf{YNN}^T\mathbf{Y}^T = \mathbf{YMY}^T \text{ and } \mathbf{B}^T\mathbf{B} = \mathbf{XN}^T\mathbf{Y}^T\mathbf{YNX}^T = \mathbf{XN}^T\mathbf{NX}^T = \mathbf{XLX}^T \tag{7}$$

with the following relations included

$$\mathbf{M} = \mathbf{NN}^T \text{ and } \mathbf{L} = \mathbf{N}^T\mathbf{N} \tag{8}$$

Based on the singular value decomposition of the compatibility matrix **B** analysis of three analytical models were made. The calculations were carried out in the Mathematica environment. The computational programs to analyse three-dimensional and two-dimensional truss were written based on finite element analysis.

4.1. White Rhino

White Rhino building was analyzed as the truss (Fig. 3) which consists of tensegrity truss Simplex (elements 1-12) with additional tension elements (elements: 13, 14, 15). If we consider only model Simplex (12-element structure) **B** is the 12x12 real matrix and the matrices eigenvalues of both $\mathbf{BB}^\mathbf{T}$ and $\mathbf{B}^\mathbf{T}\mathbf{B}$ matrices are

$$\mathbf{M} = \mathbf{L} = diag\{3.69 \quad 2.51 \quad 2.16 \quad 1.98 \quad 1.71 \quad 1.33 \quad 0.99 \quad 0.77 \quad 0.24 \quad 0.16 \quad 0.08 \quad 0\} \tag{9}$$

The zero eigenvalue presented in matrix (9) is responsible for the existence of the self-stress state defined by the eigenvector of $\mathbf{BB}^\mathbf{T}$ corresponding to this value and for the existence of the mechanism defined by the eigenvector of $\mathbf{B}^\mathrm{T}\mathbf{B}$ corresponding to this value too:

$$\mu_{12} = 0 \Rightarrow \mathbf{y}_{12} = \{0.12 \quad 0.12 \quad 0.12 \quad 0.21 \quad 0.21 \quad 0.21 \quad -0.43 \quad -0.43 \quad -0.43 \quad 0.3 \quad 0.3 \quad 0.3\} \tag{10}$$
$$\lambda_{12} = 0 \Rightarrow \mathbf{x}_{12} = \{0 \quad 0 \quad 0 \quad 0.47 \quad 0.27 \quad -0.21 \quad -0.47 \quad 0.27 \quad -0.21 \quad 0 \quad -0.54 \quad -0.21\} \tag{11}$$

Three additional elements caused that the structure had lost features which are typical for tensegrity. Analyzing the 15-elements structure we get:

$$\mathbf{M} = diag\{3.96 \quad 2.59 \quad 2.48 \quad 2.29 \quad 1.76 \quad 1.68 \quad 1.57 \quad 0.97 \quad 0.65 \quad 0.44 \quad 0.34 \quad 0.1 \quad 0 \quad 0 \quad 0\} \tag{12}$$
$$\mathbf{L} = diag\{3.96 \quad 2.59 \quad 2.48 \quad 2.29 \quad 1.76 \quad 1.68 \quad 1.57 \quad 0.97 \quad 0.65 \quad 0.44 \quad 0.34 \quad 0.1\} \tag{13}$$

which means that there is no mechanism identified but there are three self-stresses:

$$\mu_{13} = 0 \Rightarrow \mathbf{y}_{13} = \{0.2 \quad 0.1 \quad -0.2 \quad 0.4 \quad -0.2 \quad -0.1 \quad -0.2 \quad -0.4 \quad 0.4 \quad -0.2 \quad 0.3 \quad -0.04 \quad 0.4 \quad -0.1 \quad -0.3\} \quad (14)$$

$$\mu_{14} = 0 \Rightarrow \mathbf{y}_{14} = \{-0.2 \quad 0.1 \quad -0.1 \quad -0.05 \quad 0.02 \quad -0.4 \quad 0.6 \quad 0.05 \quad 0.3 \quad -0.4 \quad -0.2 \quad -0.01 \quad -0.04 \quad 0.3 \quad -0.2\} \quad (15)$$

$$\mu_{15} = 0 \Rightarrow \mathbf{y}_{15} = \{0.04 \quad -0.2 \quad -0.1 \quad -0.2 \quad -0.4 \quad 0.1 \quad -0.03 \quad 0.5 \quad 0.4 \quad -0.1 \quad -0.1 \quad -0.4 \quad 0.1 \quad -0.3 \quad 0.2\} \quad (16)$$

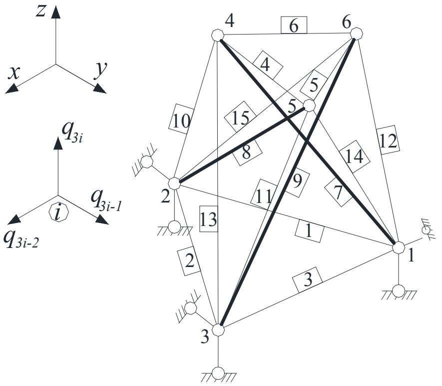

Fig. 3. Scheme of White Rhino

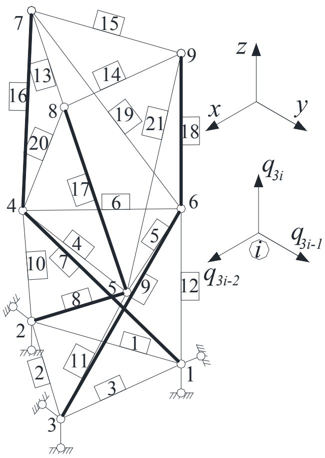

Fig. 4. Scheme of Warnow Tower

4.2. Warnow Tower

Warnow Tower is constructed of six tensegrity trusses Simplex. In this paper analysis for two models will be carried out in. The first model consists of one truss Simplex and second – two trusses Simplex (Fig. 4). For first case **B** is the 12x12 real matrix and the matrices eigenvalues of both $\mathbf{BB}^\mathbf{T}$ and $\mathbf{B}^\mathbf{T}\mathbf{B}$ matrices are

$$\mathbf{M} = \mathbf{L} = diag\{3.6 \quad 2.41 \quad 2.23 \quad 1.77 \quad 1.44 \quad 1.31 \quad 1.18 \quad 0.93 \quad 0.18 \quad 0.08 \quad 0.05 \quad 0\} \quad (17)$$

The zero eigenvalue presented in matrix (17) is responsible for the existence of the self-stress state defined by the eigenvector of $\mathbf{BB}^\mathbf{T}$ corresponding to this value:

$$\mathbf{y}_{12} = \{0.14 \quad 0.14 \quad 0.14 \quad 0.14 \quad 0.14 \quad 0.14 \quad -0.42 \quad -0.42 \quad -0.42 \quad 0.34 \quad 0.34 \quad 0.34\} \quad (18)$$

and for the existence of the mechanism defined by the eigenvector of $\mathbf{B}^\mathrm{T}\mathbf{B}$ corresponding to this value too:

$$\mathbf{x}_{12} = \{0 \quad 0 \quad 0 \quad 0.49 \quad 0.28 \quad -0.12 \quad -0.49 \quad 0.28 \quad -0.12 \quad 0 \quad -0.57 \quad -0.12\} \quad (19)$$

For the second case **B** is the 21x21 real matrix and the matrices **M** and **L** contains two zero eigenvalues. It means that two mechanism of geometrical variation

$$\mathbf{x}_{20} = \{0 \quad 0 \quad 0 \quad -0.3 \quad -0.2 \quad 0.1 \quad 0.3 \quad -0.2 \quad 0.1 \quad 0 \quad 0.4 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.1 \quad -0.4 \quad 0 \quad 0.1 \quad 0.2 \quad -0.3 \quad 0.1\} \quad (20)$$

$$\mathbf{x}_{21} = \{0 \quad 0 \quad 0 \quad 0.3 \quad 0.2 \quad -0.1 \quad -0.3 \quad 0.2 \quad -0.1 \quad 0 \quad -0.3 \quad -0.1 \quad 0.2 \quad 0.3 \quad -0.2 \quad -0.4 \quad 0 \quad -0.2 \quad 0.2 \quad -0.3 \quad -0.2\} \quad (21)$$

and two self-stress states

$$\mathbf{y}_{20} = \{-0.1 \quad -0.1 \quad -0.1 \quad -0.2 \quad -0.2 \quad -0.2 \quad 0.4 \quad 0.4 \quad 0.4 \quad -0.3 \quad -0.3 \quad -0.3 \quad -0.02 \quad -0.02 \quad -0.02 \quad 0.1 \quad 0.1 \quad 0.1 \quad -0.1 \quad -0.1 \quad -0.1\} \quad (22)$$
$$\mathbf{y}_{21} = \{0.03 \quad 0.03 \quad 0.03 \quad -0.1 \quad -0.1 \quad -0.1 \quad -0.1 \quad -0.1 \quad -0.1 \quad 0.1 \quad 0.1 \quad 0.1 \quad -0.1 \quad -0.1 \quad -0.1 \quad 0.4 \quad 0.4 \quad 0.4 \quad -0.3 \quad -0.3 \quad -0.3\} \quad (23)$$

balancing these mechanisms of truss were identified. According to this results Warnow Tower can be classified as tensegrity structure.

4.3. Two-dimensional truss

Many domes classified as tensegrity consists of interconnected two-dimensional truss showed in Fig. 5. For this case **B** is the 8x8 real matrix and the eigenvalues of both $\mathbf{BB^T}$ and $\mathbf{B^TB}$ matrices are:

$$\mathbf{M} = \mathbf{L} = diag\{1.73 \quad 1.62 \quad 1.62 \quad 1.41 \quad 1.0 \quad 0.62 \quad 0.62 \quad 0\} \quad (24)$$

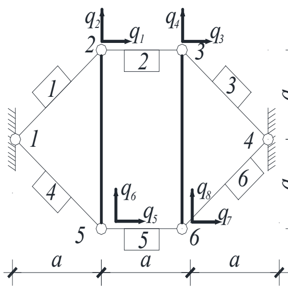

Fig. 5. The scheme of truss

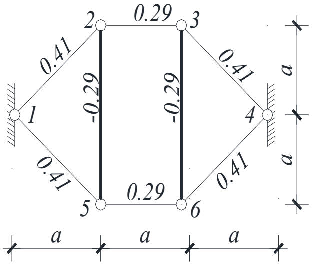

Fig. 6. The self-stress state of truss

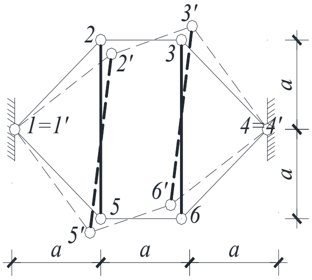

Fig. 7. The mechanism of truss

The zero eigenvalue presented in matrix (24) is responsible for the existence of the self-stress state (Fig. 6) defined by the eigenvector of $\mathbf{BB^T}$ corresponding to this value and for the existence of the mechanism (Fig. 7) defined by the eigenvector of $\mathbf{B^TB}$ corresponding to this value too:

$$\mu_8 = 0 \Rightarrow \mathbf{y}_8 = \{0.41 \quad 0.29 \quad 0.41 \quad 0.41 \quad 0.29 \quad 0.41 \quad -0.29 \quad -0.29\} \quad (25)$$
$$\lambda_8 = 0 \Rightarrow \mathbf{x}_8 = \{0.35 \quad -0.35 \quad 0.35 \quad 0.35 \quad -0.35 \quad -0.35 \quad -0.35 \quad 0.35\} \quad (26)$$

5. Conclusions

The tensegrity concept has found wide applications within architecture and civil engineering, such as towers, large dome structures, stadium roofs, temporarily structures and tents. There are many advantages of this kind of structures. The definition of tensegrity structures has evolved in last 50 years what is the reason why there are some structures which are called "tensegrity", but they don't meet requirements. In this paper a few models of structures were analyzed and checked if they are really tensegrity type.

According to a qualitative analysis of trusses we concluded that White Rhino structure should not be named tensegrity, because of three additional elements caused that the structure had lost features which are typical for tensegrity.

Based on the analysis of models consist of one trusses Simplex and two trusses Simplex, it can be assumed that Warnow Tower which consist of six tensegrity trusses Simplex has got six mechanism of geometrical variation and six self-stress states balancing these mechanisms of truss. Warnow Tower can be classified as tensegrity structure.

Moreover domes which consist of interconnected two-dimensional truss showed in Fig. 5 have got feature which are characteristic for tensegrity.

References

[1] R. B. Fuller, Tensile-integrity structures. United States Patent 3,063,521, 1962. Filed 31 August 1959, Granted 13 November 1962.
[2] W. Gilewski, A. Kasprzak, Introduction to tensegrity modules, in: Theoretical Foundations of Civil Engineering, Vol. I. Mechanics of Materials and Structures. S.Jemioło, Sz.Lutomirski, eds., Warszawa, pp. 83-94, 2012. (in Polish)
[3] V. Gómez-Jáuregui, Tensegrity Structures and their Application to Architecture. 2010: Servicio de Publicaciones de la Universidad de Cantabria.
[4] A. Hanaor, Geometrically rigid double-layer tensegrity grids. International Journal of Space Structures 9, pp. 227–238, 1994.
[5] K. Kawaguchi, S. Ohya, S. Vormus, Long-Term Monitoring of White Rhino, Building with Tensegrity Skeletons, in 5th Annual Symposium of IABSE / 52nd Annual Symposium of IASS / 6th International Conference on Space Structures. 2011: London, September 2011.
[6] V.C. Klema, The singular value decomposition: it's computation and some applications. IEEE Transactions on Automatic Control AC-25, 2, 1980, pp. 164-176.
[7] K.Miura, S. Pellegrino, Structural concepts. 1999. Draft.
[8] R. Motro, Tensegrity. Structural Systems for the Future, Kogan Page, London-Sterling, 2003.
[9] A. Pugh, An introduction to tensegrity. University of California Press, Berkeley, CA, USA, 1976.
[10] M.Schlaich, The Messeturm in Rostock– A Tensegrity Tower. Journal of the International Association for Shell and Spatial Structures, Vol. 45 (No. 2), 2004.