Rotman

GROUP Assignment Cover Sheet

Course Code:	RSM 8423	**Student Numbers**: *Please list all student numbers included in this group assignment.*	1008680312 1007396649 1008247149 1002996369 1003099862 1008654591
Course Title:	Optimizing Supply Chain Management and Logistics		
Instructor Name:	Andre Cire		
Assignment Title:	Case Study: Hot Delivery		
Date:	23rd April 2022		

Academic Integrity Compliance

In submitting this **group** work, we affirm:

- The student numbers listed above are correct and complete.
- The work is original. Due credit is given to others where appropriate and we have acknowledged the ideas, research, phrases etc. of others with accurate and proper citations.
- All members have contributed substantially and proportionally to this assignment.
- All members have sufficient familiarity with the entire contents to be able to sign off on this work as original.
- This is the final version of the assignment and not a draft.
- We have followed any specific formatting requirements set by the instructor.
- We are submitting this work for the correct course, via the specified platform/method (e.g., Quercus).
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We agree that the statements above are true. If we have concerns, we will consult the course instructor immediately.

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Complete Course Code (including Section) – GROUP – **Assignment Title**

Example: **RSM1234HF.2021-0108**-Group22-**Homework1**

Executive Summary

Uber Eats has had an increasing growth in the last few years and has surpassed $8 billion in revenues in the US by the end of 2021. However, they are making losses every year mainly due to the large costs of paying drivers according to the total distance they travel for deliveries. This stems from the lack of strategizing and optimizing delivery routes. Therefore, this report aims to propose solutions for effective optimization strategies and routes using well-developed models such as the flow/mixed-integer programming model for Uber Eats to minimize the distance traveled by the drivers in deliveries.

The analysis approach is sequential and has been done in four parts or steps. The initial part starts from a base model with simple assumptions and additional restrictions and variables are added in the following parts to make the model more applicable in reality. Data sets used include regional and distance data which is needed to evaluate the distances between each of the locations and for visualizing the routes. Orders data is also used which includes features such as the customer, restaurant, and estimated order availability.

The models used include a flow mixed-integer programming model and a heuristic. The objective for each model is to minimize the total distance traveled by the driver(s).

Firstly, the optimization model without the addition of time gives simple and quick results for the optimal route for a single driver case. However, as the aspect of time is added, there is a trade-off between the total distance the driver travels and the customer waiting time. In general, as the maximum customer waiting time increases, the optimal minimum distance decreases. This is an important trade-off for Uber Eats where they must trade-off quality customer service levels with driver costs. By incorporating delivery costs such as higher delivery costs to customers that order from far away restaurants, high costs from long distances can be addressed.

As additional drivers are considered, not only does the optimal distance traveled by the drivers decrease but so do the customer waiting times. However, additional costs related to additional vehicles and labor hired are not part of the model, which must be considered otherwise.

Since this model is not scalable, a clustering heuristic was proposed. This helped to group together orders that were in similar localities and assign them to drivers respectively. This gave good results that are quite close to the results from solving the model. For comparison, the heuristic gave an optimal solution that had a distance only about 16% higher than the actual optimal minimum distance.

Lastly, there are some limitations in the solution that must be addressed. This includes the long time taken to run the model, due to the high number of constraints and variables added to the model that made it much more complex. Also, some assumptions such as no additional costs related to an increased number of drivers as well as knowing all orders beforehand must be addressed as the solution from this model is otherwise impractical. Drivers not earning enough money will look for other platforms and would bring bad reviews in the driving community. By incorporating labor and additional vehicle costs in the model as well as using the heuristic hand-in-hand with the model, the assumptions will be addressed and the solution will be more practical to implement.

Background

Uber Eats has had impressive growth and revenue in the last few years, as seen by their high market share of 29% in the US in 2021. However, they continue to lose money every year, without reporting any actual profits. This is due to various reasons such as high investments in ads and promotions, but most importantly due to their largest source of expense which is to pay drivers based on the distance they travel. Therefore, in order to reduce their delivery costs, this report aims to propose solutions for Uber Eats to optimize their delivery routes, which will help to assign orders to drivers based on those routes. These solutions will help Uber Eats to reduce and manage its costs more effectively.

Data

In order to conduct the analysis, regions and distances datasets will be used. The regions data include the name, province, code, latitude, and longitude of each region within Toronto. The distances data includes the origin, destination, and the distance between the origin and destination for regions within Toronto. It is important to note that the distance estimates are based on centroids of Toronto neighborhoods.

The analysis will be done part-wise in a total of four parts with an incremental approach. Part one focuses on a basic model and more real-world aspects are added gradually. Therefore, the data for each part also differs as more variables are incrementally added for testing the models. Besides the regions and distances data sets, there are separate data sets for each part.

These separate data sets for each part are discussed below:

- **Part 1:** There are two datasets for testing part 1 (part1_ordersA and part1_ordersB) which include two variables, that is, the customers and the respective restaurants they have ordered from. For each data set, the optimal route needs to be determined. Each row indicates a particular order of a customer and the restaurant they have ordered from. For part1_ordersA, there is only 1 order whereas for part1_ordersB, there are a total of 5 orders.
- **Part 2:** For part 2, the main element added is time, to ensure the deliveries are done within a certain window. Therefore, the data sets for part 2 contain not only the customer and restaurants data but additionally an estimated time for the availability of the food for pick-up for each order. Part2_ordersA contains 2 orders whereas part2_ordersB contains 4 orders in total.
- **Part 3:** For part 3, two additional datasets are taken. One for drivers named part3_drivers and part3_small which contains the order information. The order table also contains the time each order would be available for pickup like the part2_ordersB in problem 2. It has 5 entries. The driver table has information about 3 drivers with their starting points and their average velocities.
- **Part 4:** For part 4, we have 10 drivers' information with their starting points and average velocities in dataset part4_drivers. The part4_large dataset has information about 15 customer orders with restauraunt and customer location data as well as information about the time the order would be ready for pickup.

Part 1

Problem

Once an order arrives in Uber Eats, it is assigned to a driver who picks it up from the restaurant and delivers it to the customer. The objective of this part is to design a model that gives the optimal delivery route for a single driver, assuming some of their future orders have already been assigned to them.

There are some assumptions made in this case that must be satisfied. Firstly, the driver starts in the Downtown Toronto (Rosedale) neighborhood and can carry multiple orders in their vehicle. Next, after all the deliveries are done, the driver parks the car in the neighborhood of the last order delivered, waiting for future orders. Overall, the objective is to minimize the total distance traveled by the driver, which will help to save costs as drivers are paid according to the distance traveled.

Model

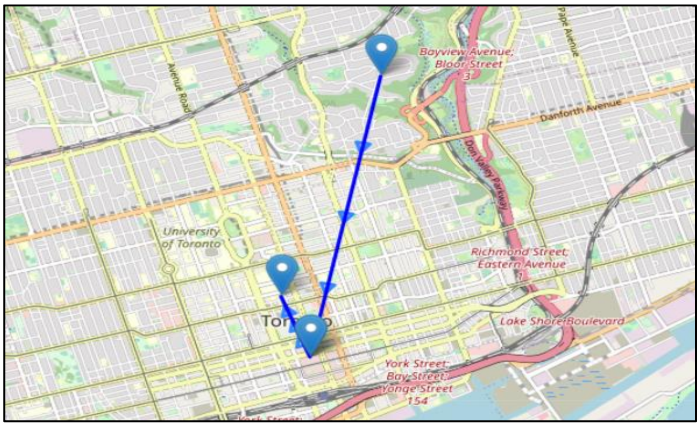
A flow mixed-integer linear programming model has been used for solving this optimization problem as the flow of the driver is considered here, visiting each location to pick up and deliver orders.

Firstly, lists were created indicating unique starting (locations-i) and ending (locations-j) locations. Additionally, a list of numbers indicating the various steps (steps-t) was also created. Unique restaurant and customers lists were also created as well as the respective restaurant and customer pairings for further use in the model.

The parameters of the model include the distances from the respective start and end locations in the problem. The only variable in the model is the ‘xvar’ which is a binary variable that equals 1 if the driver travels from location i to location j in step t, or else is zero. The model constraints include constraints to make sure each location is visited once and there is conservation of flow. Furthermore, the starting point should be Rosedale as indicated above for which additional constraints were used. Lastly, a constraint was used to ensure that a driver goes to the restaurant before delivering food to the respective customer. The model objective is to minimize the total distance traveled. The full mathematical formulation can be seen in the Appendix and the code can be viewed to see how the problem was solved using the code.

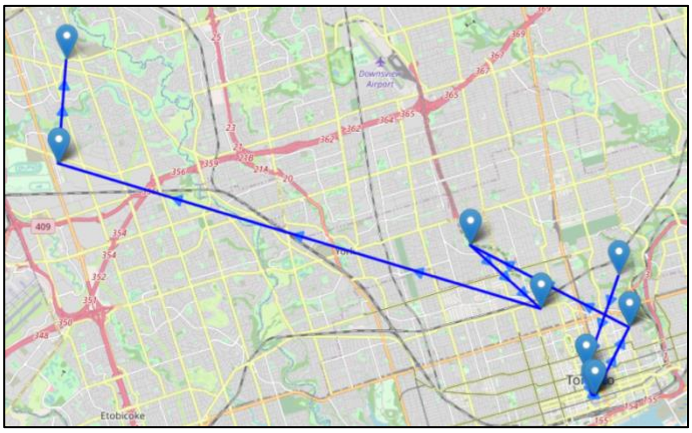
Results

The optimal delivery route for orders_part1A can be seen below with the full list of locations and steps given in the Appendix. The data for this consists of only one order with one customer and restaurant. All the constraints are satisfied and the optimized minimum distance traveled is 4.65 km.



Part1 OrdersA Route Map

Similarly, for orders_part1B, the optimal minimized distance is 33.88 km. The whole route consists of 8 steps and ends at the Etobicoke Northwest location at the top left on the map below. The optimal delivery route for orders_part1B can be seen below with the full list of each step and locations in the Appendix.



Part1 OrdersB Route Map

These solutions seem quite simple and too good to be true as they do not factor in the order timings which is an integral factor for deliveries and part of good customer service since food must be delivered on time or it is not fresh anymore. Therefore, for the next part, time will be added to the model.

Part 2

Problem

Given that time is an integral part of food delivery services for good customer service, Uber Eats must make sure that the food does not arrive later than a certain period of time, which is the maximum average waiting time, ‘W’. The total customer wait time here is defined as the time between when the food is ready for pick up and when it arrives at the customer’s neighborhood.

The velocity of the driver is assumed to be 40 km/h which will be used to determine the traveling times using the distances already given. The driver again starts from the Downtown Toronto (Rosedale) location and is assumed to spend 5 minutes on average at each location waiting for the customer to pick up their order.

By incorporating the time and also varying ‘W’, the objective is to incorporate time in the model and investigate how time impacts the solution.

Model

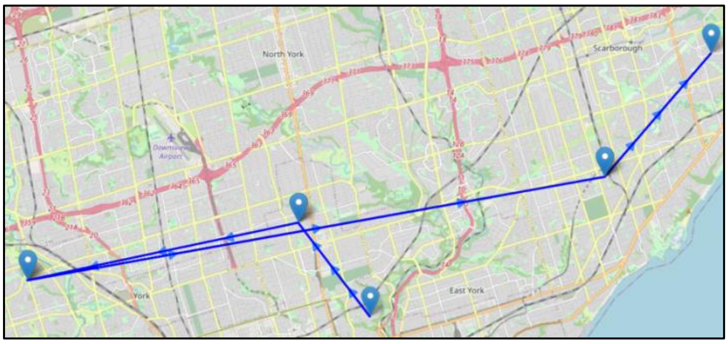
The model in this part is the same as in Part 1 with some additions made to incorporate time. Firstly, since the data now contain estimated food availability times, the model takes that into consideration. Two extra parameters are added, one for the minimal arrival time (in minutes) to each restaurant which makes sure that the orders are not picked up before the food is available. Next, a list of travel times is also generated which calculates the travel time between each location using the distances given, the 5-minute wait time, and the assumed velocity of 40 km/h. Three extra decision variables are generated; ‘dvar’, ‘wvar’ and ‘zvar’ which represent the total time of the route in minutes, the waiting time of each order for each step, and the cumulative waiting time for all the orders for each step respectively.

Additional constraints added include firstly a flow constraint to make sure that the total travel time up to each step is at least equal to the sum of the previous step’s time and the travel time for the current step. Another constraint ensures that the arrival time for a location is at least equal to the minimum arrival time. Next, to deal with waiting time, a constraint defines the waiting time for each order and the minimum arrival time which is the time when the food is ready to pick up. This is then used in the most important constraints which define the cumulative waiting times and ensure that the average waiting time of all the orders is lower than a maximum of W. The value of W is changed to see the impact it has on the optimal route and distance. This is important as food delivery times are of high priority for good customer service. The objective function, in this case, remains the same as in Part 1. For more details on the mathematical formulation and code for this model, please refer to the Appendix and Jupyter Notebook respectively.

Results

The optimal delivery route for orders_part2A can be seen below with the full list of locations and steps given in the Appendix. The data for this consists of only two orders. The route starts at Rosedale, as was assumed, and ends at Scarborough (Woburn) in a total of 4 steps.

The minimum distance occurs when W is greater than or equal to 70.5 minutes. The total travel time for this route comes to be about 2 hours and 33 minutes (153 minutes). The optimal distance came to be 42.77 km for this case.



Part2 OrdersA Route Map

The optimal delivery route for orders_part2B can be seen below with the full list of locations and steps given in the Appendix. The data for this consists of four orders. The route starts as Rosedale, as was assumed, and ends at Downtown Toronto Station A in a total of 8 steps. The minimum distance occurs when W is greater than or equal to 82 minutes. The total travel time for this route comes to be about 2 hours and 38 minutes (158 minutes). The optimal distance came to be 51.40 km.

For more details on the stepwise solution values and code for this model, please refer to the Appendix and Jupyter Notebook respectively.

Part2 OrdersB Route Map

Now, to assess the impact of changing the maximum average waiting time W, the following questions are assessed.

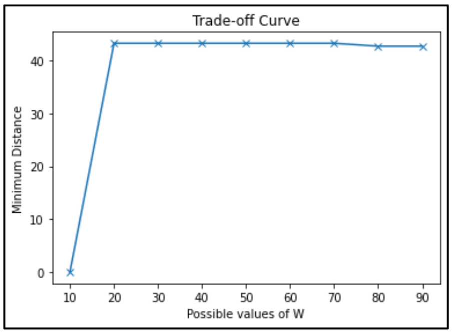
1. **How much more difficult does the problem become as W decreases or increases? Why?**

   As W increases, the problem becomes easier as there is a trade-off between traveling the shortest distances and trying to meet the maximum average waiting time for each customer in order to get a feasible solution. As W is lower, it is harder for the driver to meet some deliveries in time without sacrificing the optimal route so the distance may be higher in order to make the deliveries in time. There will be a certain W below which deliveries are not possible at all due to not enough time no matter what route is taken.

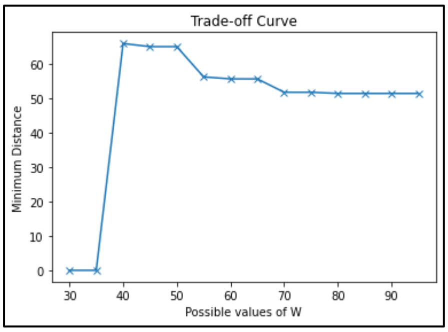
2. **Create a trade-off curve comparing W and distance for these instances. Why do you observe that particular shape? Furthermore, are they correlated? (Note: with this curve, you are trying to infer the sensitivity or elasticity of the total distance with respect to W).**

   It is clear from the graphs below of W, the maximum average waiting time, against the minimum distances, that as W increases from some threshold that yields feasible solutions, the minimum distance decreases then becomes constant. The points where the minimum distance is zero indicate values of W that gave infeasible solutions. The constraint of W is too strict that the driver is not able to deliver the food within the defined W. When W is above this feasible threshold, the minimum distance is larger with smaller values of W because of the trade-off between the distance and the customer waiting time. In this case, the costs might be higher for the company as the driver needs to forego the optimal path to make the deliveries to customers on time.

It is also shown that in general, for smaller values of W the sensitivity of the distance is higher, as slight changes in W can change the optimal path and minimum distance. However, as W is larger, it is more flexible and less sensitive as the driver has more time and can take on the optimal path as it approaches and takes a total time of less than W. In general, there is a negative correlation after the feasible threshold between W and the minimum distance which is clear and will be more prominent in routes with more orders. Therefore, for all feasible cases, the higher the maximum average waiting time, W, the lower the minimum distance and vice versa.



Trade-off Curve-Part2_Orders A



Trade-off Curve-Part2_Orders B

3. **What is the problem when considering the average waiting time? Propose and test an alternative metric to address the issue you identified, showing how the solution changed with your new measure.**

   The problem with considering the average waiting time is that it averages out the extreme cases so that different customers might get very different delivering experiences. This will lead to inconsistent service levels and possibly some very unhappy customers. Another metric that could be used that will prevent this issue is the maximum waiting time for each customer. This will ensure that each customer gets their order delivered within a certain maximum waiting time and quality customer service levels are met.

   This alternative metric was incorporated in place of W in the previous model and the results can be seen below.

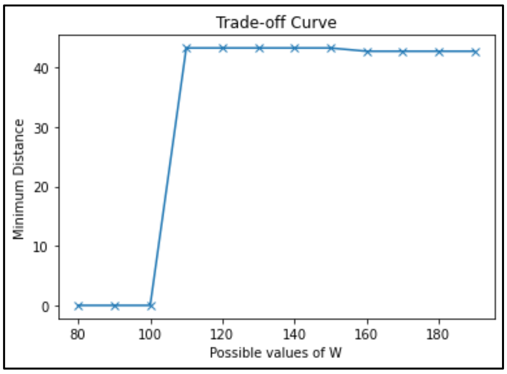
Model-2

By incorporating the maximum waiting time instead of the average for part 2, the constraints are simplified and run time will be improved with a more complicated order dataset. In this case, the optimal distance for orders A occurs when W is more than about 100 minutes. The optimal distance is 42.77 km which is the same as before and the route is also the same. The total time taken is 200 minutes which is much higher compared to before.

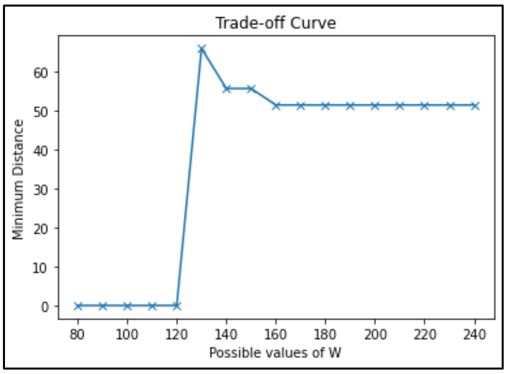
For part 2 orders B, when W is more than about 150 minutes, the minimum distance occurs. In this case, the optimal distance is 55.63 km which is slightly higher than before and the route is also different but still consists of 7 steps. The total time taken is 150 minutes which is slightly higher than before. For the complete route details, please refer to the Appendix.

The graphs below indicate how W changes with the optimal minimum distance. Here, the solution is infeasible for part 2 orders A when W is below 110 minutes and below 130 minutes for orders B. It seems like the previous graph has moved forward to the right horizontally. This implies that there is less flexibility in the model and it is quite difficult to make sure each order is delivered under a certain time period which leaves less room to make deliveries that are farther away. The optimal solutions here are possible at higher levels of W compared to previously which was an average measure. So in this case not even one delivery is allowed above W minutes. Therefore, the solution is infeasible if W is below 110 and 3 minutes respectively for part 2 orders A and part 2 orders B.

It is demonstrated that by making sure all the customers get their deliveries within a certain time period, the drivers' total travel time is higher but the distances are still similar or slightly higher. This makes it costly for delivery companies like Uber Eats as they may have to travel greater distances just so deliveries are made in time.



Trade-off Curve-Part2_Orders A



Trade-off Curve-Part2_Orders B

One solution is that if there are multiple drivers, then any driver who is close to a given restaurant and customer as well as free at a given time can deliver the respective orders even if one driver is busy. This will help to make the model and solutions more flexible and realistic. This aspect of adding multiple drivers to the model is addressed in part 3.

Part 3

Problem

There are multiple drivers and the idea is to assign orders to drivers based on the optimal routes. Each driver has their own velocities which are 40km/h, 35km/h, and 30km/h, respectively and have different starting locations. This means three routes will be in process simultaneously and the minimized total distance is a combination of these three routes, which is the most challenging part of the problem. To track the three routes, it is assumed that all three vehicles will start from Step 0, however, it doesn't mean all three vehicles depart at the same point in time. It represents the respective first step for each vehicle. The objective is the same that is to minimize the distance traversed by all drivers. The average waiting time has been calculated similarly to part 2.

Model

In this part, multiple drivers with different velocities and starting locations are considered, so the model includes a new parameter "v" in variables to represent multiple vehicles. The variables and constraints from the previous part have been modified to take "v" into account. There are three starting locations and three corresponding routes, and considering some of the starting regions are the same as the locations of restaurants and customers, it is not feasible to directly track the name of each location. Instead, in Part 3, all locations are given indices to differentiate starting regions, restaurants, and customers. For example, one location can have two indices "3" which represents "restaurant" and "8" which represents "customer".

Some additional constraints were also added which include firstly, that drivers may end their tour earlier than step T. This is because one driver is not taking all T orders in T steps and multiple drivers do not need T steps to deliver all T orders, they can finish delivering earlier before Step T. Secondly, each driver starts from their respective starting locations, and they are forced to start from Step 0. The third additional constraint is that each vehicle can only take one order at each step. The objective function remains the same as before. For more details on the mathematical formulation and code for this model, please refer to the Appendix and Jupyter Notebook respectively.

Results

By initialing just solving for a single value of W, that is 30, the optimal delivery route for part 3 can be seen below with the full list of locations and steps given in the Appendix. The data for this consists of five orders. Vehicle 0 starts from Downtown Toronto (Richmond) and ends at North York from step 0 to 5. Vehicles 1 and 2 only require 2 steps each and these can be seen below for each driver respectively. The total waiting time is about 98 minutes, and the average waiting time is about 19 minutes per customer. There are a total of 6 steps due to multiple drivers delivering simultaneously. The optimal minimum distance came to be 30.54 km. For more details on the stepwise solution values and code for this model, please refer to the Appendix and Jupyter Notebook respectively.

Part3 Route Map (Multiple Drivers)

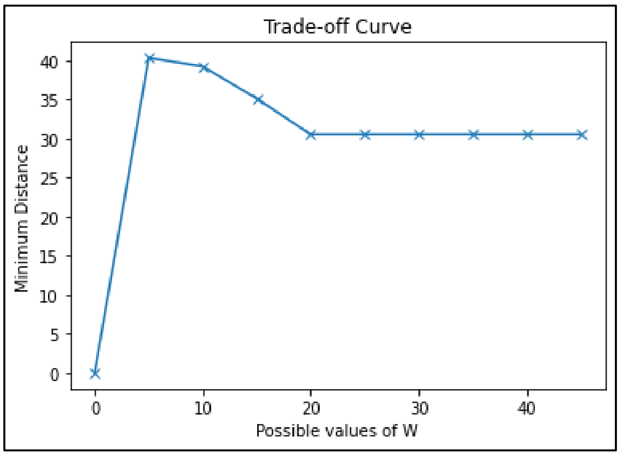
As before, to assess the impact of changing the maximum average waiting time W on the optimal solution, the following questions are addressed.

1. **How much more difficult does the problem become as W decreases or increases? Why?**

   As W increases, the problem becomes easier as there is a trade-off between traveling the shortest distances and trying to meet the maximum average waiting time for each customer in order to get a feasible solution. As W is lower, it is harder for the drivers to meet some deliveries in time without sacrificing the optimal route so the distance may be higher in order to make the deliveries in time. In this case, however, due to multiple drivers, there will be more flexibility as if one driver can not make it in time, it is possible to allocate an order to another driver.

2. **Create a trade-off curve comparing W and distance for these instances. Why do you observe that particular shape? Furthermore, are they correlated? (Note: with this curve, you are trying to infer the sensitivity or elasticity of the total distance with respect to W).**

   Similar to the previous case, there is a threshold value of W below which the solution is infeasible and above which the solution is feasible as seen in the graph below. This value is around 5 minutes. When W is below 5 minutes, the minimum distance is zero which has been coded so that if the solution is infeasible, it gives a minimum distance of zero. After that, the same trend follows as seen previously at the beginning when W <= 20, the minimum distance is larger than optimal solution. It makes sense that in order to guarantee a shorter average waiting time, drivers need to deliver to customers once they go to the corresponding restaurants, which makes the distance longer. As W increases, the minimum distance decreases until it is constant as it reaches the optimal minimum distance within the time it needs. This optimal value is 30.54 km. The negative correlation can be seen here as well after the feasible threshold.

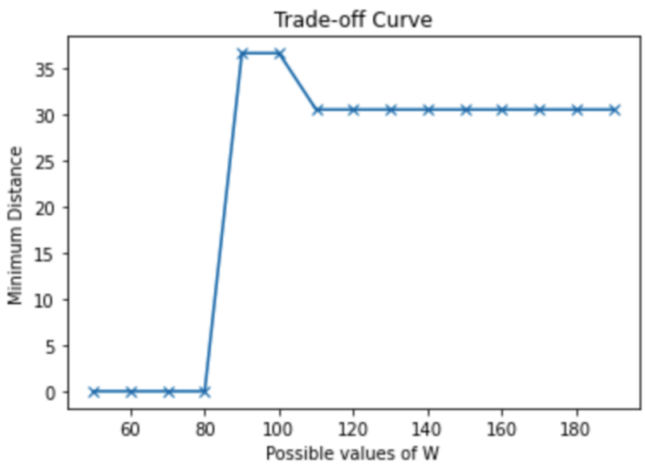


Trade-off Curve-Part3 (W=Maximum Average Waiting Time)

3. **What is the problem when considering the average waiting time? Propose and test an alternative metric to address the issue you identified, showing how the solution changed with your new measure.**

An alternative metric to be used is the maximum waiting time for each customer instead of the maximum average waiting time. This will ensure that each customer gets their order delivered within a certain maximum waiting time and quality customer service levels are met.

This alternative metric was incorporated in place of W in the previous model and the results can be seen below. The optimal minimum distance reached is much the same, yet the values of W are quite different. The threshold value of W here is about 90 minutes after which solutions are feasible, whereas this threshold was only about 5 minutes previously which is a huge difference. This is because the limitation for not exceeding W is for each customer instead of on average which is more strict. This means that drivers may need to trade-off the distance and travel more to meet deliveries on time. This also suggests that there is high variation among customer waiting times so there might be few orders with low waiting times and some with very high waiting times that are causing the threshold of W to be much higher in the individual waiting time case. If each customer’s waiting time was close to the average, then the W in this case would be the same as the average one in the previous case.



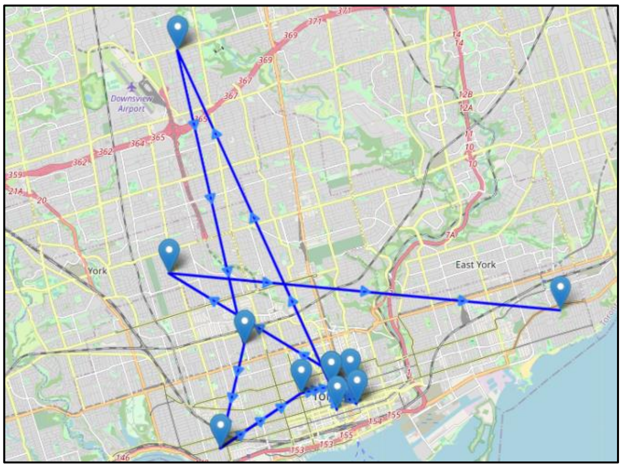
Trade-off Curve-Part3 (W=Maximum Waiting Time)

4. **How does the solution change when you consider fewer drivers, including a single one? That is, what are the benefits/disadvantages to the distance and average waiting time?**

By considering fewer drivers, the solution is likely to change. The case of a single and two drivers is considered and discussed as follows:

1-Driver

In the case of one driver, the total steps for completing the orders are now 10 which is 4 additional steps than previously. The total waiting time is 150 minutes, about 50 minutes more than before and W is taken as 30 minutes in this case for equal comparison. The optimal distance here is 51 km which is almost 20 km higher than for the multiple drivers' case. The average waiting time for each step is therefore exactly 30 minutes. It makes sense that overall the distance traveled as well as the average waiting times are higher since one driver is expected to travel to all locations while also making sure deliveries are made in time. The full route details can be seen in the Appendix.



Part3 Route Map (1 Driver)

2-Driver

As there is one more driver, the model becomes slightly more flexible as orders can now be redirected to another driver in case of time constraints. This helps to divide orders between drivers and reduce the maximum average waiting time for customers overall but this is still not as flexible as the three driver case. The total journey takes a total of 8 steps even with 2 drivers, which was only 6 steps with 3 drivers. The total waiting time is 143 minutes and on average it is about 28 minutes per customer. This is slightly lower than the one driver case but still much higher than the three driver case which was 19 minutes per customer. The distance is quite shorter as 33 km, which is very close to the three driver case with 30km.

It is clear that the optimal distance is still quite closer with 2 drivers to the 3 drivers case, yet the average customer waiting time is still a bit higher. Depending on the service level, if the decrease in the customer waiting time is important as well as the slight decrease in total distance traveled then 3 drivers should be considered, and if not then 2 drivers may be good enough.

Part3 Route Map (2 Drivers)

5. Based on your insights from 1-4, how does that relate to the current driver strategies that Uber Eats and Door Dash implement?

This is similar to what is implemented by Uber Eats and Door Dash as they assign an order to the driver that is free and within the locality of the order to minimize the distance and also will help get the order delivered as soon as possible. This will help to maintain customer service levels and deliver food on time and also reduce the costs for Uber Eats and Door Dash by minimizing the distance traveled by the driver. Also, having multiple drivers helps to make sure there are drivers in all the major areas for orders and reduces the overall maximum average waiting times and another driver is also always available in case one is busy.

Therefore, in order to minimize the total distance traveled multiple drivers are needed as well as to minimize the customer waiting times. In reality, however, more drivers may lead to increased costs and risks to the company due to car and labor costs. This must be kept in mind as this has not been factored into the model.

Part 4

Problem

Based on the findings from parts 1-3, a more complex system could be developed that is able to handle a larger number of orders. While the solution in part 3 is able to scale across multiple drivers, it becomes too computationally expensive to optimize as the number of orders is increased. Therefore, a different approach had to be used, that not only utilizes a linear programing optimization model but also uses a clustering approach to break down the problem into simpler pieces. W is fixed as 120 minutes in this case.

Model

The first step in developing a scalable approach was to examine the available data. Using the provided customer and restaurants from the part4_large.csv file, available drivers, along with the provided longitude and latitude of each of the regions, the locations of each of the restaurants, drivers, and customers could be plotted as seen below.

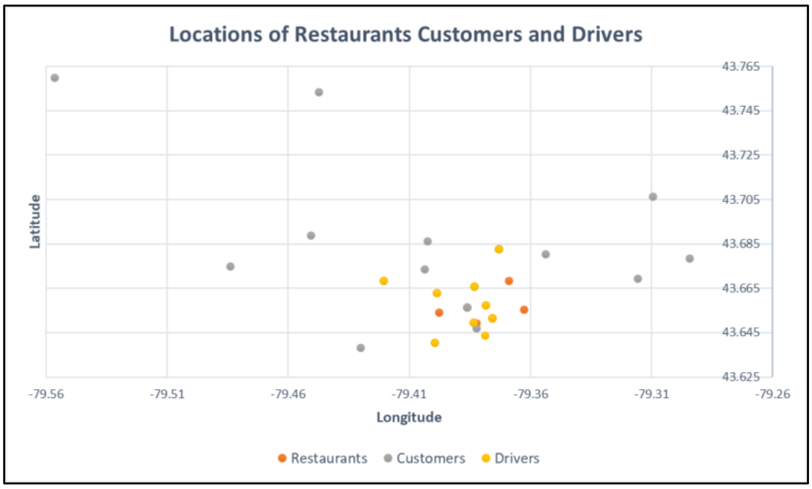


Chart showing locations of Restaurants, Customers, and Drivers

The primary insight from this graph is that the locations of the customers are much more spread out compared to the restaurants and locations of the drivers. Intuitively, it would make sense that for a driver to minimize the distance they travel, they should minimize the number of times they need to travel to the far-out locations of the customers. Additionally, if a driver does have to travel to a far-out location, it would likely make sense for them to fulfill orders along a similar route on the way. Therefore, an efficient solution will likely allocate orders to drivers such that they are all along a similar route, picking up the orders from the downtown restaurants first, before delivering them to the customers. Based on this intuition, the following approaches were attempted.

Restaurant to Customer Angle Clustering

The following plot shows each pair of restaurant and customer in the part4_large dataset, connected by lines

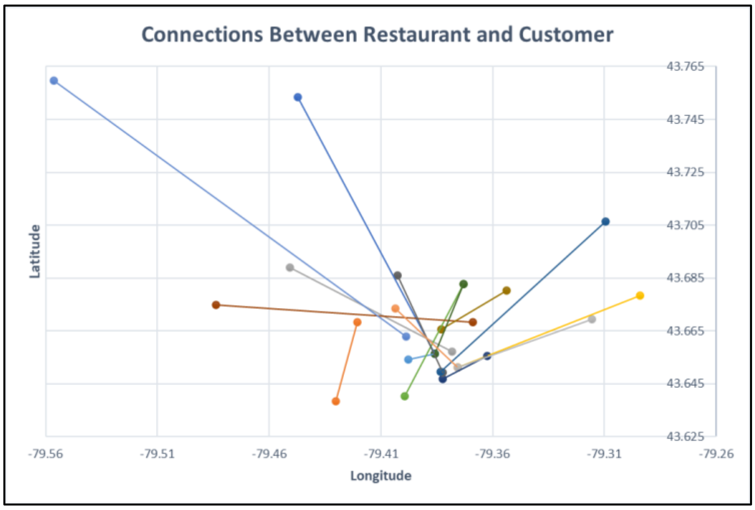


Chart showing connection between restaurants and customers

This shows that many of the customers that would intuitively be delivered to on the same trip have connection lines that are close to parallel. Therefore, it was hypothesized that clustering the pairs of restaurants and customers based on the angle of these lines, before applying the single driver route optimization as used in part 2 to each cluster, could be effective at minimizing total distance. Specifically, this clustering was performed using a K-means algorithm and was tested using a variety of cluster numbers. The algorithm was modified to set a maximum number of customers per cluster, to ensure the linear programming model optimization was able to finish successfully in a reasonable amount of time.

Restaurant Centroid to Customer Angle Clustering

Similar to the above approach, this method used a modified K-means clustering algorithm by angle. However, rather than computing the angle between each restaurant and its respective customer, the angle was computed between the centroid of the restaurants and each customer. It was hypothesized that there could be issues with the first clustering method if connections between restaurants and customers were nearly parallel, but they were far apart. Therefore, this method clusters by the angle between the centroid of the restaurant and each customer, such that customers that are far apart are more likely to be placed in separate clusters.

Radial Clustering

As an alternative approach and a source of comparison to the angular clustering approaches, clustering was also attempted based on the Euclidean distance between the centroid of the restaurants and each customer. In practice, this would mean that a given driver would fulfill orders that are at similar distances from the restaurants. However, this could potentially cause inefficiencies if there are two customers of similar distances to the restaurant centroid, but on opposite sides of the city that are clustered together.

Single Driver per Cluster Optimization

For each of these clustering methods, the dataset was first split based on the clusters, and the single driver route optimization as developed in part 2 was used. However, because a list of 10 drivers was provided, the best driver for each route had to be determined. One approach initially considered was selecting the driver closest to the centroid of the restaurants in each cluster. This would ensure that the driver for each cluster would be close to the optimal starting point of the route. However, this approach does not place any additional weight on the driver being near the first restaurant on the route. Therefore, an initial optimization was conducted to determine the optimal route for each cluster, excluding the driver's starting location. The first restaurant in each of these routes was then selected as the optimal starting location for each route. The closest driver to each of these optimal starting locations was then selected to serve that route, and the optimization was rerun to include the driver.

Multiple Drivers per Cluster Optimization

Similar to part 3, on top of the clustering results, the option to optimize upon multiple drivers becomes available. In this case, the model and constraint setups are identical to part 3, and the only difference is that some additional options of assigning drivers and deciding the number of drivers to utilize come into consideration.

In this situation, because it is under the situation of online calculation with limited computing resources and time constraints, two approaches were explored: random assignment and greedy. With random assignment, the logic is that it tries a few different combinations of assignment between drivers and clusters and takes the most optimized solution. The advantage of this method is that it is easy to implement and fast. When the total number of drivers and orders becomes exponentially large, this method is theoretically the fastest and requires the least number of resources. However, the downside is also obvious, sometimes, the randomized assignment is far from optimized. This is when the second approach comes into the scene. The greedy approach starts by finding the closest available vehicles and assigning a random number of them to the designated cluster. This way, it improves on making sure the start location of the vehicles is relatively close to the starting point of the trips, which theoretically should improve the overall optimization compared to a fully randomized assignment. This method becomes beneficial when the starting location of the vehicles is more spread out in the city, which is usually the reality.

With the two approaches tested, the findings are shown in part 4 of the Appendix. In this specific setup of the problem, because the drivers are relatively close to each other, and so are the restaurants, the greedy assignment did not show a huge amount of improvement compared to a randomized assignment. The appendix shows the minimized distance solution for all three clustering methods. As shown, the lowest optimized distance happened with the radial clustering method at only 26 kilometers, which is far better than the other two. This is likely caused by the wait time constraint as well as the situation where more drivers are driving to further customers.

Results

The following table outlines the total distance evaluated on the part4_large dataset for each of the clustering methods, and one driver per cluster using the optimal number of clusters

	Restaurant to Customer Angle Clustering	Restaurant Centroid to Customer Angle Clustering	Radial Clustering
Total Distance (km)	63.58	64.00	101.54
Average Waiting Time (minutes)	75.26	72.37	80.52

This indicates that for the case of a single driver per cluster, the algorithms that cluster based on angles seem to perform better than radial clustering. These clustering methods were also compared to the multi-driver optimization from part 3, evaluated on the part3_small dataset. For reference, the multi vehicle optimization from part 3 produced a total distance of 30.54 km, which represents the most efficient routing, and had an average waiting time of 98 minutes.

	Restaurant to Customer Angle Clustering	Restaurant Centroid to Customer Angle Clustering	Radial Clustering
Total Distance (km)	31.77	40.85	36.90
Average Waiting Time (minutes)	16.92	4.61	11.26

When comparing the results from the clustering methods to the multi-driver optimization from part 3, the results are very similar. Specifically, each of the total distance and average waiting time combinations seem to correspond to a point along the average waiting time and total distance tradeoff curve. This indicates that using clustering seems to be an effective way to produce efficient routes. However, in the case of dataset 4, the radial clustering seems to be producing better performances compared to the other two. The results are shown below:

	Restaurant to Customer Angle Clustering	Restaurant Centroid to Customer Angle Clustering	Radial Clustering
Total Distance (km)	45.826	41.03	26.89
Average Waiting Time (minutes)	82.982	108.045	111.483

Limitations

One of the primary limitations of the various strategies discussed in part 4 is the average delivery time constraint. Firstly, the models tested used an average maximum waiting time of two hours, which is likely to be too large in practice. Additionally, using the average waiting time allows some of the orders to take significantly longer than two hours if it is compensated for by shorter waiting times for other deliveries. If implemented, this would likely result in many complaints from customers who are waiting many hours for their food, which will certainly be cold and too late. Most of the strategies, when optimized, resulted in most of the restaurants being visited first, picking up all the food on the route before delivering it to all the customers. However, this likely would not be a viable strategy in practice due to some customers waiting too long and their food getting cold.

Application In Practice

For the purposes of this application, the order times and locations of each of the restaurants and customers were known ahead of time. However, in practice, it will be unknown when and where orders will arrive. To use the models discussed in part 4 in practice, certain modifications would have to be made. One potential method for applying the clustering heuristic would be to run the clustering algorithm at a fixed time interval. Then, once a cluster reaches a certain size, the route of the cluster could be optimized, and the nearest available drivers could be assigned to the cluster. This application has several tradeoffs, one of which is the minimum cluster size after which the route should be optimized and assigned. Setting the minimum cluster size low would ensure customers receive timely service, as there would be fewer stops on each route. However, this would come at the additional cost of more distance covered. Using a larger cluster size would ensure that routes are more efficient to decrease cost, but would result in longer wait times, and likely more customer complaints. This tradeoff between minimizing cost and customer satisfaction would need to be evaluated by the business in making the final decision.

Conclusion

Overall, the goal of Uber Eats was to assign drivers to orders based on the best-possible delivery routes. It is clear that time is an important factor in the model and there is a trade-off between the total distance the driver travels and the customer waiting time. This is an important trade-off for Uber Eats where they must trade-off quality customer service levels with driver costs.

As additional drivers are considered, the optimal distance traveled by the drivers and customer waiting times, both decrease. However, additional costs related to additional vehicles and labor hired is not part of the model, which must be considered otherwise.

A clustering heuristic helped to group together orders that were in similar localities and assign them to drivers respectively as the model was no longer scalable. This gave good results that are quite close to the results from solving the model. For comparison, the heuristic gave an optimal solution that had a distance only about 16% higher than the actual optimal minimum distance.

Lastly, there are some limitations in the solution that must be addressed. This includes the long time taken to run the model, which was due to the high number of constraints and variables added to the model that made it much more complex. Also, some assumptions such as no additional costs related to an increased number of drivers as well as knowing all orders beforehand must be addressed as the solution from this model is otherwise impractical. By incorporating labor and additional vehicle costs in the model as well as using the heuristic hand-in-hand with the model, the assumptions will be addressed and the solution will be more practical to implement.

Rotman

Appendix

Part I

Dataset: OrdersA

Q1. Mathematical Formulation

- Let $J \in \{$ all locations for restaurants and customers $\}$
  $I \in \{$ all locations restaurants, customers and start region (i.e. Rosedale) $\}$
- Let d_{ij} be the distance between location i and j.
- Let x_{ijt} be a binary decision variable. $x_{ijt} = 1$ if the driver travels from i to j at step t.

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=0}^{T} d_{ij} x_{ijt}$$

s.t.

$$\sum_{i=1}^{n} \sum_{t=0}^{T} x_{ijt} = 1 \qquad j \in J \qquad \text{" Every location visited once "}$$

$$\sum_{i=1}^{n} x_{ijt} = \sum_{k=1}^{n} x_{jk\,t+1} \qquad j \in J, \; t = 0, \dots, T-1 \qquad \text{" Conservation of flows "}$$

$$\sum_{j=1}^{n} x_{1j0} = 1$$

$$\sum_{j=1}^{n} x_{ij0} = 0 \qquad i = 2, \dots, n$$

" Node 1 (Rosedale) has an outflow of 1 in the first step $t=1$, others are zero. "

$$\sum_{i=1}^{n} x_{ijt} \le \sum_{t'=0}^{t} \sum_{i=1}^{n} x_{ij't'} \qquad j \in \{\text{customer locations}\}, \; t = 0, \dots, T$$
$$j' \in \{\text{restaurant locations for each } j\}$$

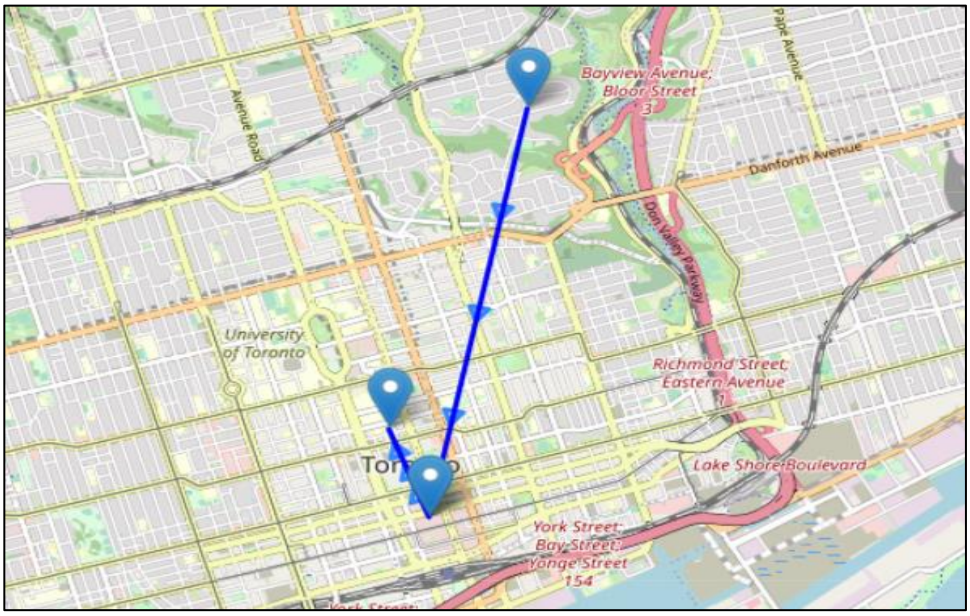
" The drive should go to the restaurant where the customer order the food before going to the customer "

$$x_{ijt} \in \{0,1\} \qquad \forall i, j, t$$

Part1 Mathematical Formulation

```
Step: 0
        Leave from Downtown Toronto (Rosedale)
        Travel to Downtown Toronto (Underground city)
Step: 1
        Leave from Downtown Toronto (Underground city)
        Travel to Downtown Toronto (Central Bay Street)
```

Part1 OrdersA Optimal Route

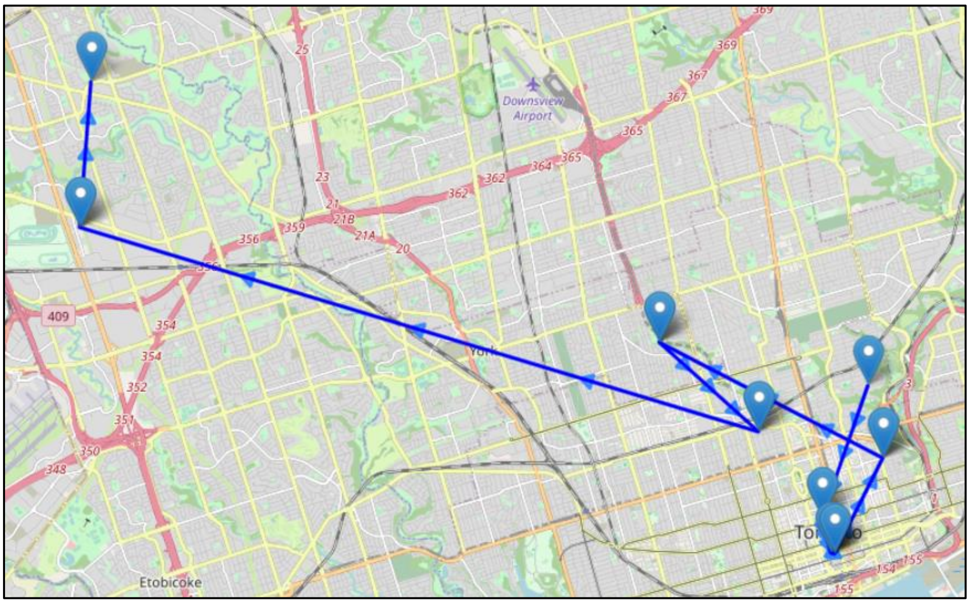


Part1 OrdersA Route Map

Dataset: OrdersB

```
Step: 0
        Leave from Downtown Toronto (Rosedale)
        Travel to Downtown Toronto (Central Bay Street)
Step: 1
        Leave from Downtown Toronto (Central Bay Street)
        Travel to Downtown Toronto (Richmond / Adelaide / King)
Step: 2
        Leave from Downtown Toronto (Richmond / Adelaide / King)
        Travel to Downtown Toronto (Underground city)
Step: 3
        Leave from Downtown Toronto (Underground city)
        Travel to Downtown Toronto (St. James Town / Cabbagetown)
Step: 4
        Leave from Downtown Toronto (St. James Town / Cabbagetown)
        Travel to York (Cedarvale)
Step: 5
        Leave from York (Cedarvale)
        Travel to Central Toronto (The Annex / North Midtown / Yorkville)
Step: 6
        Leave from Central Toronto (The Annex / North Midtown / Yorkville)
        Travel to Etobicoke Northwest (Clairville / Humberwood / Woodbine Downs / West Humber / Kipling Heights / R
exdale / Elms / Tandridge / Old Rexdale)
Step: 7
        Leave from Etobicoke Northwest (Clairville / Humberwood / Woodbine Downs / West Humber / Kipling Heights /
Rexdale / Elms / Tandridge / Old Rexdale)
        Travel to Etobicoke (South Steeles / Silverstone / Humbergate / Jamestown / Mount Olive / Beaumond Heights
/ Thistletown / Albion Gardens)
```

Part1 OrdersB Optimal Route



Part1 OrdersB Route Map

Part 2

Dataset: OrdersA

Q2. Mathematical Formulation

- Let $J \in \{$ all locations for restaurants and customers $\}$
  $I \in \{$ all locations restaurants, customers and start region (i.e. Rosedale) $\}$
- Let l_j be the minimum arrival time (in minutes) to location j. (a fixed parameter)
  l_j is the minutes from the pre-defined starting time to the "estimated availability" time for each restarant. $l_j = 0$ for $j \in \{$customers$\}$
- Let V_{ij} be the travel time from location i to location j (a fixed parameter).
- Let d_{ij} be the distance between location i and j.
- Let X_{ijt} be a binary decision variable. $X_{ijt} = 1$ if the driver travels from i to j at step t.
- Let D_t be a decision variable representing the total travel time of the tour up to step t. (in minutes in this case).
- Let W_{jt} be the customer waiting time when travelling to location j at step t.
- Let Z_t be the total customer waiting time up to step t.
- Assume $M = 500000$ (a large constant)

  W = "maximum average customer waiting time"

Rotman

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=0}^{T} d_{ij} x_{ijt}$$

s.t.

$$\sum_{i=1}^{n}\sum_{t=0}^{T} x_{ijt} = 1 \qquad j \in J$$ "Every location visited once"

$$\sum_{i=1}^{n} x_{ijt} = \sum_{k=1}^{n} x_{jk\,t+1} \qquad j \in J, \; t = 0, \dots, T-1$$ "Conservation of flows"

$$\sum_{j=1}^{n} x_{1j0} = 1$$

$$\sum_{j=1}^{n} x_{ij0} = 0 \qquad i = 2, \dots, n$$

"Node 1 (Rosedale) has an outflow of 1 in the first step $t=0$, others are zero."

$$\sum_{i=1}^{n} x_{ijt} \le \sum_{t'=0}^{t}\sum_{i=1}^{n} x_{ij't'} \qquad j \in \{\text{customer locations}\}, \; t = 0, \dots, T$$

$$j' \in \{\text{restaurant locations for each } j\}$$

"The drive should go to the restaurant where the customer order the food before going to the customer"

$$D_0 = \sum_{i=1}^{n}\sum_{j=1}^{n} l_j \, x_{ij0}$$

$$D_t \ge D_{t-1} + \sum_{i=1}^{n}\sum_{j=1}^{n} v_{ij} x_{ijt} \qquad t = 1, \dots, T$$

"Time to arrival at the location of the t-th step"

$$D_t \ge l_j \, x_{ijt} \qquad \forall i, j, t$$

"if arriving at location j, ensures time is at least l_j"

$$w_{jt} = 0 \qquad \forall t, \; j \in \{\text{restaurants}\}$$

$$w_{jt} = D_t - \sum_{i=1}^{n} l_{j'} \cdot x_{ijt} \qquad \forall t, \; j \in \{\text{customers}\}$$

$$j' \in \{\text{restaurant locations for each } j\}$$

"customer waiting time is defined as the time between when the food is ready to pick up, and when the driver arrives at the customer's location"

Rotman

$$Z_0 = 0$$

$$Z_t \geq Z_{t-1} + W_{jt} - M * (1 - X_{ijt}) \quad \forall i, j, t$$

$$Z_t \leq Z_{t-1} + W_{jt} + M * (1 - X_{ijt}) \quad \forall i, j, t$$

" Define Z_t as the total customer waiting time up to step t "

$$Z_t \leq W \cdot \text{len}(\text{customers}) \quad \forall t$$

" The average customer waiting time at each step t should not exceed W "

$$X_{ijt} \in \{0, 1\} \quad \forall i, j, t$$

$$D_t \geq 0 \quad \forall t$$

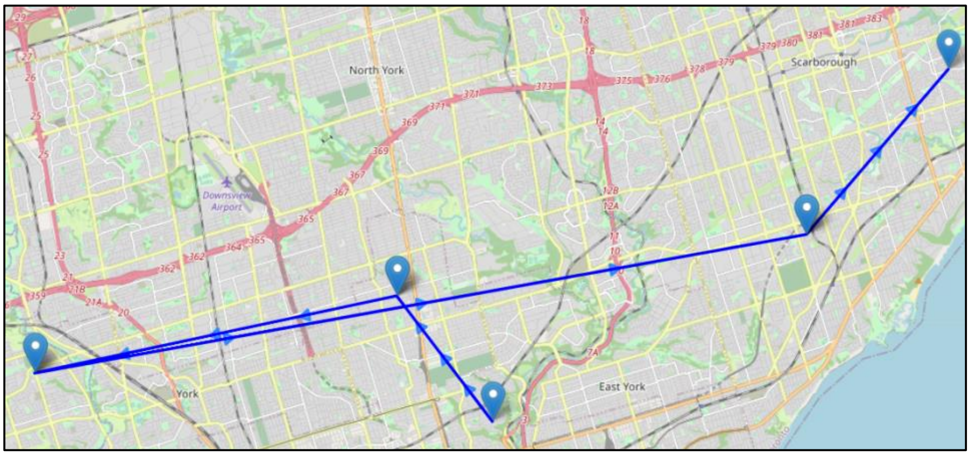
$$W_{jt} \geq 0 \quad \forall j, t$$

$$Z_t \geq 0 \quad \forall t$$

Part2 Mathematical Formulation

```
Step: 0
Total travel time (in minutes) up to this step: 90.0
        Leave from Downtown Toronto (Rosedale)
        Travel to Central Toronto (North Toronto West)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
Step: 1
Total travel time (in minutes) up to this step: 105.50979
        Leave from Central Toronto (North Toronto West)
        Travel to Etobicoke (Westmount)
        Waiting time: 15.509793
        Total waiting time up to this step: 15.509793
Step: 2
Total travel time (in minutes) up to this step: 143.3877
        Leave from Etobicoke (Westmount)
        Travel to Scarborough (Kennedy Park / Ionview / East Birchmount Park)
        Waiting time: 0.0
        Total waiting time up to this step: 15.509793
Step: 3
Total travel time (in minutes) up to this step: 152.51188
        Leave from Scarborough (Kennedy Park / Ionview / East Birchmount Park)
        Travel to Scarborough (Woburn)
        Waiting time: 125.51188
        Total waiting time up to this step: 141.02167
Average waiting time: 70.510835
```

Part2 OrdersA Optimal Route

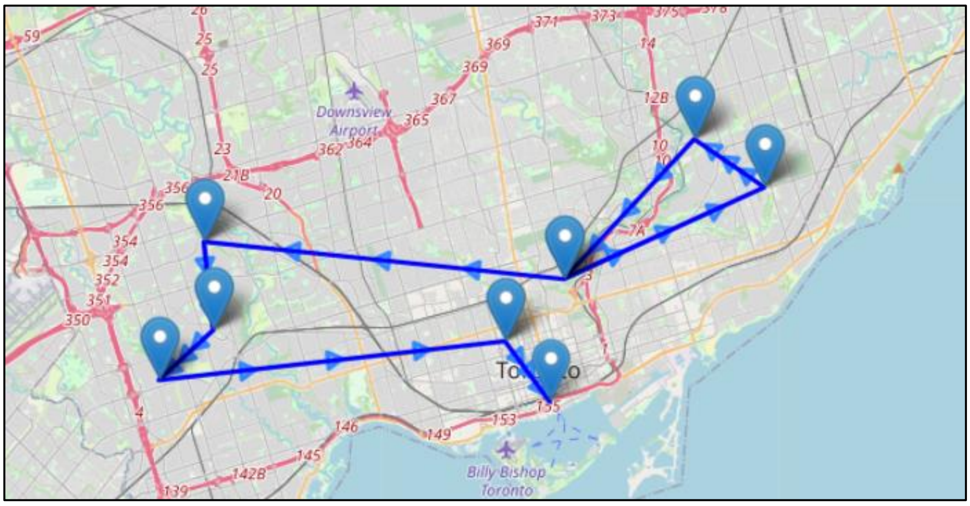


Part2 OrdersA Route Map

Dataset: OrdersB

```
Step: 0
Total travel time (in minutes) up to this step: 77.0
        Leave from Downtown Toronto (Rosedale)
        Travel to Scarborough (The Golden Mile / Clairlea / Oakridge / Birchmount Park East)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
Step: 1
Total travel time (in minutes) up to this step: 81.485021
        Leave from Scarborough (The Golden Mile / Clairlea / Oakridge / Birchmount Park East)
        Travel to North York (Sweeney Park / Wigmore Park)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
Step: 2
Total travel time (in minutes) up to this step: 92.213805
        Leave from North York (Sweeney Park / Wigmore Park)
        Travel to Downtown Toronto (Rosedale)
        Waiting time: 15.213805
        Total waiting time up to this step: 15.213805
Step: 3
Total travel time (in minutes) up to this step: 116.53413
        Leave from Downtown Toronto (Rosedale)
        Travel to Etobicoke (Westmount)
        Waiting time: 0.0
        Total waiting time up to this step: 15.213805
Step: 4
Total travel time (in minutes) up to this step: 121.34656
        Leave from Etobicoke (Westmount)
        Travel to Etobicoke (Islington Avenue)
        Waiting time: 0.0
        Total waiting time up to this step: 15.213805
Step: 5
Total travel time (in minutes) up to this step: 125.20644
        Leave from Etobicoke (Islington Avenue)
        Travel to Etobicoke (West Deane Park / Princess Gardens / Martin Grove / Islington / Cloverdale)
        Waiting time: 72.206437
        Total waiting time up to this step: 87.420243
Step: 6
Total travel time (in minutes) up to this step: 148.78483
        Leave from Etobicoke (West Deane Park / Princess Gardens / Martin Grove / Islington / Cloverdale)
        Travel to Downtown Toronto (University of Toronto / Harbord)
        Waiting time: 99.784829
        Total waiting time up to this step: 187.20507
Step: 7
Total travel time (in minutes) up to this step: 157.79493
        Leave from Downtown Toronto (University of Toronto / Harbord)
        Travel to Downtown Toronto Stn A PO Boxes 25 The Esplanade (Enclave of M5E)
        Waiting time: 140.79493
        Total waiting time up to this step: 328.0
Average waiting time: 82.0
```

Part2 OrdersB Route



Part2 OrdersB Route Map

```
Step: 0
Total travel time (in minutes) up to this step: 90.0
        Leave from Downtown Toronto (Rosedale)
        Travel to Central Toronto (North Toronto West)
        Waiting time: 0.0
Step: 1
Total travel time (in minutes) up to this step: 105.50979
        Leave from Central Toronto (North Toronto West)
        Travel to Etobicoke (Westmount)
        Waiting time: 15.509793
Step: 2
Total travel time (in minutes) up to this step: 143.3877
        Leave from Etobicoke (Westmount)
        Travel to Scarborough (Kennedy Park / Ionview / East Birchmount Park)
        Waiting time: 0.0
Step: 3
Total travel time (in minutes) up to this step: 200.0
        Leave from Scarborough (Kennedy Park / Ionview / East Birchmount Park)
        Travel to Scarborough (Woburn)
        Waiting time: 173.0
```

Part2 OrdersA Route (Model-2 with Maximum Waiting Time)

```
Step: 0
Total travel time (in minutes) up to this step: 53.0
        Leave from Downtown Toronto (Rosedale)
        Travel to Etobicoke (Westmount)
        Waiting time: 0.0
Step: 1
Total travel time (in minutes) up to this step: 60.76662
        Leave from Etobicoke (Westmount)
        Travel to Etobicoke (West Deane Park / Princess Gardens / Martin Grove / Islington / Cloverdale)
        Waiting time: 7.7666201
Step: 2
Total travel time (in minutes) up to this step: 69.626498
        Leave from Etobicoke (West Deane Park / Princess Gardens / Martin Grove / Islington / Cloverdale)
        Travel to Etobicoke (Islington Avenue)
        Waiting time: 0.0
Step: 3
Total travel time (in minutes) up to this step: 97.323659
        Leave from Etobicoke (Islington Avenue)
        Travel to North York (Sweeney Park / Wigmore Park)
        Waiting time: 0.0
Step: 4
Total travel time (in minutes) up to this step: 101.80868
        Leave from North York (Sweeney Park / Wigmore Park)
        Travel to Scarborough (The Golden Mile / Clairlea / Oakridge / Birchmount Park East)
        Waiting time: 0.0
Step: 5
Total travel time (in minutes) up to this step: 131.46008
        Leave from Scarborough (The Golden Mile / Clairlea / Oakridge / Birchmount Park East)
        Travel to Downtown Toronto (Rosedale)
        Waiting time: 54.460079
Step: 6
Total travel time (in minutes) up to this step: 140.9899
        Leave from Downtown Toronto (Rosedale)
        Travel to Downtown Toronto (University of Toronto / Harbord)
        Waiting time: 91.989901
Step: 7
Total travel time (in minutes) up to this step: 150.0
        Leave from Downtown Toronto (University of Toronto / Harbord)
        Travel to Downtown Toronto Stn A PO Boxes 25 The Esplanade (Enclave of M5E)
        Waiting time: 133.0
```

Part2 OrdersB Route (Model-2 with Maximum Waiting Time)

Part 3

Objective Function:

$$\sum_{i=0}^{N}\sum_{j=3}^{M}\sum_{v=0}^{V}\sum_{t=0}^{T} (location\_distance)_{ij} \times X_{ijvt}$$

Variables:

- X_{ijvt} 1 if the step happens, 0 if it's not $X_{ijt} \in \{0,1\}$
- $d_{v,t}$ the arriving time for vehicle v in step t
- W_{jvt} waiting time for each vehicle at each end location in each step t
- $z_{v,t}$ total Waiting time for each vehicle until step t

Rotman

Parameters

- N: all restaurants, all customers and all start regions of vehicles
- J: all restaurants and all customers
- V: all vehicles
- T: Total number of steps
- l_j: minimal arriving time in minutes to location j
  $l_{j'}$: minimal arriving time in minutes to location j' (restaurants of corresponding customer j)
  $l_j = 0$ if j is customers' locations
- r_{ijv}: traving time for each vehicle from location i to location j
- w_{jvt}: waiting time of vehicle v at location j in step t
- $z_{v,t}$: total waiting time of vehicle v until step t
- $d_{v,t}$: Time to arrive at location in the t^{th} step for each vehicle
- $(location\_distance)_{ij}$: distance between location i and location j
- $M = 10000$ a large constant
- W: the maximum average customer waiting time
  (for trade-off plot, set $W \in (0, 60)$)
- C: the other vehicles except the vehicle that's called in for loop
  e.g. if for $v=0$, $C \in \{1,2\}$

Constraints:

Constraint 1: Every location visited once by one vehicle

$\sum_{i=0}^{N} \sum_{v=0}^{V} \sum_{t=0}^{T} X_{ijvt} = 1 \qquad j = 3, 4, 5, \cdots, J$

Constraint 2: Conservation of Flow

$\sum_{i=0}^{N} X_{ijvt} \geq \sum_{k=3}^{J} X_{jkv,(t+1)} \qquad V = 0, 1, 2 \quad t = 0, 1, 2, \cdots T-1 \quad j = 3, 4, \cdots, J$

Constraint 3: First node of each driver has an outflow of 1 in the first step, others are 0

$$\Sigma_{j=3}^{J} x_{vjvt} = 1 \qquad v = 0,1,2$$

$$\Sigma_{j=3}^{J} x_{jjvt} = 0 \qquad v = 0,1,2$$

$$\Sigma_{j=3}^{J} \Sigma_{t=1}^{T} x_{vjvt} = 0$$ each driver can only start from their starting region once

$$\Sigma_{a}^{C} \Sigma_{j=3}^{J} \Sigma_{t=1}^{T} x_{ajvt} = 0$$ each driver cannot start from other locations

Constraint 4: Each vehicle can only takes at most one order in each step

$$\Sigma_{i=0}^{N} \Sigma_{j=3}^{J} x_{ijvt} \le 1 \qquad v = 0,1,2 \qquad t = 0,1,2,\cdots T$$

Constraint 5: Make sure driver goes to a restaurant before delivering food to customer

$$\Sigma_{i=0}^{N} x_{ijvt} \le \Sigma_{i=0}^{N} \Sigma_{t}^{T-1} x_{ij',vt} \qquad t = 0,1,2,\cdots T \qquad v = 0,1,2 \qquad j = 8,9,10,11,12$$ (j is customer here)

j' here is the function to find restaurant of the customer whom the drive is delivering to

Constraint 6: Time to arrive at the location of the t^{th} step:

$t=0$:

$$d_{v,t} \ge \Sigma_{i=0}^{N} \Sigma_{j=3}^{J} x_{ijv0} \times l_j \qquad v = 0,1,2 \qquad t = 0.$$

$$d_{v,t} \ge \Sigma_{i=0}^{N} \Sigma_{j=3}^{J} x_{ijvt} \times r_{ijv} \qquad v = 0,1,2 \qquad t = 0$$

$t \ge 1$:

$$d_{v,t} \ge d_{v,t-1} + \Sigma_{i=0}^{N} \Sigma_{j=3}^{J} x_{ijvt} \times r_{ijv} \qquad v = 0,1,2 \qquad t = 1,2,\cdots T$$

Constraint 7: If arriving at location j, ensures time is at least the min_arrival_time_mins

$$d_{v,t} \ge l_j' \times x_{ijvt} \qquad t = 0,1,2\cdots T \qquad i = 0,1,2,\cdots N \qquad j = 3,4,5,\cdots \qquad t = 0,1,\cdots T$$

Constraint 8: Ensures the average waiting time of the orders is lower than the maximum W

$j = 3,4,5,6,7$: $$w_{jvt} = 0 \qquad v = 0,1,2 \qquad t = 0,1,2,\cdots T$$

$j = 8,9,10,11,12$: $$w_{jvt} = d_{v,t} - \Sigma_{i=0}^{N} l_{j'} \times x_{ijvt} \qquad v = 0,1,2 \qquad t = 0,1,2,\cdots T$$

set $M = 100000$

$t=0$: $z_{v,t} = 0$

$t=1$: $w_{jvt} - M(1 - x_{ijvt}) \le z_{v,t} \le w_{jvt} + M(1 - x_{ijvt})$

$t=2\cdots T$: $z_{v,t-1} + w_{jvt} + M(1 - x_{ijvt}) \le z_{v,t} \le z_{v,t-1} + w_{jvt} - M(1 - x_{ijvt})$

$i = 0,1,2,\cdots N$
$j = 3,4,5,\cdots J$
$v = 0,1,2$

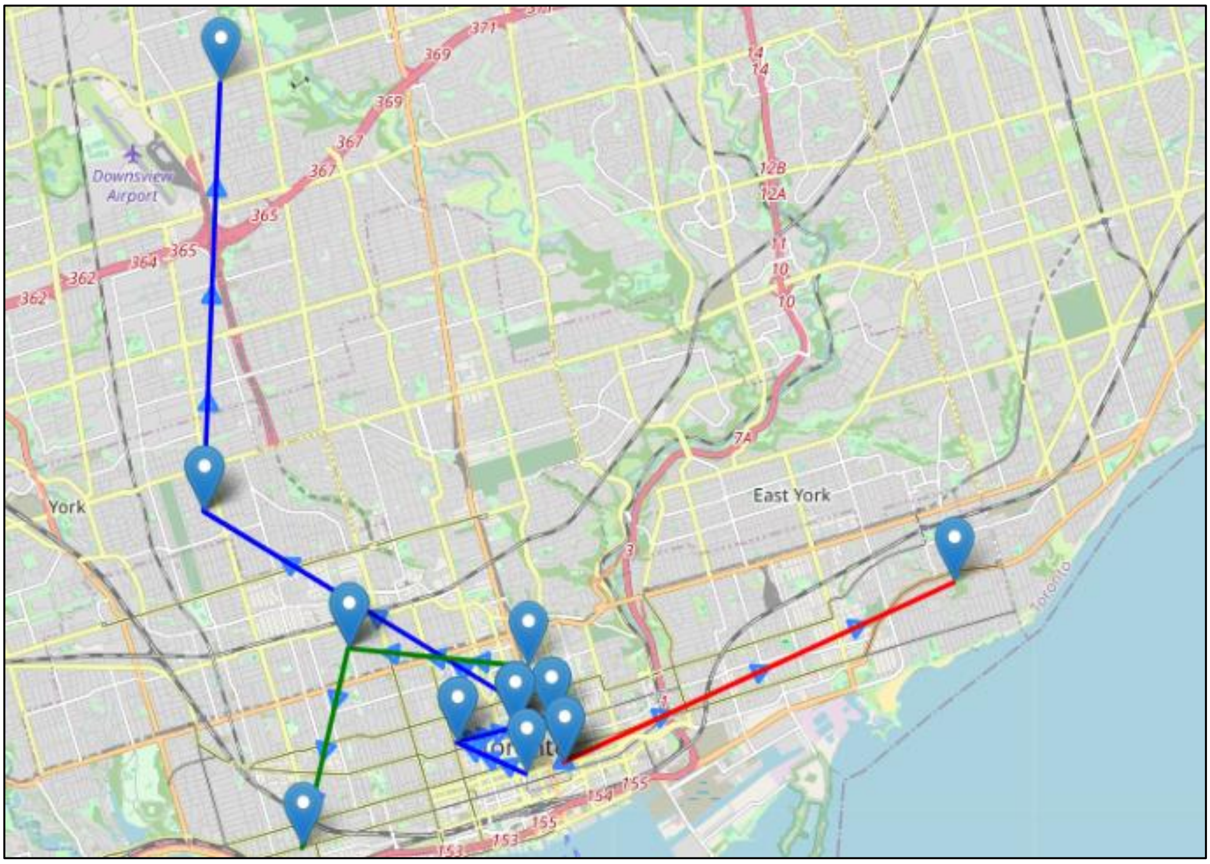
$$\Sigma_{v=0}^{V} z_{v,t} \le W \times (\text{number of customers}) \qquad t = 0,1,2,\cdots T$$

Part3 Mathematical Formulation

Rotman

```
Step: 0
        Vehicle: 0
        Leave from 0: Downtown Toronto (Richmond / Adelaide / King)
        Travel to 7: Downtown Toronto (Kensington Market / Chinatown / Grange Park)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
        Vehicle: 1
        Leave from 1: Downtown Toronto (St. James Park)
        Travel to 6: Downtown Toronto (St. James Park)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
        Vehicle: 2
        Leave from 2: Downtown Toronto (Church and Wellesley)
        Travel to 4: Downtown Toronto (Christie)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
Step: 1
        Vehicle: 0
        Leave from 7: Downtown Toronto (Kensington Market / Chinatown / Grange Park)
        Travel to 3: Downtown Toronto (Central Bay Street)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
        Vehicle: 1
        Leave from 6: Downtown Toronto (St. James Park)
        Travel to 11: East Toronto (The Beaches)
        Waiting time: 12.368678
        Total waiting time up to this step: 12.368678
        Vehicle: 2
        Leave from 4: Downtown Toronto (Christie)
        Travel to 9: West Toronto (Brockton / Parkdale Village / Exhibition Place)
        Waiting time: 6.42018
        Total waiting time up to this step: 6.42018
Step: 2
        Vehicle: 0
        Leave from 3: Downtown Toronto (Central Bay Street)
        Travel to 12: Downtown Toronto (Central Bay Street)
        Waiting time: 1.4747411
        Total waiting time up to this step: 1.4747411
Step: 3
        Vehicle: 0
        Leave from 12: Downtown Toronto (Central Bay Street)
        Travel to 5: Downtown Toronto (Ryerson)
        Waiting time: 0.0
        Total waiting time up to this step: 1.4747411
Step: 4
        Vehicle: 0
        Leave from 5: Downtown Toronto (Ryerson)
        Travel to 10: York (Fairbank / Oakwood)
        Waiting time: 10.210002
        Total waiting time up to this step: 11.684743
Step: 5
        Vehicle: 0
        Leave from 10: York (Fairbank / Oakwood)
        Travel to 8: North York (Armour Heights / Wilson Heights / Downsview North)
        Waiting time: 85.993062
        Total waiting time up to this step: 97.677806
```

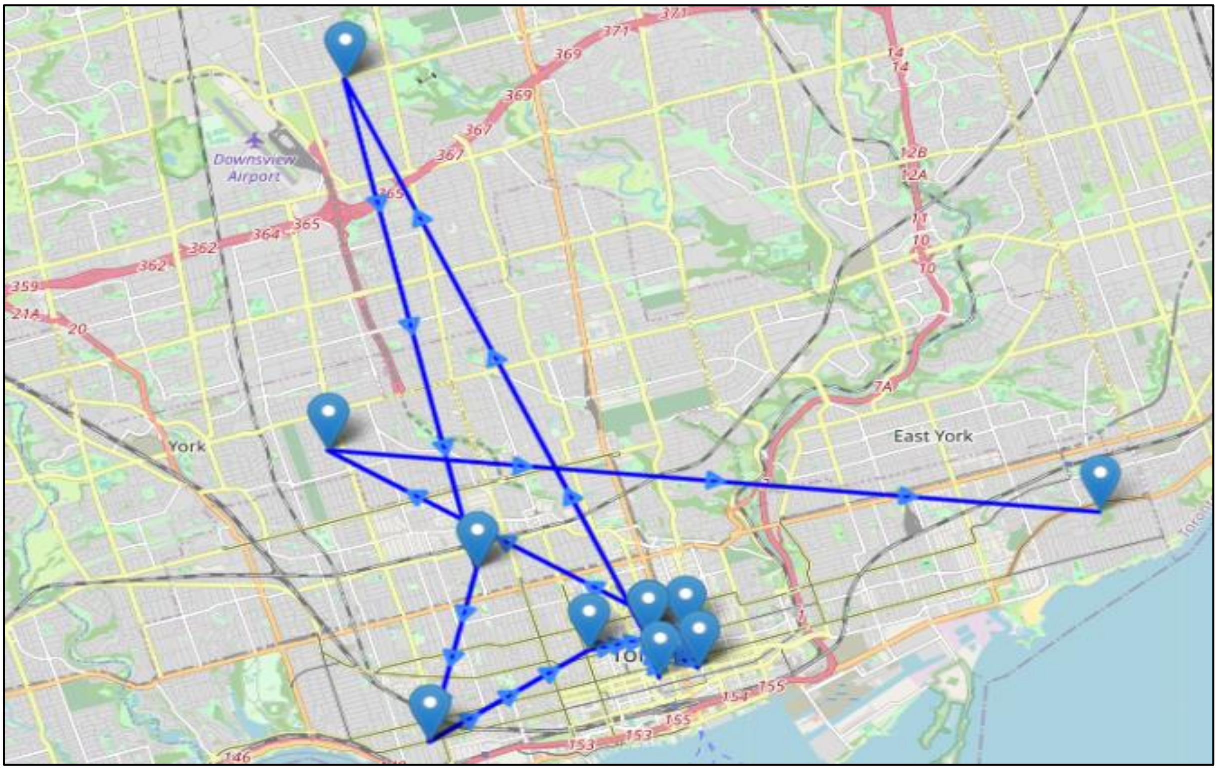
Part3 Optimal Route Description



Part3 Route Map (Multiple Drivers)

```
Step: 0
        Vehicle: 0
        Leave from 0: Downtown Toronto (Richmond / Adelaide / King)
        Travel to 1: Downtown Toronto (Central Bay Street)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
Step: 1
        Vehicle: 0
        Leave from 1: Downtown Toronto (Central Bay Street)
        Travel to 6: North York (Armour Heights / Wilson Heights / Downsview North)
        Waiting time: 17.79745
        Total waiting time up to this step: 17.79745
Step: 2
        Vehicle: 0
        Leave from 6: North York (Armour Heights / Wilson Heights / Downsview North)
        Travel to 2: Downtown Toronto (Christie)
        Waiting time: 0.0
        Total waiting time up to this step: 17.79745
Step: 3
        Vehicle: 0
        Leave from 2: Downtown Toronto (Christie)
        Travel to 7: West Toronto (Brockton / Parkdale Village / Exhibition Place)
        Waiting time: 10.504358
        Total waiting time up to this step: 28.301808
Step: 4
        Vehicle: 0
        Leave from 7: West Toronto (Brockton / Parkdale Village / Exhibition Place)
        Travel to 5: Downtown Toronto (Kensington Market / Chinatown / Grange Park)
        Waiting time: 0.0
        Total waiting time up to this step: 28.301808
Step: 5
        Vehicle: 0
        Leave from 5: Downtown Toronto (Kensington Market / Chinatown / Grange Park)
        Travel to 10: Downtown Toronto (Central Bay Street)
        Waiting time: 46.684682
        Total waiting time up to this step: 74.986491
Step: 6
        Vehicle: 0
        Leave from 10: Downtown Toronto (Central Bay Street)
        Travel to 4: Downtown Toronto (St. James Park)
        Waiting time: 0.0
        Total waiting time up to this step: 74.986491
Step: 7
        Vehicle: 0
        Leave from 4: Downtown Toronto (St. James Park)
        Travel to 3: Downtown Toronto (Ryerson)
        Waiting time: 0.0
        Total waiting time up to this step: 74.986491
Step: 8
        Vehicle: 0
        Leave from 3: Downtown Toronto (Ryerson)
        Travel to 8: York (Fairbank / Oakwood)
        Waiting time: 11.447464
        Total waiting time up to this step: 86.433955
Step: 9
        Vehicle: 0
        Leave from 8: York (Fairbank / Oakwood)
        Travel to 9: East Toronto (The Beaches)
        Waiting time: 63.566045
        Total waiting time up to this step: 150.0
Step: 10
```

Part3 Optimal Route Description (1 Driver)

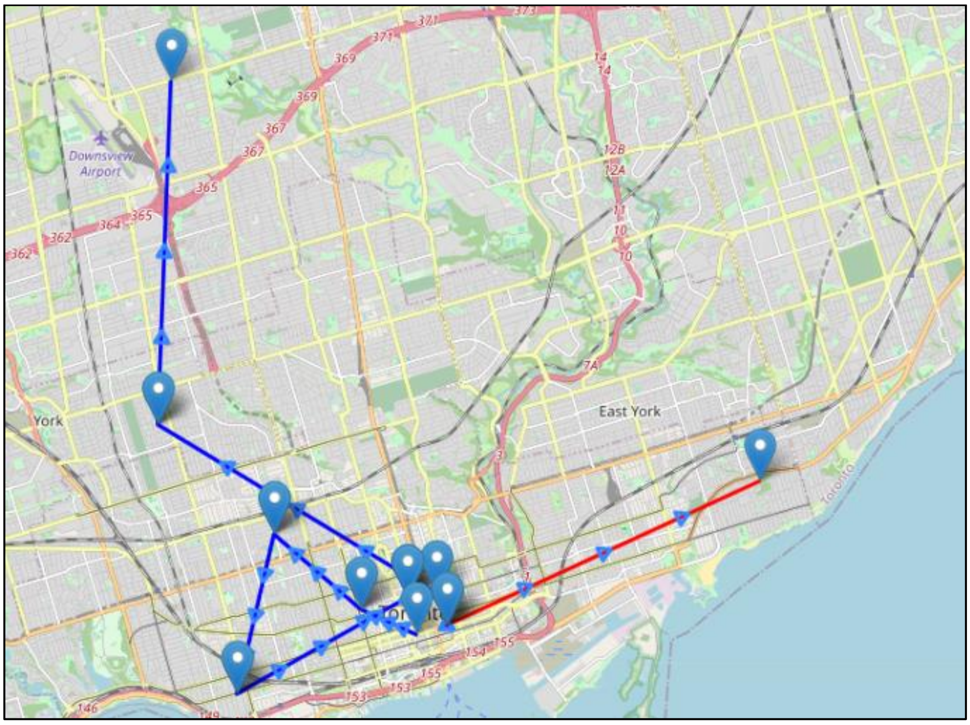


Part3 Route Map (1 Driver)

Rotman

```
Step: 0
        Vehicle: 0
        Leave from 0: Downtown Toronto (Richmond / Adelaide / King)
        Travel to 6: Downtown Toronto (Kensington Market / Chinatown / Grange Park)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
        Vehicle: 1
        Leave from 1: Downtown Toronto (St. James Park)
        Travel to 5: Downtown Toronto (St. James Park)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
Step: 1
        Vehicle: 0
        Leave from 6: Downtown Toronto (Kensington Market / Chinatown / Grange Park)
        Travel to 3: Downtown Toronto (Christie)
        Waiting time: 0.0
        Total waiting time up to this step: 0.0
        Vehicle: 1
        Leave from 5: Downtown Toronto (St. James Park)
        Travel to 10: East Toronto (The Beaches)
        Waiting time: 12.368678
        Total waiting time up to this step: 12.368678
Step: 2
        Vehicle: 0
        Leave from 3: Downtown Toronto (Christie)
        Travel to 8: West Toronto (Brockton / Parkdale Village / Exhibition Place)
        Waiting time: 5.136144
        Total waiting time up to this step: 5.136144
Step: 3
        Vehicle: 0
        Leave from 8: West Toronto (Brockton / Parkdale Village / Exhibition Place)
        Travel to 11: Downtown Toronto (Central Bay Street)
        Waiting time: 41.255209
        Total waiting time up to this step: 46.391353
Step: 4
        Vehicle: 0
        Leave from 11: Downtown Toronto (Central Bay Street)
        Travel to 2: Downtown Toronto (Central Bay Street)
        Waiting time: 0.0
        Total waiting time up to this step: 46.391353
Step: 5
        Vehicle: 0
        Leave from 2: Downtown Toronto (Central Bay Street)
        Travel to 4: Downtown Toronto (Ryerson)
        Waiting time: 0.0
        Total waiting time up to this step: 46.391353
Step: 6
        Vehicle: 0
        Leave from 4: Downtown Toronto (Ryerson)
        Travel to 9: York (Fairbank / Oakwood)
        Waiting time: 10.210002
        Total waiting time up to this step: 56.601355
Step: 7
        Vehicle: 0
        Leave from 9: York (Fairbank / Oakwood)
        Travel to 7: North York (Armour Heights / Wilson Heights / Downsview North)
        Waiting time: 85.993062
        Total waiting time up to this step: 142.59442
Step: 8
Step: 9
Step: 10
Step: 11
```

Part3 Optimal Route Description (2 Drivers)



Part3 Route Map (2 Drivers)

Part 4

Objective Function:

$\sum_{i=0}^{N}\sum_{j=3}^{M}\sum_{v=0}^{V}\sum_{t=0}^{T}$ (location_distance) $_{ij} \times X_{ijvt}$

Variables:

- X_{ijvt} 1 if the step happens, 0 if it's not $X_{ijt} \in \{0,1\}$
- $d_{v,t}$ the arriving time for vehicle v in step t
- W_{jvt} waiting time for each vehicle at each end location in each step t
- $z_{v,t}$ total Waiting time for each vehicle until step t

Rotman

Parameters

- N: all restaurants, all customers and all start regions of vehicles
- J: all restaurants and all customers
- V: all vehicles
- T: Total number of steps
- l_j: minimal arriving time in minutes to location j
  $l_{j'}$: minimal arriving time in minutes to location j' (restaurants of corresponding customer j)
  $l_j = 0$ if j is customers' locations
- r_{ijv}: traving time for each vehicle from location i to location j
- w_{jvt}: waiting time of vehicle v at location j in step t
- $z_{v,t}$: total waiting time of vehicle v until step t
- $d_{v,t}$: Time to arrive at location in the t^{th} step for each vehicle
- (location_distance)$_{ij}$: distance between location i and location j
- M = 10000 a large constant
- W: the maximum average customer waiting time
  (for trade-off plot, set $W \in (0, 60)$)

Constraints:

Constraint 1: Every location visited once by one vehicle

$$\sum_{i=0}^{N}\sum_{v=0}^{V}\sum_{t=0}^{T} X_{ijvt} = 1 \qquad j = 3, 4, 5, \cdots, J$$

Constraint 2: Conservation of Flow

$$\sum_{i=0}^{N} X_{ijvt} = \sum_{k=3}^{J} X_{jkv(t+1)} \qquad v = 0, 1, 2 \quad t = 0, 1, 2, \cdots T-1 \quad j = 3, 4, \cdots, J$$

Constraint 3: First node of each driver has an outflow of 1 in the first step, others are 0

$$\sum_{j=3}^{J}\sum_{t=0}^{T} X_{vjvt} = 1 \qquad V = 0.1.2$$

$$\sum_{j=3}^{J} X_{ijv0} = 0 \qquad i = 3.4.5.\cdots J \qquad V = 0.1.2$$

Constraint 4: Each vehicle can only takes at most one order in each step

$$\sum_{i=0}^{N}\sum_{j=3}^{J} X_{ijvt} \le 1 \qquad V = 0.1.2 \qquad t = 0,1,2,\cdots\cdots T$$

Constraint 5: Make sure driver goes to a restaurant before delivering food to customer

$$\sum_{i=0}^{N} X_{ijvt} \le \sum_{i=0}^{N}\sum_{t}^{T-1} X_{ij',vt} \qquad t = 0,1,2,\cdots T \qquad V = 0,1,2 \qquad j = 8.9.10.11.12 \quad (j \text{ is customer here})$$

j' here is the function to find restaurant of the customer whom the drive is delivering to

Constraint 6: Time to arrive at the location of the t^{th} step:

$t = 0$: $d_{v,t} \ge \sum_{i=0}^{N}\sum_{j=3}^{J} X_{ijv0} \times l_j \qquad V = 0,1,2 \qquad t = 0.$

$d_{v,t} \ge \sum_{i=0}^{N}\sum_{j=3}^{J} X_{ijvt} \times r_{ijv} \qquad V = 0.1.2 \qquad t = 0$

$t \ge 1$: $d_{v,t} \ge d_{v,t-1} + \sum_{i=0}^{N}\sum_{j=3}^{J} X_{ijvt} \times r_{ijv} \qquad V = 0,1,2 \qquad t = 1,2,\cdots T$

Constraint 7: If arriving at location j, ensures time is at least the min_arrival_time_mins

$$d_{v,t} \ge l_j' \times X_{ijvt} \qquad t = 0,1,2\cdots T \qquad i = 0,1,2,\cdots N \qquad j = 3.4.5,\cdots \qquad t = 0,1,\cdots T$$

Constraint 8: Ensures the average waiting time of the orders is lower than the maximum W

$j = 3.4.5.6.7$: $W_{jvt} = 0 \qquad V = 0.1.2 \qquad t = 0,1,2,\cdots T$

$j = 8.9.10,11.12$: $W_{jvt} = d_{v,t} - \sum_{i=0}^{N} l_{j'} \times X_{ijvt} \qquad V = 0,1,2 \qquad t = 0,1,2,\cdots T$

set $M = 100000$

$t = 0$: $Z_{v,t} = 0$

$t = 1$: $W_{jvt} - M(1 - X_{ijvt}) \le Z_{v,t} \le W_{jvt} + M(1 - X_{ijvt})$

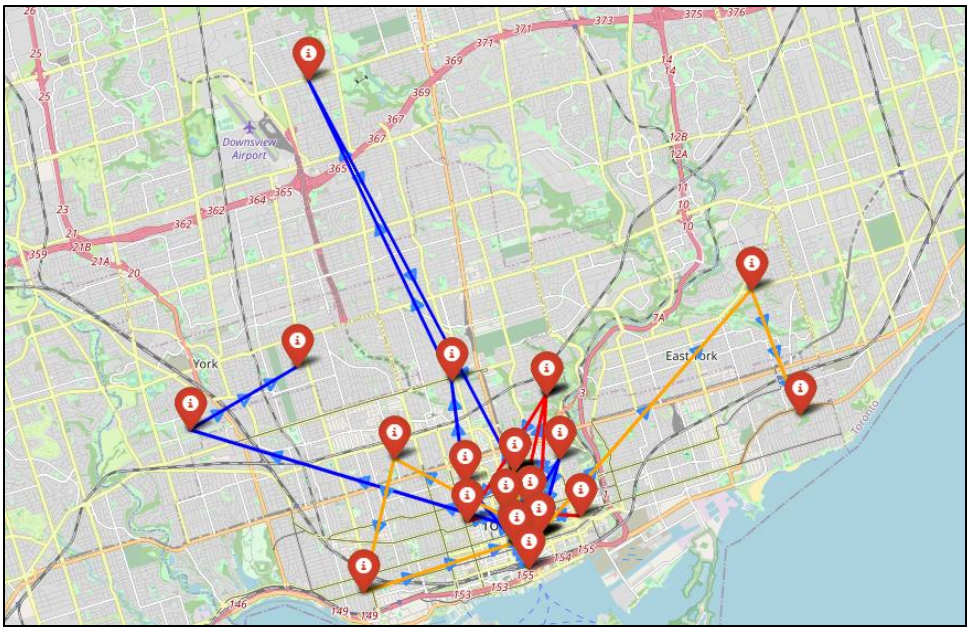
$t = 2\cdots T$: $Z_{v,t-1} + W_{jvt} + M(1 - X_{ijvt}) \le Z_{v,t} \le Z_{v,t-1} + W_{jvt} - M(1 - X_{ijvt})$

$i = 0,1,2,\cdots N$

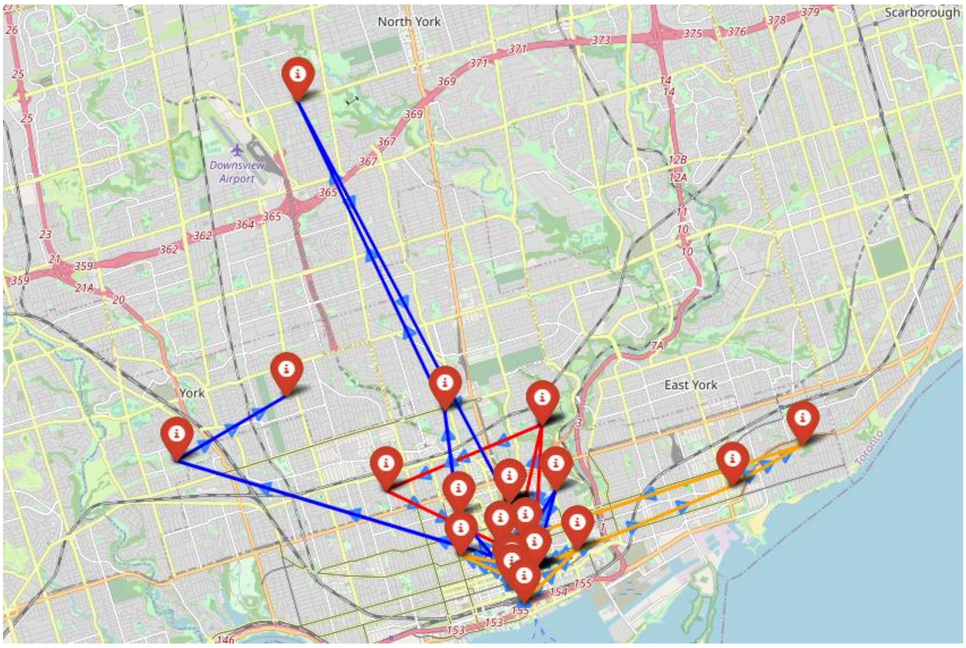
$j = 3.4.5,\cdots J$

$V = 0.1.2$

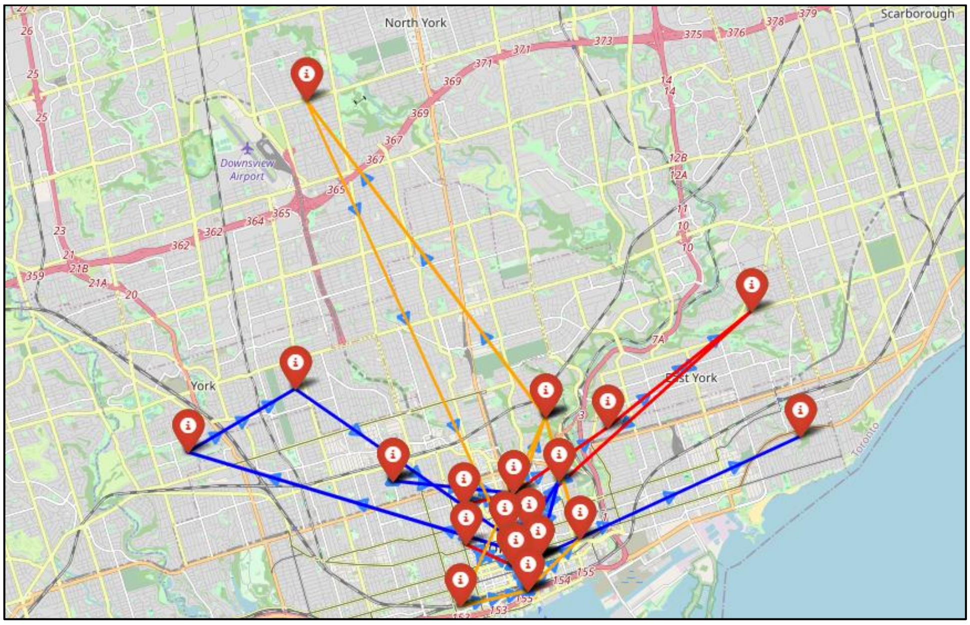
$$\sum_{v=0}^{V} Z_{v,t} \le W \times (\text{number of customers}) \qquad t = 0.1.2,\cdots T$$



Part4 Route Map (Clustering with Angle): 42kms



Part4 Route Map (Clustering with Angle and centroid of restaurants): 41kms



Part4 Route Map (Clustering with Radial Distance): 26kms

Rotman

Final Page

Grade: ____________