

- **Uninformed search** 无信息的搜索：除了问题中提供的定义之外没有任何关于状态的附加信息。
- **Informed search** 有信息的搜索：在问题本身的定义之外还可利用问题的特定知识。

Review: Last Class

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```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
  fringe ← INSERT(MAKE-NODE(INTIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT (fringe)
    if GOAL-TEST[problem] applied to STATE(node) succeeds
      return node
    fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

A strategy is defined by picking the **order of node expansion**

Variety of **uninformed** search strategies

Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

Uninformed Search Strategies

3

Uninformed search strategies use only the information available in the problem definition

- Breadth-first search （广度优先搜索）
- Uniform-cost search （代价一致搜索）
- Depth-first search （深度优先搜索）
- Depth-limited search （深度有限搜索）
- Iterative deepening search （迭代深入深度优先搜索）

Uninformed Search Strategies

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- Summary of uninformed tree (problem-solving) search:
 - ▣ Algorithms differ by method of *expansion*, or choice of *state-action* sequence for offline evaluation
 - ▣ Complexity tradeoffs, but poor (worst case or typical case) performance from all when state space is large

5 Informed Search Algorithms

Chapter 4



Outline

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- **Best-first search** (最佳优先搜索)
 - ▣ Greedy best-first search
 - ▣ A* search
- Heuristics
- Local search algorithms
 - ▣ Hill-climbing search
 - ▣ Simulated annealing search
 - ▣ Local beam search
 - ▣ Genetic algorithms

Best-First Search

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Idea: use an **evaluation function** (评价函数) $f(n)$ for each node

— estimate of “desirability”

⇒ Expand most desirable unexpanded node

Implementation:

fringe is a queue sorted in decreasing order of desirability

— priority queue (优先级队列)

Special cases:

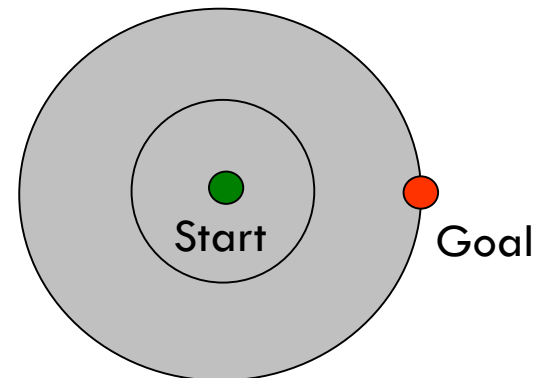
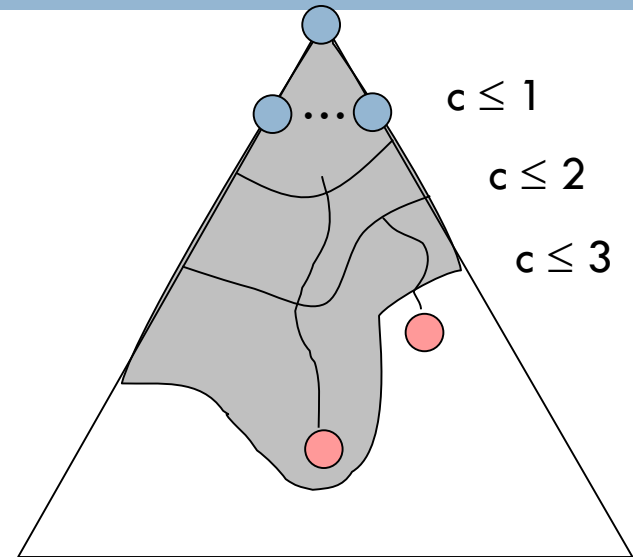
greedy search

A* search

Uniform Cost

8

- Strategy: expand lowest path cost
- The good: UCS is complete and optimal!
- The bad:
 - ▣ Explores options in every “direction”
 - ▣ No information about goal location



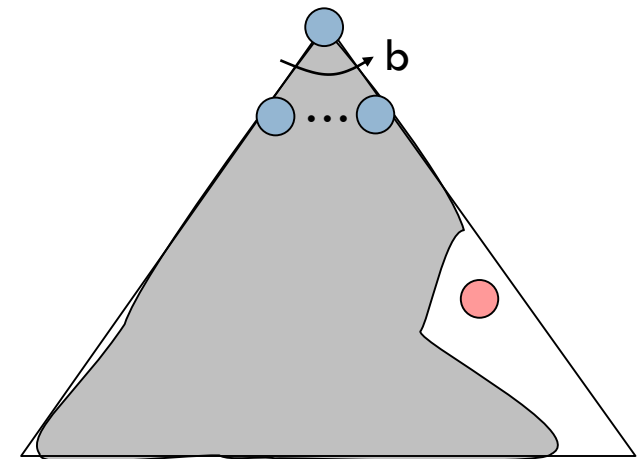
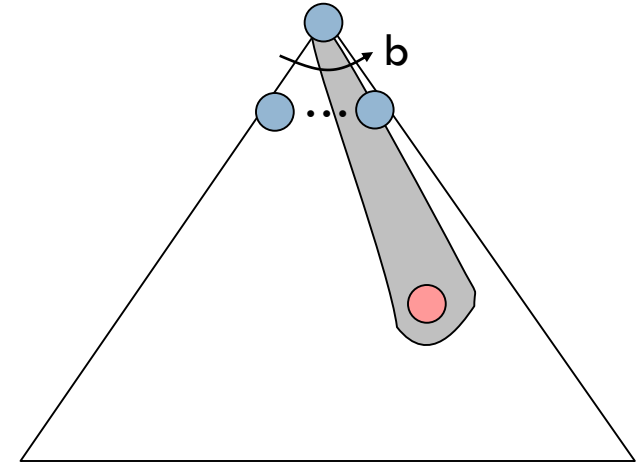
Best First

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- Strategy: expand nodes which appear closest to goal
 - ▣ Heuristic (启发函数) : function which maps states to distance

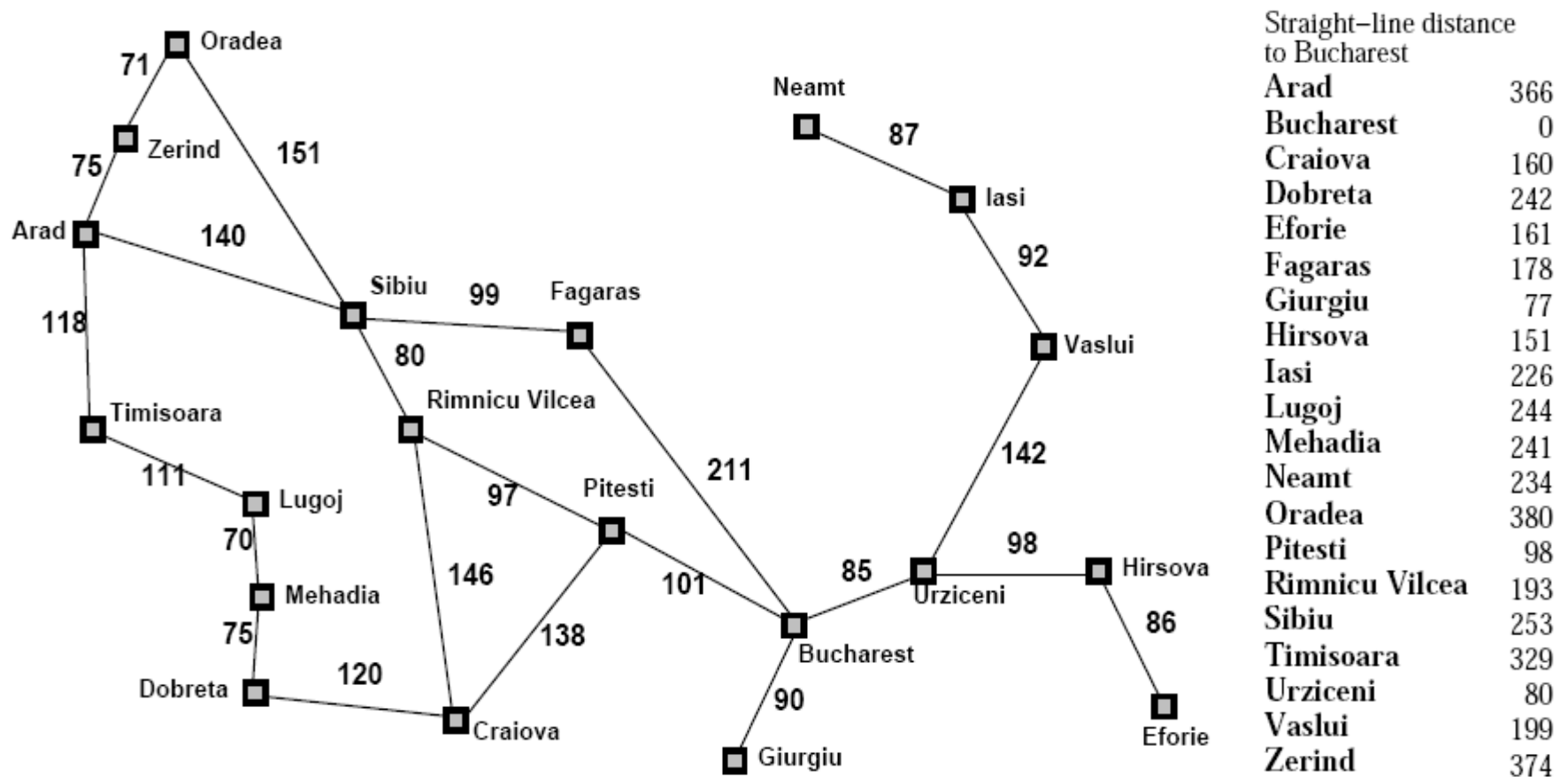
- A common case:
 - ▣ Best-first takes you straight to the (wrong) goal

- Worst-case: like a badly-guided DFS



Romania with Step Costs in Km

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Greedy Search

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Evaluation function $f(n) = h(n)$ (**heuristic function** 启发函数)

= estimate of cost from n to the closest goal

(节点 n 到目标节点的最低耗散路径的耗散估计值)

- ▣ $h(n)$ a problem-specific function
- ▣ $h(n) = 0$ if n is goal node

E.g., $h_{\text{SLD}}(n)$ = straight-line distance from n to Bucharest

Greedy search expands the node that **appears** to be closest to goal (试图扩展离目标节点最近的点)

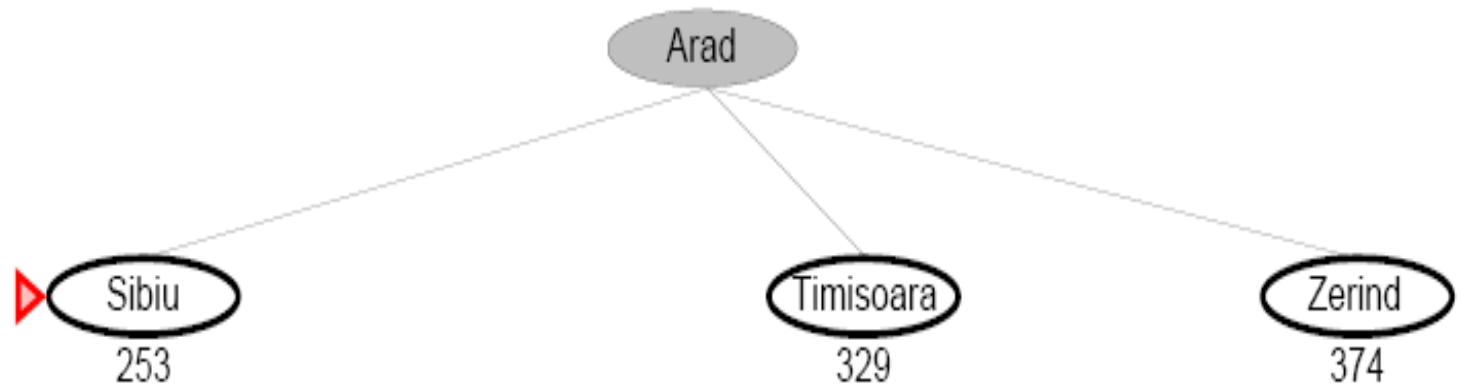
Greedy Search Example

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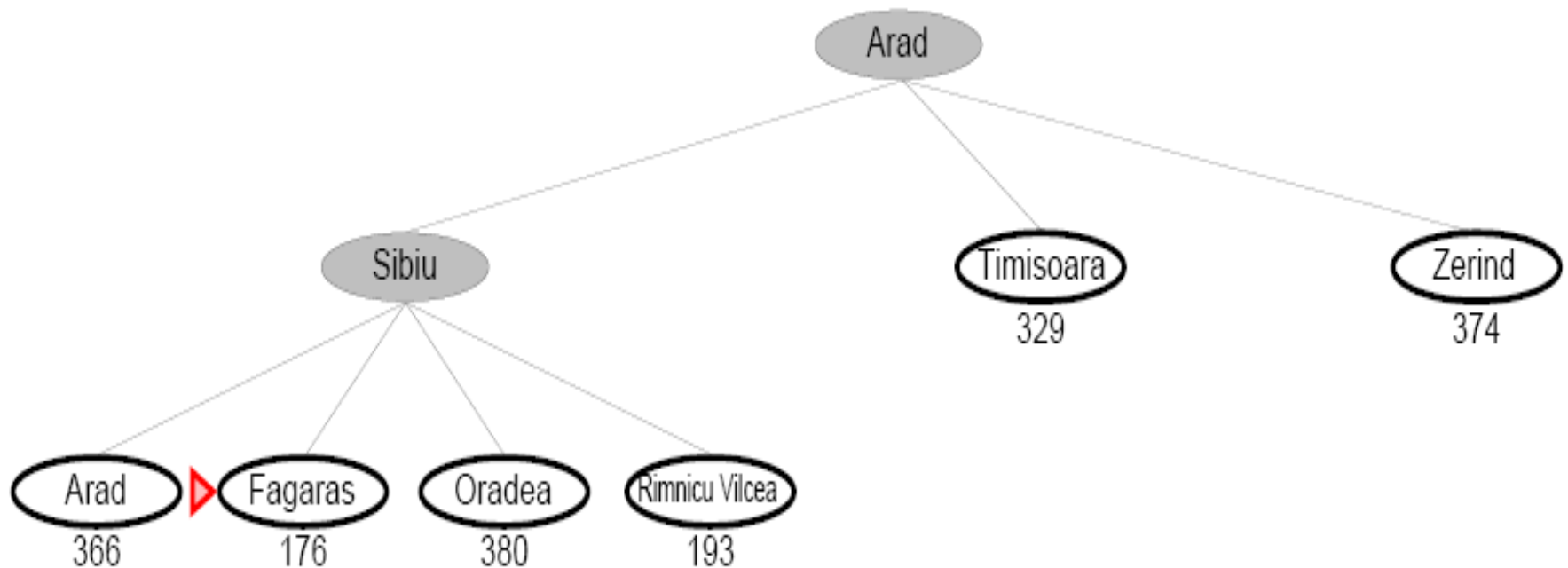
Greedy Search Example

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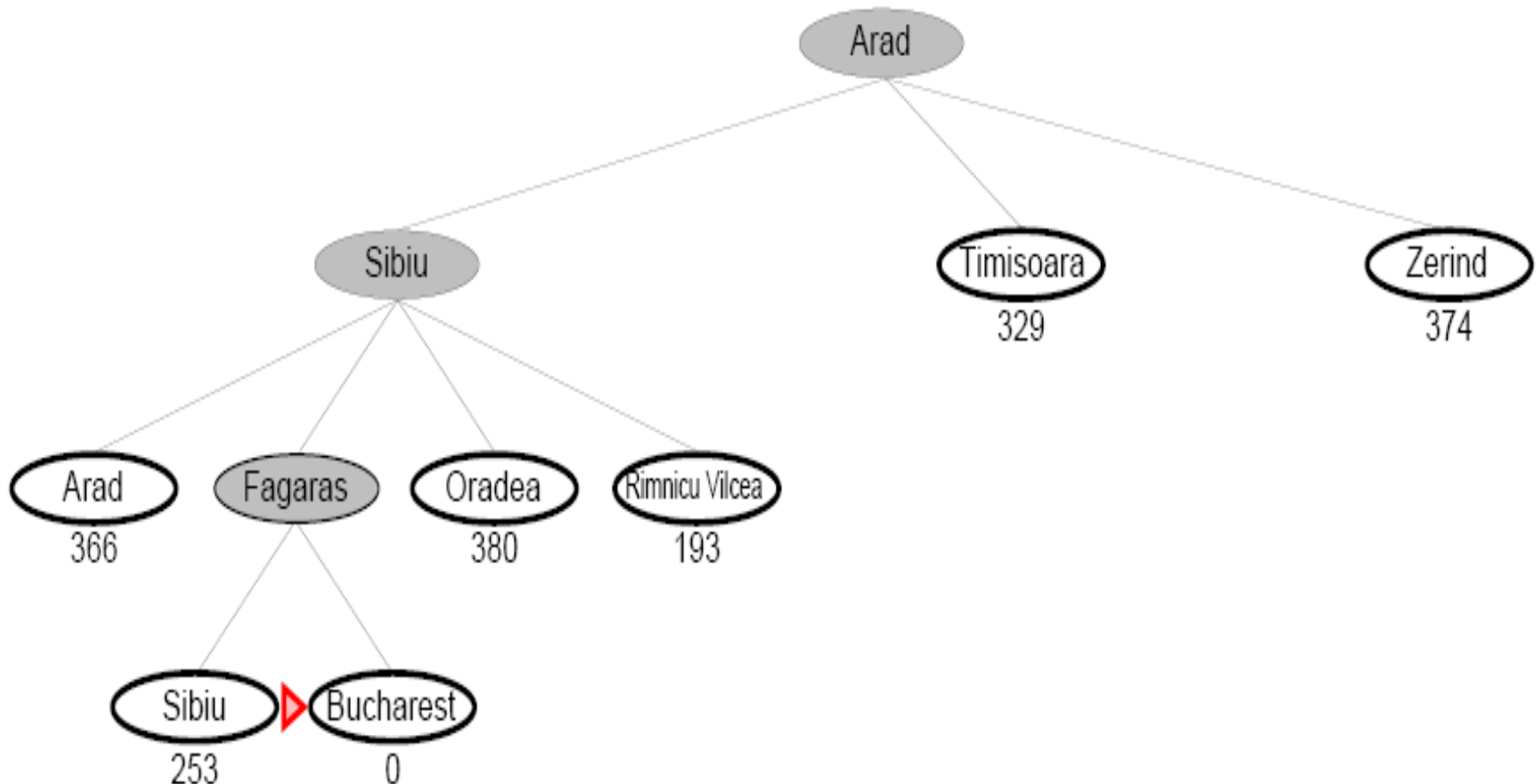
Greedy Search Example

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Greedy Search Example

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Properties of Greedy Search

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Complete??

Properties of Greedy Search

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Complete?? No — can get stuck in loops, e.g., with Oradea as goal,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time??

Properties of Greedy Search

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Complete?? No — can get stuck in loops, e.g., with Oradea as goal,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

b: Branching factor

d: Solution depth

m: Maximum depth

Properties of Greedy Search

19

Complete?? No — can get stuck in loops, e.g., with Oradea as goal,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$ — keeps all nodes in memory

b: Branching factor
d: Solution depth
m: Maximum depth

Optimal??

Properties of Greedy Search

20

Complete?? No — can get stuck in loops, e.g., with Oradea as goal,

Iasi \rightarrow Neamt \rightarrow Iasi \rightarrow Neamt \rightarrow

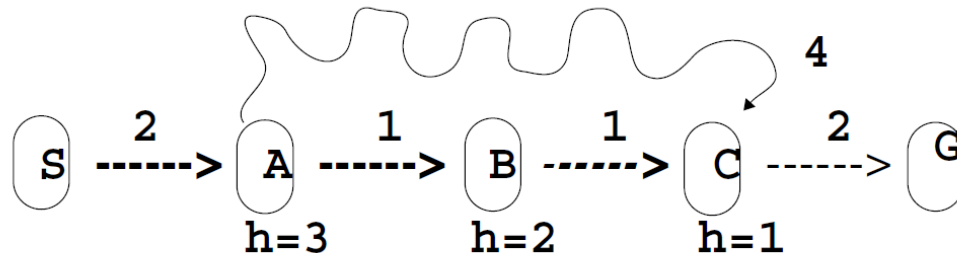
Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$ — keeps all nodes in memory

b: Branching factor
d: Solution depth
m: Maximum depth

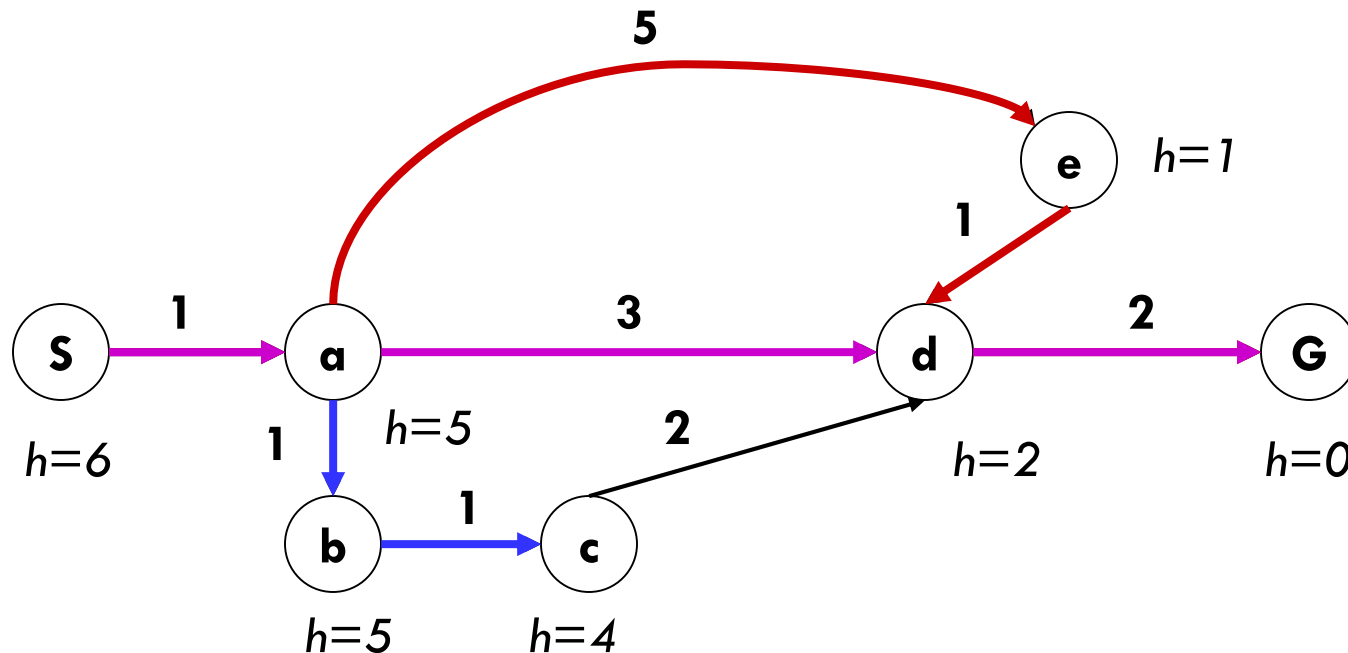
Optimal?? No



Combining UCS and Greedy

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- **Uniform-cost** orders by path cost, or *backward cost* $g(n)$
- **Greedy-search** orders by goal proximity, or *forward cost* $h(n)$



- **A* Search** orders by the sum: $f(n) = g(n) + h(n)$

A* Search

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Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = cost so far to reach n — 到达节点 n 的耗散

$h(n)$ = estimated cost to goal from n

— 启发函数：从节点 n 到目标节点的最低耗散路径的耗散估计值

$f(n)$ = estimated total cost of path through n to goal

— 经过节点 n 的最低耗散的估计函数

A* search uses an **admissible** heuristic 可采纳启发式

i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from n .

(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G .)



Theorem: A* search is optimal

E.g., $h_{\text{SLD}}(n)$ never overestimates the actual road distance (**SLD: Straight-Line Distance**)

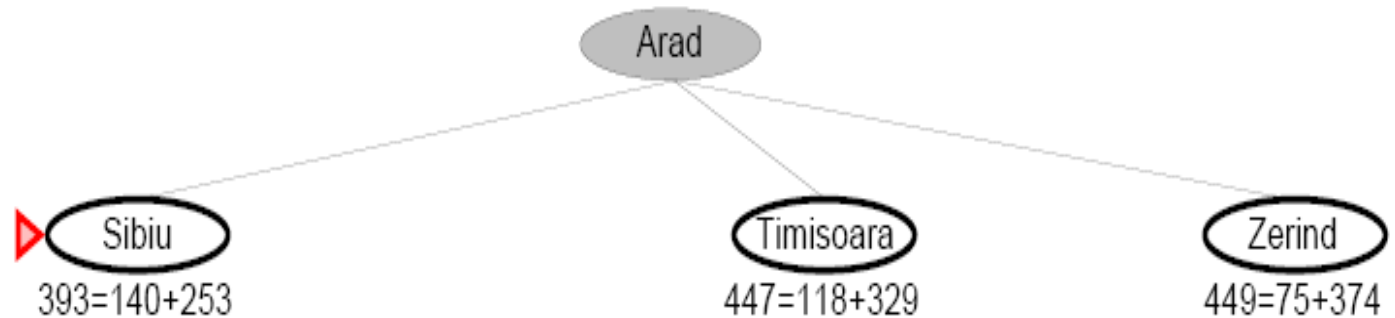
A* Search Example

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▶ Arad
 $366 = 0 + 366$

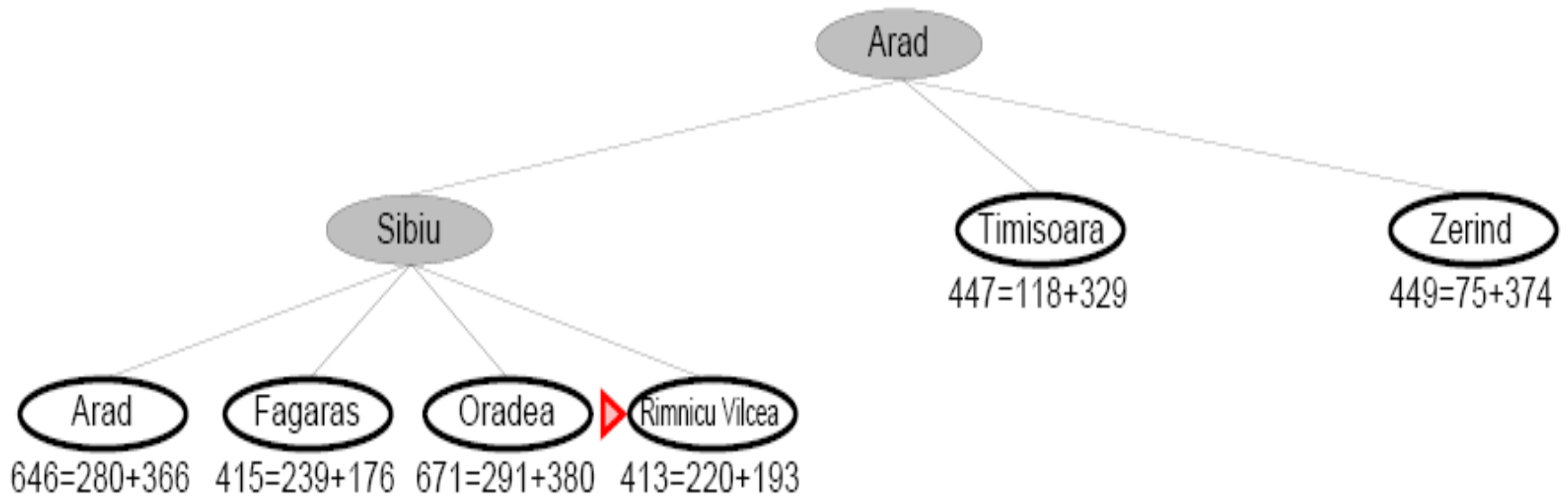
A* Search Example

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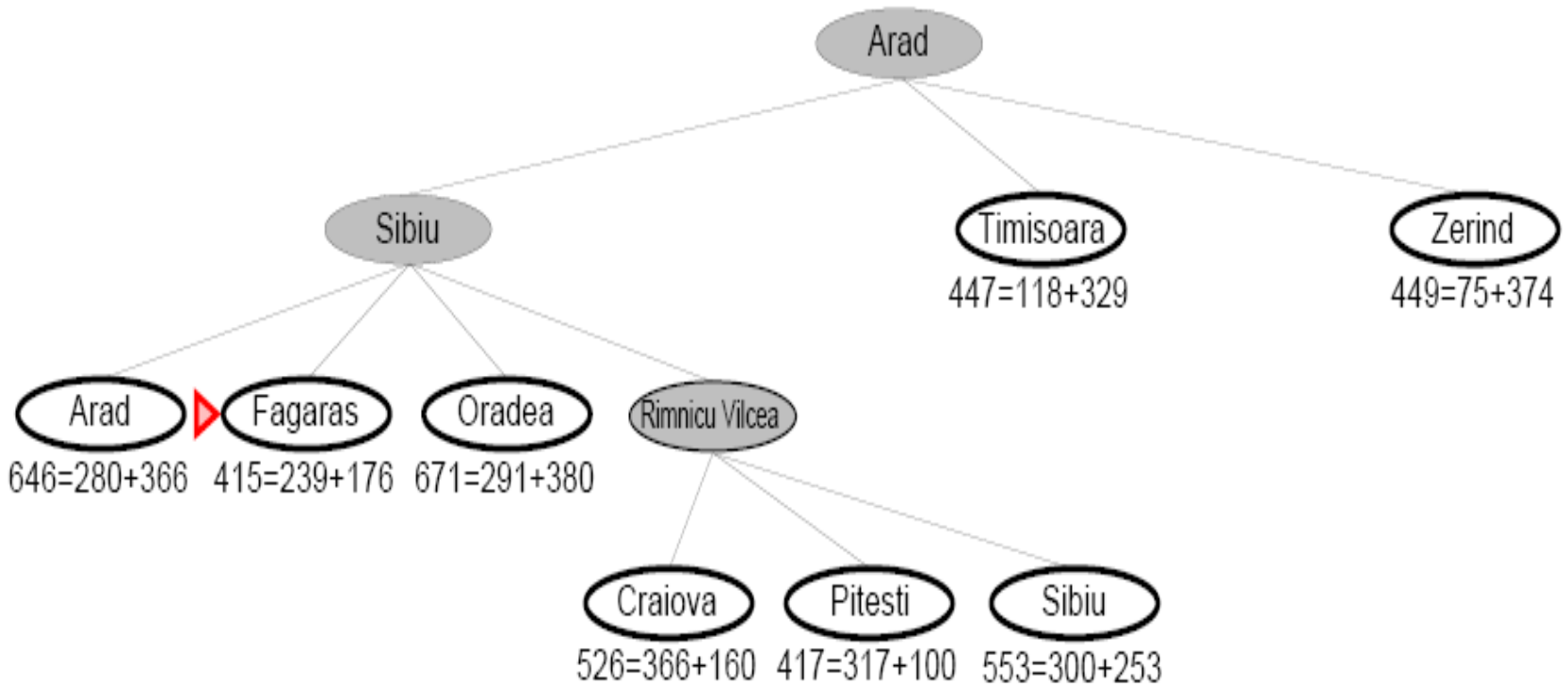
A* Search Example

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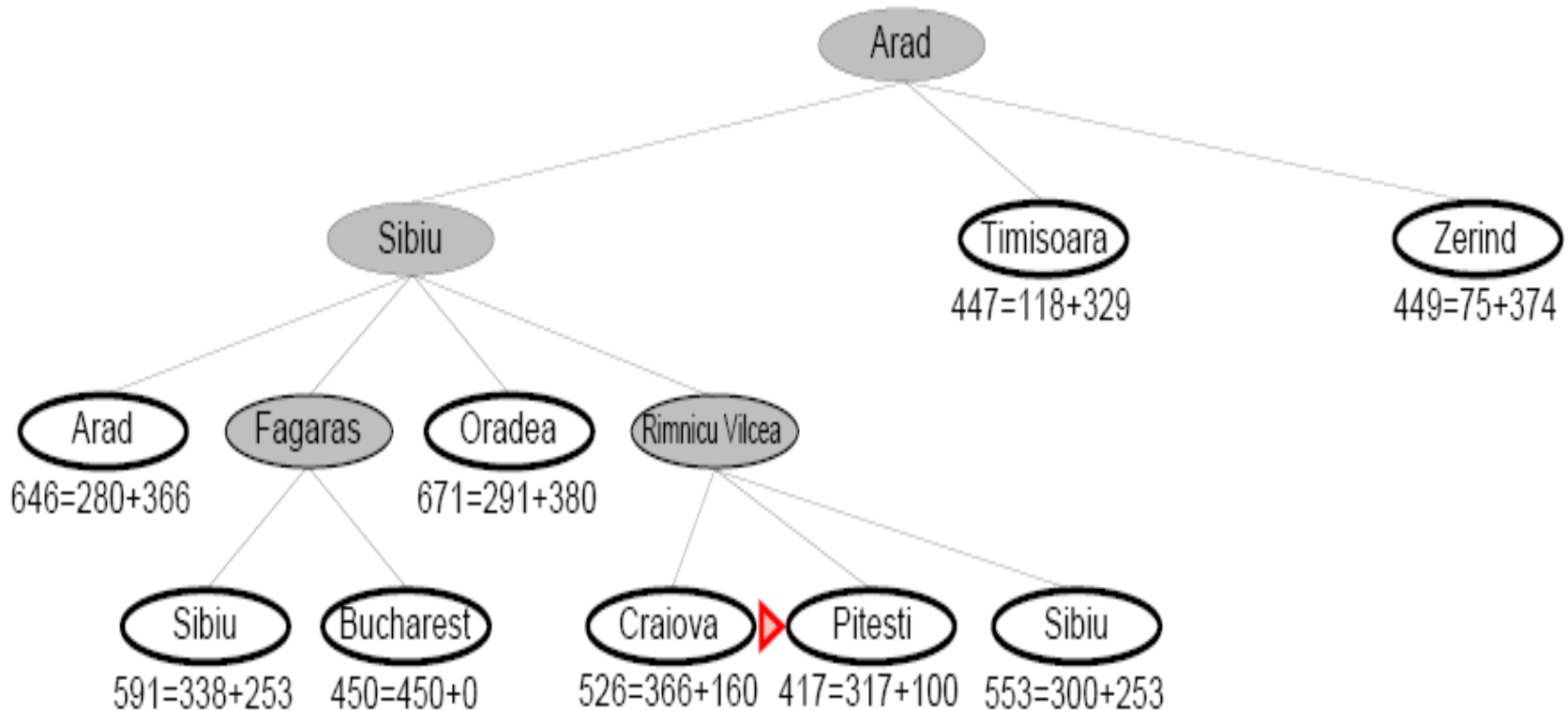
A* Search Example

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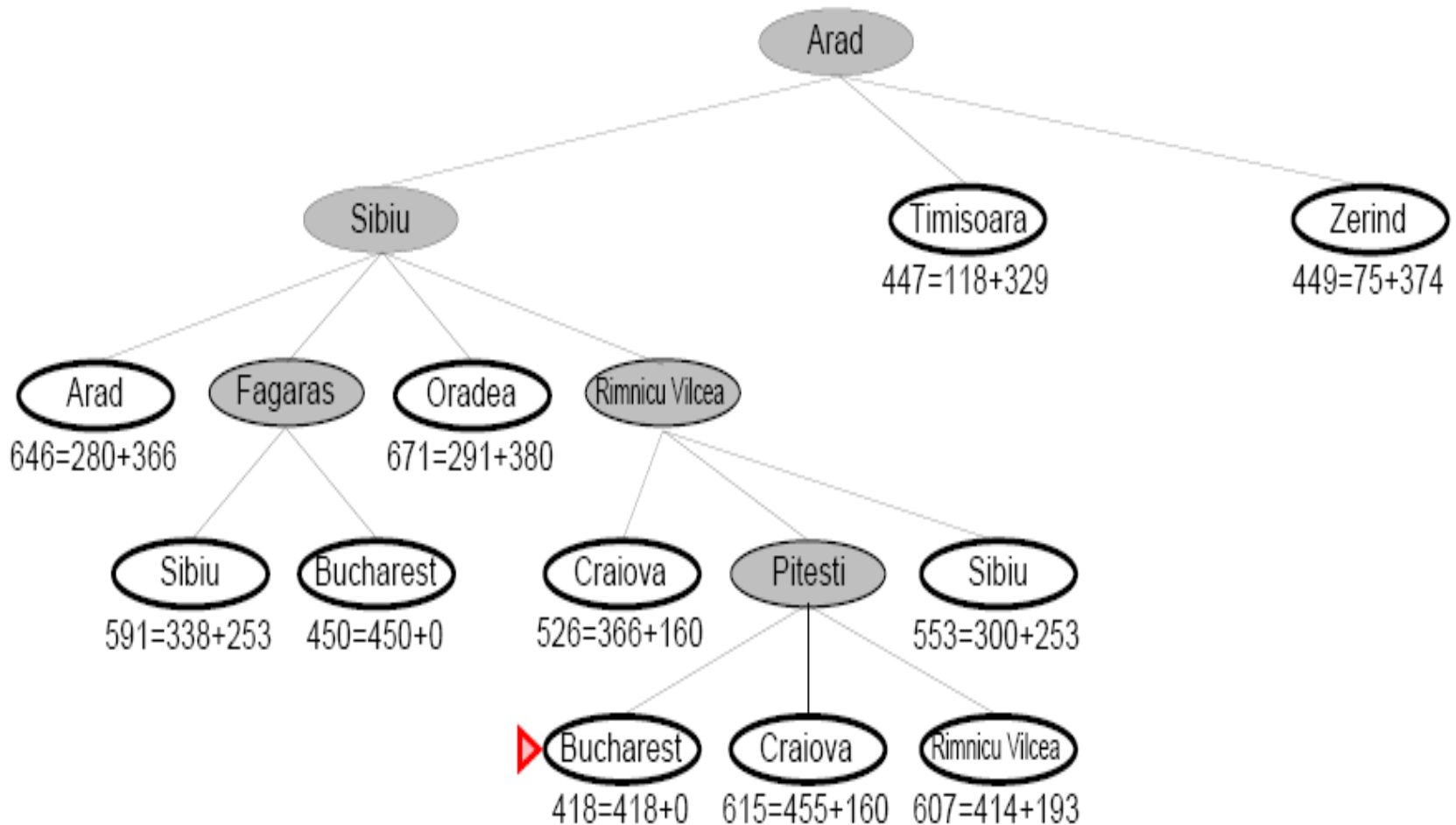
A* Search Example

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A* Search Example

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Admissible Heuristics

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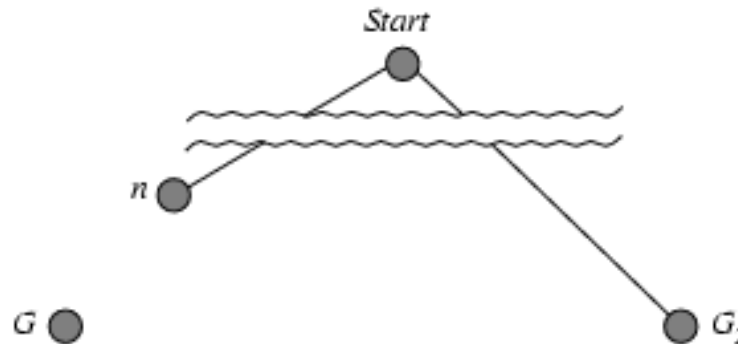
- A* heuristic $h(n)$ is **admissible** (可采纳的) if for every node n , $h(n) \leq h^*(n)$, where $h^*(n)$ is the **true** cost to reach the goal state from n .
- An admissible heuristic **never overestimates** the cost to reach the goal (从不会过高估计到达目标的耗散), i.e., it is **optimistic** (乐观的)
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- **Theorem:** If $h(n)$ is admissible, A* using TREE-SEARCH is optimal

Coming up with admissible heuristics is most of what's involved in using A* in practice

Optimality of A^* (proof)

30

Suppose some suboptimal (非最优) goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G .

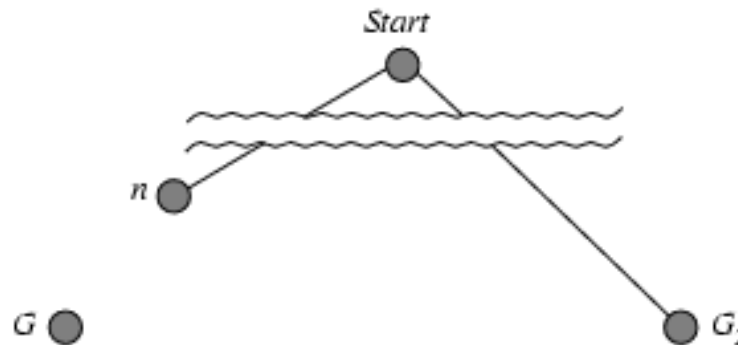


- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

Optimality of A^* (proof)

31

Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G .



- $f(G_2) > f(G)$ from above
- $h(n) \leq h^*(n)$ since h is admissible
- $g(n) + h(n) \leq g(n) + h^*(n) = g(G) = f(G)$ n is on a shortest path to an optimal goal G
- $f(n) \leq f(G)$

Hence $f(G_2) > f(n)$, and A^* will never select G_2 for expansion

Consistent Heuristics

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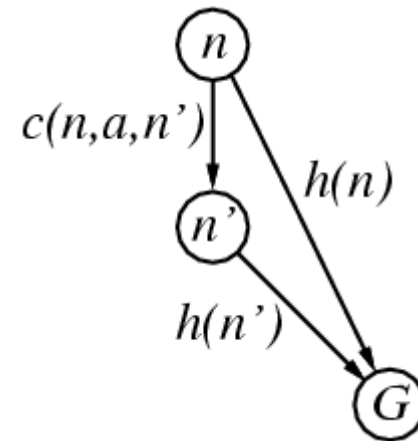
- A* heuristic is **consistent** (一致) if for every node n , every successor n' of n generated by any action a ,

$$h(n) \leq c(n,a,n') + h(n')$$

- If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

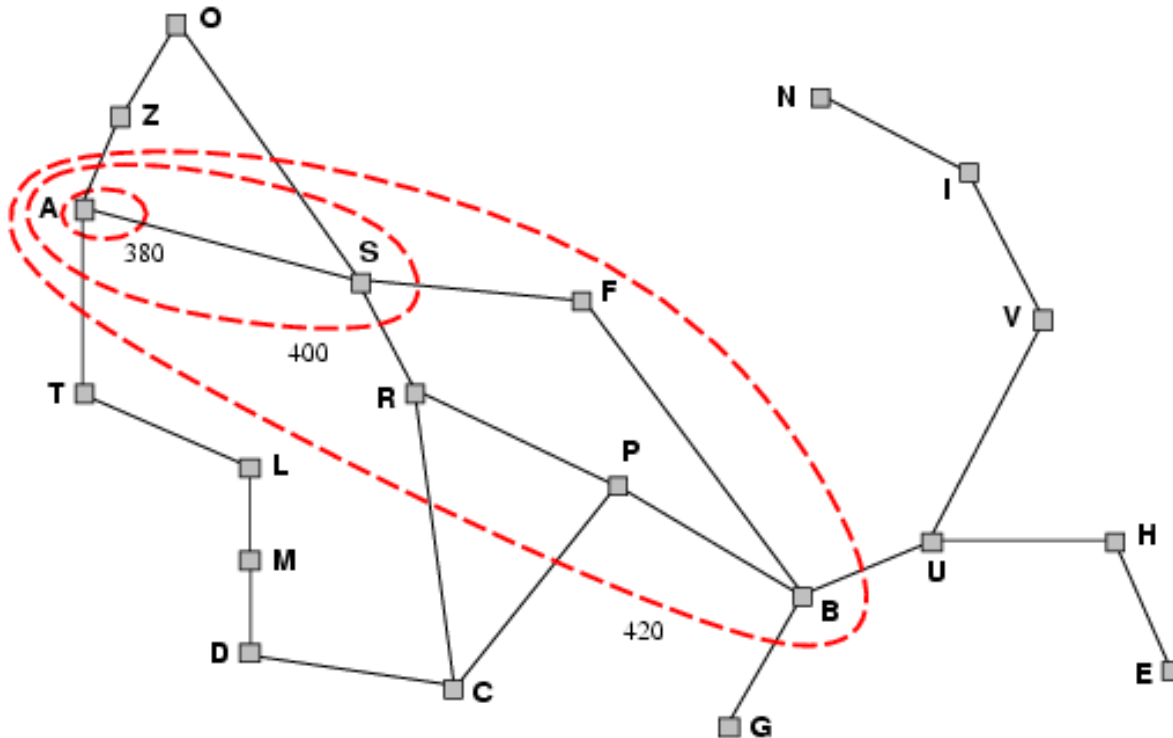
- i.e., $f(n)$ is non-decreasing along any path.
- **Theorem:** If $h(n)$ is consistent, A* using GRAPH-SEARCH is optimal



Optimality of A^*

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- A^* expands nodes in order of increasing f value
- Gradually adds " f -contours (等值线)" of nodes
- Contour i has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Properties of A*

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- Complete? Yes (unless there are infinitely many nodes with $f \leq f(G)$)
- Time? A*算法对于任何给定的启发函数都是效率最优的
But still exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

A* expands all nodes with $f(n) < C^*$

A* expands some nodes with $f(n) = C^*$

A* expands no nodes with $f(n) > C^*$

8-puzzle Revisited

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- **8-puzzle** — 把棋子水平或者竖直地滑动到空格中，直到目标局面：

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- 平均解步数是**22**步。分支因子约为3
 - ▣ 到达深度为22的穷举搜索将考虑约 $3^{22} \approx 3.1 \times 10^{10}$
 - ▣ 状态个数 $O((n+1)!)$ ，NP完全问题

Admissible Heuristics

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E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles (错位的棋子数)
- $h_2(n)$ = total Manhattan distance (所有棋子到其目标位置的水平和垂直距离和)
(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- $h_1(S) = ?$
- $h_2(S) = ?$

Admissible Heuristics

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E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles (错位的棋子数)
- $h_2(n)$ = total Manhattan distance (所有棋子到其目标位置的水平和垂直距离和)
(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- $h_1(S) = ?$ 8
- $h_2(S) = ?$ $3+1+2+2+2+3+3+2 = 18$

Dominance

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- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 **dominates** h_1 (dominate 统治、占优)
- h_2 is better for search
- Typical search costs (average number of nodes expanded):
 - $d=12$
IDS = 3,644,035 nodes
 $A^*(h_1) = 227$ nodes
 $A^*(h_2) = 73$ nodes
 - $d=24$
IDS = too many nodes
 $A^*(h_1) = 39,135$ nodes
 $A^*(h_2) = 1,641$ nodes

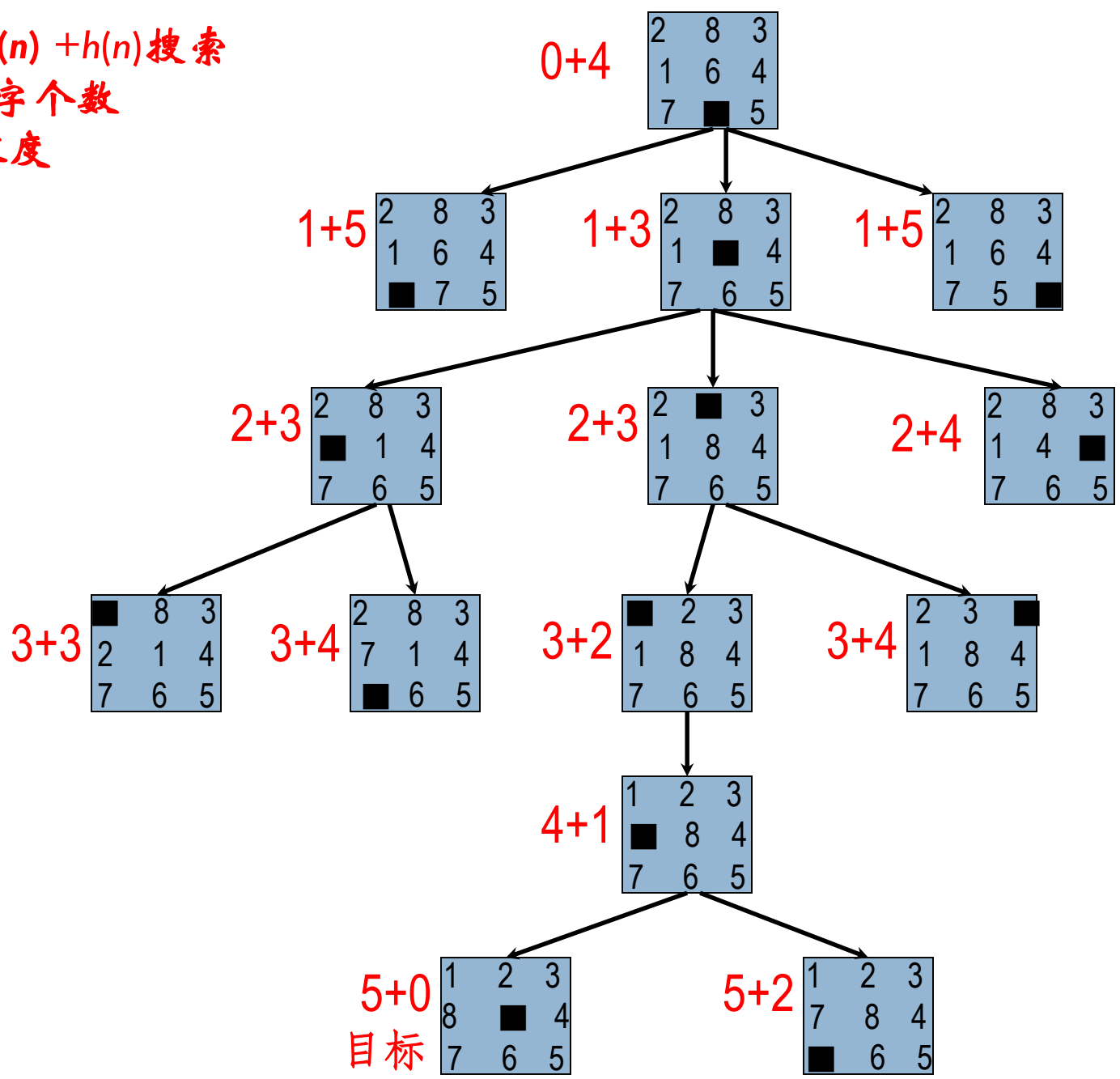
Given any admissible heuristics h_a, h_b ,

$$h(n) = \max(h_a(n); h_b(n))$$

is also admissible and dominates h_a, h_b

With A^* : a trade-off between quality of estimate and work per node!

使用 $f(n) = g(n) + h(n)$ 搜索
 $h(n)$ = 错放数字个数
 $g(n)$ = 节点深度



Most of the work is in coming up with admissible heuristics

How do we get good heuristics?

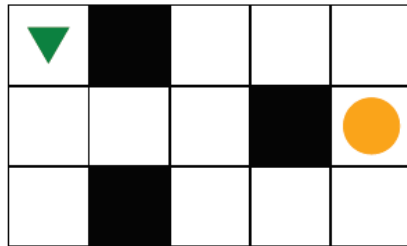
Just relax...



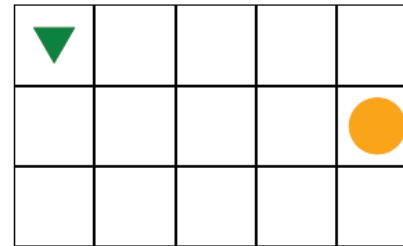
Relaxation

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Goal: move from triangle to circle



Hard



Easy

Heuristic:

$$h(s) = \text{ManhattanDistance}(s, (2, 5))$$

□ Interpretation: relax → removing constraints

Relaxed Problems

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- A problem with fewer restrictions on the actions is called a **relaxed problem** (松弛问题)
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
一个松弛问题的最优解的耗散是原问题的一个可采纳的启发式
- If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution
如果棋子可以任意移动，则 h_1 给出最短的确切步数
- If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution
如果棋子可以移动到任意相邻的位置，则 h_2 给出最短的确切步数

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Relaxed Problems

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□ 构造松弛问题

- 原问题：一个棋子可以从方格A移动到方格B，如果A与B水平或者垂直相邻**而且**B是空的
- 松弛1：一个棋子可以从方格A移动到方格B，如果A与B相邻 — h_2
- 松弛2：一个棋子可以从方格A移动到方格B，如果B是空的
- 松弛3：一个棋子可以从方格A移动到方格B — h_1

□ 如果有一个可采纳启发式的集合 $\{h_1, \dots, h_m\}$

$$h(n) = \max(h_1(n), \dots, h_m(n))$$

可采纳并比成员启发式更有优势

Evaluation Function $f(n)$

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$h(n)$ — heuristic, estimate of cost from n to the closest goal
(节点 n 到目标节点的最低耗散路径的耗散估计值)

$g(n)$ — path cost to n (初始节点到这个节点的路径损耗的总和)

Possible evaluation functions:

- $f(n) = g(n)$ \equiv **Uniform Cost**
- $f(n) = h(n)$ \equiv **Greedy**
- $f(n) = g(n) + h(n)$ \equiv **A***
*estimates the total cost of a solution path
which goes through node n*

Informed (Heuristic) Search

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- Summary of uninformed tree (problem-solving) search:
 - ▣ Algorithms differ by method of *expansion*, or *choice of state-action* sequence for offline evaluation
 - ▣ Complexity tradeoffs, but poor (worst case or typical case) performance from all when state space is large
- Improvement:
 - ▣ Exploit knowledge about which successor states are preferable
 - ▣ “Best-First” search: expand nodes based on evaluation function $f(n)$
 - ▣ $f(n) \sim$ estimate of distance to goal
 - ▣ expand node n with lowest evaluation
 - ▣ “Best-First” algorithms are characterized by choice of evaluation function

Outline

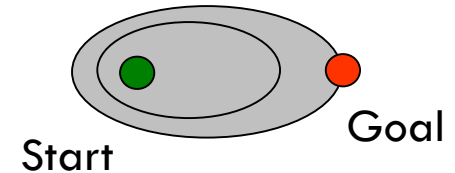
46

- Best-first search (最佳优先搜索)
 - ▣ Greedy best-first search
 - ▣ A* search
- Heuristics
- Local search algorithms (局部搜索算法)
 - ▣ Hill-climbing search (爬山法搜索)
 - ▣ Simulated annealing search (模拟退火搜索)
 - ▣ Local beam search (局部剪枝搜索)
 - ▣ Genetic algorithms (遗传算法)

Large Scale Problems with A*

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- What states get expanded?
 - ▣ All states with f-cost less than optimal goal cost
- In huge problems, often A* isn't enough
 - ▣ State space just too big
 - ▣ Can't visit all states with f less than optimal
 - ▣ Often, can't even store the entire fringe
- Solutions
 - ▣ Better heuristics
 - ▣ Greedy hill-climbing (fringe size = 1)
 - ▣ Beam search (limited fringe size)



Limited Memory Options

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- Bottleneck: not enough memory to store entire fringe
- Hill-Climbing Search:
 - ▣ Only “best” node kept around, no fringe!
 - ▣ Usually prioritize successor choice by h (greedy hill climbing)
 - ▣ Compare to greedy backtracking, which still has fringe
- Beam Search (Limited Memory Search)
 - ▣ In between: keep K nodes in fringe
 - ▣ Dump lowest priority nodes as needed
 - ▣ Can prioritize by h alone (greedy beam search), or $h+g$ (limited memory A^*)
- No guarantees once you limit the fringe size!

Local Search Algorithms

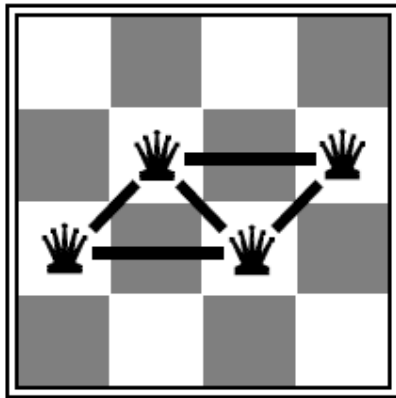
49

- In many optimization problems, the **path** to the goal is irrelevant; the goal state itself is the solution
- State space = set of “complete” configurations (完全状态)
 - ▣ Find configuration satisfying constraints, e.g., n-queens
- In such cases, we can use **local search algorithms**
- keep a single “current” state, try to improve it
 - ▣ until you can’t make it better
- Constant space, suitable for online as well as offline search
- Generally much more efficient (but incomplete)

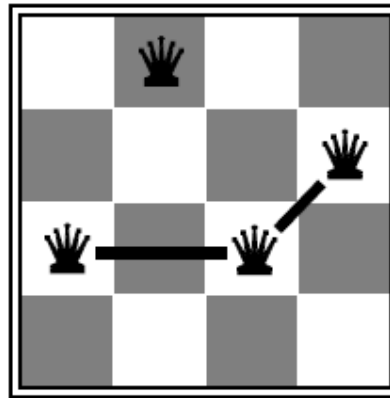
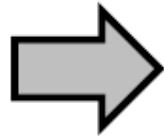
Example: n -Queens

50

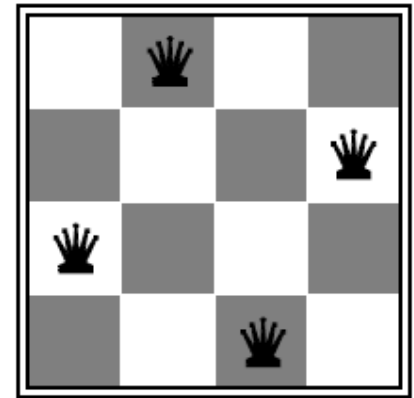
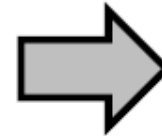
- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal



$h = 5$



$h = 2$



$h = 0$

Optimization Problems

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- Now a different setting:
 - ▣ Each state s has a score or cost, $f(s)$, that we can compute
 - ▣ The goal is to find the state with the highest (or lowest) score, or a reasonably high (low) score
 - ▣ We do not care about the path
 - ▣ This is an optimization problem
 - ▣ Enumerating the states is intractable
 - ▣ Previous search algorithms are too expensive
 - ▣ No known algorithm for finding optimal solution efficiently

Hill-Climbing Search

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- Simple, general idea:
 - ▣ Start wherever
 - ▣ Always choose the best neighbor
 - ▣ If no neighbors have better scores than current, quit
- Why can this be a terrible idea?
 - ▣ Complete?
 - ▣ Optimal?
- What's good about it?

Hill-Climbing Search

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- “Like climbing Everest in thick fog with amnesia (健忘症)”
 - ▣ Can’t see the entire landscape all at once

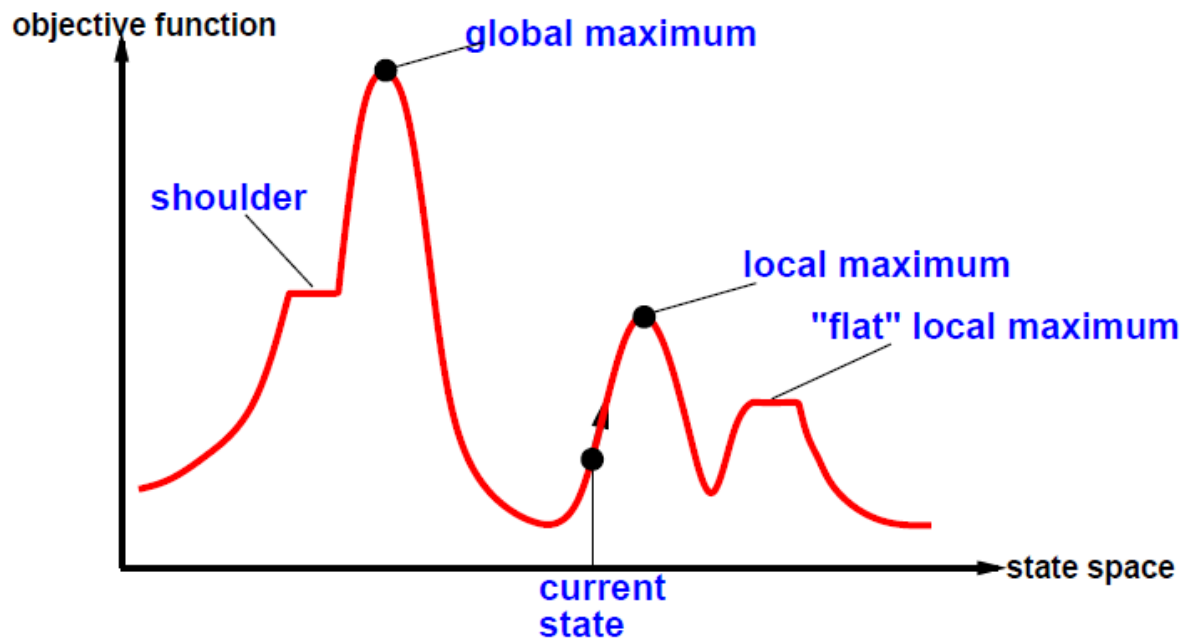
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  inputs: problem, a problem
  local variables: current, a node
                  neighbor, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
    neighbor ← a highest-valued successor of current
    if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
    current ← neighbor
```

Hill-Climbing Search

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- Problem: depending on initial state, can get stuck in local maxima (局部最大值)



Random-restart hill climbing overcomes local maxima — trivially complete
Random sideways moves escape from shoulders 😊 loop on at maxima ☹️

Hill-Climbing with Random Restarts

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□ Very simple modification:

1. When stuck, pick a random new starting state and re-run hill-climbing from there
2. Repeat this k times
3. Return the best of the k local optima

- ▣ Can be very effective
- ▣ Should be tried whenever hill-climbing is used
- ▣ Fast, easy to implement; works well for many applications where the solution space surface is not too “bumpy” (i.e., not too many local maxima)

Hill-Climbing Search: 8-Queens Problem

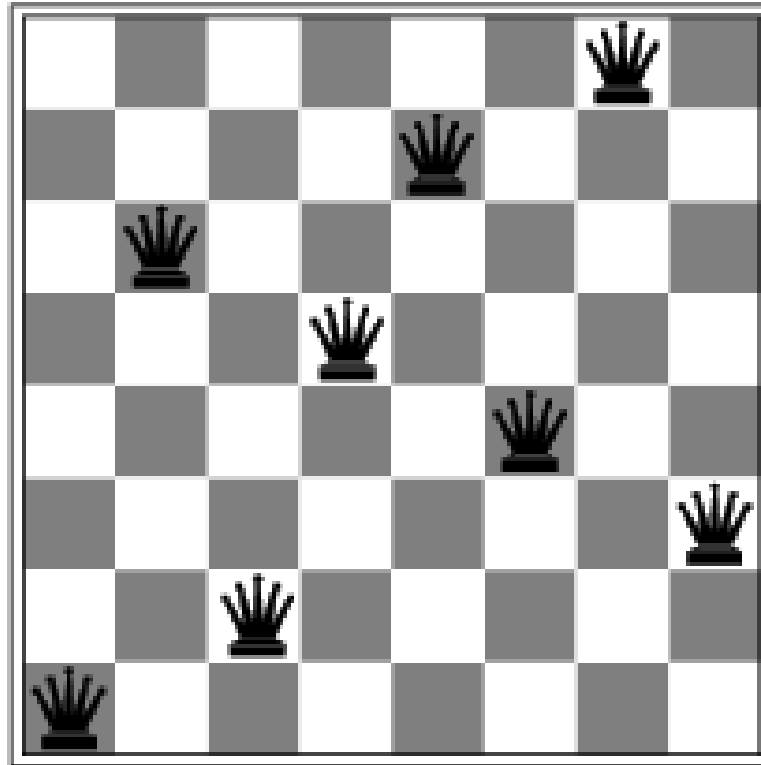
56

18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	♚	13	16	13	16
♚	14	17	15	♚	14	16	16
17	♚	16	18	15	♚	15	♚
18	14	♚	15	15	14	♚	16
14	14	13	17	12	14	12	18

- h = number of pairs of queens that are attacking each other, either directly or indirectly
- $h = 17$ for the above state

Hill-Climbing Search: 8-Queens Problem

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- A local minimum with $h = 1$

Life Lesson

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Sometimes one needs to temporarily step backward in order to move forward

- Lesson applied to iterative, local search:
 - Sometimes one needs to move to an *inferior neighbor* in order to escape a local optimum

Simulated Annealing Search

(模拟退火搜索)

59

- Idea: escape local maxima by allowing some "bad" moves but **gradually decrease** their frequency

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
           schedule, a mapping from time to "temperature"
  local variables: current, a node
                   next, a node
                   T, a "temperature" controlling prob. of downward steps

  current ← MAKE-NODE(INITIAL-STATE[problem])
  for t ← 1 to  $\infty$  do
    T ← schedule[t]
    if T = 0 then return current
    next ← a randomly selected successor of current
     $\Delta E$  ← VALUE[next] - VALUE[current]
    if  $\Delta E > 0$  then current ← next
    else current ← next only with probability  $e^{\Delta E/T}$ 
```

Properties of Simulated Annealing Search

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- One can prove: If T decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1
- Widely used in VLSI layout, airline scheduling, etc

Local Beam Search (局部剪枝搜索)

61

Keep track of k states rather than just one

Start with k randomly generated states

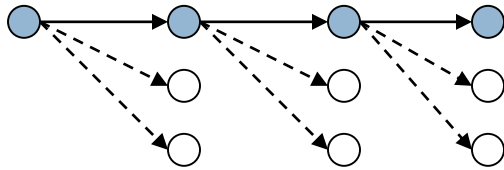
At each iteration, all the successors of all k states are generated

If any one is a goal state, stop; else select the k best successors from the complete list and repeat.

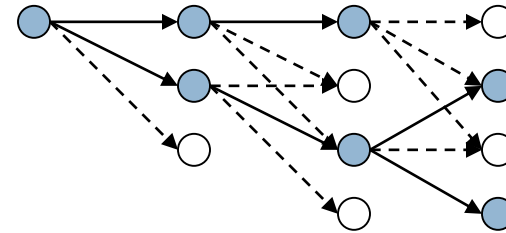
Local Beam Search

62

- Like greedy search, but keep K states at all times:



Greedy Search



Beam Search

- The best choice in MANY practical settings
- Not the same as k searches run in parallel!
- Searches that find good states recruit other searches to join them
- Variables: beam size, encourage diversity?
- **Problem:** quite often, all k states end up on same local hill
- **Idea:** choose k successors randomly, biased towards good ones

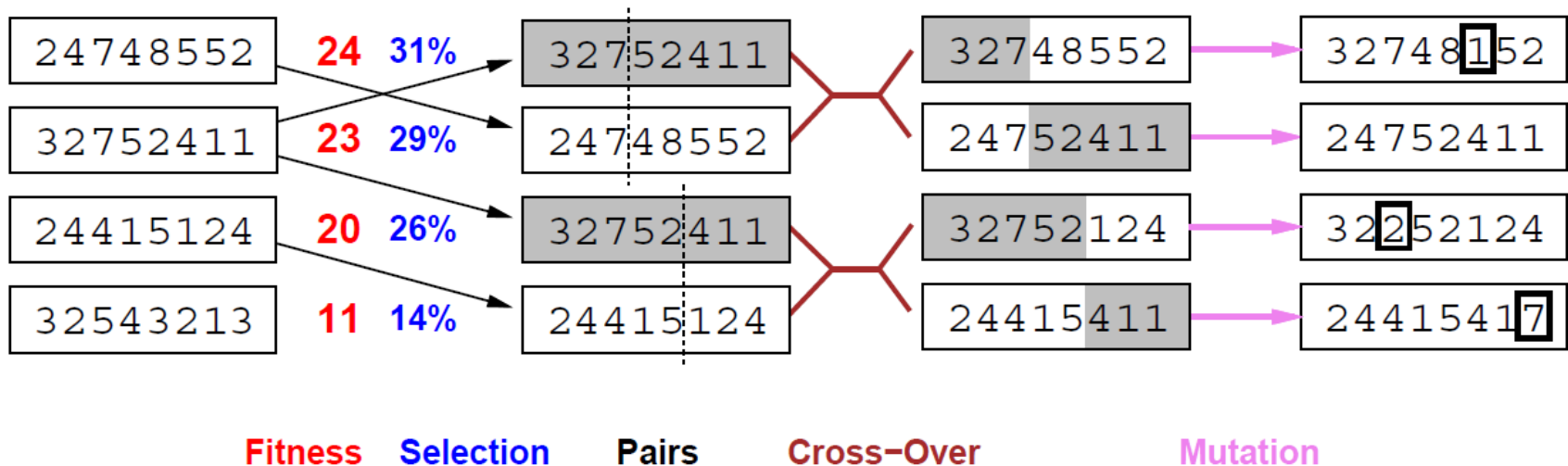
Genetic Algorithms

63

- Genetic algorithms use a natural selection metaphor
- A successor state is generated by combining two parent states
- Start with k randomly generated states (**population**种群)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (**fitness function**适应度函数). Higher values for better states.
- Produce the next generation of states by selection, crossover, and mutation (选择, 杂交, 变异)

Genetic Algorithms

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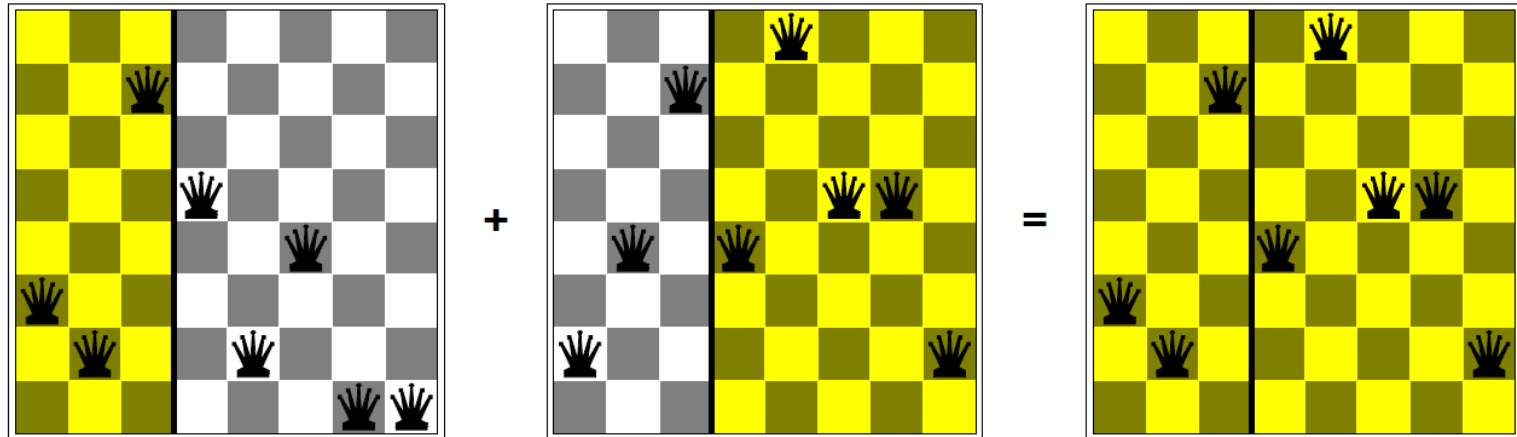
- Fitness function: number of non-attacking pairs of queens 不互相攻击的皇后对的数目 (min = 0, max = $8 \times 7/2 = 28$)

$$24/(24+23+20+11) = 31\%$$

$$23/(24+23+20+11) = 29\% \text{ etc}$$

Genetic Algorithms

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Summary 1

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Heuristic functions estimate costs of shortest paths

Good heuristics can dramatically reduce search cost

Greedy best-first search expands lowest h

— incomplete and not always optimal

A* search expands lowest $g + h$

— complete and optimal

— also optimally efficient (up to tie-breaks, for forward search)

Admissible heuristics can be derived from exact solution of relaxed problems

Summary2

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□ Local search algorithms

the **path** to the goal is irrelevant; the goal state itself is the solution

keep a single "current" state, try to improve it

▣ Hill-climbing search

- depending on initial state, can get stuck in local maxima

▣ Simulated annealing search

- escape local maxima by allowing some "bad" moves but **gradually decrease** their frequency

▣ Local beam search

- Keep track of k states rather than just one

▣ Genetic algorithms

Uninformed/Informed Search

— Main points

68

- 好的启发式搜索能大大提高搜索性能
- 但由于启发式搜索需要抽取与问题本身有关的特征信息，而这种特征信息的抽取有时会比较困难，因此盲目搜索仍不失为一种有用的搜索策略。

Uninformed/Informed Search

— Main points

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- 好的搜索策略应该
 - ▣ 引起运动——避免原地踏步
 - ▣ 系统——避免兜圈
 - ▣ 运用启发函数——缓解组合爆炸

Uninformed/Informed Search

— Main points

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- 搜索树 vs 搜索图
 - ▣ 搜索树：结点有重复，但登记过程简单
 - ▣ 搜索图：结点无重复，但登记过程复杂（每次都要查重）
 - 省空间，费时间。

作业

71

- 4.1, 4.2, 4.6, 4.7 (第二版) =
3.23, 3.25, 3.28, 3.29 (第三版)