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Combining disaggregate forecasts for inflation: The SNB's ARIMA model*

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Abstract

This study documents the SNB's ARIMA model based on disaggregated CPI data used to produce inflation forecasts over the short-term horizon, and evaluates its forecasting performance. Our findings suggest that the disaggregate ARIMA model for the Swiss CPI performed better than relevant benchmarks. In particular, estimating ARIMA models for individual CPI expenditure items and aggregating the forecasts from these models gives better results than directly applying the ARIMA method to the total CPI. We then extend the model to factor in changes in the collection frequency of the Swiss CPI data and show that this extension further improves the forecasting performance.

JEL Classification: C22, C52, C53, E37
Keywords: Swiss CPI inflation; Forecast combination; Forecast aggregation; Disaggregate information; ARIMA models; Missing data; Kalman filter

1. Introduction

Autoregressive integrated moving average (ARIMA) models are widely used for conducting short-term forecasts of economic time series. The method is used at the SNB in a wide variety of settings. This study documents the ARIMA model employed by SNB staff in the context of inflation forecasting. The model is part of a suite of models used every quarter for preparing the SNB's conditional inflation forecast.[1] The approach of using a suite of models for preparing the SNB's conditional inflation forecast is based on the idea that the various models all have their individual strengths and weaknesses. ARIMA models generally give good results for forecasts over short horizons. Therefore, the ARIMA model is traditionally given a relatively large weight in the construction of the first few quarters of the SNB's conditional inflation forecast. For longer-term forecasts, structural models are regarded to be more appropriate. Structural models also have the advantage that they can be employed for policy analysis.

The ARIMA model presented in this study is typically used for forecasting inflation over the coming 15 months. A key feature of our approach is the emphasis on disaggregation. That is, we do not apply the ARIMA methodology directly to the CPI, but to the individual expenditure items that constitute the CPI. Forecasts are then produced for each item and these are aggregated using the CPI's expenditure weights. Based on this approach, forecasts can be produced for CPI inflation, core inflation and sub-aggregates of the CPI such as goods or services.

The theoretical literature does not agree on whether or not disaggregation improves forecasts. On the one hand, Theil (1954), among others, argued that disaggregation makes it possible to exploit more information, if information is heterogeneous. For example, as shown by Lütkepohl (1984), the lag order of an aggregate process is higher than the lag order of the underlying disaggregate process. On the other hand, following Grunfeld and Griliches (1960), it has been argued that disaggregate forecasts can be contaminated by model and estimation uncertainty. These uncertainties are bigger issues at the disaggregate than at the aggregate level.

Ultimately, the forecast performance of disaggregate approaches and the optimal level of disaggregation is mainly an empirical question. The empirical evidence presented by researchers is mixed. Fritzer et al. (2002) for Austria and Bermingham and D'Agostino (2011) for the euro area and the United States find that forecasting sub-aggregates of the CPI improves forecast accuracy. Marcellino et al. (2003) find that combining country-specific forecasts for the euro area also improves forecast accuracy. By contrast, Benalal et al. (2004) and Hubrich (2005) provide evidence for the euro area that favours the aggregate approach over the disaggregate approach. For Switzerland, our experience since 2004 shows that estimating ARIMA models at a low level of disaggregation has performed fairly well relative to an aggregate approach. In our view, this is because a disaggregate approach allows us to take into account the peculiarities of the individual price series.

1 For more information about the SNB's inflation forecasting models, cf. Jordan and Peytrignet (2001), Stalder (2001), Jordan et al. (2002), Jordan and Savioz (2003), Lack (2006), Assenmacher-Wesche and Pesaran (2009) and Cuche-Curti et al. (2009).

In this study, we document and evaluate the ARIMA model used by SNB staff for forecasting inflation over short forecast horizons. The study is organised as follows. Section 2 describes the data. The methodological approach is presented in section 3. Section 4 provides the results of the model evaluation. In section 5, we propose an extension of the model that accounts for changes in the collection frequency of the various CPI items. In recent years, this frequency has been increased for many expenditure items of the Swiss CPI. As we will see, this is an important issue when dealing with Swiss CPI data. Finally, section 6 concludes.

2. Data

We estimate disaggregate models at the lowest level, for which price indices and expenditure weights are available from the Swiss Federal Statistical Office (SFSO). The number of expenditure items may change at major revision of the Swiss CPI. Currently, the CPI basket comprises 217 expenditure items. Estimating models at such a low level of disaggregation may improve forecast accuracy because there is a lot of heterogeneity across the expenditure items of the Swiss CPI. The heterogeneity has economic and methodological sources; in either case, the price data displays statistical regularities which we can exploit in our forecasting models.

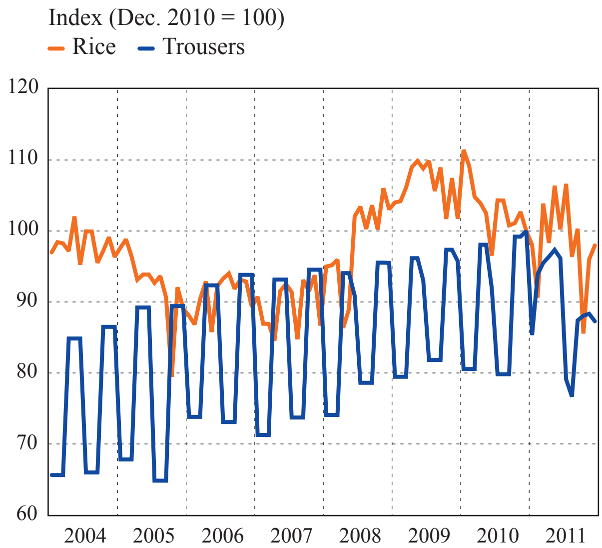
The heterogeneity may stem from the price-setting behaviour of individual firms (cf. Kaufmann, 2009). Some prices are adjusted almost every month while other prices tend to be sticky. But also, the absolute magnitude of price changes differs across firms. This does not come as a surprise, as the CPI contains prices for goods as well as services, and for imported items as well as domestic items. Some of these items are therefore influenced more strongly by sluggish domestic factors (e.g. by wages) and some by volatile foreign factors (e.g. by exchange rates and commodity prices). Finally, while most of the prices are determined by market forces, some of the prices are administered.

The heterogeneity in the price-setting behaviour of firms manifests itself in heterogeneous statistical properties of the price indices. First, the persistence, and therefore the forecastability of inflation, differs across the expenditure items of the Swiss CPI (cf. Kaufmann and Lein, 2011). Second, the volatility of some prices is especially high because they are influenced by volatile foreign intermediate inputs (e.g. oil products and foreign food items). Finally, many price indices display seasonal patterns stemming from end-of-season sales (e.g. clothing and footwear) or seasonal availability of products (e.g. domestic vegetables).

The statistical properties of the price indices are also influenced by methodological peculiarities in the data collection process. Although the SFSO publishes the CPI on a monthly basis, most prices have been collected less frequently (cf. appendix A). For these expenditure items, the SFSO carries forward the index value of the previous collection month to calculate the total CPI. Until the end of 2007, only food items, heating oil and fuels have been collected monthly. Most other items have been collected once a quarter or even less frequently. In 2008 and 2011, the collection frequency was increased to monthly for many items. Taking into account the collection frequency may improve forecast accuracy because the forecasted prices will only change in the months when the SFSO effectively collects price data.

To give an example of the heterogeneity of CPI expenditure items with respect to collection frequency and seasonality, Chart 1 displays the price indices of trousers and rice over the period from 2004 to 2011. The prices for trousers were collected only once per quarter until the end of 2007. As a result, the price series exhibits discrete steps every three months during that period. From 2008 onwards, the prices have been collected six times a year, and from 2011 onwards every month. In addition, there is a distinct seasonal pattern over the whole period, which can be attributed to end-of-season sales. In contrast, prices for rice are collected monthly and do not appear to have a seasonal pattern.

Chart 1: CPI Expenditure Items



For most of the CPI expenditure items, we use data starting in May 2000. This starting date reflects the fact that the CPI was subject to a major revision at that time (cf. SFSO, 2000). Most importantly, the SFSO increased the collection frequency for clothing and footwear from half-yearly to quarterly. Therefore, the SFSO started to collect end-of-season sales prices with a distinct seasonal pattern. Some of the series are shorter, because the SFSO introduced them after May 2000 (e.g. mobile telephony and Internet services). Some of the series are longer, because they were collected in a methodologically consistent way and on a monthly basis even before May 2000 (e.g. fresh food).

3. Model

3.1 CPI items forecast with ARIMA models

Our forecasts for the vast majority of the expenditure items are based on ARIMA models. The ARIMA methodology was developed by Box and Jenkins (1976) and represents a simple modelling technique for time series data. The methodology is based on the idea that any stationary stochastic process can be approximated well by an autoregressive moving-average (ARMA) process. Depending on the nature of this process, past observations contain information about its future development. The analyst's task then is to identify a statistical model which explains the current value of the process as a weighted sum of past values – the autoregressive (AR) part – and past error terms – the moving average (MA) part. The stochastic process is assumed to be stationary. If the process is integrated of order $d > 0$ stationarity is obtained by taking the d^{th} difference. To keep the exposition simple we omit the constant and assume that the process is integrated of order one. Thus, for a price index in logarithms, p_t, an ARIMA model of order (p, 1, q) can be written as:

$$\Delta p_t = \underbrace{\phi_1 \Delta p_{t-1} + \phi_2 \Delta p_{t-2} + \ldots + \phi_p \Delta p_{t-p}}_{AR\ part} + \varepsilon_t + \underbrace{\theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q}}_{MA\ part}, \tag{1}$$

where p and q give the number of autoregressive and moving average terms, respectively, and Δ denotes the first difference. The error term ε_t is assumed to follow a white noise process with variance σ^2.

If the price changes exhibit a seasonal pattern, we can extend the case above to a seasonal ARIMA model. For monthly price data, there is often a seasonality at lag 12 which motivates us to add a seasonal AR term with coefficient ρ to the specification. The seasonal model can then be written as:

$$\begin{aligned} \Delta p_t - \rho \Delta p_{t-12} = {} & \phi_1 (\Delta p_{t-1} - \rho \Delta p_{t-13}) + \phi_2 (\Delta p_{t-2} - \rho \Delta p_{t-14}) + \ldots \\ & \ldots + \phi_p (\Delta p_{t-p} - \rho \Delta p_{t-p-12}) + \varepsilon_t + MA\ part. \end{aligned} \tag{2}$$

Note that if $\rho = 1$ the only difference to the non-seasonal case is that we remove a seasonal unit root by seasonally differencing Δp_t. If $\rho = 0$, we are back to the non-seasonal ARIMA model.

For each expenditure item, we select a model in two steps. First, we analyse the statistical properties of each price series to determine the order of integration, decide whether it exhibits a seasonality and how often it is collected. This analysis can be tedious and is therefore done only every few years. We assume that all CPI items are integrated of order one and use them in first log-differences ($d = 1$). We test this assumption by means of two unit root tests (cf. Dickey and Fuller, 1979; Kwiatkowski et al., 1992). For very few of the more than 200 items, the tests do not support this assumption at the 5% level. Afterwards, to detect seasonal patterns in the price changes, we inspect the autocorrelation function (ACF). A seasonality at 12 months leads to a significant spike in the ACF at multiples of 12. For those items, we allow for a seasonal AR term. Finally, the collection frequency is available from the SFSO and reprinted in appendix A. For items collected on a quarterly basis, we only include lags that are multiples of three, which is equivalent to estimating a quarterly model.

Second, we choose the lag order of the models on the basis of an automatic lag selection criterion. This procedure can be easily automatised and therefore the lag order is selected every time the models are reestimated. For each expenditure item, numerous models with different lag orders are estimated. The best model is then selected based on the Schwarz information criterion

$$\text{SIC} = \log\left(\frac{\text{SSE}}{T}\right) + k\frac{\log(T)}{T}, \tag{3}$$

where SSE denotes the sum of squared errors, T the number of observations and k the number of estimated parameters. The algorithm selects the model with the smallest SIC. The SIC increases in the SSE. More lags improve the fit of the model and therefore lead to lower SSE and SIC. However, overfitting of the model (i.e. choosing an over-large lag order) can lead to inconsistency of the maximum likelihood estimator (cf. Neusser, 2009, pp. 89–91) and reduce the out-of-sample forecast performance. For that reason, the information criterion contains a penalty term which increases with the number of parameters k so that the criterion favours a more parsimonious specification.

3.2 CPI items forecast with other methods

For some of the expenditure items, we do not use the ARIMA forecast but replace it with ad-hoc assumptions. These ad-hoc assumptions include an extrapolation of the index by its most recent value, the average month-on-month growth rate or the average year-on-year growth rate. These assumptions are used in three cases. First, a visual inspection of the forecast may point to implausible and/or extreme price movements. Second, for items which have only recently been added to the CPI, the number of observations is too small to estimate a model. Third, we do not estimate a model for items that are collected less than once a quarter. We also replace the ARIMA forecast if we have information on special factors and events which affect only some of the items in the CPI basket. Some examples are electricity price increases that have been announced, or the introduction of Pigouvian taxes on tobacco and heating oil. If the resulting price effects can be quantified, we include these as add-factors in the forecast.

We use different modelling approaches for prices of oil products and rents. These items deserve special attention because they heavily influence the Swiss CPI. This is because the prices of oil products are very volatile and rents have a large expenditure weight (almost 20%). Prices of oil products are forecast using error-correction models (ECMs) because these prices are cointegrated with the crude oil spot price in Swiss francs.[2] This helps us to produce a forecast for the current month because we observe the daily crude oil spot price and the USDCHF exchange rate at the beginning of the month when the SFSO collects the price data. To produce a forecast for the subsequent months, we assume that the crude oil spot price in Swiss francs remains constant over the forecasting period, at the average of the last ten daily observations.

Rents are partly tied to mortgage rates in Switzerland (cf. Stalder, 2003) and interest rate changes usually affect rents with some lag (cf. Kaufmann and Lein, 2012). Therefore, recent mortgage rate changes may contain information about future rent changes. Because rents are only collected once a quarter, we estimate a quarterly ARIMA model in first differences. In addition, we include the fourth lag

2 Oil products comprise the expenditure items heating oil, petrol and diesel.

of the change in the mortgage rate as an exogenous variable.[3] The variable enters the specification separately for positive and negative changes to account for potential asymmetries in the extent to which mortgage rate increases and decreases are passed through to rents. The lag order of the autoregressive terms is determined by the SIC.[4] To produce a forecast, we first have to obtain a forecast for the mortgage rate. We assume that, going forward, the mortgage rate will remain constant.

4. Evaluation of the forecasting performance

This section examines the performance of the ARIMA model in real-time and compares it to three relevant benchmarks. We then produce pseudo-out-of-sample forecasts with various versions of the ARIMA model to analyse how much disaggregation is optimal, and whether the alternative modelling approaches described in the previous section perform better than their ARIMA counterparts.

4.1 Real-time forecasts

The evaluation covers the period 2004–2011. We obtained real-time vintages of the following forecasts: (i) the SNB's ARIMA forecast, (ii) the SNB's conditional inflation forecast published with the press release after the quarterly monetary policy assessment, (iii) the quarterly consensus forecast from Consensus Economics, an international survey of professional forecasters, and (iv) a simple autoregressive forecast which serves as a naive benchmark. The SNB's conditional inflation forecast is based on a fixed short-term interest rate over three years and incorporates the ARIMA model as well as about six other models (time series models as well as structural models). Therefore, we actually evaluate whether it was worth deviating from the ARIMA forecast in real time in terms of forecasting CPI inflation.[5]

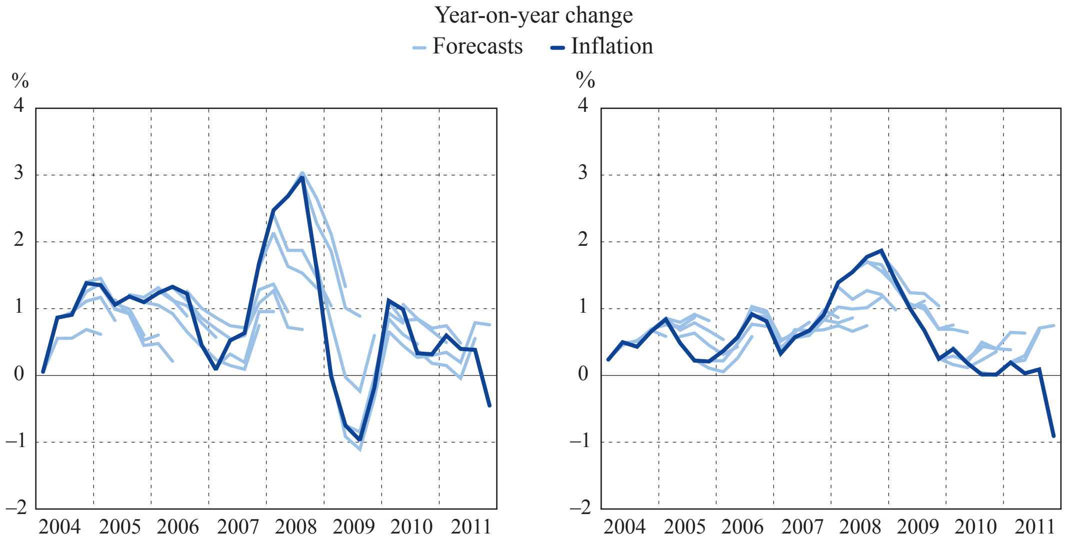
Although the ARIMA model is updated every month, we only consider those forecasts that were available at the SNB's quarterly inflation forecasting meeting, which takes place before every monetary policy assessment. The ARIMA model forecast therefore includes two observations of CPI data for the current quarter. Moreover, we constructed the autoregressive forecast on the same information set as the ARIMA forecast. The two other forecasts may only partially include this information because they are mainly based on quarterly models (SNB conditional inflation forecast) and they are usually published only a few days after the CPI release (Consensus Economics).

3 We link the average mortgage rate offered by cantonal banks in the second quarter of 2008 with the average mortgage rate published by the *Bundesamt für Wohnungswesen* (Federal Housing Office).

4 Until 2010, we used a more involved model which included mortgage rates and imbalances on the housing market. These imbalances are estimated by a stock-flow model of the Swiss housing market (cf. Steiner, 2010).

5 We may argue that we should not compare a forecast conditional on a fixed interest rate path with the unconditional ARIMA forecast. However, in the very short run the unconditional and conditional inflation forecasts usually coincide because interest rate changes are transmitted with some lag.

Chart 2: Real-time ARIMA Forecasts



(a) Total CPI   (b) Excluding oil products

Chart 2 shows the quarterly forecasts of the ARIMA model in real time, for the total CPI and for the CPI excluding oil products. In line with standard SNB practice, all forecasts are expressed as quarterly year-on-year inflation rates. The ARIMA model performed well in periods with relatively stable inflation (panel a). In 2004–2007 and in 2010, the forecast errors are small. Moreover, the mean forecast error is not significantly different from zero, i.e. the forecasts are unbiased. In periods with more volatile inflation, the forecast errors are larger. In 2008 and 2009, this is partly because of a surge in oil prices and a subsequent collapse in the wake of the financial crisis. If we exclude oil products, the forecast errors are smaller in absolute terms (panel b). A noteworthy exception is the last quarter of 2011. The strong appreciation of the Swiss franc triggered substantial price cuts for imported items and led to a large forecast error because the model does not explicitly account for exchange rate movements and their pass-through to consumer prices.

The charts suggest that a major part of the forecast errors is driven by large changes in the crude oil spot price. We can confirm this visual impression by regressing the forecast errors at a particular forecasting horizon on the changes in the crude oil spot price during that period. Because we assume that the crude oil spot price remains constant over the forecasting period, these changes correspond to forecast errors with respect to the crude oil spot price. The R^2 of such a regression tells us how much of the forecast error at a certain horizon can be attributed to oil price forecast errors. For a one-quarter ahead forecast, the R^2 turns out to be zero. This reflects the fact that the last month of the current quarter can be forecast well using the ECMs with daily information on crude oil spot prices. At longer horizons, however, oil price surprises are a major driver of the forecast errors. For a forecast horizon longer than one quarter, roughly 60% of the variation in the forecast error can be explained by oil price changes. We repeated the exercise using a trade-weighted exchange rate instead of the oil price. However, unexpected changes in the exchange rate do not seem to be a major factor driving the forecast errors in the evaluation period – with the exception of the last quarter in 2011. This may be related to the fact that, until very recently, the exchange rate pass-through has been relatively small in Switzerland since the 1990s (cf. Stulz, 2007).

To judge the accuracy of the ARIMA forecasts relative to two benchmarks more formally, we can calculate the root mean squared forecast error (RMSFE). The RMSFE for a forecast horizon *f* is defined as:

$$\mathrm{RMSFE}_f = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\pi_{t+f} - \pi_{t+f|t})^2} \ ,$$

where $\pi_{t+f|t}$ is the *f*-quarter-ahead inflation forecast, given data up to *t* for an out-of-sample forecast exercise of length *T*. The first row of Table 1 shows the RMSFE of the ARIMA model for forecasting horizons up to five quarters. The remaining rows show the relative RMSFEs for the three benchmarks. If the relative RMSFE is larger than one, the ARIMA model performs better than the benchmark. The significance of the relative forecast performance can be tested by a Diebold-Mariano-West (DMW) test.[6] Significant differences at the 10% level are denoted in bold font and the corresponding *p*-values are given in brackets.

The table shows that the ARIMA model performs relatively well compared to the three benchmarks. For all forecast horizons and all benchmarks, the relative RMSFEs are larger than one. The ARIMA model outperforms the benchmarks mainly in the short run. The forecasting performance is significantly better for the current quarter (all benchmarks) and for the following quarter (only Consensus Economics and autoregressive forecast). For the current quarter, the RMSFEs of the benchmarks are more than twice as large as the RMSFE of the ARIMA model. This is related to the very accurate forecast of the ARIMA model for the current quarter and can be explained by the fact that the ARIMA forecast already includes two months of the CPI data for that quarter. Notice that the ARIMA forecast traditionally has a relatively large weight in the construction of the SNB's conditional inflation forecast in the short term. With the benefit of hindsight, the results suggest that, at least for the current quarter, the SNB should have given even more weight to the ARIMA forecast.

4.2 Optimal degree of disaggregation

To evaluate whether full disaggregation is superior to partial disaggregation, we produce pseudo-out-of-sample forecasts using different versions of the ARIMA model. These versions comprise an aggregate model, and models with six, 14, and 182 sub-aggregates. A list of the sub-aggregates is available in appendix B. The six sub-aggregates group the expenditure items according to origin and product type. The 14 sub-aggregates roughly correspond to the divisions according to the COICOP

6 cf. Diebold and Mariano (1995) and West (1996). Following Harvey et al. (1997), we adjust the DMW test statistic by a correction factor and compare it to the critical values of a Student's *t*-distribution with *T*–1 degrees of freedom. According to their Monte Carlo simulations, this should improve the power of the test in small samples.

Table 1: Real-time forecasting performance

	Forecast horizon				
	1	2	3	4	5
Real-time ARIMA	0.04	0.36	0.66	0.83	0.96
SNB conditional inflation forecast	**2.40**	1.25	1.13	1.11	1.07
	(0.003)	(0.170)	(0.280)	(0.530)	(0.680)
Consensus Economics	**3.43**	**1.32**	1.04	1.04	1.01
	(0.001)	**(0.028)**	(0.465)	(0.582)	(0.918)
AR(1)	**3.37**	**1.19**	1.08	1.08	1.06
	(0.004)	**(0.087)**	(0.287)	(0.223)	(0.292)

Note: Absolute RMSFE (first row) and relative RMSFE, with p-value from DMW test in brackets. Bold figures denote significance at the 10% level. A relative RMSFE exceeding one implies that the real-time ARIMA forecast is more accurate than the benchmarks.

classification.[7] The 182 sub-aggregates mimic full disaggregation. However, because of changes in the CPI basket, we had to aggregate some expenditure items to a higher level to obtain a consistent data set throughout the evaluation period. Moreover, some items that were introduced after May 2000 had to be dropped. Nevertheless, the 182 items cover 93% of the CPI basket at 2010 expenditure weights, and the differences between the official CPI and our constructed data set are usually small (cf. Chart 4 in appendix B). Note that the evaluation is still based on forecast errors with respect to the official CPI inflation rate. To replicate the information set available at the SNB's quarterly inflation forecasting meeting, we assume that two months of the CPI data for the current quarter are known. For oil products and rents, we use an ARIMA forecast rather than the models described in section 3. The latter are evaluated in the next subsection. As the Swiss CPI is not subject to revisions, the pseudo-out-of-sample exercise is actually close to a real-time evaluation.

Full disaggregation performs at least as well as the higher disaggregation levels. Table 2 shows the RMSFE of the fully disaggregated model in the first row and the relative RMSFEs for other levels of disaggregation. The relative RMSFEs are always larger than one. However, the forecast accuracy is only significantly better compared to the aggregate model and to the model with six sub-aggregates at horizons of two to three quarters. For the current quarter, full disaggregation does not significantly pay off. But this is because, in all models, two months of the current quarter are already known and so the forecast error is small by construction.

Surprisingly, perhaps, the RMSFE of the real-time ARIMA forecast given in Table 1 are lower than the RMSFE of the pseudo-out-of-sample forecasts with 182 sub-aggregates in Table 2. First, this may be because the pseudo-out-of-sample data set does not cover the complete CPI basket. Second, this may be because of add-factors that were introduced by the analyst in real time. Third, this may be because of the performance of the alternative approaches for oil products and rents. The first two factors are hard to evaluate. We therefore restrict the analysis to an evaluation of the alternative approaches for oil products and rents in the next subsection.

7 cf. unstats.un.org/unsd/cr/registry/regcst.asp?Cl=5

Table 2: Optimal level of disaggregation

	Forecast horizon 1	2	3	4	5
ARIMA (182)	0.08	0.41	0.72	0.92	1.04
ARIMA (14)	1.01	1.19	1.04	1.04	1.01
	(0.970)	(0.182)	(0.254)	(0.242)	(0.898)
ARIMA (6)	1.03	**1.17**	1.05	1.05	1.03
	(0.844)	**(0.002)**	(0.236)	(0.215)	(0.625)
ARIMA (1)	1.27	**1.60**	**1.40**	1.43	1.29
	(0.220)	**(0.002)**	**(0.007)**	(0.112)	(0.317)

Note: Absolute RMSFE (first row) and relative RMSFE, with p-value from DMW test in brackets. Bold figures denote significance at the 10% level. A relative RMSFE exceeding one implies that the ARIMA (182) forecast is more accurate than the benchmarks.

4.3 Forecasts for prices of oil products and rents

To evaluate the forecasting models for prices of oil products and rents, we produce pseudo-out-of-sample forecasts. The exogenous variables are assumed to be available up to the first day of the last month of the current quarter. We then compare these forecasts with two benchmarks, a pseudo-out-of-sample ARIMA forecast with the lag length selected by the SIC and the real-time forecast.

The ECM helps to forecast prices of oil products. Table 3 shows that the relative RMSFEs of the ARIMA specification are larger than one for all forecast horizons. For the one and two-quarter-ahead forecasts, the forecast accuracy of the ECM is significantly better. The real-time forecast differs from the ECM forecast mainly because more daily observations for the exogenous variables were usually available in real time. Moreover, add-factors were sometimes imposed on these forecasts, which we ignore in our pseudo-out-of-sample exercise. For the current quarter, the real-time forecast is slightly, but not significantly, better than the ECM forecast. By contrast, for a forecasting horizon of five quarters, the pseudo-out-of-sample forecast outperforms the real-time forecast significantly.

We can forecast rents more accurately if we add mortgage rates to the ARIMA specification. For the current quarter, the RMSFE is zero, which reflects the fact that rents are always collected in the second month of the quarter, and we assume that two months of the current quarter are known. Especially for longer forecast horizons, the model including the mortgage rate outperforms the ARIMA specification. The real-time forecast is almost as accurate as the pseudo-out-of-sample forecast.

5. Extension: Accounting for time-varying collection frequency

In recent years, the collection frequency has been increased for many expenditure items in the Swiss CPI. To show this, we calculated the share of the 182 sub-aggregates in our pseudo-out-of-sample data set with more collection months in 2011 than in 2001. The accumulated weights of these items exceed one-third of the CPI basket.

Table 3: Performance of the models for prices of oil products and rents

	Forecast horizon				
	1	2	3	4	5
Oil products (ECM)	1.14	6.72	12.49	15.25	17.10
Oil products (ARIMA)	**1.34**	**1.30**	1.13	1.07	1.04
	(0.030)	**(0.034)**	(0.256)	(0.497)	(0.710)
Oil products (real-time)	0.70	1.10	1.08	1.06	**1.04**
	(0.133)	(0.268)	(0.151)	(0.189)	**(0.040)**
Rents (ARIMA with mortgage rate)	0.00	0.24	0.46	0.56	0.61
Rents (ARIMA)	–	1.18	**1.44**	**1.72**	**1.90**
	(–)	(0.148)	**(0.005)**	**(0.000)**	**(0.000)**
Rents (real-time)	–	1.07	1.01	1.01	1.10
	(–)	(0.670)	(0.975)	(0.967)	(0.793)

Note: Absolute RMSFE (first row in each panel) and relative RMSFE, with p-value from DMW test in brackets. Bold figures denote significance at the 10% level. A relative RMSFE exceeding one implies that the ECM and ARIMA with mortgage rate forecasts are more accurate than the benchmarks.

These additional observations may help to identify the true lag structure of the process and therefore to improve the forecast performance. However, the quarterly ARIMA models ignore these observations. This section proposes an extension to the ARIMA model that factors in changes in the collection frequency and evaluates its forecasting performance. For a technical discussion of the methodology cf. appendix C.

5.1 Model and estimation

To factor in changes in the collection frequency, it is useful to set up the forecasting model in state-space representation (cf. Hamilton, 1994, chapter 13). The first part of this representation is a measurement equation which relates the unobserved month-on-month log-changes to the observed data. The measurement equation uses the fact that the log-change since the last collection period is simply the sum of the unobserved month-on-month log-changes. As an example, let us assume that the SFSO collected price data on a quarterly basis. Therefore, the last collection took place three months ago. Then, the measurement equation can be written as

$$\Delta^3 p_t = 3\mu + \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \Delta p_t - \mu \\ \Delta p_{t-1} - \mu \\ \Delta p_{t-2} - \mu \end{bmatrix} \tag{4}$$

where $\Delta^3 p_t = p_t - p_{t-3}$, μ is a constant to be estimated and $\{p_{t-1}, p_{t-2}, p_{t-4}, p_{t-5}, \ldots\}$ are unobserved. The measurement equation is allowed to change over time as the SFSO changes the collection months. For example, if the SFSO collects the data every month, the measurement equation changes to

$$\Delta p_t = \mu + \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta p_t - \mu \\ \Delta p_{t-1} - \mu \\ \Delta p_{t-2} - \mu \end{bmatrix} \tag{5}$$

The second part of the state-space representation is a law of motion for the unobserved month-on-month changes, which is called the transition equation. We assume that these unobserved month-on-month log-changes are stationary and that they follow an AR(p) process, possibly with a seasonality at lag 12.[8] As an example, the transition equation for an AR(2) can be written as:

$$\begin{bmatrix} \Delta p_t - \mu \\ \Delta p_{t-1} - \mu \\ \Delta p_{t-2} - \mu \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta p_{t-1} - \mu \\ \Delta p_{t-2} - \mu \\ \Delta p_{t-3} - \mu \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ 0 \end{bmatrix}, \tag{6}$$

where the variance of ε_t is σ^2. The parameters of the measurement and transition equations, $\{\mu, \phi_1, \phi_2, \sigma^2\}$, are estimated by maximum likelihood, with the likelihood function being derived from the Kalman filter (cf. Hamilton, 1994, pp. 385–386). After estimating the parameters we can use the transition equation to produce forecasts for the unobserved month-on-month changes.

Extending the ARIMA models to time-varying collection frequency has several advantages. First, if we plug the forecast for the unobserved month-on-month changes into the measurement equation, we get a forecast for the irregularly collected price data. Second, we can estimate the models over longer sample periods taking into account the monthly information whenever the collection frequency has been increased.[9] Third, we can obtain an estimate of the unobserved month-on-month changes using the Kalman smoother and therefore interpolate the missing data (cf. Hamilton, 1994, pp. 394–397). As an example, Chart 3 shows the irregularly collected series for trousers, the interpolated series and a forecast. Up to 2007, the series was collected quarterly. Afterwards, the SFSO increased the collection frequency to six times a year. We see that the new collection frequency is reflected in the forecast. Moreover, in periods where a collection took place, the interpolated and the official series coincide.

5.2 Evaluation of the forecasting performance

The question arises, as to whether this extension significantly improves forecast accuracy for the total CPI. We therefore compare the extended ARIMA forecast with the forecasts presented in the previous section. We estimate the extended ARIMA model on a longer sample starting in 1993. In the evaluation period, changes in the collection frequency were sometimes known in advance. Nevertheless, we use the

8 Because of time constraints, we only estimated autoregressive processes for the individual expenditure items. Appendix C shows how to estimate ARMA models.

9 The approach also allows us to handle measurement errors by choosing the measurement equation accordingly. Sometimes, tiny price changes falsely suggest that a collection occurred. In the measurement equation, we ignore price changes that are smaller than 10^{-3}.

Chart 3: Irregular collection frequency 'trousers'

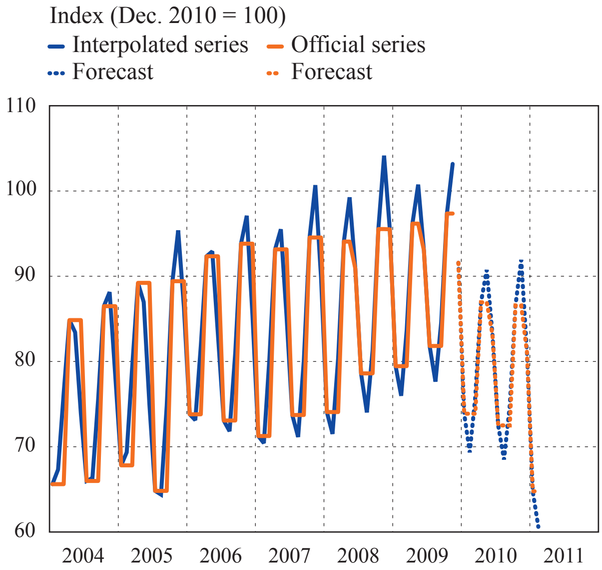


Table 4: Performance of the extended ARIMA model

	Forecast horizon				
	1	2	3	4	5
Extended ARIMA (182)	0.06	0.32	0.64	0.86	1.00
ARIMA (182)	1.18 (0.278)	**1.28** **(0.067)**	1.11 (0.157)	1.07 (0.184)	1.04 (0.438)
Real-time ARIMA	**0.57** **(0.003)**	1.12 (0.321)	1.02 (0.466)	0.97 (0.258)	0.95 (0.203)

Note: Absolute RMSFE (first row) and relative RMSFE, with p-value from DMW test in brackets. Bold figures denote significance at the 10% level. A relative RMSFE exceeding one implies that the extended ARIMA (182) forecast is more accurate than the benchmarks.

collection months observed in the previous year to produce a forecast. This allows us to automatise the procedure and requires little knowledge about the CPI collection frequency in real time.

It turns out that the extension to time-varying collection frequency improves forecast accuracy. The relative RMSFE of the first benchmark, ARIMA (182), are all larger than one. For two-quarter-ahead forecasts, the improvement is significant at the 10% level. However, the extended ARIMA model still performs significantly worse than the real-time ARIMA forecast for the current quarter. We attribute this to the fact that we do not use the alternative approaches for prices of oil products and rents in the extended ARIMA model.

6. Conclusion

This study documented the SNB's ARIMA model based on disaggregated CPI data and evaluated its forecasting performance. This model is used to produce inflation forecasts over the short-term horizon. The strengths of the model are that it considers monthly data, the heterogeneity of the CPI data, and that we can include other information as add-factors. Moreover, we can forecast arbitrary sub-aggregates of the CPI as well as core inflation rates.

The evaluation results showed that the ARIMA model yields fairly accurate inflation forecasts for the short run and outperforms relevant benchmarks. Also, the results showed that estimating ARIMA models for expenditure items of the CPI and aggregating the forecasts from these models gives better results than directly applying the ARIMA method to the total CPI. This supports the view that the heterogeneous movements of the underlying price series contain useful information that can be exploited in the context of forecasting CPI inflation. Overall, by using the disaggregated ARIMA approach, price dynamics, seasonal patterns and the data collection frequency can be better modelled and predicted, since the model specifications may vary between individual expenditure items of the CPI.

Appendix

A Collection frequency of Swiss CPI data

Table 5: Collection frequency 2000–2005

Main groups / expenditure items	Collection months											
	J	F	M	A	M	J	J	A	S	O	N	D
Food and non-alcoholic beverages	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Biscuit products			x			x			x			x
Tinned products			x			x			x			x
Frozen products			x			x			x			x
Sweets			x			x			x			x
Other food			x			x			x			x
Beverages			x			x			x			x
Alcoholic beverages and tobacco			**x**			**x**			**x**			**x**
Clothing and footwear		**x**			**x**			**x**			**x**	
Housing and energy		**x**			**x**			**x**			**x**	
Heating oil	x	x	x	x	x	x	x	x	x	x	x	x
Charges	Aperiodic											
Natural gas, electricity, remote heating	Aperiodic											
Furnishings, household equipment and routing house maintenance			**x**			**x**			**x**			**x**
Health	**x**			**x**			**x**			**x**		
Medical services	Aperiodic											
Other health services	Aperiodic											
Hospital services	Aperiodic											
Transport	**x**			**x**			**x**			**x**		
Fuels	x	x	x	x	x	x	x	x	x	x	x	x
Public transport	Aperiodic											
Communication			**x**			**x**			**x**			**x**
Postal services	Aperiodic											
Telecommunication	Aperiodic											
Recreation and culture			**x**			**x**			**x**			**x**
Sporting events (soccer)						x						
Sporting events (ice hockey)									x			
Sports and leisure activities (public swimming baths)						x			x			
Sports and leisure activities (skating rinks)									x			
Radio and television licences	Aperiodic											
Education			**x**						**x**			
Restaurants and hotels	**x**			**x**			**x**			**x**		
Camping				x								
Alternative accommodation facilities	x						x					
Miscellaneous goods and services		**x**			**x**			**x**			**x**	

Source: SFSO (2000)

Table 6: Collection frequency 2006–2007

Main groups / expenditure items	Collection months											
	J	F	M	A	M	J	J	A	S	O	N	D
Food and non-alcoholic beverages	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Biscuit products		x			x			x			x	
Tinned products		x			x			x			x	
Frozen products		x			x			x			x	
Sweets		x			x			x			x	
Other food		x			x			x			x	
Beverages		x			x			x			x	
Alcoholic beverages and tobacco			**x**			**x**			**x**			**x**
Clothing and footwear	**x**			**x**			**x**			**x**		
Housing and energy		**x**			**x**			**x**			**x**	
Heating oil	x	x	x	x	x	x	x	x	x	x	x	x
Charges	Aperiodic											
Natural gas, electricity, remote heating	Aperiodic											
Wood				x						x		
Furnishings, household equipment and routing house maintenance			**x**			**x**			**x**			**x**
Health	**x**			**x**			**x**			**x**		
Medical services	Aperiodic											
Other health services	Aperiodic											
Hospital services	Aperiodic											
Transport	**x**			**x**			**x**			**x**		
Fuels	x	x	x	x	x	x	x	x	x	x	x	x
Public transport	Aperiodic											
Communication			**x**			**x**			**x**			**x**
Postal services	Aperiodic											
Telecommunication	Aperiodic											
Recreation and culture			**x**			**x**			**x**			**x**
Sports equipment	x			x			x			x		
Sporting events (soccer)						x			x			
Sporting events (ice hockey)									x			x
Sports and leisure activities (public swimming baths)						x			x			
Sports and leisure activities (skating rinks)									x			x
Radio and television licences	Aperiodic											
Education			**x**						**x**			
Life-long learning			x			x			x			x
Restaurants and hotels	**x**			**x**			**x**			**x**		
Alternative accommodation facilities	x						x					
Miscellaneous goods and services		**x**			**x**			**x**			**x**	

Source: SFSO (2006)

Table 7: Collection frequency 2008–2010

Main groups / expenditure items	Collection months											
	J	F	M	A	M	J	J	A	S	O	N	D
Food and non-alcoholic beverages	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Alcoholic beverages and tobacco	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Tobacco			x			x			x			x
Clothing and footwear	**x**			**x**		**x**	**x**			**x**		**x**
Upkeep of textiles and garment alterations	x			x			x			x		
Shoe repairs	x			x			x			x		
Housing and energy	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Rent		x			x			x			x	
Charges						Aperiodic						
Natural gas, electricity, remote heating						Aperiodic						
Furnishings, household equipment and routing house maintenance	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Furniture, furnishings and floor coverings	x		x			x	x		x			x
Household cleaning services			x			x			x			x
Health						Aperiodic						
Medicines and medical products	x	x	x	x	x	x	x	x	x	x	x	x
Therapeutic appliances	x			x			x			x		
Dental services	x			x			x			x		
Transport	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Repair services and work	x			x			x			x		
Other services in respect of personal transport equipment	x			x			x			x		
Public transport						Aperiodic						
Taxi	x			x			x			x		
Communication						Aperiodic						
Telephone equipment	x	x	x	x	x	x	x	x	x	x	x	x
Recreation and culture	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Musical instruments			x			x			x			x
Sports equipment	x			x		x	x			x		x
Sporting events (soccer)						x			x			
Sporting events (ice hockey)									x			x
Sports and leisure activities (public swimming baths)						x			x			
Sports and leisure activities (skating rinks)									x			x
Cinema, theatre, concerts, leisure-time courses			x			x			x			x
Radio and television licences						Aperiodic						
Books, newspapers and stationery			x			x			x			x
Education									**x**			**x**
Life-long learning			x			x			x			x
Restaurants and hotels	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Canteens	x			x			x			x		
Alternative accommodation facilities	x			x			x			x		
Miscellaneous goods and services	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Hairdressing establishments		x			x			x			x	
Social protection services		x			x			x			x	
Insurance						Aperiodic						
Financial services		x			x			x			x	

Source: SFSO (2008)

Table 8: Collection frequency since 2011

Main groups / expenditure items	Collection months											
	J	F	M	A	M	J	J	A	S	O	N	D
Food and non-alcoholic beverages	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Fruits (various)	Seasonal											
Vegetables (various)	Seasonal											
Alcoholic beverages and tobacco	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Tobacco			x			x			x			x
Clothing and footwear	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Upkeep of textiles and garment alterations	x			x			x			x		
Shoe repairs	x			x			x			x		
Housing and energy		**x**			**x**			**x**			**x**	
Charges	Aperiodic											
Heating Oil	x	x	x	x	x	x	x	x	x	x	x	x
Natural gas, electricity, remote heating	Aperiodic											
Wood	x	x	x	x	x	x	x	x	x	x	x	x
Furnishings, household equipment and routing house maintenance	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Household cleaning services			x			x			x			x
Health	Aperiodic											
Medicines	x	x	x	x	x	x	x	x	x	x	x	x
Therapeutic appliances	x			x			x			x		
Dental services	x			x			x			x		
Transport	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Repair services and work	x			x			x			x		
Other services in respect of personal transport equipment	x			x			x			x		
Public transport	Aperiodic											
Taxi	x			x			x			x		
Communication	Aperiodic											
Telephone equipment	x	x	x	x	x	x	x	x	x	x	x	x
Recreation and culture	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Musical instruments			x			x			x			x
Winter sports equipment	Seasonal											
Veterinary services for pets			x			x			x			x
Sporting events (soccer)								x				
Sporting events (ice hockey)			x						x			
Sports and leisure activities (public swimming baths)						x						
Theatre, concerts									x			
Mountain railways, ski lifts						x						x
Radio and television licences	Aperiodic											
Education									**x**			**x**
Life-long learning			x						x			
Restaurants and hotels	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Canteens	x			x			x			x		
Alternative accommodation facilities	x			x			x			x		

Table 8 continued

Main groups / expenditure items	Collection months											
	J	F	M	A	M	J	J	A	S	O	N	D
Miscellaneous goods and services	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**	**x**
Hairdressing establishments		x			x			x			x	
Social protection services		x			x			x			x	
Insurance	Aperiodic											
Financial services		x			x			x			x	

Source: SFSO (2011)

B Sub-aggregates of Swiss CPI

Table 9: Sub-aggregates of Swiss CPI

Model	Sub-aggregates	Weights (2010)
ARIMA (1)	Total CPI	100.000
ARIMA (6)	Domestic durable goods	16.875
	Domestic private services	27.818
	Rents	19.499
	Public services	8.711
	Imported goods and services excl. oil products	23.447
	Oil products	3.650
		100.000
ARIMA (14)	Food and non-alcoholic beverages	11.063
	Alcoholic beverages and tobacco	1.764
	Clothing and footwear	4.454
	Housing and energy (excl. oil products and rents)	4.972
	Oil products	3.650
	Rents	19.499
	Furnishings, household equipment and routine maintenance of the house	4.635
	Health	13.862
	Transport (excl. oil products)	8.643
	Communication	2.785
	Recreation and culture	10.356
	Education	0.669
	Restaurants and hotels	8.426
	Miscellaneous goods and services	5.222
		100.000
ARIMA (182)	Rice	0.046
	Flour	0.062
	Bread	0.522
	Small baked goods	0.169
	Viennese pastries and other pastry products	0.305
	Biscuit products	0.261
	Pasta	0.172
	Other cereal products	0.186

Table 9 continued

Model	Sub-aggregates	Weights (2010)
	Beef	0.364
	Veal	0.119
	Pork	0.317
	Lamb	0.072
	Poultry	0.364
	Other meat	0.214
	Sausages	0.549
	Processed and cooked meat	0.471
	Fresh fish	0.188
	Frozen fish	0.073
	Tinned and smoked fish	0.075
	Whole milk	0.158
	Other type of milk	0.139
	Hard and semi-hard cheese	0.542
	Fresh, soft and melted cheese	0.304
	Other dairy products	0.357
	Cream	0.114
	Eggs	0.154
	Butter	0.138
	Margarine, fats and edible oils	0.145
	Citrus fruit	0.116
	Stone fruit	0.105
	Pome fruit	0.156
	Bananas	0.083
	Other fruits	0.240
	Dried, frozen and tinned fruits	0.182
	Fruiting vegetables	0.225
	Root vegetables	0.138
	Salad vegetables	0.228
	Brassicas	0.055
	Onions	0.055
	Other vegetables	0.057
	Potatoes	0.082
	Dried, frozen and tinned vegetables	0.134
	Potato products	0.121
	Jam and honey	0.108
	Chocolate	0.363
	Sweets and chewing gum	0.110
	Ice-cream	0.104
	Sugar	0.036
	Soups, spices and sauces	0.539
	Ready-made foods	0.242
	Coffee	0.289
	Tea	0.059
	Cocoa and nutritional beverages	0.033
	Natural mineral water	0.173
	Soft drinks	0.273
	Fruit and vegetable juices	0.177
	Spirits and brandies	0.095
	Liqueurs and aperitifs	0.049
	Swiss red wine	0.156
	Foreign red wine	0.421

Table 9 continued

Model	Sub-aggregates	Weights (2010)
	Swiss white wine	0.120
	Foreign white wine	0.057
	Sparkling wine	0.083
	Beer	0.132
	Cigarettes	0.619
	Other tobaccos	0.032
	Coats and jackets (Men)	0.190
	Suits (Men)	0.137
	Trousers (Men)	0.219
	Shirts (Men)	0.112
	Sweaters (Men)	0.191
	Underwear (Men)	0.100
	Coats and jackets (Women)	0.053
	Skirts (Women)	0.230
	Trousers (Women)	0.339
	Jackets (Women)	0.348
	Blouses (Women)	0.124
	Jumpers (Women)	0.459
	Underwear (Women)	0.259
	Coats and jackets (Children)	0.026
	Trousers and skirts (Children)	0.064
	Jerseys (Children)	0.068
	Baby clothes	0.090
	Hosiery and underwear (Children)	0.049
	Sportswear	0.022
	Garment fabrics	0.027
	Haberdashery and knitting wool	0.018
	Garment alterations	0.066
	Upkeep of textiles	0.454
	Footwear (Women)	0.271
	Footwear (Men)	0.122
	Footwear (Children)	0.022
	Shoe repairs	0.241
	Rents	19.499
	Rental of garages and parking spaces	0.685
	Products for housing maintenance and repair	0.086
	Services for housing maintenance and repair	1.167
	Natural gas	0.506
	Electricity	1.931
	Heating oil	1.282
	Remote heating	0.086
	Furniture for living room	0.729
	Furniture for bedroom	0.485
	Furniture for kitchen and garden	0.259
	Furnishings	0.310
	Floor coverings and carpets	0.082
	Bed linen and household linen	0.228
	Curtains and curtain accessories	0.093
	Major household appliances	0.362
	Smaller electric household appliances	0.324
	Kitchen utensils	0.143
	Tableware and cutlery	0.111

Table 9 continued

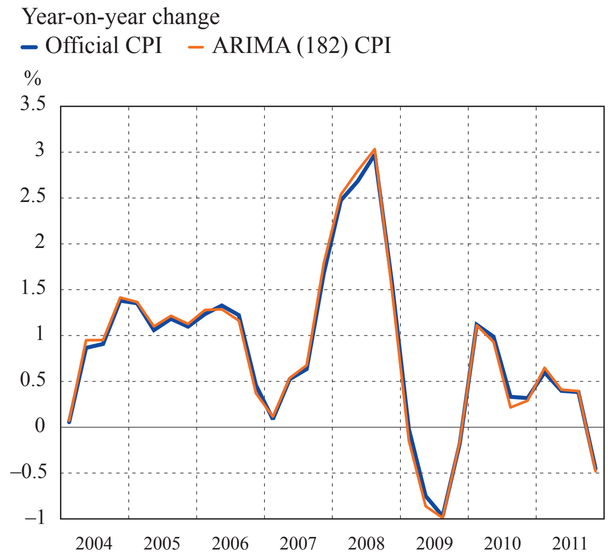
Model	Sub-aggregates	Weights (2010)
	Motorised tools for DIY and garden	0.094
	Tools for house and garden	0.077
	Detergents and cleaning products	0.339
	Cleaning articles	0.019
	Other household articles	0.223
	Medicines	2.313
	Medical products	0.439
	Therapeutic appliances	3.305
	Medical services	1.492
	Dental services	0.946
	Other health services	5.321
	Hospital services	0.046
	New cars	2.910
	Motorcycles	0.250
	Bicycles	0.269
	Spare parts	0.051
	Tyres and accessories	0.223
	Fuels	2.368
	Repair services	1.165
	Other services in respect of personal transport equipment	0.434
	Public transport: direct service	1.014
	Public transport: combined services	0.574
	Taxi	0.071
	Postal services	0.107
	Telephone equipment	0.098
	Telecommunication	2.580
	TV and audiovisual appliances	0.521
	Photographic, cinematographic equipment and optical instruments	0.129
	PC hardware	0.463
	Computer software	0.029
	Recording media	0.236
	Repair and installation	0.039
	Games, toys and hobbies	0.423
	Sports equipment	0.387
	Plants and flowers	0.513
	Pets and related products	0.360
	Sporting events	0.052
	Sports and leisure activities	0.500
	Mountain railways and ski lifts	0.226
	Cinema	0.115
	Theatre and concerts	0.350
	Radio and television licences	0.879
	Photographic services	0.112
	Leisure-time courses	0.630
	Books and brochures	0.328
	Newspapers, purchased singly	0.132
	Newspapers, by subscription	0.481
	Writing and drawing materials	0.168
	Package holidays	2.889
	Education	0.669
	Meals taken in restaurants and cafés	3.190

Table 9 continued

Model	Sub-aggregates	Weights (2010)
	Wine taken in restaurants and cafés	0.646
	Beer taken in restaurants and cafés	0.321
	Spirits and other alcoholic drinks taken in restaurants and cafés	0.106
	Coffee and tea taken in restaurants and cafés	0.604
	Mineral water and soft drinks taken in restaurants and cafés	0.513
	Other non-alcoholic beverages taken in restaurants and cafés	0.047
	Meals in canteens	0.546
	Beverages in canteens	0.155
	Hotels	0.707
	Alternative accommodation facilities	0.275
	Hairdressing establishments	0.889
	Soaps and foam baths	0.074
	Hair-care products	0.120
	Dental-care products	0.059
	Beauty products and cosmetics	0.609
	Paper articles for personal hygiene	0.226
	Personal care appliances	0.144
	Watches	0.459
	Other personal effects	0.243
		92.788

Source: SFSO, authors' calculations

Chart 4: Official CPI vs. ARIMA (182) CPI



C Forecasting integrated processes with missing data

This appendix proposes a procedure to produce forecasts for integrated processes with missing data. Because the observations are missing in levels of the time series, we observe only the changes since the last collection period, not the sequential changes. We show how to estimate an ARMA model for these unobserved sequential changes and produce a forecast using the Kalman filter.

C.1 Model

Let $\{P_t\}$ denote an integrated process. The data for this process are collected irregularly. If we calculate log-changes of this integrated process, we obtain changes since the last collection period rather than sequential changes. Let h_t denote the number of periods since the last collection took place in period t. First differences of the logs yield $\Delta^{h_t} p_t$, where $p_t = \log(P_t)$ and $\Delta^{h_t} p_t = p_t - p_{t-h_t}$.

The log-change since the last collection equals, by definition, the sum of the sequential log-changes. Therefore, we can write the measurement equation as:

$$\begin{aligned} \Delta^{h_t} p_t &= A_t\mu + H_t'\xi_t + W_t, \text{ where} \\ R_t &= \mathbf{E}(W_t W_t'). \end{aligned} \tag{7}$$

The state vector ξ_t contains demeaned, current and lagged unobserved sequential log-changes ($\Delta p_t - \mu$) and unobserved errors (ε_t). The measurement equation states that the observed change depends on a constant ($A_t\mu$), the true underlying sequential changes ($\Delta p_t - \mu$), and a measurement error (W_t). The system matrices, A_t and H_t, are time-varying; due to the irregular collection frequency, we have to consider two cases.

(i) A collection takes place. Then, $H_t' = [1 \quad 1 \quad \dots \quad 0 \quad 0]$ sums up the unobserved sequential changes since the last collection period. By definition, this sum is equal to the observed log-change since the last collection period,

$$\Delta^{h_t} p_t = \sum_{j=0}^{h_t - 1} \Delta p_{t-j}.$$

Also, $R_t = 0$ since we observe the true value and thus the measurement error is zero. Finally, $A_t = h_t$ sums up the underlying constant of the process since the last collection period.

(ii) No collection takes place. Then, $H_t' = [0 \quad 0 \quad \dots \quad 0 \quad 0]$ and $R_t = r$, where r is an arbitrary positive constant (cf. Neusser, 2009, pp. 239–240).

The two cases can be written more compactly:

$$\begin{aligned} A_t &= h_t \times 1\{\Delta^{h_t} p_t \neq 0\}, \\ H_t' &= [1_{(1\times h_t)} \times 1\{\Delta^{h_t} p_t \neq 0\}\ 0 \ \cdots \ 0], \\ R_t &= r \times 1\{\Delta^{h_t} p_t = 0\}, \end{aligned} \tag{8}$$

where $1\{\}$ denotes an indicator function which equals 1 (0) if the statement in curly brackets is true (false) and $1_{(m \times n)}$ denotes an $(m \times n)$-matrix of ones.

As an example, let us assume that no collection took place for two consecutive periods. Then, the number of periods since the last collection is $h_t = 3$ and the system matrices are given by:

$$\begin{aligned} A_t &= 3, \\ H_t' &= \begin{bmatrix} 1 & 1 & 1 & 0 & \cdots & 0 \end{bmatrix}, \\ R_t &= 0. \end{aligned} \tag{9}$$

Now we derive a law of motion for the unobserved states. The transition equation is given by:

$$\begin{aligned} \xi_t &= F\xi_{t-1} + V_t, \text{ where} \\ Q &= \mathbf{E}(V_t V_t'). \end{aligned} \tag{10}$$

The law of motion will be a seasonal ARMA process. So the unobserved states follow

$$\Phi(L)\Gamma(L)(\Delta p_t - \mu) = \Theta(L)\varepsilon_t, \tag{11}$$

where

$$\begin{aligned} \Phi(L) &= 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p, \\ \Theta(L) &= 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_q L^q \text{ and} \\ \Gamma(L) &= 1 - \rho L^{12}. \end{aligned}$$

The example shows an ARMA(2, 2) process without seasonality:

$$\begin{aligned} \xi_t &= [(\Delta p_t - \mu)\ (\Delta p_{t-1} - \mu)\ \varepsilon_t\ \varepsilon_{t-1}]', \\ F &= \begin{bmatrix} \phi_1 & \phi_2 & \theta_1 & \theta_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \\ V_t &= [\varepsilon_t \quad 0 \quad \varepsilon_t \quad 0]', \\ Q &= \begin{bmatrix} \sigma^2 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & 0 \\ \sigma^2 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{12}$$

C.2 Estimation

The likelihood function is derived by the Kalman filter (cf. Hamilton, 1994, pp. 385–386). To maximise the likelihood function, we use the code by Sims (1999). In practice, we estimate the model subject to the following restriction. We require the process to be causal and invertible with respect to ε_t, that is, all roots of the polynomials $\Phi(z)$ and $\Theta(z)$ lie outside the unit circle. For the seasonal model, the restrictions are imposed on the polynomial $\Phi(z)\Gamma(z)$.

C.3 Forecasting

After we have estimated the parameters $\hat{\mu}$ and $\hat{F}$, the Kalman filter delivers a recursion for producing a forecast:

$$\begin{aligned} \Delta^{h_{t+f|t}} p_{t+f|t} &= A_{t+f|t}\hat{\mu} + H'_{t+f|t}\xi_{t+f|t}, \\ \xi_{t+f|t} &= \hat{F}^f \xi_t, \end{aligned} \tag{13}$$

where $\Delta p_{t+f|t}$ denotes the conditional expectation of Δp_{t+f} given information at time t. The starting values for this recursion are given in Hamilton (1994), pp. 378–379. With appropriate $A_{t+f|t}$ and $H_{t+f|t}$ we either obtain a forecast for the unobserved sequential changes ($\Delta p_{t+f|t}$) or a forecast for a series with specific collection frequency ($\Delta^{h_{t+f|t}} p_{t+f|t}$). The latter requires the analyst's best guess about the future collection frequency ($h_{t+f|t}$).

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