Problem 1

- **Current Stock Price $165**
- **Strike Price $165**
- **Current Date 03/13/2022**
- **Options Expiration Date 04/15/2022**
- **Risk Free Rate of 4.25% ivol=0.2**
- **Continuously Compounding Coupon of 0.53%**

Implement the closed form greeks for GBSM. Implement a finite difference derivative calculation.

Compare the values between the two methods for both a call and a put.

Implement the binomial tree valuation for American options with and without discrete dividends. Assume the stock above:

- **Pays dividend on 4/11/2022 of $0.88**

Calculate the value of the call and the put. Calculate the Greeks of each.

What is the sensitivity of the put and call to a change in the dividend amount?

Answer:

- Implement the closed form greeks for GBSM. Implement a finite difference derivative calculation. Compare the values between the two methods for both a call and a put.

	Delta	Gamma	Vega	Theta	Rho	Carry_Rho
GBSM_Greeks_call	0.534009	0.040038	19.71018	24.898522	7.583586	7.966246
GBSM_Greeks_put	-0.465512	0.040038	19.71018	18.786997	-7.277011	-6.944416

	Delta	Gamma	Vega	Theta	Rho	Carry_Rho
FD_CENTRAL_call	0.534009	0.040038	19.71018	24.898522	7.583554	7.966246
FD_FORWARD_call	0.534009	0.040038	19.71018	24.898462	7.624676	7.966250
FD_BACKWARD_call	0.534009	0.040038	19.71018	24.898583	7.542433	7.966241
FD_CENTRAL_put	-0.465512	0.040038	19.71018	18.786997	-7.277045	-6.944416
FD_FORWARD_put	-0.465512	0.040038	19.71018	18.786936	-7.229205	-6.944412
FD_BACKWARD_put	-0.465512	0.040038	19.71018	18.787058	-7.324884	-6.944420

Delta : Change S. The Delta of call is positive, while the Delta of put is negative.

Gamma: Change the Delta. The Gamma of call is same as the Gamma of put.(both positive)

Theta: Change the Time to Maturity. The Theta of call is larger than the Theta of put. (both positive)

Vega: Change the implied Volatility. The Vega of call is same as the Vega of put. (both positive)

Rho: Change the interest rates. The Rho of call is positive, while the Rho of put is negative.

Carry Rho: Change the Continuously Compounding Coupon. The Carry Rho of call is positive, while the Carry Rho of put is negative.

The difference between two methods(Finite difference and GBSM) is slight, which means the formula can be used to get the accurate Greeks just like the finite difference method. And using Finite difference allow us to explore the shape of the option values' curve by comparing the

FORWARD value, BACKWARD value and CENTRAL value.

- Implement the binomial tree valuation for American options with and without discrete dividends. Calculate the value of the call and the put. Calculate the Greeks of each. What is the sensitivity of the put and call to a change in the dividend amount?

	American_call	Americal_put
with Dividend	4.087667	4.156962
without Dividend	4.262316	3.749667

	Delta	Gamma	Vega	Theta	Rho	Carry_Rho
call_withDiv	0.542042	-2.664535e-09	19.809411	25.247964	7.383045	7.711754
put_withDiv	-0.465269	2.664535e-09	20.010136	18.639114	-7.776422	-7.438290
call_noDiv	0.534269	1.065814e-08	19.860711	25.595451	8.108879	8.494273
put_noDiv	-0.469999	3.345479e-01	19.833980	18.987976	-6.501017	-6.215980

When change the dividend to $1.88, the results:

	American_call	Americal_put
with Dividend	4.013830	4.670929
without Dividend	4.262316	3.749667

The sensitivity of put and call to the change of dividend amount shows: When dividend rises $1, the price of American call option will decrease $0.074 and put price will go up $0.51. It means that the put price has positive relationship with dividend amount while the call price has the negative relationship with dividend. And the put price is more sensitive to the dividend amount.

Problem 2

Using the options portfolios from Problem3 last week (named problem2.csv in this week's repo) and assuming :

- **American Options**
- **Current Date 03/03/2023**
- **Current AAPL price is 151.03**
- **Risk Free Rate of 4.25%**
- **Dividend Payment of $1.00 on 3/15/2023**

Using DailyPrices.csv. Fit a Normal distribution to AAPL returns – assume 0 mean return. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Calculate VaR and ES using Delta-Normal. Present all VaR and ES values a $ loss, not percentages.

Compare these results to last week's results.

Answer:

- Using DailyPrices.csv. Fit a Normal distribution to AAPL returns – assume 0 mean return. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES.

	mean	Historical VaR	T_ES
Call	1.240663	6.780229	14.810037
CallSpread	0.081295	4.570418	4.954072
CoveredCall	-0.963165	30.387928	13.388662
ProtectedPut	0.752599	9.040000	16.660398
Put	0.483702	4.845232	12.237758
PutSpread	0.133314	3.006191	4.714099
Stock	0.304093	34.435854	21.673194
Straddle	1.724365	1.374592	10.988316
SynLong	0.756961	35.336083	21.448579

Last week:

	mean	Historical VaR	T_ES
Call	0.366747	6.740472	13.281575
CallSpread	-0.216709	4.531848	5.247354
CoveredCall	-0.852272	24.112747	12.584760
ProtectedPut	0.402856	9.465640	14.540918
Put	0.870526	4.817052	9.970494
PutSpread	0.369605	2.982425	4.084320
Stock	-0.303199	28.153272	20.206244
Straddle	1.237273	1.390729	9.317490
SynLong	-0.503779	28.451658	20.479827

It shows that the American option has higher mean payoff and higher VaR and ES, which means American option has higher return and higher risk.

- Calculate VaR and ES using Delta-Normal. Present all VaR and ES values a $ loss, not percentages.

$$\frac{dR}{dr_i} = \frac{P_i}{PV} \sum_{j=1}^{m} h_j \delta_j \qquad VaR(\alpha) = - PV * F_X^{-1}(\alpha) * \sqrt{\nabla R^T \Sigma \nabla R}$$

	Call	CallSpread	CoveredCall	ProtectedPut	Put	PutSpread	Stock	Straddle	SynLong
VaR	9.510842	3.891792	10.105183	11.432654	-8.189718	-3.760763	17.617450	1.321124	17.700560
ES	11.926980	4.880464	12.672307	14.337009	-10.270237	-4.716148	22.092993	1.656743	22.197217

It basically share the same result with the previous results.

Problem 3

Use the Fama French 3 factor return time series (F-F_Research_Data_Factors_daily.CSV) as well as the Carhart Momentum time series (F-F_Momentum_Factor_daily.CSV) to fit a 4 factor model to the following stocks.

AAPL	FB	UNH	MA
MSFT	NVDA	HD	PFE
AMZN	BRK-B	PG	XOM
TSLA	JPM	V	DIS
GOOGL	JNJ	BAC	CSCO

Fama stores values as percentages, you will need to divide by 100 (or multiply the stock returns by 100) to get like units.

Based on the past 10 years of factor returns, find the expected annual return of each stock.

Construct an annual covariance matrix for the 10 stocks.

Assume the risk free rate is 0.0425. Find the super efficient portfolio.

Answer:

- Use the Fama French 3 factor return time series (F-F_Research_Data_Factors_daily.CSV) as well as the Carhart Momentum time series (F-F_Momentum_Factor_daily.CSV) to fit a 4 factor model to the following stocks. Based on the past 10 years of factor returns, find the expected annual return of each stock.

```
AAPL    -0.047237
UNH      0.067766
MA       0.044072
MSFT    -0.041534
NVDA     0.118125
HD      -0.036259
PFE     -0.072617
AMZN    -0.345505
BRK-B    0.001816
PG      -0.087081
XOM      0.467954
TSLA    -0.118513
JPM     -0.009752
V        0.067799
DIS     -0.250480
GOOGL   -0.282832
JNJ      0.016012
BAC     -0.216139
CSCO    -0.054989
dtype: float64
```

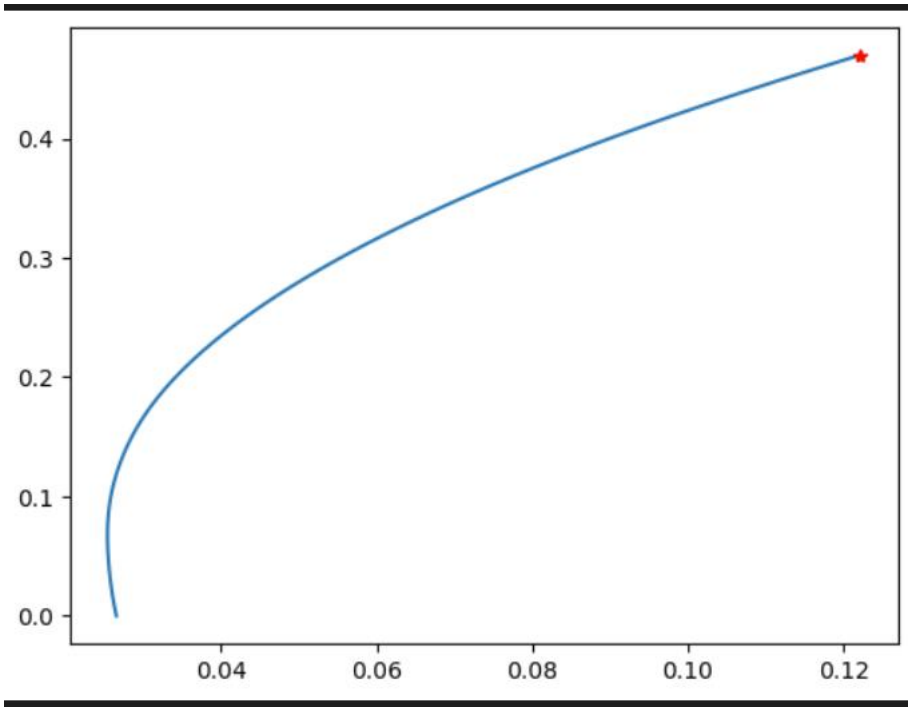
- Construct an annual covariance matrix for the 10 stocks

	AAPL	UNH	MA	MSFT	NVDA	HD	PFE	AMZN	BRK-B	PG	XOM	TSLA	JPM	V	DIS	GOOGL	JNJ	BAC	CSCO
AAPL	0.127744	0.037874	0.081766	0.103490	0.172015	0.066697	0.033043	0.123223	0.055802	0.037198	0.037997	0.157247	0.058782	0.071905	0.088549	0.112502	0.022985	0.066470	0.066917
UNH	0.037874	0.061370	0.031471	0.036753	0.047234	0.026443	0.032208	0.035741	0.028378	0.028116	0.026958	0.039947	0.033441	0.029949	0.023007	0.030068	0.023065	0.034755	0.028938
MA	0.081766	0.031471	0.096534	0.080164	0.138169	0.057116	0.033863	0.097499	0.047893	0.031244	0.031317	0.098893	0.058673	0.083025	0.078133	0.079451	0.017521	0.063903	0.052137
MSFT	0.103490	0.036753	0.080164	0.128660	0.176648	0.071344	0.035514	0.135221	0.052949	0.034021	0.031487	0.132702	0.056885	0.068511	0.088881	0.120954	0.020508	0.065465	0.061248
NVDA	0.172015	0.047234	0.138169	0.176648	0.405429	0.112417	0.047061	0.223450	0.084893	0.041958	0.055732	0.294965	0.098772	0.119021	0.159458	0.188335	0.022115	0.114205	0.098848
HD	0.066697	0.026443	0.057116	0.071344	0.112417	0.098598	0.033793	0.097838	0.042570	0.034786	0.015986	0.078667	0.043865	0.050505	0.064605	0.069903	0.022672	0.046909	0.048892
PFE	0.033043	0.032208	0.033863	0.035514	0.047061	0.033793	0.070919	0.038237	0.031429	0.027676	0.020150	0.022882	0.032057	0.030981	0.025023	0.029602	0.027791	0.031071	0.029685
AMZN	0.123223	0.035741	0.097499	0.135221	0.223450	0.097838	0.038237	0.246883	0.066451	0.030754	0.037914	0.189252	0.071497	0.084653	0.126130	0.151189	0.023089	0.086104	0.072463
BRK-B	0.055802	0.028378	0.047893	0.052949	0.084893	0.042570	0.031429	0.066451	0.050381	0.024982	0.034152	0.062338	0.047193	0.042399	0.051806	0.056577	0.019351	0.050967	0.040747
PG	0.037198	0.028116	0.031244	0.034021	0.041958	0.034786	0.027676	0.030754	0.024982	0.049146	0.004657	0.024013	0.029579	0.029570	0.027929	0.028256	0.023704	0.028624	0.035161
XOM	0.037997	0.026958	0.031317	0.031487	0.055732	0.015986	0.020150	0.037914	0.034152	0.004657	0.121664	0.043925	0.029385	0.024531	0.036022	0.031521	0.006564	0.032981	0.023381
TSLA	0.157247	0.039947	0.098893	0.132702	0.294965	0.078667	0.022882	0.189252	0.062338	0.024013	0.043925	0.467924	0.068298	0.092291	0.130909	0.144512	0.012897	0.084646	0.067785
JPM	0.058782	0.033441	0.058673	0.056885	0.098772	0.043865	0.032057	0.071497	0.047193	0.029579	0.029385	0.068298	0.083100	0.053403	0.062154	0.059692	0.018958	0.083011	0.047412
V	0.071905	0.029949	0.083025	0.068511	0.119021	0.050505	0.030981	0.084653	0.042399	0.029570	0.024531	0.092291	0.053403	0.083639	0.068241	0.067478	0.016892	0.059195	0.045559
DIS	0.088549	0.023007	0.078133	0.088881	0.159458	0.064605	0.025023	0.126130	0.051806	0.027929	0.036022	0.130909	0.062154	0.068241	0.140522	0.096579	0.015422	0.073320	0.053546
GOOGL	0.112502	0.030068	0.079451	0.120954	0.188335	0.069903	0.029602	0.151189	0.056577	0.028256	0.031521	0.144512	0.059692	0.067478	0.096579	0.162566	0.019896	0.070482	0.065602
JNJ	0.022985	0.023065	0.017521	0.020508	0.022115	0.022672	0.027791	0.023089	0.019351	0.023704	0.006564	0.012897	0.018958	0.016892	0.015422	0.019896	0.031630	0.018017	0.022220
BAC	0.066470	0.034755	0.063903	0.065465	0.114205	0.046909	0.031071	0.086104	0.050967	0.028624	0.032981	0.084646	0.083011	0.059195	0.073320	0.070482	0.018017	0.101094	0.049190
CSCO	0.066917	0.028938	0.052137	0.061248	0.098848	0.048892	0.029685	0.072463	0.040747	0.035161	0.023381	0.067785	0.047412	0.045559	0.053546	0.065602	0.022220	0.049190	0.088593

- Assume the risk free rate is 0.0425. Find the super efficient portfolio.

```
the weight for AAPL is 2.420748073698874e-16
the weight for UNH is 0.0
the weight for MA is 8.442582376194067e-16
the weight for MSFT is 5.549112683300147e-16
the weight for NVDA is 1.27675647831893e-15
the weight for HD is 0.0
the weight for PFE is 5.470343209967563e-17
the weight for AMZN is 9.135160705264643e-16
the weight for BRK-B is 0.0
the weight for PG is 0.0
the weight for XOM is 0.999999999999996
the weight for TSLA is 1.4242381938512915e-15
the weight for JPM is 0.0
the weight for V is 0.0
the weight for DIS is 1.8572480342759932e-15
the weight for GOOGL is 1.4048272101740358e-15
the weight for JNJ is 0.0
the weight for BAC is 1.2178526631538422e-15
the weight for CSCO is 1.196830607865205e-18
```

```
Portfolio return: 0.4679539766998212
Portfolio variance: 0.12166351936190853
Tangent Portfolio Sharpe Ratio: 1.2197544178572859
```



With these weights, we can get the best portfolio with 0.46795 return and 0.12166 variance, which leads the sharpe ratio=1.21975. This portfolio has high return as well as high risk.