Problem 1

Assume you a call and a put option with the following

- **Current Stock Price $165**
- **Current Date 03/03/2023**
- **Options Expiration Date 03/17/2023**
- **Risk Free Rate of 4.25%**
- **Continuously Compounding Coupon of 0.53%**

Calculate the time to maturity using calendar days (not trading days).

For a range of implied volatilities between 10% and 80%, plot the value of the call and the put. Discuss these graphs. How does the supply and demand affect the implied volatility?

Answer:

- The time to maturity = 14 days

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The time to maturity using calendar days of the option is 14 days
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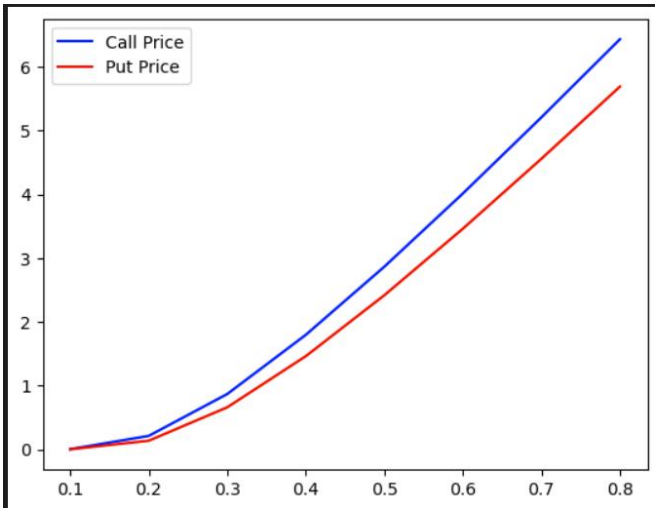
- Plot the value of the call and the put

I set :

strike price for call = 175

Strike price for put = 155



- Discuss these graphs. How does the supply and demand affect the implied volatility?

I set :

Strike price for call = 175

Strike price for put = 155

When the implied volatility increases, the prices of the option(both call and put) get higher.
And the value of call option always higher than put option.

When the demand increase(transcend the supply), the price of the option increase, making the higher implied volatility.

When the supply increase(transcend the demand), the price of the option decrease, making the lower implied volatility.

Problem 2

Use the options found in AAPL_Options.csv

● Current AAPL price is 151.03

● Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

Calculate the implied volatility for each option.

Plot the implied volatility vs the strike price for Puts and Calls.

Discuss the shape of these graphs. What market dynamics could make these graphs?

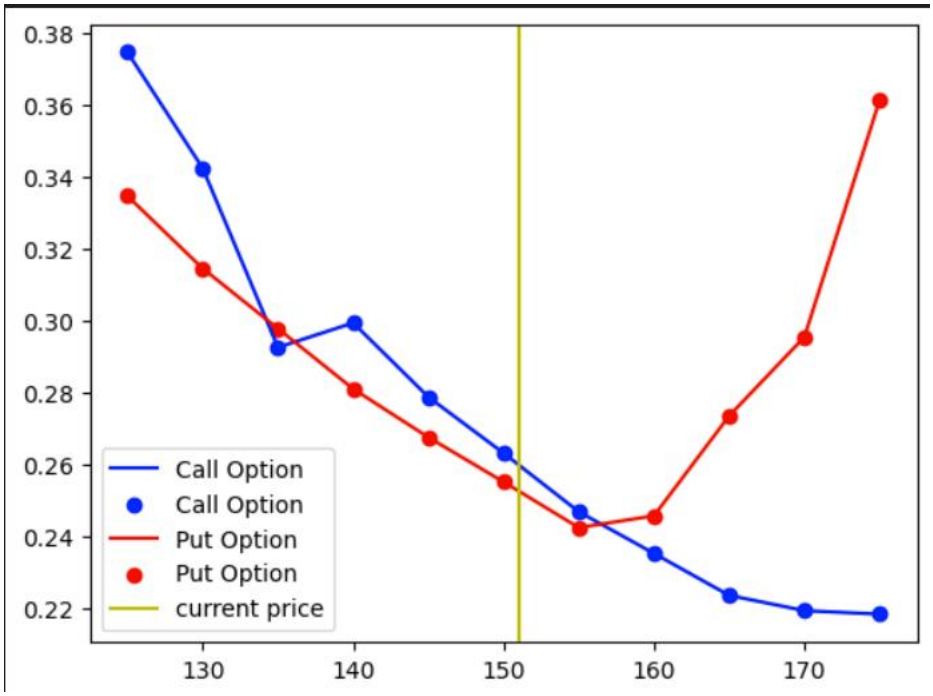
There are bonus points available on this question based on your discussion. Take some time to research if needed.

Answer:

- Calculate the implied volatility for each option.

	Stock	Expiration	Type	Strike	Last Price	Current_Date	TTM	implied_vol
0	AAPL	4/21/2023	Call	125	27.300	2023-03-03	49 days	0.374597
1	AAPL	4/21/2023	Call	130	22.575	2023-03-03	49 days	0.342351
2	AAPL	4/21/2023	Call	135	17.750	2023-03-03	49 days	0.292522
3	AAPL	4/21/2023	Call	140	13.850	2023-03-03	49 days	0.299358
4	AAPL	4/21/2023	Call	145	9.975	2023-03-03	49 days	0.278743
5	AAPL	4/21/2023	Call	150	6.700	2023-03-03	49 days	0.263141
6	AAPL	4/21/2023	Call	155	4.050	2023-03-03	49 days	0.246828
7	AAPL	4/21/2023	Call	160	2.210	2023-03-03	49 days	0.235242
8	AAPL	4/21/2023	Call	165	1.035	2023-03-03	49 days	0.223567
9	AAPL	4/21/2023	Call	170	0.460	2023-03-03	49 days	0.219339
10	AAPL	4/21/2023	Call	175	0.195	2023-03-03	49 days	0.218342
11	AAPL	4/21/2023	Put	125	0.405	2023-03-03	49 days	0.334615
12	AAPL	4/21/2023	Put	130	0.665	2023-03-03	49 days	0.314473
13	AAPL	4/21/2023	Put	135	1.120	2023-03-03	49 days	0.297772
14	AAPL	4/21/2023	Put	140	1.840	2023-03-03	49 days	0.280994
15	AAPL	4/21/2023	Put	145	3.010	2023-03-03	49 days	0.267532
16	AAPL	4/21/2023	Put	150	4.750	2023-03-03	49 days	0.255134
17	AAPL	4/21/2023	Put	155	7.150	2023-03-03	49 days	0.242417
18	AAPL	4/21/2023	Put	160	10.575	2023-03-03	49 days	0.245700
19	AAPL	4/21/2023	Put	165	14.925	2023-03-03	49 days	0.273493
20	AAPL	4/21/2023	Put	170	19.425	2023-03-03	49 days	0.295414
21	AAPL	4/21/2023	Put	175	24.625	2023-03-03	49 days	0.361243

- Plot the implied volatility vs the strike price for Puts and Calls.

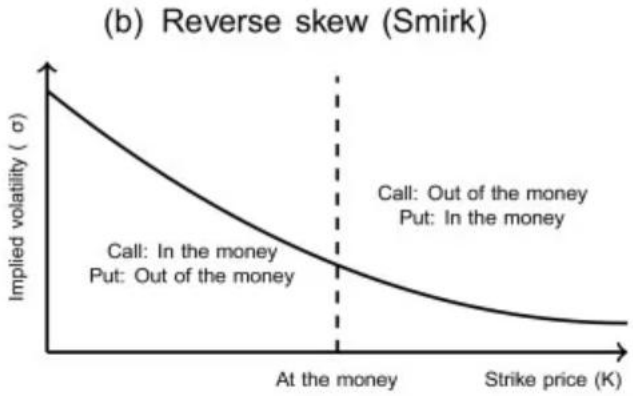


➢ Discuss the shape of these graphs. What market dynamics could make these graphs?

The shape of the graphs look like **smiles**, especially the put's graph. It is what we called "volatility smile".

Implied volatility smile implies that: for the options with same expiration date but different strike prices, the closer the strike price deviates from the underlying price, the lower the implied volatility, while the farther the strike price deviates from the underlying price, the higher the implied volatility. To sum up, **at-the-Money (ATM) has lower volatility, In-the-Money and Out-of -the-Money have higher volatility**.

However, the graph which is showed above display a **reverse skew(smirk)** characteristic. Like this:



It means that the implied volatility of the lower strike price option is higher the higher strike price option. This pattern illustrates that in-the-money calls and out-of-the-money puts are more expensive than out-of-money calls and in-the-money puts.

One explanation for this phenomenon is investors worry about the market collapse and **then buy puts for protection**. (It was not until 1987 that stock options had a reverse skew).

Another explanation is in-the-money calls is becoming popular as they **provide leverage comparing to buying stocks directly**. So it **increases the demand** of in-the-money calls, which leads to the increment of implied volatility at lower strike price.

Problem 3

Use the portfolios found in problem3.csv

● Current AAPL price is 151.03

● Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

For each of the portfolios, graph the portfolio value over a range of underlying values. Plot the portfolio values and discuss the shapes. Bonus points available for tying these graphs to other topics discussed in the lecture.

Using DailyPrices.csv. Calculate the log returns of AAPL. Demean the series so there is 0 mean. Fit an AR(1) model to AAPL returns. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss.

Hints:

● you will need to calculate the implied volatility - might not be the same as #2

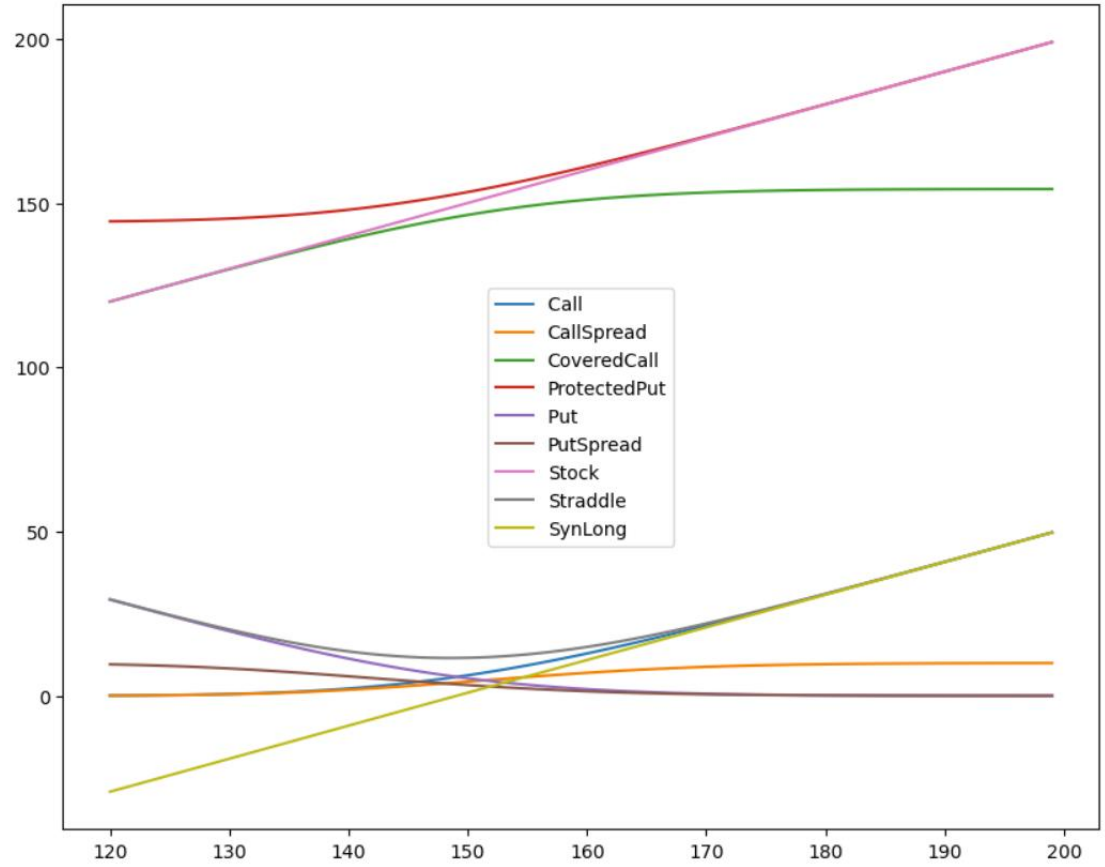
● you need to take into account the change in dates for option valuations. You are simulating forward in time and options valuations are a function of time

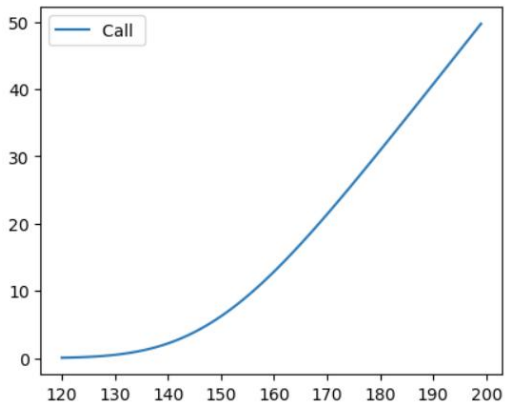
● Calculate the PL from the current portfolio value using Current Date

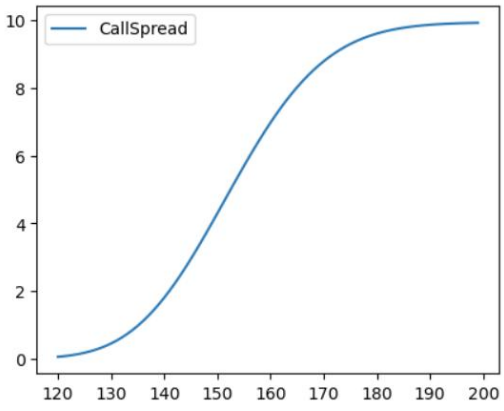
Answer:

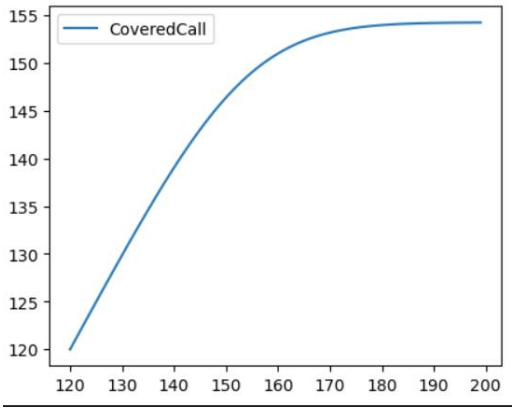
- graph the portfolio value over a range of underlying values

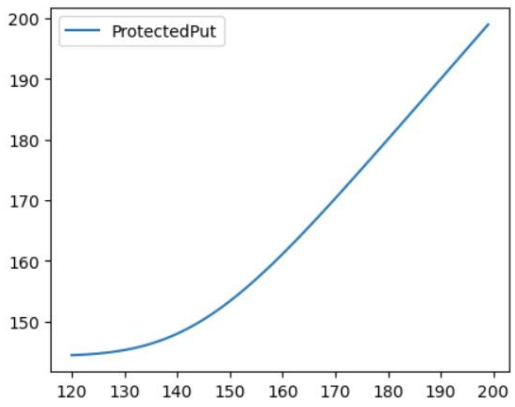
Portfolio	Call	CallSpread	CoveredCall	ProtectedPut	Put	PutSpread	Stock	Straddle	SynLong
120	0.059849	0.058214	119.989827	144.410459	29.279583	9.553442	120.0	29.339432	-29.219734
121	0.076352	0.073999	120.986177	144.448055	28.294175	9.479922	121.0	28.370527	-28.217823
122	0.096641	0.093293	121.981400	144.492978	27.312150	9.396334	122.0	27.408791	-27.215509
123	0.121389	0.116676	122.975207	144.546285	26.334154	9.301999	123.0	26.455543	-26.212765
124	0.151343	0.144778	123.967255	144.609107	25.360911	9.196306	124.0	25.512254	-25.209568
...	...	...	...	...	...	...	...	...	...
195	45.730986	9.911934	154.233478	195.004829	0.011953	0.009510	195.0	45.742938	45.719033
196	46.727723	9.917019	154.238015	196.004008	0.009988	0.007959	196.0	46.737710	46.717735
197	47.724842	9.921328	154.241955	197.003322	0.008333	0.006650	197.0	47.733175	47.716510
198	48.722290	9.924968	154.245387	198.002749	0.006941	0.005547	198.0	48.729231	48.715349
199	49.720019	9.928036	154.248386	199.002272	0.005773	0.004619	199.0	49.725791	49.714246

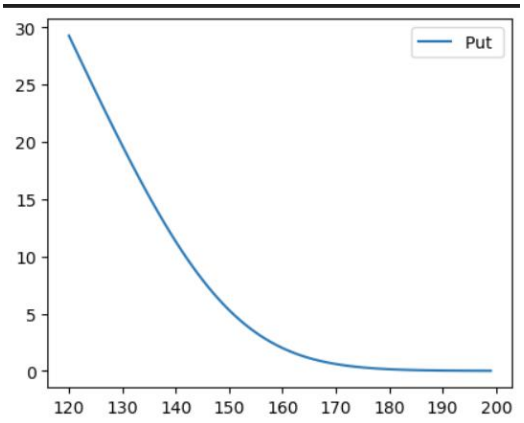

Call
CallSpread
CoveredCall
ProtectedPut
Put
PutSpread
Stock
Straddle
SynLong

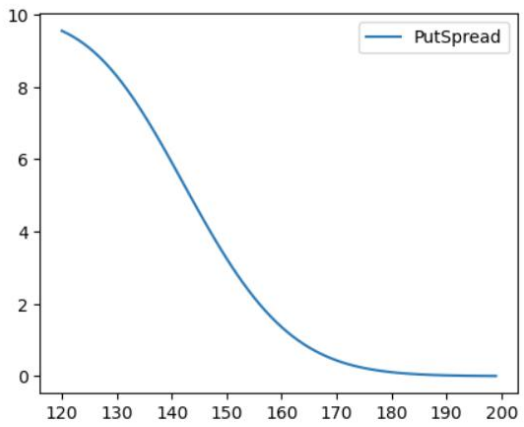

Call

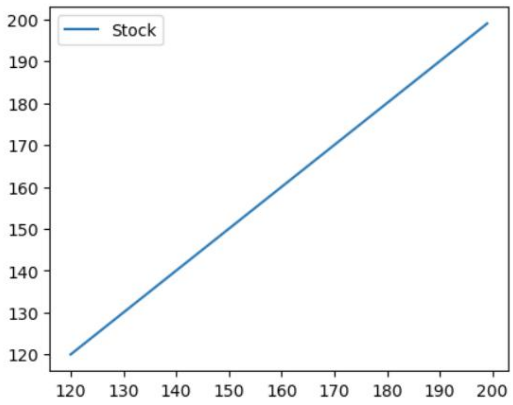

CallSpread

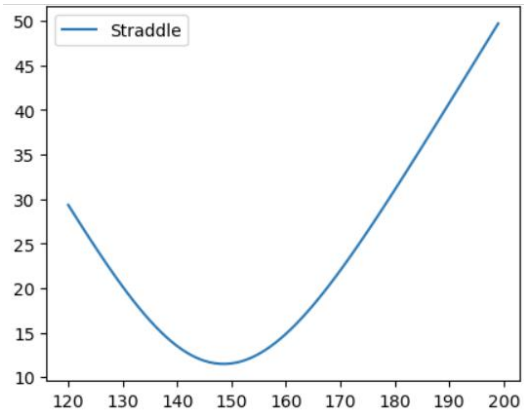

CoveredCall

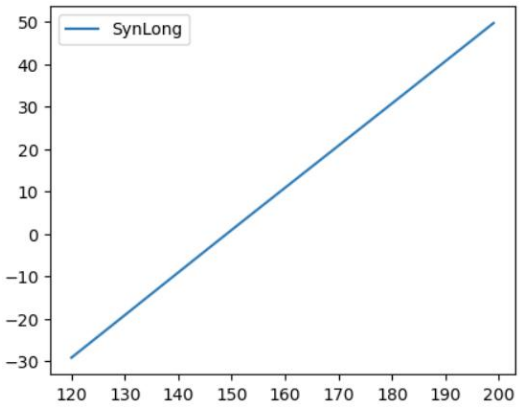

ProtectedPut











<1> Call: has the same shape with ProtectedPut.

Max profit=stock price at expiration date-strike price - call option price (-> unlimited)

Max loss=call option price

<2> ProtectedPut: has the same shape with Call. It contained a put and an underlying stock.

Max profit= stock price at expiration date - (Underlying price+ put option price)

(-> unlimited)

Max loss= put option price+(underlying price - strike)

It means that: one underlying asset+ one put has the similar result as one call.

<3> CallSpread: reverse with PutSpread. It contains one long call option(lower strike price) and one short call(higher strike price)

Max profit= the difference between two strike price -option price(long)+ option

price(short)

Max loss= option price(long)-option price(short) + the difference between two strike prices

<4> PutSpread: reverse with CallSpread. It contains one long put option(higher strike price) and one short put(lower strike price)

Max profit=the difference between two strike price -option price(long)+ option price(short)

Max loss= option price(long)-option price(short) + the difference between two strike prices

<5> Stock: has the same shape as SynLong. (larger risk)

Max profit= stock price at expiration date - underlying price (-> unlimited)

Max loss= stock price at expiration date - underlying price (-> unlimited)

<6> SynLong: has the same shape as Stock. It contains one long call and one short put at the same strike price. (larger risk)

Max profit= stock price at expiration date - strike - option price(long) - option price (short) (-> unlimited)

Max loss (-> unlimited)

It means that: longing one call + shorting one put has the similar result as longing one underlying asset.

<7>Covered Call: reverse with Put. It contains one long stock + one short call.

Max profit=option price(short)+strike price - underlying price

Max loss=underlying price- stock price at expiration date -option price(short)

<8> Put: reverse with Covered Call.

Max profit=- put option price+strike price - stock price at expiration date

Max loss=put option price

<9> Straddle: it contains a long call and long put with same strike price

Max profit=abs(stock price at expiration date - strike)- call price -put price (-> unlimited)

Max loss= call price + put price

- Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss.

Portfolio	Call	CallSpread	CoveredCall	ProtectedPut	Put	PutSpread	Stock	Straddle	SynLong
152.91549350998588	7.211781	4.974692	148.673722	154.921651	3.551307	2.443681	152.915494	10.763087	3.660474
165.85516511332585	17.223184	8.339353	153.068191	166.182831	0.703550	0.544624	165.855165	17.926734	16.519635
138.13926293756455	1.271025	1.130928	137.686459	146.828443	12.431325	6.671295	138.139263	13.702350	-11.160300
141.36564086964108	2.033694	1.738463	140.535532	148.023635	9.945167	5.708069	141.365641	11.978861	-7.911473
150.57527290372278	5.837821	4.243864	147.342077	153.216480	4.512476	3.013646	150.575273	10.350297	1.325345
...	...	...	...	...	...	...	...	...	...
148.46818381226527	4.740758	3.599257	145.995024	151.804457	5.522180	3.580061	148.468184	10.262938	-0.781423
144.25587020635584	2.954466	2.419016	142.911832	149.371969	7.959778	4.821080	144.255870	10.914244	-5.005312
142.1282238457043	2.253676	1.906015	141.180493	148.353618	9.397865	5.474251	142.128224	11.651542	-7.144189
139.95433411746254	1.667767	1.452340	139.311630	147.461287	10.999962	6.136293	139.954334	12.667728	-9.332195
147.01169219632132	4.061819	3.170800	144.985253	150.902477	6.301946	3.996001	147.011692	10.363765	-2.240127

	mean	Historical VaR	T_ES
Call	0.366747	6.740472	13.281575
CallSpread	-0.216709	4.531848	5.247354
CoveredCall	-0.852272	24.112747	12.584760
ProtectedPut	0.402856	9.465640	14.540918
Put	0.870526	4.817052	9.970494
PutSpread	0.369605	2.982425	4.084320
Stock	-0.303199	28.153272	20.206244
Straddle	1.237273	1.390729	9.317490
SynLong	-0.503779	28.451658	20.479827

I calculate historical VaR and Expected shortfall fitted T distribution.

<1> Stock & SynLong: both have negative mean payoff and similar large VaR and ES. As what we discussed above that they have larger risks than other portfolio.

<2> Call & ProtectedPut: both have similar positive mean payoff and similar medium VaR and ES.

<3> Straddle: has largest mean payoff, and small VaR and ES. It just like what we analyzed before that Straddle has large payoff with limited risk.

<4> Call & Put: both have positive mean payoff and similar medium VaR and ES.

<5> CallSpread & PutSpread: both have relatively small VaR and ES. Low risk. PutSpread has positive mean payoff while CallSpread has negative mean payoff.

<6> CoveredCall: lowest mean payoff and relatively large VaR and ES. High risk with low profit.